Measurements of optical turbulence with higher-order structure functions

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D. L. Walters

Higher-order structure functions have been used to extract atmospheric optical $C_n^2$ profiles from a vertical sequence of temperature data collected by a single probe carried by a meteorological balloon. This technique circumvents trends and fluctuations in the atmospheric mean temperature and simplifies the equipment and complexity of measurement collection compared with traditional, horizontal differential-probe pair systems.

1. Introduction

Atmospheric optical turbulence degrades the performance of ground-based imaging and laser systems. Typically, random phase perturbations limit the resolution of large optical telescopes operating in the visible to that of a 5–10-cm aperture. Consequently, a variety of adaptive optical techniques are under development to ameliorate these effects. Knowledge of the atmospheric turbulence profile $C_n^2$ and wind speed versus altitude are helpful during the development of adaptive optical systems, as well as in the design and deployment of large, 4–10-m aperture, optical instruments.

Over the past two decades, balloon-borne microthermal probes have collected high-resolution atmospheric $C_n^2$ profiles. Typically, the $C_n^2$ data come from temperature differences measured by fluctuations in the resistance of two, fine-wire, platinum or tungsten probes separated horizontally by approximately 1 m. Collecting $C_n^2$ profiles with microthermal probe pairs carried by a balloon tends to be awkward and expensive. Consequently, this procedure has been used infrequently. Figure 1 shows a typical atmospheric-temperature profile versus altitude. The mean temperature varies with altitude and numerous temperature inversions exist that alter the local lapse rate. In general, gradients in the temperature, particularly near inversions, overwhelm the fine, <0.1-K, turbulent temperature fluctuations, which are hidden by the width of the line. Figure 2 shows a power spectrum of the temperature fluctuations of the data, shown in Fig. 1, in the stratosphere above 13 km. The spectra has an $f^{-2}$ power spectral density, although a decrease in slope appears above 0.05 cycles/m. Correcting for the adiabatic temperature change with altitude through the use of the potential temperature doesn’t change these results significantly as seen in Fig. 3.

To simplify atmospheric $C_n^2$ vertical profile measurements, I have investigated the use of vertical temperature differences obtained from a vertical sequence of data collected from a single temperature probe carried by a balloon. To circumvent large-amplitude, low-frequency trends in the vertical temperature data, the data-reduction procedure involved higher-order structure parameters applied to the sequence of temperature measurements. This combination of a single probe with higher-order structure function signal processing provides results that are equivalent to those collected with conventional, horizontal, differential temperature probes, but the measurements are obtained with an order of magnitude less hardware and complexity.

2. Kolmogorov Structure Functions

Structure functions $D_X$ of the form

$$D_X(r_1, r_2) = |X(r_1) - X(r_2)|^2,$$

where $X$ is an atmospheric scaler parameter such as the temperature or index of refraction, $r_1$ and $r_2$ are position vectors of two points in space, and $<\cdot\cdot\cdot>$ implies an ensemble average, arise in the theory of wave propagation through random media. Although Eq. (1) is a tensor, it is frequently possible to assume local isotropy and homogeneity so that $D$ is

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a function of the scalar difference between \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \)
\[
D_X(\mathbf{r}_1, \mathbf{r}_2) = D_X(|\mathbf{r}_2 - \mathbf{r}_1|)
\]  
(2)

The structure function may follow a power law allowing one to write
\[
D_X(|\mathbf{r}_2 - \mathbf{r}_1|) = C_X^2 r^m,
\]  
(3)

where
\[
|\mathbf{r}_2 - \mathbf{r}_1| = r
\]  
(4)

and \( 0 < m \leq 2 \). The structure parameter \( C_X^2 \) is a proportionality constant. Over sufficiently small regions, of the order of millimeters to meters in size, which are locally homogeneous and isotropic, the atmosphere can have a Kolmogorov structure function dependence with \( m = 2/3 \).

Within these constraints, a passive additive such as the atmospheric index of refraction structure parameter becomes
\[
C_n^2 = \frac{|n(\mathbf{r}_2) - n(\mathbf{r}_1)|^2}{r^{2/3}}.
\]  
(5)

Similarly, the atmospheric-temperature structure parameter is
\[
C_T^2 = \frac{\langle |T(\mathbf{r}_2) - T(\mathbf{r}_1)|^2 \rangle}{r^{2/3}}.
\]  
(6)

At visible wavelengths, the atmospheric index of refraction depends on the atmospheric density, which in turn depends on the pressure \( P \) and the temperature \( T \) by approximately
\[
n = 1 + 0.01 \frac{P}{T} + \cdots,
\]  
(7)

where the pressure is in pascal and the temperature is in kelvin. (One atmosphere is approximately \( 1.01 \times 10^5 \) Pa). For isobaric turbulence, the optical structure parameter is proportional to the temperature structure parameter
\[
C_n^2 = \left[ 0.01 \frac{P}{T^2} \right] C_T^2,
\]  
(8)

where \( P(\mathbf{z}) \) is the pressure at altitude \( \mathbf{z} \). Atmospheric temperature is not a conservative passive additive; rather, one should use the adiabatic, potential temperature\(^6\)
\[
\Theta = T(\mathbf{z}) \left( \frac{P_0}{P(\mathbf{z})} \right)^{R/c_p},
\]  
(9)

where \( P(\mathbf{z}) \) is the pressure at altitude \( \mathbf{z} \), \( P_0 \) is a convenient reference pressure, \( R \) is the universal gas constant, and \( c_p \) is the atmospheric heat capacity at constant pressure \((R/c_p = 0.28)\). For turbulence measurements, the reference pressure in Eq. (9) is local to the turbulent volume. Approximately the pressure change with altitude as \( P(\mathbf{z}) = P_0 \exp(-z/z_0) \) where \( z_0 \) is 8 km, a power series expansion of Eq. (9) gives the local potential temperature as
\[
\Theta \approx T \left( 1 + \frac{dz}{z_0} \right) \approx T + 0.0098 \text{dz}.
\]  
(10)
Equations (5) and (6) are valid as long as we avoid the inner-scale region where dissipation of the passive additive occurs by diffusion or conduction rather than viscosity and the distance \( r \) in Eq. (4) is less than the outer scale.

A key assumption required by Eqs. (1)-(5) is that

\[
\langle X_{r_2} - X_{r_1} \rangle = 0, \tag{11}
\]

which implies that the atmosphere must be homogeneous. The vertical temperature and potential temperature profile of the atmosphere almost never satisfy this requirement. Of course, we could redefine the structure function as

\[
D_x = \langle |x_2 - \bar{x}_2| - |x_1 - \bar{x}_1|^2 \rangle \\
= \langle |x_2 - x_1|^2 \rangle - \langle x_2 - x_1 \rangle^2, \tag{12}
\]

and include the mean gradient term if it is significant, but this does not address the underlying problem of nonhomogeneous data.

A. \( n \)-th-Order Structure Functions of Yaglom

A fundamental problem is that first-order vertical differences such as Eq. (1) only remove a constant from the data. Higher-order trends may still remain. One way to handle this problem is to use temperature data with a very high vertical-temperature resolution so that we can ignore the mean gradient term in Eq. (12). Another approach is to use some form of moving average or other trend-removal technique. Both of these approaches are arbitrary, and they are difficult to achieve in practice. There is a better alternative.

Recently Yaglom introduced generalized structure functions for \( n \)-th-order stationary random processes that incorporate \( n \)-th-order differences. An \( n \)-th-order difference removes the lower \( n - 1 \) polynomial components of the data, inherently. Rather than one’s resorting to arbitrary, trend-removal techniques, higher-order structure functions avoid trends in a robust manner.

Following Yaglom, an \( n \)-th-order, self-similar, stationary, random process \( F(r) \) has an \( n \)-th-order difference expressed in terms of the binomial coefficients and increments \( \rho \):

\[
\Delta_n^m F(r) = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} F(r - k \rho). \tag{13}
\]

The mean square difference produces an \( n \)-th-order structure function of the form

\[
\langle |\Delta_n^m F(r)|^2 \rangle = D_n^m(0; \rho, \rho) = D_n^m(\rho). \tag{14}
\]

Equation (14) assumes that \( F(\rho) \) is real, and the one-dimensional symmetric form \( D_n(\rho) \) was introduced rather than retain Yaglom’s more general cross-structure function.

Significantly, when the \( n \)-th-order difference is mean zero

\[
\langle \Delta_n^m F(r) \rangle = 0, \tag{15}
\]

the \( D_n^m \) term may have the form

\[
D_n^m(\rho) = C_n^m |\rho|^{m}. \tag{16}
\]

If Eq. (15) is not satisfied, then Eq. (14) could be rewritten as

\[
\langle |\Delta_n^m F(r)|^2 \rangle = \langle |\Delta_n^m F(r) - \Delta_n^m F(0)|^2 \rangle = D_n^m(0). \tag{17}
\]

The \( C_n^m \) are generalized \( n \)-th-order structure parameters analogous to Eq. (3). If \( 0 < m < n \) and the corresponding spectral distribution function \( d(k) \) of \( F(r) \) is absolutely continuous, \( dF^m(\kappa) = f(\kappa) d\kappa \), then the corresponding spectral representation of an isotropic, \( n \)-th-order structure function is

\[
C_n^m |\rho|^{m} = 2 \int_{-\infty}^{\infty} \{ 1 - \cos \kappa |\rho|^{m} f(\kappa) d\kappa, \tag{18}
\]

where \( f(\kappa) \) is the one-dimensional spectral density. One sees from Eq. (18) that the \( n \)-th-order differences act like high-pass-filter functions that emphasize the higher spatial frequencies of the turbulent spectrum with increasing \( n \).

Equation (16) implies that, for Kolmogorov turbulence where \( m = 2/3 \), the \( n \)-th-order structure functions also have a \( r^{2/3} \) dependence. Each \( D_n^m \) has a corresponding structure parameter \( C_n^m \) that is analogous to \( C_2 \) of Eq. (3). Significantly, the set \( C_n^m \) for \( n = 1, 2, 3, \ldots \), are related to each other. For example, the second-order structure function is

\[
\langle F_2 - 2F_1 + F_0 \rangle = \langle |F_2 - F_1| - |F_1 - F_0| \rangle = \langle F_2 - F_1 \rangle^2 - \langle F_1 - F_0 \rangle^2, \tag{19}
\]

If one uses the algebraic relation

\[
|a - b| = \sqrt{a^2 + b^2 - 2ab} \tag{20}
\]

to rewrite the cross-product term in Eq. (19) and if one further assumes homogeneity and uniform spacing of the data samples and a Kolmogorov \( r^{2/3} \) structure function, then

\[
\langle F_2 - 2F_1 + F_0 \rangle = 2.413 |F_1 - F_0|^2. \tag{21}
\]

Equation (21) shows that the second-order, Yaglom–Kolmogorov structure function is approximately 2.4 times larger than the conventional Kolmogorov first-order structure function. By induction, an \( r^m \) structure function \( C_n^m \) is larger than \( C_1^1 \) by

\[
C_n^m = C_1^1 \sum_{j=0}^{n-1} \binom{n}{j} |j-n|^{m-j} \tag{22}
\]

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Table 1. High-Order Structure Parameter Coefficients Relative to the First-Order for Kolmogorov Turbulence

<table>
<thead>
<tr>
<th>$C^2/C^1$</th>
<th>$C^3/C^1$</th>
<th>$C^4/C^1$</th>
<th>$C^5/C^1$</th>
<th>$C^6/C^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.412</td>
<td>7.555</td>
<td>25.67</td>
<td>90.84</td>
<td>329.14</td>
</tr>
</tbody>
</table>

Table 1 shows the $n$th-order structure parameters normalized to the first order for $n = 2, 3, 4, 5, 6$ for a Kolmogorov structure function with an infinite outer scale and zero inner scale. The higher-order structure parameters increase with $n$ because of the $n$ dependence of the binomial coefficients.

For atmospheric vertical-temperature data, the utility of Eq. (14) arises from the fact that the $D^n$ removes the lower $(n-1)$-order derivatives of the temperature data set. It is not necessary to use trend-removal procedures nor even to use potential temperatures for orders $n > 1$. In general, it is easier to satisfy the mean zero requirement of Eq. (15) for $n > 1$. Using relations such as Eq. (21) and Table 1, one can infer the atmospheric structure parameters $C_T^2$ and $C_n^2$ needed for optical-propagation applications from the higher-order $D^n$ because they are related to each other for a Kolmogorov atmosphere.

For a measurement system, the thermal-response time of the temperature probe must be included in Eq. (18), as well as the averaging time of the data-acquisition system. If $\tau_1$ were used for the probe $e^{-1}$ response time and $\tau_2$ for the averaging time, and if Taylor's hypothesis was assumed, a measured $n$th-order structure function would involve

$$C_{\text{meas}}^{n|r|^{2/3}} = 2^n \int_{-\infty}^{\infty} \left(1 - \cos \frac{\kappa r}{\zeta} \right)^n \frac{1}{1 + (\kappa v t)^2} \sin^2(\kappa v t) \frac{1}{(\kappa v t)^{5/3}} d\kappa,$$

where $r$ is the separation distance between data points and $v$ is the speed of the airflow past the probes. The Lorentzian term follows from the $\exp(-t/\tau)$ probe response time and the integrating analog to digital converter produces the sinc$^2$ term. Following Tatarks, I introduced the one-dimensional spatial coordinate $r$ and the corresponding wave number $\kappa$ in Eq. (23). For $\tau_1 > 0$ or $\tau_2 > 0$ some high-frequency loss will occur, and the actual ratios corresponding to Table 1 are slightly smaller.

B. Noise Characteristics of Higher-Order Structure Functions

One might expect that applying higher-order differences to experimental data would degrade the signal-to-noise ratio. This effect is small for $n$th-order structure functions because they are higher-order differences, not higher-order derivatives. For a random Gaussian signal, the $n$th-order structure function is the sum of the square of the binomial coefficients

$$D_{\text{noise}}^n = \sum_{k=0}^{n} \binom{n}{k}^2 = \frac{2n}{n},$$

From Eq. (22) and Table 1 the structure functions $D^n$ also increase with $n$. Table 2 shows that the ratio of structure functions for Gaussian noise and a Kolmogorov signal remains near 2.5 for $n = 2, 3, 4$. As long as the probe has adequate high-frequency response and the dominant contribution to Eq. (18) does not come from the inner-scale region, the signal-to-noise ratio does not change significantly for higher-order structure functions and is similar to a simple first-order structure function. On the other hand, higher-order differences require more data samples to perform the difference, and this can extend the vertical extent of the measurement beyond the outer scale. In general, one must not violate either the inner- or outer-scale constraints.

### 3. Implementation

I have successfully used this approach with a single temperature probe to collect $C_T^2$ and $C_n^2$ profiles. The temperature sensor was a single, $125-\mu$m, negative-resistance, bead thermistor attached with 18-µm leads to a commercial, digital, meteorological microsonde system made by the VIZ Manufacturing Company. The microsonde system was designed for high-resolution resistance probes and provided approximately 0.02% resolution and 0.1% accuracy over a resistance range of 10 kΩ to 10 MΩ. The temperature resolution of the thermistor and sonde–telemetry package range was 0.002–0.005 K. The balloon digitization and telemetry system and thermistor combination had a noise-equivalent temperature of 0.005–0.008 K rms, slightly poorer than the 0.002–0.004-K noise of a research-grade, analog digitization and telemetry system.

Table 2. Gaussian Noise Variance Relative to a Kolmogorov Signal for the First Six $n$th-Order Structure Functions

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^n$ Gaussian noise</td>
<td>2</td>
<td>6</td>
<td>20</td>
<td>70</td>
<td>252</td>
<td>924</td>
</tr>
<tr>
<td>$D^n$ Kolmogorov signal</td>
<td>1</td>
<td>2.41</td>
<td>7.58</td>
<td>25.7</td>
<td>90.8</td>
<td>329</td>
</tr>
<tr>
<td>$D^n$ noise/signal</td>
<td>2</td>
<td>2.49</td>
<td>2.65</td>
<td>2.73</td>
<td>2.77</td>
<td>2.80</td>
</tr>
</tbody>
</table>

The temperature sensor was a single, 125-µm, negative-resistance, bead thermistor attached with 18-µm leads to a commercial, digital, meteorological microsonde system made by the VIZ Manufacturing Company. The microsonde system was designed for high-resolution resistance probes and provided approximately 0.02% resolution and 0.1% accuracy over a resistance range of 10 kΩ to 10 MΩ. The temperature resolution of the thermistor and sonde–telemetry package range was 0.002–0.005 K. The balloon digitization and telemetry system and thermistor combination had a noise-equivalent temperature of 0.005–0.008 K rms, slightly poorer than the 0.002–0.004-K noise of a research-grade, analog digital oscilloscope set up to average 1000 scans. The response time for the glass-coated thermistors was approximately 40 ms with a 4-m/s air flow, and it was 30 ms for a bare bead. Because of the reduction in heat transfer with altitude, the probe response time increased by a factor of approximately 3 at 15 km. The 200 µm × 160 µm thermistor beads were ellipsoidal in shape and were supported by two opposing 20-µm-diameter platinum–irridium wires extending along the major axis that were 5 mm long. The wire leads actually reduce the bead-
thermistor response time by acting as fins. Independently, Fuehrer et al.\textsuperscript{11} has found that the heat transfer between the air and the long support leads of the thermistors decreases the bead response time by approximately a factor of two.

The microsonde package provides digital telemetry for eight data channels every 1.25 s. Four of these eight channels were dedicated to conventional time, pressure, temperature, and humidity measurements, and the remaining four were available for external probe use. I used the latter four channels in a variety of configurations. One procedure connected a single thermistor to all four spare channels. This technique provided a burst of four measurements separated by 30–50 ms that repeated every 1.25 s. A second alternative was to space two bead thermistors 1 m apart horizontally and sample them in an A-B sequence that repeated every 1.25 s. This two-probe arrangement allowed me to compute a conventional 1-m-separation structure function to compare with the vertical-difference results. On some later flights, one of the remaining spare channels was used for a completely separate analog, differential-temperature \(C_{p}^{2}\) sensor that used 12-µm thermocouple wires horizontally separated by 1 m in addition to the thermistors.

With the balloon data samples being separated by 1.25 s and with a 4-m/s ascent rate, the vertical-temperature samples were nearly 5 m apart. Using all four channels for a single probe provided a burst of four measurements spaced 25 cm apart, but because of the binary-counting scheme used in the digital sonde analog-to-digital conversions, this spacing could vary by a factor of 2. Consequently because of the variable spacing and the probe’s heating between consecutive samples, most of the data was processed with the fixed 1.25-s (5-m-separation) rate.

The thermistor resistance to temperature conversion used a Steinhart–Hart\textsuperscript{12} relation

\[
\frac{1}{T} = a + b \ln R + c \ln R^{2} + d \ln R^{3},
\]

with the quadratic term retained. Rather than calibrate the tiny thermistor before launch, it was calibrated in situ. A least-squares fit of the bead-thermistor resistance values to the calibrated balloon temperature sensor collected during a flight provided

the temperature-calibration coefficients. It is noteworthy that the potential temperature [Eq. (9)] is important only for the first-order \(D^{1}\) structure function and that \(T\) is sufficient for the second- and higher-order differences, because linear (and higher) trends are removed by these operations.

4. Results

Figure 1 illustrates the problem of trends in vertical-temperature-difference data. Changes in the mean temperature were orders of magnitude larger than the 0.01–0.1-K turbulent fluctuations. I have investigated the use of conventional first-order differences with a number of trend-removal techniques, such as running means. In general, none of these techniques worked satisfactorily. The results changed with the technique. Temperature inversions in which the gradients changed rapidly were particularly troublesome. Employing second- and higher-order temperature structure functions has worked very well. Typical results for second-, third-, and fourth-order vertical-temperature-difference computations of the atmospheric coherence length\textsuperscript{13} and isoplanatic angle\textsuperscript{14} are shown in Table 3 and compared with optical modulation transfer-function measurements from the Fourier transform of a 35-cm telescope, stellar point-spread function and temperature differences between two thermistor probes separated horizontally by 1 m for data collected on the same balloon launch on the evening of 15 November 1991. The optical isoplanatic angle measurements were collected with an apodized-aperture, scintillation-based isoplanometer, reported elsewhere.\textsuperscript{15}

Table 3 shows that the second-, third-, and fourth-order differences produce \(C_{p}^{2}\) results that are consistent with optical measurements, whereas the first-order procedure overestimates the turbulence, which then underestimates the optical values. This particular evening had a jet stream overhead that increased turbulence near the tropopause, reducing both \(r_{0}\) and \(\theta_{0}\).

Another test of the single-probe high-order vertical-temperature-difference procedure involved a comparison of the vertical-difference results with data from an independent sensor. A balloon flight on 12 March 1993 included a single thermistor temperature probe and a separate high-speed, analog, differential-temperature sensor that used a pair of 12-µm thermo-

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\[\text{Table 3. Comparison of Vertical-Temperature-Difference, Horizontal Differential-Temperature, and Ground-Based Optical-Instrument Measurements of } r_{0} \text{ and } \theta_{0}^{c}\]

<table>
<thead>
<tr>
<th>Vertical Temperature Measurements</th>
<th>Horizontal Temperature Measurements\textsuperscript{a}</th>
<th>Ground-Based Optical Data\textsuperscript{c}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st Order</td>
<td>2nd Order</td>
</tr>
<tr>
<td>(r_{0}) (mm)</td>
<td>20.0</td>
<td>28.6</td>
</tr>
<tr>
<td>(\theta_{0}) (µrad)\textsuperscript{d}</td>
<td>1.9</td>
<td>3.1</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Vertical and horizontal temperature measurements were taken during a balloon flight on 15 November 1991 at 23:00 Pacific time.

\textsuperscript{b}Taken between two thermistors with a 1-m horizontal separation.

\textsuperscript{c}Atmospheric modulation transfer function and isoplanatic angle measurements collected from ground instruments during the balloon’s flight.

\textsuperscript{d}At 500 nm.
couples separated by 1 m with an analog differential amplifier for comparison.

Figure 4 shows five point-averaged $C_n^2$ results for the data of Fig. 1. Figure 5 shows a comparison of the unaveraged $C_n^2$ data computed from second-order vertical temperature difference and from 1 m, horizontal temperature differences for this same flight. The comparison results are similar, although differences arise from statistical sampling.

Table 4 shows the average $C_n^2$ for the data in Fig. 4, which is between 14 and 16 km as computed from the first-through-fourth-order vertical differences compared with a thermocouple-based, horizontal, differential-temperature sensor with 12-µm probes spaced by 1 m. The first-order vertical-temperature difference is 2 orders of magnitude too large, whereas the second-through-fourth-order differences are similar, although they are nearly a factor of 1.6 larger than the horizontal differential-temperature sensor. This is a characteristic of all higher-order vertical-difference data collected to date and may reflect aliasing from abrupt changes in the temperature between layers unresolved by the 4–5-m separation of the data points. It may also reflect real differences in the horizontal and vertical isotropy of the atmosphere. The data in Table 4 include corrections for the wind speed and separation between data points because of the pendulum-swing motion of the sonde package that introduces a horizontal wind component that is added to the nominal 4-m/s vertical-ascent rate.

Using optical telescopes, I have observed that the balloon–sensor packages typically swing ±30° with a somewhat random pendulum oscillation frequency of approximately 12 s. This corresponds to ±15 m amplitude and a corresponding peak pendulum horizontal-velocity component of 8.6-m/s, which is larger...
than the 4-m/s ascent velocity. The probability distribution of a sine wave forces the balloon to spend most of its time near an extremum. The weighted average horizontal velocity would be 4.1 m/s so the average wind speed including the vertical and horizontal components was approximately 5.7 m/s. Although the horizontal component can be reduced by a tail or other aerodynamic drag surface, the horizontal wind component must be considered when the probe is placed to avoid wake effects from the sensor-electronics package.

In these measurements, the vertical separation of the data samples was approximately 5 m and was limited by the telemetry rate of the microsonde system. Ideally, a closer separation of approximately 1–2 m or less between data samples would reduce the potential for one to approach the outer scale when computing the higher-order differences across thin turbulent regions.

Higher-order differences are useful for handling other artifacts in turbulence data. Atmospheric horizontal or vertical temperature data with linear trends such as in the stratosphere can have a structure function proportional to $r^2$. The corresponding $f^3$ power spectra at low frequencies do not represent turbulence. The second- and higher-order difference algorithms remove this artifact.

5. Conclusions

I have successfully utilized single-probe, vertical-temperature-difference measurements and higher-order structure functions to collect vertical profiles of the atmospheric, optical, $C_n^2$ distribution. Higher-order structure functions avoid inhomogeneous trends in the atmospheric-temperature profile elegantly. The comparison between optical $r_0$ and isoplanatic angle $\theta_0$ measurements is very good. In the stratosphere, the $C_n^2$ values derived from second- through fourth-order vertical differences taken with 5-m separations were approximately 1.6 times larger than the $C_n^2$ values derived from a horizontal differential temperature sensor taken with a 1-m separation. This is probably an artifact from the 1.25-s sample rate of the balloon-telemetry system that introduces aliasing from the 5-m separation of the data points, although it may signify real differences between horizontal and vertical fluctuations in the stratosphere.

The simplicity of this vertical-difference technique reduces the cost and complexity of the collection of atmospheric $C_n^2$ by approximately a factor of 20. The procedure is sufficiently simple that it could be implemented on routine meteorological atmospheric balloon soundings.

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References


Table 4. Average $C_n^2$ Computed from Vertical-Temperature-Difference Measurements Compared with Simultaneously Collected Conventional Differential-Temperature Sensor Measurements

<table>
<thead>
<tr>
<th>Vertical Temperature Measurements</th>
<th>1st Order</th>
<th>2nd Order</th>
<th>3rd Order</th>
<th>4th Order</th>
<th>Horizontal Temperature Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_n^2$ (m$^{-2/3}$)</td>
<td>$4.13 \times 10^{-15}$</td>
<td>$4.17 \times 10^{-17}$</td>
<td>$4.06 \times 10^{-17}$</td>
<td>$3.98 \times 10^{-17}$</td>
<td>$2.49 \times 10^{-17}$</td>
</tr>
</tbody>
</table>

*Measurements were taken during a balloon flight on 12 March 1993 (see Fig. 1). Probes in the conventional sensor were separated horizontally by 1 m.