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Abstract

Most internal wars end on the battlefield. In comparison to inter-state conflicts, a relatively small percentage of these fights are concluded at the negotiating table. While significant attention has been given to how and why internal wars begin and even how they begin to evolve, little attention has been given to how they are concluded. What research has been done on this subject has focused on the problems that stand in the way of achieving a negotiated outcome, not how these conflicts are resolved by force. The purpose of this article is to evaluate the dynamics of the end game struggle and the differing ways in which states and insurgencies "win" and "lose" internal wars.

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THINGS FALL APART
The 'Endgame' Dynamics of Internal Wars*

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All internal wars come to an end, even if -- from the vantage point of the combatants and those who are caught in the crossfire -- they sometimes seem to go on forever. The subject of just how these conflicts end is an important one. There have been almost 300 internal wars initiated since 1945. At this writing 250 of these have come and gone.[1] The human and material costs of these conflicts have been incalculable, much greater than the combined costs of the inter-state wars fought over the same period.[2] Despite the cost and frequency of internal wars, however, we still do not have a close understanding of how they are resolved. What research has been conducted on this subject has focused almost exclusively on the subject of negotiated outcomes.[3] Very little attention, by contrast, has addressed the complementary question of how organized internal conflicts end in the absence of a meaningful negotiated settlement, which is to say, how they are concluded on the battlefield. More than 80% of these wars, it turns out, were resolved by force.[4] This stands in contrast to inter-state conflicts since 1945 which, according to one recent estimate, have had a better than 50% chance of ending in a negotiated compromise.[5]

Given the frequency of internal wars and the small chance, on average, that these conflicts will end in a compromise settlement, recent interest in the subject of negotiated outcomes and how the chances of such an outcome can be improved, is not surprising. What is surprising, perhaps, is that these same numbers have not led to similar interest in the subject of how these conflicts are so frequently resolved on the battlefield. This issue is of interest, not only because most organized internal conflicts are fought down to their logical conclusion, but because "winning" and "losing" is an essential part of the conflict process, whether the war is actually resolved by force or not. It is frequently the inability to win that drives the two sides to the negotiating table and a compromise settlement. Understanding just how this occurs, and, indeed, what winning and losing actually mean, can give us insights into what we can expect from internal wars and how close to the end a conflict might be at any particular point in time. It can also give us insights into when the players in an internal war might be open to a compromise settlement, and when they are more likely to try to bring the conflict to a unilateral conclusion on the battlefield.

We will begin to address some of these issues by developing an informal and then a formal framework for understanding the "endgame" dynamics of an internal war. Our discussion, in this case, is couched in terms of the final period dynamics of a classical insurgency, although our model can be generalized to other types of organized internal conflict. Throughout the paper we raise and offer preliminary answers to a number of questions: Does the final period of struggle between a state and internal challenger follow

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a predictable pattern? How do we know that an insurgency is over in the absence of an agreement that the end has arrived? What do the dynamics of an insurgent defeat look like in the face of a successful counterinsurgency campaign? What do the dynamics of state collapse look like in the wake of a successful insurgency? What explains these dynamics? Can insurgencies ever end in a stalemate in the absence of a "clean win"? Is there a point of no return in an insurgency, where the outcome of the war has already been decided but the fighting continues? All things being equal, what influence can the dynamics of these conflicts have on the chances for a compromise settlement?

1. Win, Lose, or Draw

An insurgency is a struggle for power (over a political space) between a state (or occupying power) and one or more organized, popularly based internal challengers. The objective of the state is to retain power and defeat or displace its competitor(s). The insurgency's objective is to expand its popular base of support and defeat or displace the state. The strategic logic of the struggle follows from the relative position of the two players at the beginning of the game. The state, as the incumbent player, is a *force in being*. It enters the game with an inertial advantage -- it will remain in power unless it is forcefully replaced. The insurgents, by contrast, are a *force in development*.[6] As Mao once noted, they "must start from nothing and grow."[7] To win, the rebels must reverse the force ratio that defines the two sides at the outset of the conflict to the point where they can either openly defeat their opponent or force him to concede or withdraw. How far the insurgents must evolve to reach this point depends on the strength of the state. The stronger the state, the stronger they must become to prevail. The side that is able to first eliminate the other as an organized political-military challenge "wins" the game.

Because of the force asymmetry that defines the two sides at the beginning of the game, successful insurgencies must evolve in size and complexity over time. As this occurs, the scope and intensity of the conflict will naturally increase, reflecting the fact that the insurgency is gradually getting stronger and the state is getting weaker. Mao, among others, suggested that this progression must unfold in three stages: an opening phase, in which the insurgents are on the strategic defensive, an intermediate phase, or period of strategic parity, in which neither side has a clear battlefield advantage, and a terminal phase, in which the strategic advantage has shifted against the state and the insurgents are able to move on to the offensive.[8] This, of course, looks at the problem from the perspective of the challenger. The state's position during this evolution is the complement of its opponent -- when the state is on the offensive, the insurgents are on the strategic defensive -- when it is on the defensive, it is because the prevailing force ratio has changed to the point where the rebels have been able to assume the offensive. Each side's relative position can ebb and flow over time until the fight is concluded.

At a general level, insurgencies can end in one of three ways short of a negotiated settlement; with a win by the state, a win by the insurgents, or an effective stalemate in which neither side is able to dominate the other and the conflict gradually grounds down

INSURGENT CONFLICTS SINCE 1945		
Total Insurgent-State Dyads Since 1945[a]	**278**	**100.00%**
Continuing Insurgent-State Dyads[b]	**44**	**15.83%**
Concluded Insurgent-State Dyads[c]	**234**	**84.17%**
State Losses[d]	18	7.69%
Insurgent Losses[e]	104	44.44%
Nominal Settlements (State Loss)[f]	16	6.84%
Nominal Settlements (Insurgent Loss)[g]	27	11.54%
Real Negotiated Settlements[h]	47	20.09%
Other/NA End Games[i]	22	9.40%

a: This figure represents the sum of all continuing and concluded insurgent-state dyads since 1945. "Insurgent-state dyad" is the term we use to designate a conflict between a state and an individual insurgent group resulting in at least 25 recorded battle-deaths per year. This dyad is our unit of analysis for counting purposes. A larger war can involve one or more dyads.

b: This figure includes all ongoing insurgent-state conflict dyads resulting in 25 battle-deaths or more per year, as of August, 2006.

c: This figure represents all insurgent-state conflict dyads that have been concluded since 1945. It sums all state losses, insurgent losses, nominal and real negotiated settlements, and all other cases from 1945 to the present.

d: A state loss is recorded when an insurgent group defeats or displaces the state.

e: An insurgent loss is recorded when an insurgent group is defeated or displaced by the state, i.e., it is no longer an effective fighting force. This is said to occur when the conflict it is engaged in falls below the 25 battle-death threshold.

f: A nominal settlement (state loss) is recorded when the insurgent group defeats or displaces the state (or occupier), assumes control of the government, and this win is codified in "negotiations" with the state. In this case, the insurgent group has won by force. Negotiations are a formality that do not result in a meaningful compromise between the political objectives of the state and the insurgency.

g: A nominal settlement (insurgent loss) is recorded when the regime defeats or displaces the insurgent group, retains control of the state, and this win is codified in "negotiations" with the insurgent group. In this case, the state has won by force.

h: A real negotiated settlement is recorded when both the insurgent group and the state negotiate a meaningful compromise to end the conflict, despite competing objectives. Real negotiated settlements are divided into three "types."

i: This category is used to code insurgent-state conflict dyads which do not have a specific endgame or settlement. This category is also used to designate cases in which the endgame is unknown due to a lack of data.

Source: Internal War Database; Department of Defense Analysis, NPS, Monterey, CA.

Table 1: Insurgent Conflicts Since 1945

REAL NEGOTIATED SETTLEMENTS SINCE 1945			
Total Real Negotiated Settlements	**47**	% of Total	% of all Concluded Insurgent-State Dyads
Type 1 Settlements[a]	8	17.02%	3.42%
Type 2 Settlements[b]	17	36.17%	7.26%
Type 3 Settlements[c]	22	46.81%	9.40%

a: Real negotiated settlements in which a breakaway faction of the insurgency decides to carry on the conflict in defiance of the agreement.

b: Real negotiated settlements in which a compromise agreement is reached between the state and one or more insurgent groups but a larger conflict continues against one or more other internal challengers (insurgent-state dyads).

c: Real negotiated settlements resulting in a stable peace.

Source: Internal War Database, Department of Defense Analysis, NPS, Monterey, CA.

Table 2: Real Negotiated Settlements Since 1945

to a halt, leaving both sides with some degree of control over the contested "state space". As a practical matter, as we show in Table 1, the great majority of insurgent conflicts are resolved on the battlefield, either directly, with a unilateral win by one side or the other, or indirectly, in a nominal settlement that reflects one side's clear dominance over the other. Internal conflicts that appear to end on their own in a division of power are seldom resolved, unless the cessation of hostilities and the de facto division of the spoils at the time the fighting stops is codified in a follow-on political settlement. The apparent lull in the action, in most cases, represents nothing more than a temporary respite, until one or both sides believe it is once again in their interests to take up where they left off. Most of these cases, in this respect, are not endings, but fighting pauses that are sensitive to any parametric change that gives one player a temporary advantage over the other.

If we look more closely at the statistics presented in Table 1 we see that of the 234 insurgencies that have been initiated and resolved since 1945, only 112 or roughly 48% were concluded with even a *nominal* negotiated settlement. On the face of it, this looks better than what we might expect; it appears that there is almost a fifty-fifty chance of resolving an insurgency at the negotiating table rather than on the battlefield. A more careful look at these individual cases, however, reveals a quite different story. Most of these settlements, it turns out, were little more than a cover story for victory and defeat. The resulting agreement simply codified what had already been achieved by force. Only 47 or 20% of these cases, we estimate, were concluded by *substantive* agreement; one defined by a meaningful division between the interests of the warring parties.

Turning to Table 2, we see that even these numbers do not give us a full picture of what is going on. Eight or 17% of these cases turned out to be "Type 1" settlements, in which a breakaway faction of the insurgency decided to carry on the war in defiance of the agreement. Seventeen or 36% were "Type 2" settlements, in which a compromise agreement was arrived at between the state and one or more belligerent(s), but the war

continued against one or more internal challengers. It appears that only 22 or roughly 47% of the substantive agreements negotiated were "Type 3" settlements that resulted in a stable internal peace. This is fewer than 10% of all the insurgencies begun and ended since 1945. In 212 or 90% of the cases, some aspect of the war was resolved by force.

Our primary interest here is with the "endgame" struggle of the fighting units of organization in insurgent conflicts, rather than with the end of such conflicts, per se. In the case of a two player game, of course, the one is synonymous with the other -- the conflict can be said to "end" when one side wins. The endgame, in this case, is the final phase of the final period of the conflict resulting in a win by one player and the end of the other. In the event of a multiplayer game, however, in which the state faces more than one challenger, the issue is somewhat more involved. Where the state faces multiple competitors which are also fighting among themselves, the larger conflict will not end until all but one of the players are defeated or displaced. Each dyad involves its own endgame fight. Similarly, where the state is competing against a coalition of players, the larger conflict cannot be said to end until the state is either defeated or each coalition member is beaten. Each of these actions again involves its own endgame struggle. As long as the coalition holds, an insurgent win will only involve a single final struggle against the state. The state, for its part, must beat every member of the coalition.

A player can be said to defeat or displace his opponent when the latter is pushed below his "breakpoint." Following the standard literature on combat modeling, this is defined as the point at which a combatant is no longer "combat effective." This will occur when the losing player either "breaks" or "breaks off" the engagement.[9] In the first case, he is no longer able to offer further organized resistance; in the latter case he withdraws from the fight rather than incur the costs of continuing. In either case, the winning contestant is the only player that is still in the game when the smoke clears. While the notion of a breakpoint has been traditionally used to define the point at which one side wins and another loses a conventional force-on-force engagement, it is also applicable to the more complex problem of winning and losing an internal war. Each player is an organized belligerent. Their ability to compete, much less go on to win, depends on their capacity to mobilize, transform and employ a diverse array of human and material resources against the other for a strategic purpose. The first side that is no longer able to do this can no longer compete and can be said to "lose" the game.

In considering the issue of winning and losing in an insurgency (or other type of internal war), however, we can distinguish between a "strong win" and a "weak win."[10] As noted above, an insurgency is a contest for control over a common political space. While one side cannot gain control over this space without defeating or displacing the other, it is sometimes the case that a player will succeed in defeating or displacing his opponent without first establishing full political control. A player that is not only able to push his opponent across his breakpoint but also establishes effective control over the political space they were fighting over in the first place, can be said to achieve a strong win. His opponent, in this case, is not only out of the game, he has no opportunity to come back. A player who succeeds in pushing his opponent below his breakpoint but is unable or unwilling to extend his control over this contested space, can be said to achieve

a weak win. The war is over, in the sense that the enemy no longer poses an organized threat, but he may still have a political foothold that he can use to reorganize himself to make a comeback at a later time. The fighting has been concluded, for now, but the winning side cannot preclude the possibility of a follow-on fight down the road.

2. Patterns of State and Insurgent Collapse

While the measure of victory and defeat for the two sides in an insurgent conflict, we suggest, is the same, the empirical record reveals that states and insurgent organizations decline and approach their respective breakpoints in very different ways. States generally pass a tipping point and enter their end games and begin to decay at an *accelerating* rate. This is often an indicator that the final period of the struggle has begun. Between the time the conflict enters this phase and the time the state disintegrates, the conflict "speeds up." The result, as illustrated in Figure 1A, is a parabolic trajectory of decay that approaches and crosses the state's breakpoint at a fairly high angle of attack. The end, in such cases, is typically decisive, sudden and often violent. Examples include the defeat and downfall of the Nationalist government in China (1949), the collapse of the Batista regime in Cuba (1959), the collapse of the Lon Nol regime in Cambodia (1975), the end of the Somoza government in Nicaragua (1979), the collapse of the Mengistu regime in Ethiopia (1991), and the downfall of the Siad Barre regime in Somalia (1991). Even when, in retrospect, it may be easy to see that the conflict was drawing to a rapid end, many witnesses to the event at the time did not see the point of collapse coming until it was upon them.

Insurgencies, by contrast, tend to decline historically at a *decelerating* rate. While their rate of decline may initially be fairly steep, between the time the conflict enters its end game and the insurgency collapses, the conflict typically "slows down." The result, as illustrated in Figure 1B, is an asymptotic trajectory of decline that crosses the insurgents breakpoint at a low angle of attack. The size and associated operational tempo of the insurgency continues to deteriorate at a declining rate until the group finally reaches the point that it can no longer pose an organized military challenge. Because most insurgencies fail, examples of these endgames are quite numerous, ranging from such well known cases as the Hukbalahop in the Philippines (1954), the so-called Mau Mau insurgency in British Kenya (1956), the Malaya National Liberation Army (1960), the Tupamaros in Uruguay (1972), the Polasario Front in Morocco (1989), and the Shining Path in Peru (1993), to such obscure cases as the North Kalimantan Liberation Army in Brunei (1962), and the Eritrean Islamic Jihad Movement in Ethiopia (2003). The end, in such cases, is almost always indecisive, in the sense that there is seldom a climactic engagement that marks the terminal point of the insurgency. The insurgency that comes in like a lion, as the saying goes, may go out like a lamb. Indeed, by the time the end comes, many observers will have assumed it has already come and gone.

The different way in which states and insurgencies fall apart is due to the basic information asymmetry that characterizes insurgent-counterinsurgent conflicts.[11] As the incumbent player, the state is comprised of a visible set of governing institutions. Its visibility confronts the defending regime with a classic "air defense" problem.[12] The

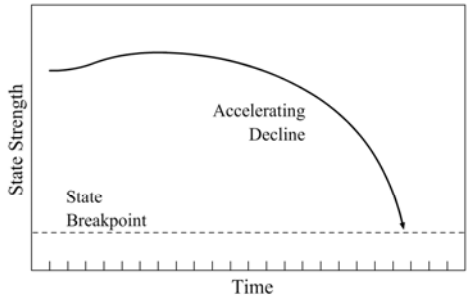

Figure 1A. State Endgame

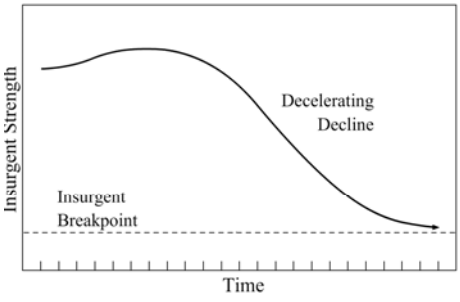

Figure 1B. Insurgent Endgame

insurgents get to choose what they will attack, while the state, not knowing where the insurgency will strike, must either try to protect everything of value poorly, or a few things well. As the challenging player, the insurgency faces no such constraint, a fact which allows it to operate with a high degree of anonymity in the face of the state's efforts to force it into the open. Its relative invisibility, furthermore, is generally reinforced by its popular origins, which provide it with a natural degree of cover and concealment. It is also due to its small size, at least in the early period of the struggle, which limits its level of exposure. As long as the rebels do not become too big too early, they can operate with a small profile that is easily masked by their environment. This places the state at a disadvantage. While the insurgency, under these conditions, can engage any target that it is strong enough to hit, the state must first hunt down the opposition before it can be engaged. This is hard to do if it is too small to see.

The nature of this asymmetry has important implications for both sides. For the state, it means that state losses to the insurgency are directly proportional to insurgent strength. The state will suffer more (or fewer) casualties as the insurgency increases (or decreases) in size. This is not true for the insurgency. Because the state must first find the enemy to eliminate him, insurgent losses are not simply proportional to the size of the state but will be influenced by the number of possible "interactions" between the combatants. It will depend, in other words, not only on the number of "shooters" but also on the number of potential "targets." By way of example, let us suppose that the state is employing four counterinsurgents, John, Paul, George, and Ringo, to hunt down three insurgents, Larry, Moe and Curly. Each hunter, in this case, has three chances of adding to his body count. If John is on patrol, he might come across Larry, he might uncover Moe, and he might find Curly. There are three possible interactions in this case. Each of the other members of the band has the same opportunity, giving us 3 x 4 = 12 possible interactions over all. If the number of insurgents increases, all other things being equal, the number of possible interactions will increase. This will result in an increase in the insurgents' attrition rate. If the number of insurgents declines, it will result in a reduction in the number of possible interactions, resulting in a corresponding decrease in the insurgents' attrition rate.

What this means as a practical matter is that there is a time to grow, a time to avoid growing, and even a time to reduce the organization's footprint and operating profile. As the insurgency expands it will gradually sacrifice its opening information advantage -- it will be easier to find and, therefore, easier to engage. Any increase in numbers, in this

respect, has two effects, each offsetting the other; a simple "force effect" that makes it stronger -- by increasing its resources, and a more subtle "information effect" that makes it weaker -- by increasing its visibility. In the face of this tradeoff, an insurgency will grow with the expectation that its increasing strength (and the declining strength of the state) will more than offset the fact that it is becoming easier to see. If true, its net advantage over the regime will increase and it will continue to move forward. If it is not true, its position will actually begin to deteriorate. While the insurgency's growing numbers will increase its physical capacity to engage the state, this advantage will be dominated by the state's growing ability to find, fix, and engage the insurgents. The insurgency's initial success, in this case, will result in its subsequent decline.

There are a number of important ramifications associated with this attrition model. One of the most interesting, for our purposes, is that as the state succeeds in reducing the insurgency, it will become increasingly difficult for it to continue to do so at a constant rate. The opposition's vulnerability to attack, in this respect, will tend to decrease as its numbers decline. This is true for the same reasons that explain why insurgent groups tend to become more vulnerable as they grow. Just as a growing insurgency increases its exposure to the state, a contracting insurgency reduces its exposure -- by reducing the number of possible enemy initiated contacts and allowing the insurgents to gradually reclaim their information advantage. This, in turn, makes it increasingly difficult for the government to capitalize on its success. The state cannot hit what it cannot see, and the insurgency will become more difficult to see as it declines, a fact that will make it harder and harder to engage. Most counterinsurgency campaigns, in this respect, are subject to *diminishing returns*. This is in stark contrast to counter-conventional campaigns, which are generally subject to *increasing returns* -- success, in such cases, reinforces success. While this dynamic is a liability to a growing insurgency, it works to its advantage as it begins to contract.

This explains why insurgent endgames consistently look the way they do. The only way that the state can continue to reduce an insurgency at a continuous rate, under these circumstances, is to either dramatically and continuously increase its force size at a faster rate than the decline in insurgent numbers, or otherwise compensate for a static or slowly growing search and engagement force by significantly and continuously improving the efficiency with which the force it does have does its job. This leads to the paradox that the state often requires a larger force to eliminate an insurgency that is in decline than it does to push it into decline in the first place once it begins to grow. The mathematical relationships that define these requirements are discussed in Section 1 of the Appendix and in previous work by one of the authors.[13] It is sufficient for our purposes here to point out that this is very difficult to accomplish. In the absence of doing so, the best the state can hope to do is push the insurgents into decline at a declining rate, eventually winning the war in slow motion as organized resistance slowly disappears. This decline may well begin, as in Figure 1A, with a relatively steep drop in insurgent numbers, but the rate of decline will eventually decrease as their numbers are reduced over time.

A common consequence of this is what we refer to as an *equilibrium trap*. The state, in this instance, is able to suppress the insurgency at a decreasing rate up to a point, but is

not able to mobilize the marginal advantage necessary to push it below its breakpoint. The insurgents, for their part, retain sufficient room for maneuver to stay in the game, but are unable to expand their presence given the state's prevailing level of effectiveness. An internal war, under these circumstances, can continue indefinitely, without either side gaining a decisive advantage over the other. Classic examples of such conflicts can be found in Afghanistan, Angola, Burma, Colombia, and Ethiopia, to name just a few cases. In a purely internal struggle, the fight can be expected to continue until the players either reach a negotiated conclusion (a statistically unlikely outcome) or the circumstances of the conflict change in a way that gives one side a winning advantage over the other. In the case of an insurgency against an intervening power, an equilibrium trap -- if it is not resolved -- typically ends in withdrawal and a *de facto* "win" for the insurgents.[14]

3. The 'Small World' of Insurgent Conflicts

While the manner in which states and insurgencies collapse is highly consistent, the speed with which these two quite different patterns reveal themselves can vary widely. As suggested at the outset, this is not only due to each side's general growth and attrition dynamics, it is also due to the internal structures of the players themselves, which have a significant influence over what these dynamics look like in the first place. How states and insurgencies are organized, in short, have implications for the outcome of the campaign and the speed with which they fall apart in the final period of the conflict.

The question of how states and insurgencies organize themselves is complex and goes far beyond the scope of this article. To make some general observations about the subject we found it useful to define each side as a "networked organization" and examine some of the ways in which their network characteristics influence their end game dynamics. Recent discussion on this subject have advanced the idea that the network architecture of states and those of their internal challengers, to include insurgent groups, are structurally distinct. Our own observations suggest that, contrary to this perspective, the types of (immature) states that have proven to be most vulnerable to popular internal challenges and the (mature) insurgent groups that challenge them, are actually typologically similar. As a general rule, each is characterized by a so-called "small world," scale free structure. The first attribute is defined by a small average path length between any two randomly selected vertices or nodes within the system, combined with a high degree of vertex clustering; the second is defined by a "power law distribution" that allows for a small number of highly connected vertices that help hold the larger system together.[15]

The reasons for this are easy to understand when we consider that both insurgencies and the types of regimes that generally spawn organized and violent internal challenges are formed and maintained through preexisting social networks and preferential growth. It is now well understood that insurgencies and other types of underground organizations form around established social networks, based on kinship ties, friendship ties, or other kinds of communal or associational linkages.[16] The same is essentially true of the weakly institutionalized regimes of the Third World, where little if any distinction can be made between the institutions of the state and the social groupings from which they emerge.[17]

Formed, as they are, out of naturally generated social networks, both the state and the insurgency step into the political arena with well defined small world properties; each side is characterized by a relatively high level of social connectivity, resulting in a short average "distance" between any two randomly selected members within each system. Once established, this small world is naturally carried forward in time through a process of preferential growth, in which well connected "vertices" are more likely to attract new members than those which have fewer and more distant ties to the network. This is the only attribute necessary to create and maintain a scale free network structure.[18]

One area in which states and insurgent groups do tend to distinguish themselves is in their authority structures. As a general rule, state authority within weak Third World regimes is *centralized*, often residing in the person of a single individual or ruling clique. While the ability of such regimes to effectively command from the center is inevitably imperfect, the authority to do so is generally concentrated in a small political elite. The political center, in this case, exercises a monopoly over the resources needed to confront the insurgents -- resources are mobilized nationally and employed locally. Insurgents, by contrast, tend to develop *decentralized* authority structures as they mature, even where they originally form around a single charismatic individual or centralized charter group. This is due, in large measure, to the circumstances under which they develop. Where the state is constructed from the top down, insurgencies are built from the bottom up. They not only emerge locally, they remain tied to (an increasingly distributed) local base of support as they grow. The resources needed to prosecute the struggle, in this sense, are mobilized at the local level and employed nationally. The local insurgent leaders that control these resources are able to exercise a measure of authority over the larger campaign that is not shared by their state counterparts, who are center-dependent.

The emergence of local authority centers is reinforced by the insurgents' need to preserve their anonymity in the face of the state's counterinsurgency campaign. As a general rule, this is incompatible with a centralized command structure, particularly as the insurgency increases in size and the "signals signature" of the center expands. All other things being equal, the state's ability to identify individual vertices within the insurgent network is directly related to their level of communications. Security and communications, in short, are inversely related; the more an element within the system communicates, the easier it will be to see.[19] While the opposition may well be able to improve this tradeoff through encryption, hand-to-hand communications, the use of covered communications, etc, they cannot escape the tradeoff itself. To help resolve this problem, growing insurgencies tend to develop decentralized command and control structures. Operational authority, in this case, is distributed across the network, rather than located in a single vertex. Groups that choose not to adopt such structures do so at their own risk. It is not only easier for the state to find and eliminate the center of the insurgency under these circumstances, the consequences of doing so are much greater than the loss of a single authority center in a decentralized system of equal size.

Each side's network structure, to include its authority structure, has important implications for its competitive strength, offensively and defensively. As discussed in greater detail in the Appendix, we can evaluate the "network strength" of the state and the

insurgency at any particular point in time by examining the size of the network, defined by the number of vertices, each side's network typology, the degree to which each side's authority structure remains intact, and each player's ability to discriminate between structurally significant and insignificant targets on the other side. Expressions (2) and (3) define the way in which these considerations relate to each other to determine player strength. As the conflict progresses, the strength of each player is subject to change. The size of each network will change through growth and attrition (including defection), the effect of these changes will vary depending on the significance of the vertices that are lost and added to the overall effectiveness of the network, and each player's ability to carry out *directed* attacks against the other side will vary directly with its force ratio. Changes in each side's strength, as we discussed above, will influence its ability to compete in the next time period, which will influence its strength index, in turn.

The small world, scale free typologies of the players in this game give each side a certain structural resiliency to non-directed attacks carried out by the other. The original work on this subject was carried out by Albert, Jeong, and Barabási, who showed that scale free networks are less vulnerable to random flaws than "exponential networks," where the probability to finding a highly connected node decreases exponentially as the number of vertices increases.[20] There are no highly clustered nodal points in exponential networks, which means that every node in the system plays a greater role in carrying the whole. The random loss of any one of them, therefore, is likely to have a greater impact on the strength of the system than a random attack on a scale free network of similar size. Scale free networks, by contrast, are highly vulnerable to directed attacks, where one's opponent is able to pick out his targets for the effect they will have on network strength. Because the overall strength of a scale free system is dependent on a relatively small number of well connected nodes, the loss of any one of them can have a significant effect on the strength of the network as a whole. Certain nodes in such a system, in short, are much more significant than others, and if a player is able to attack these nodes rapidly and systematically he can expect to push his opponent below his breakpoint.

Both the insurgency and the state are also clearly vulnerable to any loss of authority through the death, capture, or defection of key operational or inspirational leadership. Where these authority figures also serve as some of the network's most significant scale free "connectors," which is frequently the case, their loss is even more destructive. What impact the loss of authority providers will have on the strength of a network will depend on their significance to the system and the system's ability to replace them. In a decentralized system, the security of the whole is protected by the parts. These networks, all things being equal, are more likely to lose authority providers throughout the conflict, but are less vulnerable to the loss of any single one. Each loss has less of an impact on the strength of the overall system and the fact that authority is distributed across the network makes any single setback easier to overcome. In a centralized system, by contrast, the parts of the system are intended to be secured by the whole.[21] These networks depend on their ability to protect the dependent elements of the system by successfully defending the center. When they succeed in doing so, this choice can pay off. When they lose their center, however, the impact can be devastating. Historically, insurgents and poorly institutionalized states have difficulty recovering from a sudden loss of authority.

4. Things Fall Apart

All of this has implications for the speed and "tactical" picture of the endgame and each side's ability to save itself once it has begun to go into decline. For one thing, it turns out that while the trajectory of decline in the case of the insurgency and the state follow the characteristic concave and convex paths discussed above, a higher resolution picture of these collapse paths show that they do not fail smoothly but in "chunks." If the insurgency and the state were defined as random networks of largely undifferentiated vertices this would not be true; their path of decline would be smooth and predictable. The loss of any group of vertices in a given time period, in this case, would be no more significant than the loss of any other; the only question of interest would be the rate at which they were being reduced. Because the vertices that comprise insurgent and state networks, however, do not equally contribute to the whole, this is not the case. The loss of a large number of low level vertices within a system may not be as significant as the loss of a single "hub" or authority center. This not only makes each of these systems subject to periodic shocks, it also introduces an interesting stochastic dimension to the struggle as the end draws near. The conflict may appear to be on a predictable glide path in one period, only to take a significant turn for the worse (or the better) in the next.

The players' ability to effectively discriminate between high value and low value targets will tend to change, we have said, as a function of their changing force ratio. The stronger they are, all other things being equal, the better they will be at eliminating key (hardened or hidden) vertices in their opponent's network. For reasons we have discussed however, this will not influence the battlefield prospects of each side in the same way. While the state's ability to look for and destroy high value targets within the insurgent system will increase as its force advantage grows, the insurgents will *also* become more difficult to find as their numbers decline and they fade back into the woodwork. These two effects are *offsetting*. The first, if nothing else were to change, would allow the state to take the insurgency apart at an faster rate by hitting targets of greater value. The second, as we discussed earlier, works in the opposite direction, making the insurgents harder to locate and, therefore, harder to attack as their numbers begin to decline. Historically, the second effect dominates the first, which is why insurgent networks characteristically decline at a decelerating rate as they approach their breakpoint.

The state, as a general rule, is not so lucky. The effects of decline, in this case, are *reinforcing*. The rate at which the insurgents are able to engage the regime, as we have said, depends on the number of attackers; it does not depend on the number of defenders. As the balance of power shifts in its favor, therefore, the insurgency will find that it is able to hit the state at an increasing rate, thereby increasing its rate of decline. The effect of this shift, however, is not simply *quantitative*, it is also *qualitative* in nature. As their relative advantage grows the insurgents will be increasingly able to attack the state's network structure as a whole by discriminating between high and low value targets. It is not a matter at this point of only going after targets that they are able to eliminate, but targets that they increasingly choose to eliminate because of their significance to the

larger network. The first of these effects reinforces the second. Once it passes its tipping point, the regime will not only begin to lose more and more "vertices" to insurgent action, the vertices it loses are likely to be more and more important to its continuing ability to resist. This, in turn, will further accelerate its decline, improve the insurgents' force ratio, and make it more vulnerable to high value attack in the next time period.

The stochastic and volatile quality of the endgame struggle is reinforced by the fact that people's propensity to support one player over another throughout the conflict is highly sensitive to their expectations concerning which side is likely to win. Popular expectations are, in turn, sensitive to what other people are perceived to be doing. This is not only true for the players' popular base of support, but their internal membership. As the balance of power between the players begins to change, popular perceptions of the likely outcome of the fight will begin to change, accordingly. The expected benefits of aligning with one side will increase, and the expected benefits of backing the other side will decline. As this occurs, a growing number of observers will "switch sides". This, in turn, will induce others to adjust their expectations of the future and follow suit. What is interesting about this dynamic, of course, is that as people begin to abandon the "loser" and rally around the "winner" in response to their beliefs concerning which side is likely to prevail, they are increasing the probability of experiencing their prediction. If the number of people who make this switch passes a critical threshold, the losing side can experience a rapid, negative bandwagon effect that pushes it below its breakpoint.

This dynamic is one example of a broad class of strategic problems in which the best course of action depends on the choices made by others. While the payoff associated with either holding firm or switching sides depends on the outcome of the conflict, what this outcome looks like can ultimately depend on what other people decide to do. The interdependent nature of this decision problem will become increasingly acute as the conflict enters its endgame. As this occurs, the choices made by those associated with the declining side will have increasing influence over the dynamics of the conflict. The players' vulnerability to defection and unraveling will depend on their specific network structures and the "loyalty" of their individual membership. The resiliency of each system will also vary depending on the point in the network that the failure occurs. An otherwise robust network may be able to easily absorb the defection of numerous "foot soldiers" (i.e. low order vertices) without experiencing a chain reaction that cascades through the system. The effects of a high order defection, by contrast, are more serious, increasing the probability of a self-generating reaction that brings the system down.

The consequences of all of this, once more, are particularly significant for the state. We can see, historically, that insurgencies are also susceptible to negative bandwagon effects but that they are clearly less sensitive to these effects than incumbent regimes. This is due, in part, to the fact that they become less vulnerable to attack as they decline. While battlefield reversals may well induce marginal supporters of the insurgency to begin to withdraw from the conflict, reducing the group still further, this decline has a self-arresting quality in the absence of an extraordinary increase in effort by the state. Things get easier for the insurgents, in this sense, as they get harder, which helps reduce the pressure to defect. It is also easier to leave the insurgency than to leave the state,

reducing the need to do so until it is finally clear which way the wind is blowing. Most insurgents are part timers to begin with and it is often a simple matter to step in and out of the fight. The decision to abandon the war effort, in this respect, is not a bifurcated choice; it does not require actually ending one's involvement in the conflict, but simply cutting back. For these and other similar reasons insurgent defections more likely to "follow" than "lead" battlefield trends, making the insurgency less prone to panic.

The opposite, of course, is true of the state. As long as the incumbent regime hangs together, it may have a fighting chance. Once it begins to unravel, however, it can result in a self-reinforcing dynamic that can be hard to stop. As this threshold is approached, latent pressure within the government and security forces to abandon the system will gradually increase. In contrast to the insurgency, however, where it is generally easy to move in and out of the conflict at will, leaving the state is typically a one-way choice. Those closest to the regime, furthermore, will tend to be heavily invested in its survival. There are significant costs associated with leaving which help hold the system together until the end. The first sign of trouble typically occurs within the armed forces, which will experience increasing rates of desertion and a decreasing willingness to engage the enemy. As conditions on the battlefield continue to deteriorate, a growing number of individuals associated with the regime will begin preparing for the worst; moving their bank accounts out of the country, making sure that their passports and visas are in order, and sending their families abroad "on holiday." The state is now poised for collapse. It is now vulnerable to any battlefield shock that leads a critical mass of its supporters to conclude that the end has finally arrived and it is time to leave. In acting on this belief, they will set off a domino effect that results in a state defeat and an insurgent win.

5. Case 1: Insurgent End Game -- The Hukbalahap

The Hukbalahap movement was formed in 1942 around a collection of leftist peasant associations in Central Luzon, a 6,000 square mile region of 1.5 million people north of Manila. The region is bordered on the east by the Sierra Madre range and the Candaba swamp and on the west by the Zambales Mountains. The organization was formed to protect the interests of oppressed peasant sharecroppers against the Japanese. Soon after the Japanese invasion, the Huks formed an anti-Japanese alliance with the Philippine Communist Party (PKP).[22] The two most important leaders of the subsequent postwar insurgency emerged at this time, Luis Taruc, an activist peasant and member of the PKP, and Castro Alejandrino, a PKP member and former landlord. Between 1942 and 1945 the Huks became the largest anti-Japanese resistance in Luzon. At their peak, they had an estimated strength of 15,000 guerrillas and over 25,000 auxiliaries and supporters.[23] Their rapid reemergence after the war as a communist insurgency posed a significant threat to the newly independent government of the Philippines. After a decade long campaign, marked by numerous initial setbacks, the government was able to push the insurgency across its breakpoint along a decelerating decay path that mirrors our discussion.

5. 1. The Rise of the Huk Insurgency (1946-1950)

The Hukbalahap disbanded after the Japanese surrender but began to reorganize in 1946 in the face of attacks by local landlords and Philippine authorities and the decision on the part of Philippine and U.S. officials to exclude them from the newly formed interim government.[24] Events came to a head with the Congressional election of April 1946 when Luis Taruc and five other Huk representatives from Central Luzon, elected on the Democratic Alliance (DA) ticket, were prevented from taking their seats in the new Congress.[25] In response to this action, a meeting was called in May 1946 between the former Huk leadership and their political allies within the PKP where the decision was made to resurrect the Hukbalahap organization and initiate a popular uprising. In June 1946, Taruc and other Huk leaders divided the organization into a Central Luzon and Southern Luzon command, each operating independently of the other. The authority center of the insurgency was established on Mt. Arayat in Pampanga, where a number of the Huk leadership had close family and community ties and the movement enjoyed a strong natural base of support dating back to its origins in the anti-Japanese struggle.[26] The first directed actions against the government were conducted in summer 1946.

The newly-formed Philippine Armed Forces (PAF) were not prepared to deal with a peasant uprising. With independence, the government had rapidly demobilized the majority of the 132,000 men that had been brought under arms during the war years. By the time the Huks launched their first operations in 1946, Philippine security forces had been reduced to no more than 37,000 men. Twenty-four thousand of these were attached to the poorly led, trained, and equipped Philippine Constabulary (PC).[27] Prepared or not, in August 1946, President, Manuel Roxas, declared that he would "use any measures to force the peasants to give up their guns."[28] In response to this declaration, he launched his "iron fist" offensive against the guerrillas and predicted that the rebellion would be crushed within 60 days. Roxas's policy called for the use of open terror to destroy the guerrillas and their popular base. These efforts proved to be ineffectual, forcing the regime to follow up with a new, three-pronged offensive against the Huks in March 1947. PAF and PC units attempted to move simultaneously into the Central plains of Luzon, against Huk headquarters on Mt. Arayat, and into the vast wasteland of the Candaba swamp.[29] The 1947 campaign failed decisively. The Huks were able to inflict significant losses on the government and avoid any significant losses in return. PAF and PC forces were eventually forced to withdraw from central Luzon, leaving the Huks in control.

The government campaign fueled peasant support for the Huks without making any offsetting inroads against the insurgency. Huk operational capabilities grew accordingly. During this first period of the conflict, PC "neñita" units became notorious for randomly robbing and killing innocent peasants, making it safer to be an armed guerrilla than an unarmed bystander. PC units established curfews in barrios within the region and then forced local residents to buy passes in order to move around their own communities. In response to the growing and systematic abuse by undisciplined government forces, the Huks and their support base within the population expanded rapidly. By early 1948, the insurgency had established a well constructed and defended base area in and around Mt. Arayat and the provinces of Nueva Ecija, Bulacan, Pampanga, and Tarlac, an area that

came to be known in and out of the movement as "Huklandia." Taruc estimated that the Huks had 10,000 guerrillas under arms by 1948. Having secured their rear areas, the insurgency assumed the offensive, refocusing its efforts on Southern Luzon and the capital.

The Huk insurgency reached its peak between 1949 and 1950. President Roxas had died in 1948 and was replaced by Elpidio Quiriño. Quiriño managed to negotiate a cease fire with the insurgents in the summer of 1948, but the agreement quickly failed when the government could not compel the security forces to honor the truce.[30] In the wake of this failure, the insurgents launched a succession of large scale operations against government forces and their landed civilian allies on the edges of Central Luzon, extending their zone of control and bringing a larger peasant constituency under their authority. By 1949 the Philippine military estimated that the Huks had over 12,000 men under arms and over 110,000 active supporters.[31] PC leader Napoleon Valeriano estimated that the insurgents enjoyed active support from 150,000 people in Central Luzon alone, apart from the support it could increasingly claim from its expanded operating areas in the south.[32] In acknowledgement of its growing strength, in early 1950 the Huk were designated to be the military arm of PKP, which called on the movement's supporters to "prepare for the final offensive."[33] According to one source, with PKP support, the Hukbalahap forces increased their operations "ten fold" between the end of 1949 and the end of 1950.

In a Huk-PKP plan drafted in 1950, insurgent leaders laid out their vision of the future. The plan called for the movement to increasingly activate its popular base in an effort to double the number of men under arms every three months until mid-1951. If it could meet this mobilization schedule it would be able to field an estimated 172,000 fighters by the late summer of 1951 and would be ready to launch their final assault on November 1. While the government still outmanned and outgunned the guerrillas, it was clearly on the defensive. The PAF and the PC, as one source has noted, were "dispirited and discouraged" and "seemingly unwilling to seek encounters with the Huks."[34] Efforts to bring the insurgency to a rapid conclusion with force had not only failed, they had played into the insurgents' hands, demonstrating that the regime was not only cruel but incompetent. A PAF report published in the 1960s reveals that the military at the time not only believed that the insurgency was becoming stronger, but that at their current rate of advance they would be in a position to seize power sometime in the mid-1950s. By 1950, Huk elements had begun to operate in and around the outskirts of Manila and the PAF had begun to establish a defensive perimeter to keep the city from being overrun.[35]

5.2 Tipping Point and End Game (1950-1956)

In retrospect, it is clear that the turning point of the Huk insurgency came with the appointment of Ramón Magsaysay as Secretary of Defense in early 1950. Within a year he was able to fundamentally transform Manila's counterinsurgency strategy and force structure, halt the Huk advance, and ultimately reverse the course of the war. One of his first acts was to reduce the ineffectual and highly corrupt Philippine Constabulary to a secondary, supporting role in the counterinsurgency campaign under Army control.[36] In

its place he began to reorganize the Army into battalion combat teams (BCTs) that were specifically trained for local, small unit operations. Attached to each BCT was a civil affairs and psychological warfare element to serve as a liaison with local communities.[37] Magsaysay also implemented an innovative intelligence collection program designed to allow the peasants to provide the authorities with information about the Huks without fear of being identified. By the end of 1950, government forces were able to move on to the offensive and significantly step up their operational tempo. For the first time, also, they were able to pursue the Huks deep inside their mountain safe zones. One U.S. government source estimated that within months of deploying the first BCTs, PAF casualties had decreased by 25% and Huk casualties had increased by 12%.[38]

Magsaysay's campaign took a significant step forward in 1951 with two important initiatives. First, he was decisive in persuading Quiriño to implement a comprehensive land reform program in a successful effort to appeal directly to peasant interests. The government launched the Economic Development Corps (EDCOR) program in late 1950. The plan called for the military to buy and develop land in communities outside of the guerrillas' "hot zone" in Central Luzon and use this to resettle landless peasants and former Huk sympathizers. The first settlement was opened in Kapatagan, in eastern Mindanao, in spring 1951. This was followed by a series of additional developments in Mindanao in late 1951 and Northern Luzon in 1954.[39] The EDCOR program helped to diffuse the "land tenure" issue that had contributed to the initial rise of the insurgency, and played a significant role in physically separating the insurgents from their popular base. Magsaysay's second major initiative took place on the intelligence front, in the wake of an October 1950 raid on PKP's underground headquarters in Manila. The raid proved to be an intelligence coup, uncovering the names of the PKP's entire central leadership, hundreds of Huk cadre, and thousands of the group's supporters. Over 1,100 individuals were subsequently arrested based on the results of this operation. The Huks never recovered from this setback. In the words of one observer, "the information from this raid was significant in the subsequent breaking of the insurgent movement."[40]

The 1951 Congressional elections proved to be another setback for the insurgency. Magsaysay believed, correctly, that legitimate elections would encourage the peasantry to view the government in a new light and help undermine support for the insurgents. To help ensure a fair election, he provided a means for the population to report electoral fraud and publicly threatened to prosecute any government official who abused his responsibilities.[41] The elections proceeded smoothly and succeeded in bolstering the government's credibility. By the PKP's own assessment, the elections proved to be a major step forward for the state and a significant political defeat for the insurgents. In an after-action report written in December 1951, a PKP observer noted that the widely perceived success of the election "caused people to doubt the immediate need for armed struggle, especially if we consider the fact that the great majority of the masses greatly favor peace and order even though they suffer poor living conditions."[42] Magsaysay's military and political reforms were beginning to have a measurable effect, extending and legitimating the authority of the central government and undermining the guerrilla's popular base. It is clear that by the end of 1951 the Huk movement had reached and

passed its tipping point. The only thing that could possibly save them, at this point, "would have been a complete reversal" of the state's battlefield fortunes.[43]

By the end of 1951 latent divisions within the Hukbalahap network had emerged as various Huk leaders began blaming each other for their mounting setbacks. Taruc and a number of other leaders wanted to open negotiations with Magsaysay, while many of the organization's field commanders and many within the PKP leadership wished to fight on. Huk forces in late 1951 mounted a series of counteractions designed to deflect the government's increasing incursions into their territory and the growing success of the resettlement program, but without success. By the end of the year, Magsaysay's efforts to systematically reclaim central Luzon was beginning to pay off and the guerrillas were increasingly being pushed back into their core zone of support around Mt. Arayat.[44] The tables had clearly turned and the Huks were now on the defensive. The fight to extend their authority was over; they were now struggling to hold on to what they had. Desertion and "no show" rates among the guerrillas climbed steadily in late 1951 early 1952. Many of those who surrendered were lured out of the jungle by the government's offer of amnesty, the EDCOR program, and related government reintegration initiatives.[45] These defections, in turn, provided the Army with the intelligence it needed to continue to follow the guerrillas as their numbers declined and they became harder to see.

These trends continued into 1952 as the guerrillas lost the initiative. By the Spring of 1952, the government had 26 expanded BCTs of over 1000 soldiers a piece tightening the noose around a shrinking guerrilla zone in Central Luzon. In response to this pressure, and their own declining numbers, the insurgents were increasingly forced to withdraw into their core areas, where they still retained a strong popular base, or into inaccessible parts of the island, where they could be neither found nor pursued.[46] While these figures cannot be independently confirmed, according to official reports, between 35% and 40% of those on the active rolls of the Hukbalahap in 1950 had been either killed, captured, or induced to leave the group by Spring 1952.[47] The political winds continued to shift against the Huks in 1953 when Magsaysay succeeded Quiriño as President, winning 70% of the vote in Central Luzon. Magsaysay used his new authorities to accelerate his land reform program and introduce new initiatives to improve the economic life of local communities that had been brought under government control. Insurgents, meanwhile, continued to surrender in large numbers through 1953. Countless others quietly withdrew from the fight and "slipped back into the rhythm of village life."[48] By the end of 1953, the government estimated that there were fewer than 2,700 Huk insurgents still in the field and reported no major encounters with the guerrillas. The Huk network was still out there, but it was being pushed toward its breakpoint as an organized rebellion.

By early 1954, Huk support outside of their core area in central Luzon had almost completely disappeared. The guerrillas, at this point, were fighting to simply stay in the game. Alejandrino, who had taken the fight to southern Luzon in 1950 was forced back into his original base area in 1954. In May 1954, Taruc surrendered to the authorities, to receive a 15 year prison term. While the surviving Huk leadership denounced him as a traitor and vowed to make a comeback, his surrender marks the conventional end of the insurgency. By 1955 it was estimated that some 6,000 guerrillas had been killed in the

conflict, 4,700 had been captured, and almost 10,000 had accepted amnesty. Many others had simply backed out of the fight and returned to their farms. Despite Taruc's surrender, a number of key Huk leaders, such as Alejandrino, Jesus Lava, and Silvestre Liwanag remained in the field into the 1960s, engaging in a declining number of sporadic anti-government attacks.[49] Their base of support, however, continued to dissipate, as it became increasingly clear that the Huk movement was largely beaten. In 1956 the PAF withdrew from Central Luzon and major counterinsurgency efforts came to an end. Responsibility for subsequent mopping up operations was handed over to the PC.[50]

The final years of political unrest took place between 1956 and 1965. This period of the conflict has been referred to as the "smoldering period." "The war was long since over; the insurgency was not active, but not dead; it smoldered in the swamps and rice-lands of Huklandia still."[51] By the early-1960s, however, the Huk organization had melted away. The PC reported that there were only 122 core Huk guerrillas and no more than 1,600 supporters left in 1961. By the mid-1960s, even these were effectively gone. Those who survived were no longer a political presence and had largely turned to a life of crime, eking out a living exploiting the peasants they once claimed to represent.[52]

6. Case 2: State Endgame -- The Fall of the Batista Regime

On July 26, 1953, Fidel Castro and 165 supporters attacked the Moncada Army Barracks in eastern Oriente province in a failed attempt to initiate a popular uprising against General Fulgencio Batista, who had seized power in Cuba in a coup in 1952. Moncada was the second largest military post in Cuba, with 1,000 forces in garrison. Castro's plan was to seize the garrison and its stockpile of weapons, armored vehicles, and supplies and then take the local radio station, which he would use to broadcast the success of his attack and call on the citizens of Oriente to join him to depose Batista. While the attack failed, it marked the beginning of the Cuban revolutionary struggle.[53] Castro commemorated the event by forming the "July 26 Movement" (M-26), which formed the principal opposition to the Batista regime until its final collapse on January 1, 1959.[54] After a difficult beginning, in which Fidel's forces were almost decimated by Batista on several separate occasions, the movement stabilized itself and went on the offensive. The subsequent downfall of the Batista government, once it reached and crossed its tipping point, follows the accelerating path of decline discussed above.

6.1 The Rise of the Cuban Insurgency (1956-1958)

After his failure at Moncada, Castro was sentenced to 15 years in prison but was released in a general amnesty in May 1955, after serving only 19 months of his term.[55] With his release, he departed for Mexico to begin to prepare for his next attempt. To assist him, he recruited the colorful, Colonel Alberto Bayo, a former officer in the Spanish Foreign Legion, who had extensive experience fighting guerrillas in Spanish Sahara and had been involved in a series of coup plots throughout the Caribbean. Over the next 15 months, under Bayos's tutelage, Castro recruited and trained 82 men,

including 'Che' Guevara, at a ranch located outside of Mexico City.[56] On November 25, 1956, in what must be described as one of the most unlikely events in revolutionary history, Castro and his following set sail in the 58 foot *Granma* to "invade" Cuba.[57] Their destination was the eastern province of Oriente, where Fidel intended to take refuge in the Sierra Maestra mountains and begin to forge an alliance with the local peasantry.[58] A week later, after surviving a violent storm at sea, the boat foundered on the edge of a mangrove swamp, resulting in the loss of most of his supplies. Word of his impending "invasion," meanwhile, had leaked out, and when Fidel and his small band of optimists reached dry land they were met by gunfire from regime forces that had been waiting for their arrival. Only 12 of his original 82 mean reached the safety of the mountains.[59]

While the official history of the Cuban revolution naturally gives center stage to Castro's rural *foco*, opposition to the Batista regime was already coalescing around a number of different groups, notably the *Organazión Auténtico* (OA), a small but well-financed group led Manuel Antonio de Varona and supported by deposed President Carlos Prío Socarrás, the *Acción Nacional Revolucionaria* (ANR), probably the strongest and best organized opposition group, led by the talented student leader Frank País, and the *Directorio Revolutionario* (DR), which consisted largely of militant students and had a strong presence in Havana and several other urban centers.[60] At the time of Castro's homecoming, M-26 and ANR had formed a coalition, with the weak support of DR, to establish a common front against Batista.[61] While the nature of the relationship between these groups varied widely during the next two years, their often competitive efforts played a common role in destabilizing the regime. The dispersed, independent character of the resistance forced Batista to divide his efforts in a dual rural-urban campaign across the length and breadth of the island. The government was never able to concentrate its limited assets against one trouble spot without giving its opponents the opportunity to generate new problems somewhere else. By mid-1957, Batista was already in trouble.

Between December and June, Fidel focused his efforts on building a bridge to the villagers of the Sierra and laying the logistical groundwork to support the coming campaign. His first act was to establish a remote, permanent base of operations, from which he could oversee the conflict and assemble the means necessary to carry it out. Castro's guerrilla base is said to have soon "acquired the characteristics of a village." A base administration was established "which regulated the life of its inhabitants" and the group's interactions with local villagers, who were enlisted to provide Fidel's growing following with meat, bread, milk, tobacco, and other necessities in exchange for cash and guerrilla-issued "bonds."[62] Local residents were also enlisted as couriers in the group's underground postal system and to smuggle in weapons and supplies from his urban support network.[63] By the early summer of 1957 all of this was largely in tact and Fidel had begun to initiate his first actions against government forces in southern Oriente. His core force at this time was still quite small, probably numbering no more than 100.[64] This faced a regime force of around 1,000 men in the Sierra Maestra alone, supported by a combined force of approximately 30,000 troops elsewhere across the country.[65] Despite these odds, by mid-1957 M-26 was emerging as a threat to the regime.

During the Fall of 1957 and the Winter of 1958, Fidel continued to slowly increase his influence throughout eastern Cuba and step up the frequency of his attacks. The DR, at this time, was also establishing a name for itself by creating a second rural guerrilla front in the Escambray Mountains, in southern Las Villas Province, in central Cuba. By the Spring of 1958, DR forces in Las Villas had grown to several hundred men. Fidel's numbers, meanwhile, were also growing and in March he divided his force to establish a second *foco,* commanded by his brother Raúl, in the Sierra Cristal mountains in north Oriente.[66] In retrospect, we can see that the tide was beginning to turn against Batista. The government's demonstrable inability to contain either the growing rural insurgency or increasing unrest in the cities inspired more and more people to support the resistance. This, in turn, further undermined Batista's authority and effectiveness, which naturally encouraged more of the population to rally to the insurgents over time. Fidel, for his part, was keenly aware of the psychology of this dynamic and consciously used it to his advantage. While the number of forces under his command was still quite small, he was beginning to cast a long political shadow. By early Summer of 1958, the momentum of the struggle was clearly beginning to shift in favor of Castro and the resistance.

Batista launched a final offensive against Castro in May 1958 in an effort to capture his base areas.[67] He ordered his forces to "spare nothing to put an end to Fidel Castro."[68] Batista believed that it was essential to show the population that "the days of the Sierra were numbered."[69] Castro, for his part, believed that if he could beat the regime in the mountains it would have a demoralizing effect on the security forces and swing public opinion to his side. Castro's plan was to engage in a fighting retreat and bleed the Army as he lured it ever deeper into the Sierra. The offensive began in mid-June and almost immediately ran into resistance. Close to 7,000 of Batista's 12,000 forces were hastily-trained peasant conscripts, with no stake in the fight and unprepared for the harsh conditions they confronted in the mountains.[70] Desertion rates skyrocketed and by early August four out of seventeen battalions had all but disappeared. These problems were exacerbated by the first case of officer defections, resulting, in some instances, in the wholesale disintegration of the units under their command. According to one source, "Batista's high command, now a demoralized gaggle of corrupt, cruel, and lazy officers without combat experience, began to fear total extinction from an enemy" which they knew very little about and could neither find or effectively engage.[71] In the face of collapse, the Army withdrew from the Sierra in late August. It never returned.

6.2 Tipping Point and Endgame (Fall, 1958)

The failure of the 1958 offensive was the tipping point for the Batista government, pushing it into its endgame. In four months, it would be gone. Castro's success in the mountains had two immediate effects on the struggle. The first was the effect it had on the regime's already narrow base of support among the country's small political elite. As one source has noted, "after Batista proved that he was unable to control the situation, many members of the upper class withdrew their support from the government and tried to establish contact with the insurrectionists."[72] The second, quite opposite effect, was the unifying influence it had on the opposition, which until this point had been marked by a

high degree of jealousy and internal factionalism. In response to Fidel's early show of dominance in the summer offensive, most elements within the opposition agreed to form a "united front" in a meeting of Cuban opposition leaders in Venezuela. This agreement was codified in the Caracas Pact of July 1958, which called for a two-pronged strategy against the regime. The government would be defeated by the combined pressure of a rural envelopment of the capital and a general strike that would neutralize Batista's control over the cities. By September, the insurgents had seized the initiative.

With Batista's withdrawal from the Sierra, Fidel now moved on to the offensive with a plan to exploit the growing disarray within the Army and cut the country in half. The first part of his plan called for his forces to move out of the mountains and establish control over the rest of Oriente and the provincial capital of Santiago de Cuba.[73] This effort was supported by Raúl Castro, who moved out from his Sierra Cristal base area southward. A third force, under the command of Che Guevara and Cienfuegos, was sent into the Escambray mountains to link up with DR forces that had been operating in the area since Spring.[74] Their mission was to sever the regime's communications links between the western and eastern parts of the island and position themselves to take the strategic city of Santa Clara. By early October, Che, Cienfuegos, and a guerrilla column of some 140 men crossed into Las Villas and were almost immediately confronted by a superior force.[75] The Army, however, refused to fight. Numerous troops reportedly took advantage of the ensuing confusion to abandon their units and join the guerrillas. Fidel, meanwhile, was moving to consolidate his control over Oriente. The only areas within the province that he did not effectively dominate by early December 1958 were Santiago and a handful of other major towns. These last objectives would fall within weeks.[76]

Inspired by the success of Fidel's efforts in the mountains, the urban underground stepped up its attacks and armed demonstrations against the regime through the Fall. Many Cubans living in and around the cities offered open assistance to the insurgents. By the summer of 1958, the urban opposition mobilized the support of 10,000 followers. By the late Fall, this number is estimated to have swelled to 30,000.[77] Most observers agree that Castro's forces were expanding rapidly during this period as well. While Fidel's core force never grew above 450-550 men, he could increasingly draw on the support of thousands of auxiliaries, independent activists, and sympathizers, who were flocking to the insurgency.[78] One source suggests that this number may have grown to as many as 7,000 people by mid-December; which would have been at least a fifteen or sixteen-fold increase in strength in a 3 months period.[79] While it seems unlikely that Castro would have been able to effectively incorporate numbers of this magnitude into his organization in such a short period of time, there is no question that support for the insurgency was expanding dramatically. Many of those who rallied to the cause at this late date were certainly sympathizers, who now found it safe to express their true political sentiments; others, however, were clearly opportunists, who knew that the Batista regime was going down and wanted to be standing in the winners' circle when the game ended.

By the late Fall of 1958, Batista was showing clear signs of desperation. Under intense pressure from the United States, he held a postponed Presidential election in November. True to form, however, he rigged the vote to ensure the victory of his own

proxy, Rivero Agüero, who was scheduled to assume office in February.[80] Few Cubans paid any attention; the future of the country was now being decided on the battlefield, where a growing number of towns and villages were falling to the guerrillas without a fight. In early December, in a final, unsuccessful effort to regain the initiative, Batista made a number of key changes to the Army's command structure. It was too little too late. At a mid-December meeting with senior military officers, General Dolz Tabernilla, one of Batista's most trusted associates, stated that "the soldiers are tired and the officers do not want to fight….nothing more can be done."[81] Tabernilla's statement suggests that the Army had already come to the conclusion that the end of the regime was imminent. These thoughts were echoed by the U.S. Ambassador, Earl Smith, on December 17, who urged Batista to resign in a last effort to stem a guerrilla takeover. On December 23rd a group of high ranking officers sent an emissary to Fidel in an attempt to negotiate a separate peace.[82] The regime was beginning to disintegrate from within.

There were good reasons to expect the worst. By December 23rd, the Las Villas *foco* under Che and Cienfuegos effectively controlled over 150,000 people and many towns. The second largest city in the province, Yaguajay, had been taken without firing a shot. On December 29, the guerrillas encircled the provincial capital of Santa Clara. Their plan called for Cienfuegos to attack from the north, and Che to attack from the south. Fewer than 500 guerrilla fighters were involved in the assault. These were pitted against 6,500 regulars and a newly arrived armored train carrying 400 hastily gathered reinforcements. In the midst of the guerrilla attack elements of the Army mutinied. Many troops had not been paid in over three months and refused to fight. Similar defiance was shown within the air force, where pilots refused to bomb or strafe guerrilla positions. Many defending units surrendered to the insurgents en masse, others crossed over to guerrilla lines. When the city fell on December 30th the guerrillas were openly welcomed by many soldiers who still remained in the city as "brothers," to the cheers of thousands of Santa Clara residents who turned out into the streets to celebrate their "liberation."[83] The guerrillas encountered only sporadic resistance. Most of the Army had simply melted away.

On December 29th Batista had his children secretly flown to the United States, burned his private records and correspondence, and made arrangements for his own escape.[84] Two days later, on December 31st, he held his annual New Years Eve celebration for his friends and associates, political advisors, and the leadership of the armed forces. This was the event of the season, and everyone who was anyone in the Cuban elite was in attendance. In a case of "fiddling while Rome burned," the party continued in full swing into the night until, at 2am, Batista asked the band to stop playing, brought his guests together, and announced that they had all had enough fun for one night and it was now time to flee for their lives. With this bit of news, Cuba's assembled elite headed for the exits. Within the hour Batista had escaped to the Dominican Republic. An armor unit, meanwhile, was called in to establish a defensive perimeter around the airport as planes carrying Batista supporters and whatever they could carry departed through the early morning hours. When the airport was finally closed around noon on January 1, those who had not yet managed to make their escape sought asylum in various Latin American embassies or fled from the island by sea. It was a party for the history books.[85]

Havana's underground was taken by surprise by the speed of Batista's departure but managed to establish control over the city by the end of January 1. Fidel was also taken by surprise and recognized that his forces had to reach the capital as soon as possible to claim his share of the prize. Che and Cienfuegos were ordered to move on Havana with all possible speed. They arrived in the city early on January 2 with a small band of bearded guerrillas driving a fleet of commandeered Fords and Chevrolets. With the news of Batista's escape, regime forces around the island surrendered or shed their uniforms and disappeared. Santiago de Cuba was surrendered to Castro on January 1 and the rest of the island was brought under effective control of the insurgents by January 3.[86] Castro embarked on a victory march to Havana to offer his "support" for the newly appointed President of Cuba, Manuel Urrutia, and begin to jockey for power. He was appointed Prime Minister in February and President in July, after forcing Urrutia to resign.[87]

7. Concluding Remarks

This study provides a framework for evaluating the "endgame" struggle in internal wars. Most of the literature on how internal wars end, as we point out, has focused on how these conflicts are concluded at the negotiating table, with particular attention to why they do not end in a negotiated settlement more frequently. In this article, by contrast, we are interested in the complementary question of how such conflicts are concluded on the battlefield. We have given specific attention to the final period dynamics of insurgent conflicts, which constitute 94% of the internal wars that have come and gone since 1945. The endgame, we find, looks quite different for each side. States, we show, generally lose decisively but win by degrees, typically pushing the insurgents below their breakpoint at a decelerating rate of decline. Reciprocally, insurgents tend to lose by degrees but win decisively, pushing the state below its breakpoint at an increasing rate. We investigate the underlying reasons for these historical patterns and the strategic implications they pose for both sides.

This discussion is extended and formalized in the Appendix. The formal model we develop is then used to simulate the endgame struggle for the state and the insurgency under varying parametric conditions. While the tactical path of this final struggle can vary widely, often randomly, there is a deep underlying order to victory and defeat.

Appendix:
The Mathematics of Insurgency

While the general manner in which states and insurgencies collapse is highly consistent, the speed with which these patterns reveal themselves can vary widely. The model of insurgent-counterinsurgent conflict developed below suggests that who wins, who loses, and how an insurgency ends in the absence of a negotiated settlement is determined, in large measure, by two sets of considerations. The first set of factors, defined in expression (1), considers the complex interaction of a small number of (macro) variables that influence each player's growth and attrition dynamics. The second set of factors, defined in expressions (2) and (3), consider a set of (micro) variables that, in the first case, measure the changing strength of the players as a function of their size and "graph" structures and, in the second case, consider the players' changing ability to discriminate and attack high value targets on the other side as a function of changes in their force ratio. These factors are brought together in a "nested" model of insurgency that considers how the interaction of the players influence each player's strength, and how changes in the players' strength, in turn, influence their ability to compete.

1. A Dynamic Systems Model of Insurgency

Consider the following model of an insurgency by McCormick and Giordano[88]

$$g_{t+1} = a\left(1 - \frac{g_t}{g_{\max}}\right)g_t - br_t + g_t$$
$$r_{t+1} = c\left(1 - \frac{r_t}{r_{\max}}\right)r_t - dr_t g_t + r_t \qquad (1)$$

where

- g_t is a measure of the strength of the state at time t on a scale from 0 to $g_{\max}$,
- r_t is a measure of the strength of the insurgency at t on a scale from 0 to $r_{\max}$,
- $g_{\max}$ is the maximum potential strength of the state,
- $r_{\max}$ is the maximum potential strength of the insurgency,
- a is the unrestricted growth coefficient for the state,
- b is a measure of the effectiveness of the insurgency at eliminating state forces,
- c is the unrestricted growth coefficient for the insurgency, and
- d is a measure of the effectiveness of the state at eliminating insurgent forces.

With some initial values, g_0 for the strength of the state and r_0 for the strength of the insurgency, values of g_t and r_t can be computed over time for each $t \in \{1,2,3,...\}$.

1.1 Model Details

Each term plays an important role in shaping the dynamics the conflict. The term

$$a\left(1-\frac{g_t}{g_{\max}}\right)g_t$$

describes state growth, using a standard logistic growth model. When g_t is much less than $g_{\max}$, the term $1 - g_t / g_{\max} \cong 1$, and so what remains is the classic exponential growth model ag_t. Until g_t gets large enough to make $1 - g_t / g_{\max}$ significantly different from 1, government growth is approximately proportional to its strength. As g_t gets larger, $1 - g_t / g_{\max}$ gets smaller, reducing the rate of growth. If g_t reaches $g_{\max}$, there is no further room for growth in g_t; the state will have reached its mobilization limits. The value $g_{\max}$ is generally defined by the structural context of the struggle. The term

$$c\left(1-\frac{r_t}{r_{\max}}\right)r_t$$

models the growth dynamics of the insurgency in a very similar way.

Turning to the question of attrition, the term $-br_t$ models state losses resulting from the actions of the insurgents. We can see her, that state losses in each time period are proportional to the strength of the insurgency. The term $-dr_tg_t$, by contrast, models the reduction in the size of the insurgency that results from the actions of the state. As we discussed earlier, this term is different than that used to model state attrition; there is a clear asymmetry here. The state, as we have said, must first be able to *find* the insurgents before they can be engaged and eliminated. The result is a multiplicative model in which the state's capacity to reduce the opposition is conditioned by the number of possible interactions between government and insurgent forces, i.e., the multiple of the size of the rebel force r_t and the size of the government force g_t that is trying to hunt it down.

Because of this asymmetry, the parameters b and d are not directly comparable. Because the insurgents' attrition coefficient b is multiplied by r_t only, while the state's attrition coefficient d is multiplied by $g_t r_t$, the two parameters are not even in the same "units". Insurgent effectiveness measures the number of losses they are able to inflict on the state per time period per unit of rebel force. State effectiveness, on the other hand, measures the number of rebel losses it is able to inflict per time period *per potential interaction* between the opposing forces. Accordingly, b will typically be much larger than d, even in a conflict that is otherwise "balanced" between the two players.

1.2 Interpreting the Model

As we discussed earlier in the main body of this article there are some interesting ramifications associated with the insurgents' attrition model. The most important one is

that *as the state is successful at reducing the strength of the insurgency, it will be harder and harder for it to continue to reduce the insurgency at a constant rate.* This is in stark contrast to most military operations where success enables continued success.

A simple example may help motivate this idea. Consider the case where the total number of combatants (insurgents plus government forces) remains constant, *i.e.*, $g_t + r_t = k$. To simplify the discussion, let's choose $k = 100$. When $g_t = r_t = 50$, the government's ability to reduce the insurgency is $dr_t g_t = d(50)(50) = 2500d$. Suppose $d = 1/500$. This suggests that the government will eliminate 5 insurgents in the current time period. Suppose now that after continued success, the government strength grows to $g_{t'} = 80$ and the insurgency is reduced to $r_{t'} = 20$. The government, at this point, can now expect to eliminate only $dr_t g_t = 1/500(20)(80) = 3.2$ insurgents. A more extreme case is even clearer. If $g_{t''} = 99$ and $r_{t''} = 1$, the reduction in the insurgency is only 99/500 = 0.198.

In fact, subject to the constraint $g_t + r_t = k$, $g_t r_t$ is maximized when $g_t = r_t$. You can think of the government's ability to reduce the insurgency in time period t as being proportional to the area of a rectangle with sides g_t and r_t. That area declines whenever either g_t or r_t declines. The smaller either g_t or r_t becomes, the more pronounced the effect. This can be seen in the Figure A-1 below which shows the magnitude of $dr_t g_t$ with $d = 1/500$ and $g_t + r_t = k = 100$. Changing k dilates the picture horizontally and changing d does so vertically, but the underlying shape remains the same.

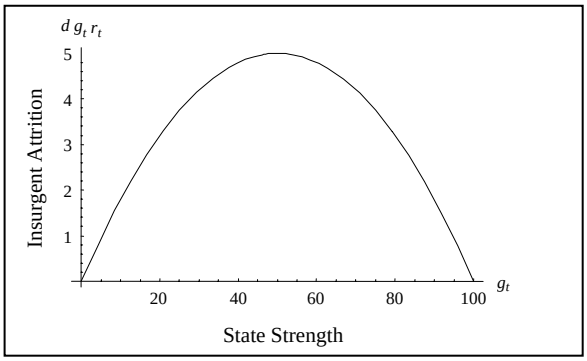

Figure A-1

A related question is to consider the state strength needed to maintain a constant reduction in the insurgency as a function of insurgent strength. Recall the reduction in the insurgency is modeled as $dr_t g_t$. Suppose again that both the government and the insurgency start with a strength of 50, and for illustration let's use $d = 1/2500$. Then the constant reduction in insurgent strength is 1/2500(50)(50) = 1. Figure A-2 below shows the state strength needed to obtain a constant reduction of 1 as a function of insurgent strength. Paradoxically, once again, *the state requires fewer forces to maintain a constant rate of attrition as the insurgency grows -- and a larger force as it declines.*

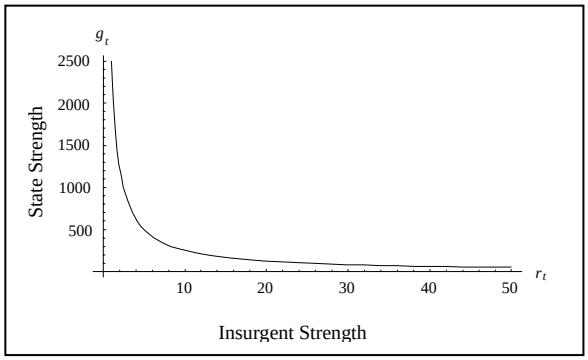

Figure A-2

Figure A-3 shows the same scenario, but gives the size of the state force needed to maintain a constant reduction of 1 in a declining insurgency as a function of time.

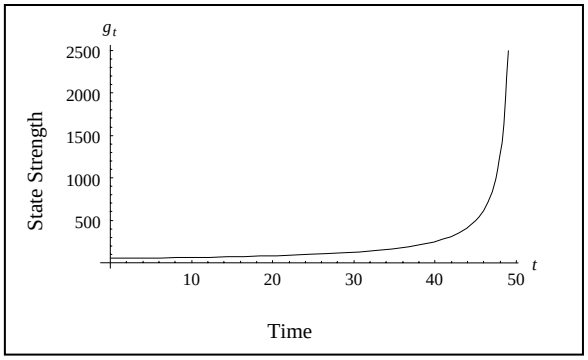

Figure A-3

2. A Graph Theoretic Model of Insurgency

The systems model above provides the context for a more detailed "graph" model that defines the internal structure of each player in the conflict. As suggested above, the graph structures of the combatants, in turn, influence each side's ability to compete.

A *graph* $G = (V, E)$ consists of a set V of *vertices* and a set E of pairs of vertices called *edges*. If $i \in V$ is a vertex of G, then $\deg(i)$ is the number of edges incident with i. If i and j are vertices, then $d(i, j)$ is the *distance* from i to j (note $d(i, j) = d(j, i)$) which is the number of edges in a shortest path from i to j. The interested reader should consult the relevant literature for more information about graphs and graph theory.[89]

Our work in this section is motivated in part by recent literature on so-called "scale free" networks.[90] Recent studies have observed that many real-world networks, including social networks, can be modeled as networks with certain statistical properties. As noted earlier, these properties include the "small world" phenomenon where two randomly

selected vertices in a network tend to be close to one another in terms of the distance metric. Another property of interest is a power law distribution of vertex degrees, that is a defining characteristic of scale-free networks. In this section, we generally follow procedures described in Albert, Jeong, and Barabási to create scale free graphs.[91] These graphs will, in turn, be used to model (mature) insurgent and (immature) state networks which, as discussed earlier, evolve out of naturally formed social networks.

Consider the following mathematical model of an insurgency and the incumbent regime that opposes it. We represent each side in the conflict, hereafter called the *state* and the *rebellion*, with a graph as described above. The vertices of the graph represent the entities each side has mobilized to participate in the conflict. The entities may be thought of as individuals or groups. There is an edge between two vertices in the graph whenever the entities that the vertices represent are in direct contact with one another.

Our general approach is as follows. Given a graph for the state and another graph for the rebellion, we will compute a measure of the current *strength* of each side. Based on these strengths, and our system dynamics model of insurgency stated above, another computation will determine changes to each graph for the current turn. These changes, which we call *updates*, normally either call for the addition of new vertices and edges to the graph, or the deletion of some vertices and their incident edges. These updates represent growth and attrition. The current updates are then implemented, resulting in new graphs for each side in the conflict. The procedure is then repeated as desired.

Each side in the conflict is presumed to have entities called *authority providers*. These are modeled by vertices of the same name. These authority providers represent elements in the state or rebellion that enable the actions of other elements. This can model higher headquarters providing directives to subordinates, or supply centers providing material resources to operational cells, or purely inspirational leaders, etc. Authority providers are presumed to broadcast their information or resources to all vertices in the graph. Every vertex in the graph contributes to the strength of the graph, but vertices that are close to authority providers (in terms of the distance metric) can contribute more if all other factors are equal. For the purposes of this discussion, we assume that all authority providers for a given side are identical, although their individual significance to the network varies depending on how many such vertices there are; a natural extension of this model might investigate a relaxation of this assumption.

2.1 Computing Strength

We can now begin to define a strength measure for each side. Vertices of a graph contribute to its overall strength based on two things; proximity to authority centers, and vertex degree. Let A be the set of authority providers. Then we can compute the strength of the graph with authority provider set A, $S(G, A)$ by evaluating

$$S(G,A) = \frac{\sum_{i \in A} \sum_{j \in V} \frac{\deg(j)}{1 + d(i,j)}}{|A_0|}. \qquad (2)$$

The notation $|A_0|$ refers to the number of elements in A_0, *i.e.*, the number of authority providers in the state or insurgent network at the beginning of the simulation.

The rationale for this model is as follows. Each authority provider broadcasts to the entire graph (including itself and the other authority providers). Therefore it is natural to sum over all authority providers and over all vertices. The contribution to graph strength for each authority provider/vertex pair is proportional to the degree of the vertex, and inversely proportional to the distance between the authority provider and the vertex plus 1. All of these contributions are added together, and the result is divided by the *initial* number of authority providers. We divide by the number of authority providers to make graph strength relatively insensitive to this number: if a graph initially has only a few authority providers, we assume they are each relatively powerful. Such a graph would reflect a centralized authority structure. Similarly, cases with many authority providers are considered to have a more decentralized authority structure. As the simulation progresses, we continue to divide by the *initial* number of authority providers to reflect the impact of losing leadership (and other authority providers) to enemy action.

2.2 Graph Growth and Reduction

When vertices and edges are added to a graph, we say that the graph *grows*. Graphs grow when an update calls for the addition of new vertices. Computationally, graph growth occurs one vertex at a time, even though updates will often call for more than one vertex to be added in any given time period. When a vertex is added, a number of edges are added connecting the new vertex to other vertices in the graph. The number of new edges added is 1 plus a Poisson random variable with a parameter value established by the modeler (since Poisson random variables can realize the value 0, we add 1 to avoid adding an isolated vertex, since such a vertex would make no contribution to strength). The government growth parameter is denoted by μ_g and the rebellion growth parameter is denoted by μ_r. Each of these new edges has one end at the newly formed vertex, and its other end at some other vertex in the graph. This endpoint is also determined randomly. Each vertex (other than the new one) has a probability proportional to its degree of being the endpoint for a new edge. In this way, existing vertices of relatively high degree tend to "attract" edges from newly created vertices more than existing vertices of relatively low degree. This growth procedure, which we referred to as *preferential growth*, is similar to that found in Albert, et.al. for creating graphs with the scale free property.[92]

Graph reduction occurs as a result of updates. An update that calls for a reduction specifies a number of vertices to be deleted. This represents a variety of events such as direct military action, recruitment to join the other side, defection, etc. It also considers the likelihood that an individual or group can be quickly replaced if they are removed. When a vertex is deleted, any edges incident with it are also deleted. The reader should note that the elimination of a vertex reduces the strength of the graph in two ways. The destroyed vertex itself is gone, so its direct contribution in expression (2) goes to zero. In addition, a deleted vertex may cause edges to be deleted that results in some distances

between surviving vertices to increase. This increases the term in the denominator of (2) resulting in a further decrease in graph strength through a decrease in efficiency.

2.3 High Value Targets

The model includes a mechanism for representing the idea that as the force ratio between the two sides changes, it becomes more able to detect and kill "good" targets. Similarly, as the force ratio shifts in the other direction, the disadvantaged side is less able to hit good targets. Note when we say a "good" target, we don't necessarily mean the "best" target. Authority providers might be considered the best targets (certainly eliminating all of them for a given side would end the conflict immediately), but here we will model the idea that highly connected vertices might be more likely to be discovered and offer a lucrative alternative to waiting for the best possible target.[93] Furthermore, authority providers may insulate themselves from the fight by using nearby highly connected entities to broadcast their authority – this seems to be the case, for example, with Osama Bin Laden. In the following paragraph, we formalize this idea.

For each vertex to be eliminated as a result of an update, there is a chance it will be a (uniformly) randomly selected vertex, and a chance it will be a vertex of highest degree. If, for example, the government graph is being reduced, the measure of the chance that the deleted vertex will be one of highest degree instead of a randomly selected one is *rqual*, the term measuring the likelihood that the rebels will be able to choose a *high value target*. In accordance with the previous paragraph, a quality target is defined as a vertex of highest degree. If *rqual* = 0, a government vertex marked for deletion will be a randomly selected one. If *rqual* = 1, a government vertex marked for deletion will be one of highest degree (the vertex of highest degree and lowest label is selected). The value of *rqual* can be thought of as a probability. This term is meant to represent the degree to which the rebellion is able to select its targets to be those whose destruction has a great impact on state strength. The value of *rqual* at time step *t* is computed as follows:

$$rqual = p_r\left(s_r\left(\ln\left(\frac{r_t}{g_t}\right)\right)^3 + 1\right). \qquad (3)$$

As a practical matter, of course, the value of *rqual* will always be between 0 and 1. Whenever (3) returns an *rqual* that exceeds 1, we reset the value to 1. Similarly, when *rqual* is evaluated to be less than 0, we reset it to 0.

This function is selected to have several properties. We first consider its behavior when $p = 0.01$ and $s = 1$. When the force ratio $r_t/g_t = 1$, *rqual* = 0.01. For this set of values of the parameters p_r and s_r, *rqual* = 0.01 represents the baseline probability that, when the two sides in the conflict are equal in strength, a vertex deleted from the government graph will be a high value target. As the force ratio shifts in favor of the rebellion, *rqual* increases from this baseline. As it shifts in favor of the government, *rqual* decreases. Of course it is possible that even a randomly selected vertex is a quality

target, or an authority provider, or both. Thus even with *rqual* = 0 it is possible for the rebellion to "get lucky" and hit critical state targets, but if *rqual* is relatively large they get an "extra chance" to obtain a quality hit. The parameters p_r and s_r allow the user to modify the baseline probability of a quality hit (p_r) and the slope (s_r) of the curve that measures how much of an increase in *rqual* results from a given shift in force balance. The function *gqual*, which measures the state's probability of discriminately targeting high value enemy vertices is analogous, as are the associated parameters p_g and s_g.

The values of *rqual* and *gqual* are computed during each update. Note that while *rqual* and *gqual* are computed and implemented symmetrically, the real world reasons for these values being high or low are different for the two sides in the conflict. While *gqual*, in a sense, represents the state's capacity to discriminate between targets it can already locate, *rqual* is a measure of the insurgency's ability get through the state's defenses and effectively attack high value targets that they are already able to discriminate.

3. Running the Simulation

The conduct of the simulation follows the macro dynamic model described at the beginning of this paper. First the modeler has to make some decisions to establish the initial conditions. In doing so, he must first establish values for the parameters of the macro model *a*, *b*, *c*, and *d*, as well as $g_{\max}$ and $r_{\max}$. The modeler also sets the values of the parameters μ_G and μ_R, as well as p_r, s_r, p_g and s_g. Next, the number of vertices that will comprise the initial graphs for the state and the rebellion is established by the user. At this point, the initial graphs are created from some base graph using the graph growth procedure described above. The final step in the initialization procedure is determining which vertices of the newly created initial graphs will be labeled as authority providers. One natural choice for doing this in a notional network is to use the set of *center* vertices of the graph. A precise definition and description of the center of a graph requires be brief technical sojourn. This discussion can be skipped without loss of continuity.

Given a vertex $i \in V$, the eccentricity of *i*, denoted by *e*(*i*), is the distance to a vertex that is farthest from *i* in *G*. We can write this as follows:

$$e(i) = \max_{j \in V} d(i, j).$$

Then the *center* of a graph *C*(*G*) is the set of vertices of minimum eccentricity.

A simple example can help make these concepts clear. Consider the graph in Figure A-4 below. The eccentricity of v_1 is 5, since $d(v_1,v_{12}) = 5$ and no vertex is farther than 5 from v_1. Similarly, we can compute all the eccentricities in the graph. The eccentricity of v_1, v_3, v_4, v_5, v_6, v_{10}, v_{11}, v_{12}, and v_{13} is 5, the eccentricity of v_2, v_9, v_{15}, and v_{16} is 4, and the eccentricity of v_7, v_8, and v_{14} is 3. The minimum of all the eccentricities is 3, so $C(G) = \{v_7, v_8, v_{14}\}$. Note that vertices in the center are not necessarily vertices of high degree, and that vertices in the center might not be "close" to each other.

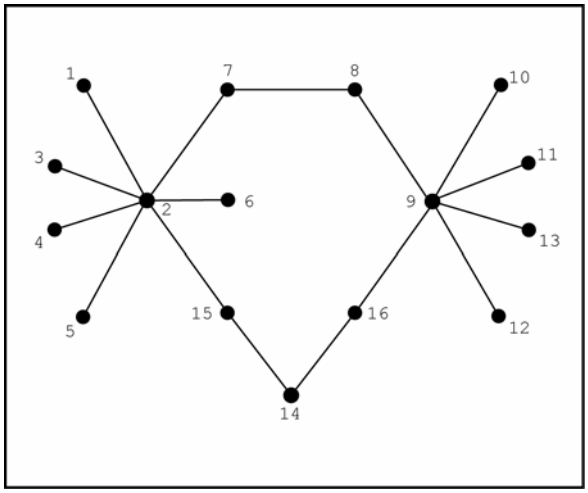

Figure A-4

Once the initialization procedure is complete, our simulation runs in a sequence of updates. Update number t starts by determining state graph strength g_t and that of the rebellion, r_t. The difference equation model (1) is then used to compute g_{t+1} and r_{t+1}. The differences $g_{t+1} - g_t$ and $r_{t+1} - r_t$ determine the outcome of the current update phase and, in turn, translate directly to a number of vertices that were gained or lost for each player. Any differences that are not integers are rounded up or down randomly. For example, if $g_{t+1} - g_t = 2.4$, there is a 40% chance the update will add two vertices to the government graph and a 60% chance it will add three vertices. Similarly, if $r_{t+1} - r_t = -1.7$, there is a 30% chance the update will delete one vertex from the rebellion graph and a 70% chance it will delete two vertices. The update instructions are then implemented; the vertices to be added or deleted are determined in accordance with the procedures we have described, and each of the graphs (each side's network structure) is updated accordingly. The next update phase then begins. This procedure is repeated and the results are analyzed.

4. Experiments and Results

The purpose of our work here is to introduce a simple mathematical model of insurgency and to illustrate some of the ways this model can be used to evaluate how insurgent conflicts end. Our experimental design is not intended to be comprehensive. We seek only to demonstrate some of the behaviors of the model. With this in mind, in the following discussion we consider several groups of illustrative simulations.

4.1 Baseline Results

We first established a set of baseline results to use to evaluate the influence of subsequent parametric changes in the model. Because we our specifically interested in the endgame, we began our simulations at the mid-point in the conflict where the rebellion has already managed to achieve relative force parity with the state. We performed five runs of thirty updates each, with the following parameter settings.

State growth	a	0.67
Rebel effectiveness	b	0.33
Rebel growth	c	0.16
State effectiveness	d	0.0015
Maximum state strength	g_{max}	100
Maximum rebel strength	r_{max}	80

Table A-1. Macro Baseline Parameter Values

Initial state graph order	$\|V_g\|$	50
Initial rebel graph order	$\|V_g\|$	40
State quality hit slope parameter	s_g	1
Rebel quality hit slope parameter	s_r	1
State quality hit probability parameter	p_g	0.01
Rebel quality hit probability parameter	p_r	0.01
State vertex degree growth parameter	μ_g	0.6
Rebel vertex degree growth parameter	μ_r	0.6
Initial number of state authority providers	$\|A_0\|$	4
Initial number of rebel authority providers	$\|A_0\|$	8

Table A-2. Micro Baseline Parameter Values

For these five runs, the results show some random variation, but they are in general very consistent from case to case. Each run resulted in state growth and rebel decline initially. Due to the finite upper limit on state strength ($g_{max} = 100$) and the nature of the macro model (1), however, the state's opportunities to continue to grow gradually decline as it continues to expand. Similarly, rebel losses decline over time as the insurgency is reduced in size. As discussed earlier, as insurgencies go into decline, they become more and more difficult to engage. Our results, in this respect, are similar to what we see in the empirical record. A single case that is typical of the behavior of this group of runs is depicted in Figure A-5 below. Notice that near update 18 there is an unusual drop in state strength due to a successful rebel attack; the overall trend, however, is as we describe it. In terms of the outcome of the conflict, the randomness built in to the model appears to be relatively insignificant. This is in contrast with the results of the next section.

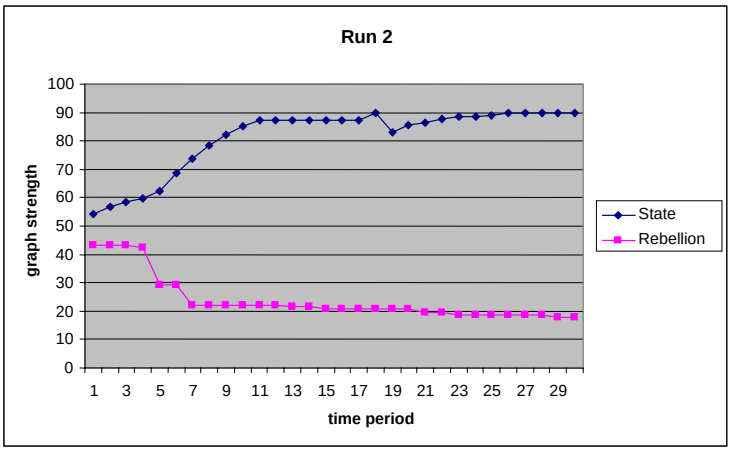

Figure A-5

4.2 Changes to Macro Parameters

In this section we consider the effect of changing certain parameter values of the macro model (1) on the both course and outcome of our baseline simulation.

4.2.1 *Increased Rebel Effectiveness*

The first parameter setting that we changed was rebel effectiveness, *b*, which was changed from $b = 0.33$ to $b = 0.41$. Once again five runs of thirty updates each were performed. The most interesting aspect of these runs is that the consistency of results we saw in our baseline case is no longer present. Some of the runs in this section are not unlike those above; they show an increase in regime strength and a rebel decline to what appears to be a fairly stable state. Figure A-6 demonstrates this. Others, however, end completely differently. In Figure A-7, for example, the conflict goes the other way, resulting in a decisive win for the rebellion and a catastrophic defeat for the state. This particular set of parameter values appears to define a "knife edge" position where either side can win and where *randomness* plays a very important role in the outcome.

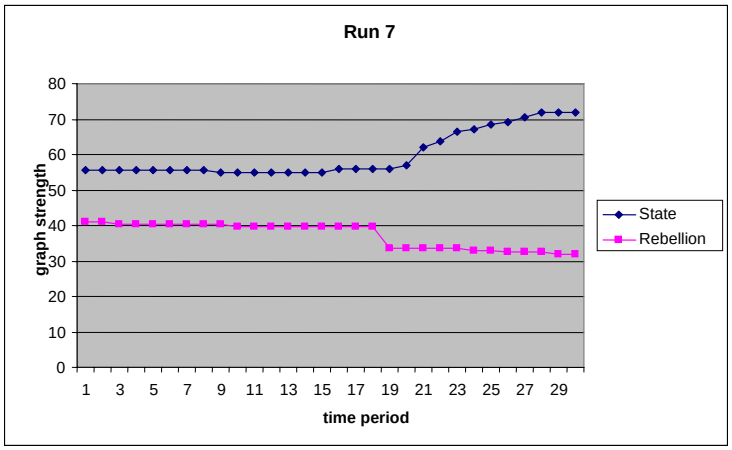

Figure A-6

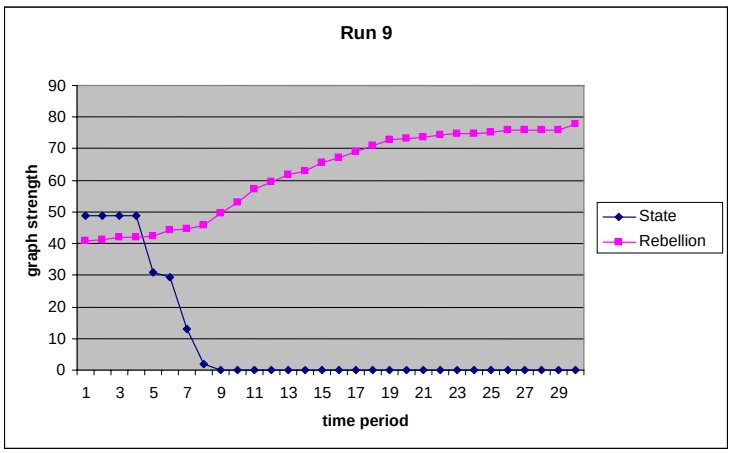

Figure A-7

4.2.2 *Increased Maximum State Strength*

The second parameter setting we changed from our baseline case was g_{max}, which we increased from $g_{max} = 100$ to $g_{max} = 200$. All other parameters remained the same. Again we carried out five runs with thirty updates each. These runs exhibit the same general behavior of our initial runs. An increase in g_{max} can be interpreted as an increase in the absolute segment (or percentage) of the population that is reasonably subject to state mobilization. In an ethnically or otherwise communally divided society, for example, an increase in g_{max} can be interpreted as an increase in the segment of the population that is aligned with the state. With r_{max} constant, we increased the size of the state's potential base of support. The state, in this case, eventually grows to an overwhelmingly great strength. The rebellion, however, eventually gets so small that it can "hide" from the attrition process modeled by the $-dr_t g_t$ term in our macro model and is able to stay in the game. While the scale in Figure A-8 makes it difficult to see that the rebels have not been completely eradicated, they are in fact still present with a strength of about 0.9833. Whether the rebels, in this case, have been hit hard enough to push them below their breakpoint is an empirical question that depends on the group and its environment.

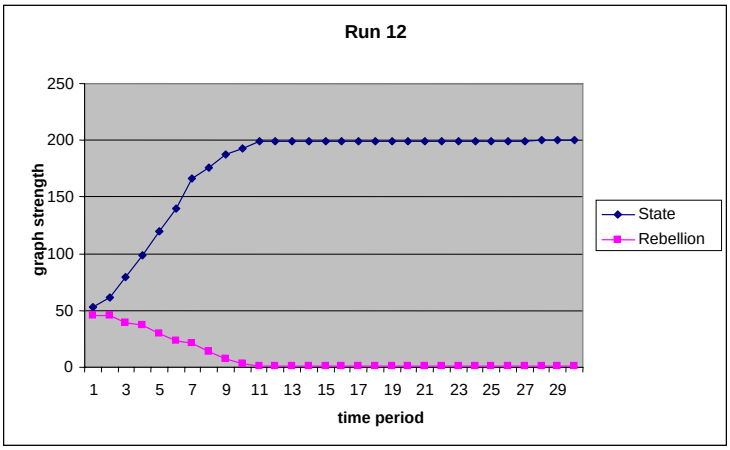

Figure A-8

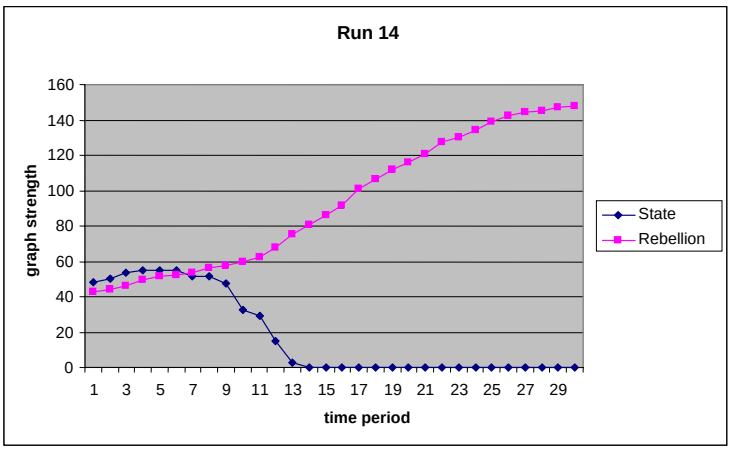

Figure A-9

4.2.3 *Increased Maximum Rebel Strength*

In our next set of runs we kept all of our baseline parameters the same with the exception of maximum rebel strength, which we increased from $r_{max} = 80$ to $r_{max} = 160$. With g_{max} constant, we increased the size of the rebellion's potential base of support. As illustrated in Figure A-9, this has a deciding influence over the outcome of the struggle. The insurgents, in these cases, win decisively and the state collapse catastrophically.

4.3 Changes to Micro Parameters

In this discussion we consider the effect of changing certain parameter values of our "micro" model, *i.e.*, the graph-theoretic model defined by expressions (2) and (3).

4.3.1 *Increased Rebel Quality Hit Probability*

Once again, any of the parametric values in Table 2-A can be changed. To illustrate how this works we consider the impact of increasing the baseline probability that a rebel hit against the state will be a guaranteed "quality hit" versus a randomly selected one. In this case, we increased p_r 500% from 0.01 to 0.05. Despite the relatively big increase in p_r, there was no significant difference in our simulation results. Each run conducted with these settings gave results that were very similar to those of our baseline case. The course and outcome of the endgame fight, in these cases, are dominated by the other parameters in the model. The rebels decline slowly, although not to extinction in 30 time steps, and the state approaches its maximum size. Figure A-10 gives representative results.

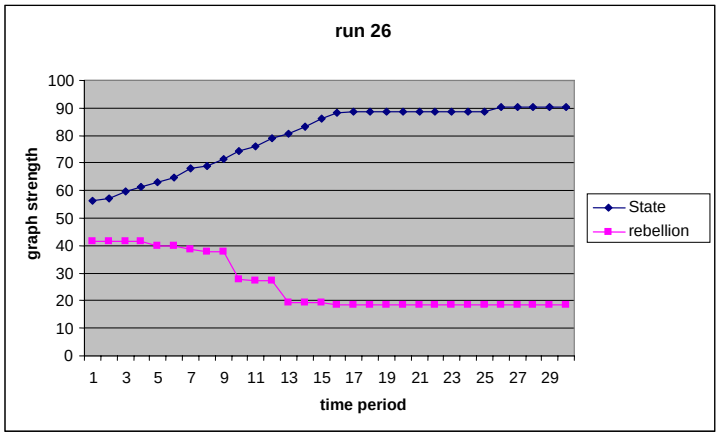

Figure A-10

4.3.2 *Increased Government Quality Hit Probability*

Next we consider the impact of increasing the baseline probability that a government hit against the rebellion will be a guaranteed quality hit versus a randomly selected one. Similar to the last section, we have increased p_g 500% from 0.01 to 0.05. Otherwise, the

parameter settings are identical to those of our baseline case. As in the previous section, the big increase in p_g seems to have had a minimal impact on the outcome, but local differences are present. Due to the larger p_g, and also due to the fact that the force ratio of government strength to rebel strength is relatively large, the rebellion is reduced in bigger chunks than they were in the runs of our baseline case. These drops represent "quality hits" with high singular impact on the group. As the rebellion is reduced, however, the "information effects" described earlier take over, and further hits (of any type) against the rebellion are hard to obtain, even given many opportunities.

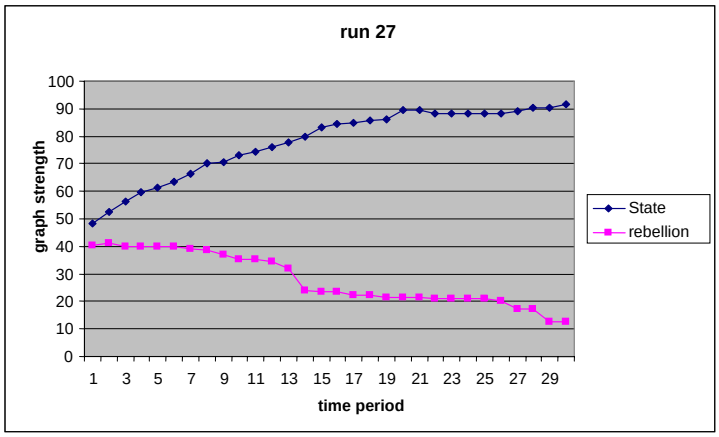

Figure A-11

4.4 A Scenario for a State "Win"

Finally, we consider a way in which the conditions of the model can be changed to deliver a government win where the insurgency is completely eliminated. The fact that rebel attrition is proportional to the product of state strength and rebel strength makes it difficult to completely eliminate an insurgency. As we discussed earlier, any initial success by the state against the insurgents makes it harder for it to continue to succeed. There are, however, a number of circumstances in which the state can decisively defeat an insurgency. We examine one such set of circumstances here. A more elaborate discussion of these and other circumstances will be offered in subsequent work.

First, if the initial number of rebel authority providers is small and difficult to replace, a small number of well-placed attacks can have a disproportionate effect on the overall effectiveness of the opposition. The idea here is to destroy the insurgency from the "inside out" rather than the "outside in" with a protracted war of attrition. All things being equal, this will clearly be easier to accomplish if authority within the rebellion is centralized, rather than decentralized among a large number of authority providers. If the targeted leadership are also vertices of high degree within the insurgent network, the impact of any such attack will be even more significant. Directed leadership attacks, in this case, will not only eliminate the single most important set of figures within the opposition, but the vertices that play the largest role in holding the graph together.

To accomplish this reliably the state must also satisfy two requirements. First, in terms of the model, the regime's "quality hit probability" must be high; the state must be

very good at discriminating between high value and low value targets and choose its shots accordingly. Second, the state must succeed in striking the insurgency decisively while it is still big enough to see, i.e., $-dr_t g_t$ must be big enough to be significant. As discussed in some detail earlier, if the insurgency (is allowed) to get too small, the state's growing ability to discriminate between any targets that it can see will be more than offset by the fact that the insurgency will become increasingly difficult to see in the first place. If it wishes to destroy the insurgent network decisively, it must do so while it is still big enough to locate its leadership element and any non-leadership vertices of high degree. As the network declines, this will eventually be impossible, except by accident, and the likelihood of such an accident occurring will decline at an increasing rate.

To simulate these effects we made three changes to our baseline parameters. First, we changed the number of rebel authority providers from 8 to 3, "centralizing" the rebellion. Next, we made these authority providers correspond to vertices of highest degree in the rebel graph. A more centralized insurgent leadership, in this sense, is managing the rebellion directly, rather than working through "cutouts" to improve its security. Finally, we changed the value of the state's "quality hit probability" p_g from $p_g = 0.01$ to $p_g = 1$. Every hit for the state, in this case, is a good hit. All other parameters were constant.

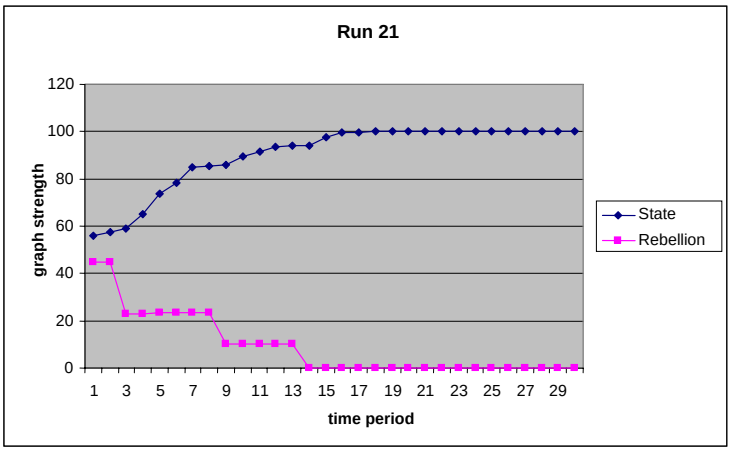

Figure A-12

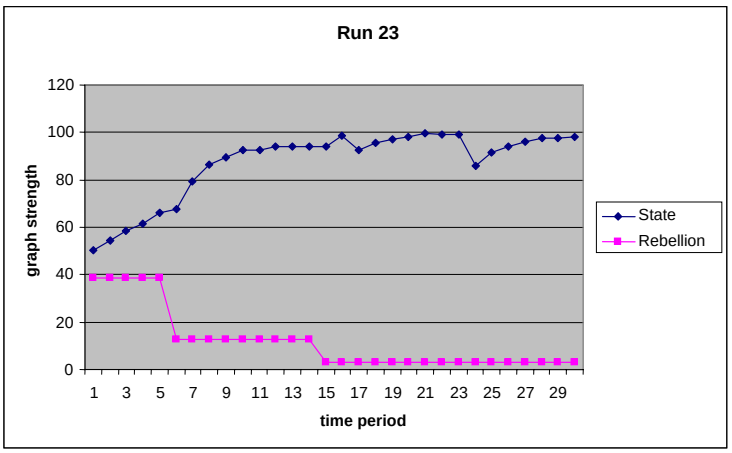

Figure A-13

Two sets of results from these runs are illustrated in Figures A-12 and A-13. As one can see, despite conditions that are highly favorable to the state a decisive "win" is still not guaranteed, at least within the 30 step time horizon of this simulation. Figure A-12 gives us the good news first. In this case, the government is clearly able to knock the insurgents out of the game. It is interesting to note, however, that even in this case, the rebels' decline follows the characteristic decelerating endgame path discussed above. Figure A-13 gives us a different outcome based on the same parameters settings. In this case the rebels are hit hard at time step 5 but do not experience any further net decline until step 14. At this point, it seems, they disappear below the regime's "radar" and experience no further net decline through the end of the simulation. They have clearly been hurt, but they are still present. If we were to extend this run in time we would be able to see how the fight ends. However it ends, it is already a protracted war.

5. Concluding Remarks

This appendix extends the McCormick-Giordano insurgency model to consider some of the ways in which the internal structures of states and insurgent groups influence their ability to compete and how this competition, in turn, influences their internal structures. Applying this model to the study of insurgent-counterinsurgent endgames we offer a theoretical explanation for the observed pattern of state and insurgent defeat. We explain why it is that insurgencies, once reduced below a certain level of visibility, are very hard to eliminate and what the state must be able to do to push them below their breakpoint. In doing so, we find it is often better to have a small influence over the *right* variables than a large influence over variables that are less likely to shape the outcome of conflict. We show, finally, how randomness (i.e., "luck" or even "factors beyond the control of decision makers") can play a critical role in some circumstances, but almost no role in others.

Finally, it is important to note that this study gives illustrative results, it does not comprehensively consider all the parameters present in the model. It also ignores many other phenomena that shape the course and outcome of insurgent conflicts that could be studied with a similar methodology. These are evaluated in forthcoming work.

Endnotes

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[31] Bohannon, p 126; and Kerkvliet, p 203; and Cable, p 50.

[32] Asprey, p 755.

[33] Bohannon, p 126; and Kerkvliet, p 187-218; and Smith, p 79.

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[45] *Undergrounds in Insurgent, Revolutionary, and Resistance Warfare*, p 329.

[46] Cable, p 60; and Kerkvliet, p 236-237.

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[73] Ibid, p 238-240; 266-271.

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