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Plant / Controller Optimization with applications to Integrated Surface Sizing and Feedback Controller Design for Autonomous Underwater Vehicles (AUVs) *

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Abstract

This paper describes a solution to the following plant-controller optimization (PCO) problem: given an autonomous underwater vehicle (AUV) - with a fixed baseline body configuration - that is required to operate over a finite number of representative trimming conditions in the vertical plane, determine the optimal size of the bow and stern control surfaces so that a weighted average J of the power required at trimming is minimized, subject to the conditions that: i) a given set of open loop requirements are met, and ii) stabilizing feedback controllers can be designed to meet desired time and frequency closed loop performance requirements about each trimming point. The solution proposed is rooted in the theory of Linear Matrix Inequalities (LMIs) and leads to efficient PCO algorithms that build on a recently released LMI Toolbox.

1 Introduction

This paper addresses the problem of integrated design of AUV plant parameters and feedback controllers to optimize the vehicle's performance over a set of operating conditions arising in the course of a given mission scenario. This research topic has been motivated by the fact that significant energy savings and increased dynamic performance can be obtained if the process of control system design is integrated with the design of the vehicle itself, thus departing considerably from the classical approach where the plant structure is essentially fixed a priori.

As a contribution towards the solution of that general problem, this paper formulates and solves the following simplified problem: given an AUV - with a fixed baseline body configuration - that is required to operate over a finite set of representative trimming conditions in the vertical plane, determine the optimal size of the bow and stern control surfaces so that a weighted average of the power required at the trimming conditions is minimized, subject to the conditions that: i) open loop requirements are met and ii) stabilizing feedback controllers can be designed to meet time and frequency closed loop performance requirements about each trimming point. Open loop requirements include the possibility of achieving trim at each operating point and meeting a desired degree of open-loop stability. Closed loop requirements include maneuverability specifications in response to depth commands, hard limits on surface deflections, and actuator bandwidth constraints.

The paper introduces a new methodology to solve the above combined plant / controller optimization problem and describes its application to the selection of the optimal size of the bow and stern surfaces for a prototype AUV. This work has been strongly influenced by and extends previous work described in [4] where the authors studied the problem of combined plant-controller optimization in the related field of aircraft control. The methodology proposed is firmly rooted in the field of control systems theory and borrows heavily from the areas of Linear Matrix Inequalities and Convex Optimization, which are the subject of current research [1].

The key idea in the new methodology for combined plant-controller optimization is to cast the problem in the form of a new constrained optimization problem where the cost J to be minimized is the average power required at trimming, and the search is done over the set of feedback controllers which meet open loop and closed loop requirements. From physical considerations, it follows that the cost J can be written explicitly in terms of the control surface sizes. Furthermore, the open loop and closed loop requirements considered can be expressed as Linear Matrix Inequalities (LMIs) [1] that are also functions of the surface sizes. Thus, one is left with the problem of minimizing a certain function of the surface sizes, while satisfying a finite set of LMI constraints. This problem is solved numerically by resorting to efficient convex optimization algorithms that are the basis of a recently released LMI Toolbox for use with Matlab[1].

The paper is organized as follows. Section 2 describes a general model for a prototype AUV in the vertical plane. Section 3 defines the concept of trimming trajectories, derives the vehicle linearized model at trimming and computes the average propulsion power required to perform a given mission scenario. Section 4 discusses open loop and closed requirements and derives the corresponding LMI formulation. Finally, Section 5 introduces the constrained optimization problem that is the main focus of this paper, indicates its numerical solution and describes a design example.

Due to space limitations, the presentation is brief and

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[1]Matlab^{TM} is a trademark of the Mathworks Inc.

many details are omitted. The reader is referred to [7] for a complete treatment of the plant / controller optimization problem summarized here.

2 AUV Model

This section introduces the dynamic model of the INFANTE AUV depicted in Figure 1, which is a modification of the prototype MARIUS AUV described in [5]. The vehicle is 4.5 m long, 1.1 m wide, and 0.6 m high. Propulsion is assured by two main back propellers. Two rudders for vehicle steering in the horizontal plane are mounted directly aft the thrusters. For diving maneuvers, the vehicle is equipped with two pairs of all moving control surfaces (bow and stern planes). Following

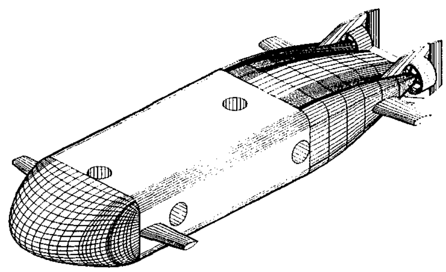

Figure 1: The INFANTE AUV

standard practice, the equations of motion of the AUV are developed using a global coordinate frame $\{I\}$ and a body-fixed frame $\{B\}$ that moves with the vehicle. For simplicity of presentation, we restrict the vehicle motion to the vertical plane. The following notation is required:

$\mathbf{p} = [x, z]'$ - position of the origin of $\{B\}$ measured in $\{I\}$;

$\mathbf{v} = [u, w]'$ - velocity of the origin of $\{B\}$ relative to $\{I\}$, expressed in $\{B\}$ (i.e., body-fixed linear velocity);

θ - pitch angle that describes the orientation of frame $\{B\}$ with respect to $\{I\}$;

q - angular velocity of $\{B\}$ relative to $\{I\}$;

$\dot{\mathbf{q}}_v' = [u, w, q]'$ - extended vehicle velocity vector in the vertical plane.

Let $\boldsymbol{\delta} := [\delta_b, \delta_s]'$ denote the vector of bow and stern plane deflections, and let T denote the thrust generated by the main propellers. With this notation, the relevant dynamics and kinematics of the AUV in the vertical plane can be written in compact form as [6]

$$M_{RB_v}\ddot{\mathbf{q}}_v + C_{RB_v}(\dot{\mathbf{q}}_v)\dot{\mathbf{q}}_v = \boldsymbol{\tau}_v(\ddot{\mathbf{q}}_v, \dot{\mathbf{q}}_v, \theta, \boldsymbol{\delta}, T) \quad (1)$$

$$\dot{\theta} = q; \;\; \dot{z} = -usin\theta + wcos\theta \quad (2)$$

where $\boldsymbol{\tau}_v$ denotes the vector of external forces and moments, and M_{RB_v} and C_{RB_v} denote the rigid body inertia matrix and the matrix of Coriolis and centrifugal terms, respectively. The vector $\boldsymbol{\tau}_v$ can be further decomposed as

$$\boldsymbol{\tau}_v(\ddot{\mathbf{q}}_v, \dot{\mathbf{q}}_v, \theta, \boldsymbol{\delta}, T) = \boldsymbol{\tau}_{v_{rest}}(\theta) + \boldsymbol{\tau}_{v_{add}}(\ddot{\mathbf{q}}_v, \dot{\mathbf{q}}_v) + \boldsymbol{\tau}_{v_{surf}}(\dot{\mathbf{q}}_v, \boldsymbol{\delta}) + \boldsymbol{\tau}_{v_{body}}(\dot{\mathbf{q}}_v, \boldsymbol{\delta}) + \boldsymbol{\tau}_{v_{prop}}(T)$$

where $\boldsymbol{\tau}_{v_{rest}}$ denotes the (restoring) forces and moments caused by gravity and buoyancy, and $\boldsymbol{\tau}_{v_{add}}$ is the added mass term. The term $\boldsymbol{\tau}_{v_{surf}}$ captures the effects of the deflections of the control surfaces, $\boldsymbol{\tau}_{v_{body}}$ consists of the hydrodynamic forces and moments acting on the vehicle's body, and $\boldsymbol{\tau}_{v_{prop}}$ represents the forces and moments generated by the main propellers. The following notation will be used in the text: $v_t = (u^2 + w^2)^{1/2}$ is the the absolute value of the linear velocity vector, $\alpha = arcsin(w/v_t)$ is the angle of attack, and $\gamma = \theta - \alpha$ is the flight path angle.

The problem addressed in this paper required that the vehicle model be parameterized in terms of the sizes of the bow and stern planes. This was done by assuming that:

- the cord c and length d of the control surfaces are such that their aspect ratio $AR = d/c$ is constant.
- the control surfaces have a constant profile.

A parameterized model of the vehicle has been computed by modifying the original model for the MARIUS AUV described in [6] that was derived from first principles of physics and experimental hydrodynamic data obtained in tank tests with a Planar Motion Mechanism (PMM). The methodology adopted consisted of subtracting the estimated effect of the original surfaces and adding the estimated effect of the new ones. The estimates were based on theoretical predictions using thin airfoil theory and experimental airfoil data.

3 Trim points. Linearized vehicle models

Given the nonlinear model of the vehicle in the vertical plane, it is important to compute the corresponding set of equilibrium (also denoted trim) points, that is, the set of input and state variables for which the net sum of the forces and moments acting on the vehicle is zero. From (1), an equilibrium point is a vector $(\dot{\mathbf{q}}_{v0}, \delta_{b0}, \delta_{s0}, \theta_0, T_0)$ that satisfies $C_{RB_v}(\dot{\mathbf{q}}_{v0})\dot{\mathbf{q}}_{v0} - \boldsymbol{\tau}_v(0, \dot{\mathbf{q}}_{v0}, \theta_0, \boldsymbol{\delta}_0, T_0) = 0$. In the equation, $\dot{\mathbf{q}}_{v0} = (u_0, w_0, q_0)'$. It is assumed that the inputs are restricted by physical constraints that are known in advance (e.g. the stall angles for the control surfaces and the maximum thrust available as a function of forward speed).

It is straightforward to show that the only equilibrium points of the AUV in the vertical plane are those that correspond to straight line trajectories, which can be parameterized in terms of the total speed v_t and flight path angle γ [6]. Given desired values of v_{t_0} and γ_0 it is possible, using equation (1), to determine if a corresponding trimming condition is achieved and in the affirmative to compute the corresponding surface deflections $\boldsymbol{\delta}_0$ and thrust T_0. Due to the existence of two control surfaces, however, the solution is not unique and therefore an additional constraint must be imposed on the state or input variables. One solution consists of fixing the angle of attack α. This is of little practical utility, since the angle of attack is hard to measure in practice. The preferred approach is to set the deflection of the bow planes to zero at trimming, since this condition is easily enforced in practice by including a "washout" in that variable during the control design phase. In the first case, straightforward algebraic manipulations show that the values of the bow and stern plane deflections can be written as

$$\delta_{b_0} = \frac{1}{\zeta_b}\mathcal{K}_{\delta_b}(\alpha_0, v_{t_0}, \gamma_0)\ ;\ \delta_{s_0} = \frac{1}{\zeta_s}\mathcal{K}_{\delta_s}(\alpha_0, v_{t_0}, \gamma_0)$$

where $\mathcal{K}_{\delta_b}(.)$, and $\mathcal{K}_{\delta_s}(.)$ are nonlinear functions of the state variables and ζ_b and ζ_s denote the sizes (areas) of the bow and stern planes, respectively. Notice how the deflection at trimming varies with the inverse of the surface sizes.

In the second case, the dependence of the stern plane deflection on the trimming point can still be written in closed form as $\delta_{s_0} = \bar{K}_{\delta_s}(\gamma_0, v_{t_0}, \zeta_b, \zeta_s)$, where $\bar{K}_{\delta_b}(.)$ is a nonlinear function of the trimming variables γ_0 and v_{t_0} and the control surface areas ζ_b and ζ_s. However, the control surface sizes do not factor out in as simple a manner as in the previous case. Close exam of the above equation reveals that it can be locally approximated by a first order Taylor expansion in the variable ζ_s^{-1} only, to obtain

$$\delta_{s_0} \simeq \mathcal{K}^0_{\delta_s}(\gamma_0, v_{t_0}, \zeta_{b_0}, \zeta_{s_0}) + \frac{1}{\zeta_s}\mathcal{K}^1_{\delta_s}(\gamma_0, v_{t_0}, \zeta_{b_0}, \zeta_{s_0}) \quad (3)$$

where ζ_{b_0} and ζ_{s_0} are nominal values about which the expansion is done. Figure 2 compares the actual and approximate values of the stern plane deflection at trimming for the case where $v_{t_0} = 1.5$ m/s and $\gamma_0 = 15$ deg. The solid line is the approximation computed about the nominal values $\zeta_{b_0} = \zeta_{s_0} = 0.4\,\mathrm{m}^2$. The dashed lines are practically coincident and represent the actual function obtained for $\zeta_{b_0} \in \{0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7\}\,\mathrm{m}^2$. As seen

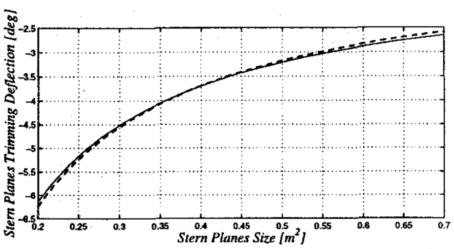

Figure 2: Stern plane deflection at trimming and its first order Taylor approximation.

in the figure, the approximation is quite good even over a large interval of stern plane areas and is fairly independent of the bow plane area. Identical behaviour is obtained for different trimming points. Using this approximation, a constraint on the stern surface area for a given trimming point $\delta_s < \delta_{s_{\max}}$ can be locally written as

$$\begin{aligned} R^+_{\mathrm{trim}}(\zeta_s) &:= \zeta_s(\mathcal{K}^0_{\delta_s}(\gamma_0, v_{t_0}, \zeta_{b_0}, \zeta_{s_0}) - \delta_{s_{\max}}) \\ &< -\mathcal{K}^1_{\delta_s}(\gamma_0, v_{t_0}, \zeta_{b_0}, \zeta_{s_0}) \end{aligned}$$

which is a linear inequality in the stern control surface area ζ_s. An identical linear inequality $R^-_{\mathrm{trim}}(\zeta_s)$ is obtained for the additional constraint $\delta_s > \delta_{s_{\min}}$.

Consider now a mission scenario where it is required that the AUV go through a finite set of trimming trajectories, each trajectory being allocated a time duration that is a given percentage of the complete mission time. Then, it is important to compute the average power that is spent in trimming the vehicle at those cruising conditions as a function of the control surface sizes. By definition, and assuming that the vehicle's angle of attack is sufficiently small, the thrust power P_t at trimming equals $P_t(\gamma_0, v_{t_0}, \zeta_b, \zeta_s) = T_t(\gamma_0, v_{t_0}, \zeta_b, \zeta_s)v_{t_0}$ where, in light of the development above, the thrust T_t at trimming can be approximated by

$$T_t(\gamma_0, v_{t_0}, \zeta_b, \zeta_s) \simeq \mathcal{F}_T(\gamma_0, v_{t_0}, \zeta_b, \zeta_s, \zeta_b^{-1}, \zeta_s^{-1})$$

where $\mathcal{F}_T$ *is linear in the variables* ζ_b, ζ_s, ζ_b^{-1}, *and* ζ_s^{-1} and depends on the nominal values of ζ_{s_0} at which the Taylor expansion for δ_{s_0} is done. The average propulsion power J corresponding to a given mission is then computed as

$$J(\zeta_b, \zeta_s) = \sum_i p_i P^i_t(\gamma^i_0, v^i_{t_0}, \zeta_b, \zeta_s),$$

where $P^i_t(\gamma^i_0, v^i_{t_0}, \zeta_b, \zeta_s)$ is the power required to trim the vehicle at the flight condition i specified by γ^i_0 and $v^i_{t_0}$, and p_i is the percentage of total mission time that is spent at that trim condition. From the above discussion, the approximation to $J(\zeta_b, \zeta_s)$ is linear in the variables ζ_b, ζ_s, ζ_b^{-1}, and ζ_s^{-1}.

For control system design purposes, we now derive the linearized equations of motion of the AUV about its trimming points. In preparation for the sections that follow we parameterize the linearized models in terms of the bow and stern control surface sizes ζ_b and ζ_s, respectively. In what follows, it is assumed that the AUV forward speed v_{t_0} is maintained constant by a dedicated controller. In this case, the AUV vertical plane model can be written in the form $\dot{\mathbf{x}}_v = F_v(\mathbf{x}_v, \mathbf{u}_v, \zeta_b, \zeta_s)$,where F_v is a nonlinear function, $\mathbf{x}_v = (\alpha, q, \theta, z)'$ is the state vector, and $\mathbf{u}_v = (\delta_b, \delta_s)'$ is the input vector. The above equations can be linearized about an AUV trimming point defined by the flight path angle γ_0 and total speed v_{t_0} to obtain

$$\dot{\mathbf{x}}_v = A(\gamma_0, v_{t_0}, \zeta_b, \zeta_s)\mathbf{x}_v + B(\gamma_0, v_{t_0}, \zeta_b, \zeta_s)\mathbf{u}_v$$

where $A(.)$ and $B(.)$ are matrices that depend on the trimming point and on the control surface sizes. Furthermore, using the approximations introduced in the previous section it can be shown that the linearizations can be re-written as

$$\dot{\mathbf{x}}_v\,[A_0 + \zeta_{\delta_b}A_1 + \zeta_{\delta_s}A_2]\,\mathbf{x}_v + [B_0 + \zeta_{\delta_b}B_1 + \zeta_{\delta_s}B_2]\,\mathbf{u}_v$$

where $A_i = A_i(\gamma_0, v_{t_0}, \zeta_{b_0}, \zeta_{s_0})$; $B_i = B_i(\gamma_0, v_{t_0}, \zeta_{b_0}, \zeta_{s_0})$. Notice the important fact that the approximate linearizations show a *linear dependence with the variables* ζ_b *and* ζ_s.

4 Open and closed loop requirements: an LMI formulation.

We now tackle the problem of AUV control about the operating points that arise in the course of a given mission scenario. The methodology adopted for control system design was $\mathcal{H}_\infty$ [3]. The key step in this methodology is to convert the usual specifications for command tracking, controller bandwidth, disturbance rejection, and robust stability specifications into the requirement that the gain ($\mathcal{H}_\infty$ -norm) of a conveniently defined weighted closed loop operator be bounded by a given fixed number. In

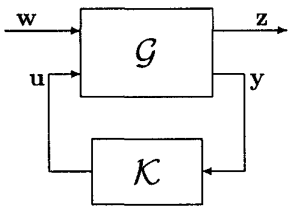

Figure 3: Feedback interconnection.

what follows, the general set-up and nomenclature in [3] is adopted, leading to the standard feedback system of Figure 3. In the figure, **w** is the input vector of exogenous signals, **z** is the output vector of errors to be reduced, **y** is the vector of measurements that are available for feedback, and **u** is the vector of actuator signals. The generalized plant $\mathcal{G}$ consists of the plant to be controlled, together with appended weights that shape the exogenous and internal signals. Suppose that the feedback system is well-posed, and let $\mathcal{T}_{\mathbf{zw}}$ denote the closed loop transfer matrix from **w** to **z**. The $\mathcal{H}_\infty$ synthesis problem consists of finding, among all controllers that yield a stable closed loop system, a controller K that makes the infinity norm $\|\mathcal{T}_{\mathbf{zw}}\|_\infty$ of the operator $\mathcal{T}_{\mathbf{zw}}$ less than a given number $\gamma > 0$, We remind the reader that $\|\mathcal{T}_{\mathbf{zw}}\|_\infty$ equals $\sup\{\sigma_{\max}(\mathcal{T}_{\mathbf{zw}}(j\omega)) : \omega \in \Re\}$, where $\sigma_{max}(.)$ denotes the maximum singular value of a matrix. The design of a controller to achieve a required closed loop H_∞ (if at all possible) can be solved in an elegant manner using the theory of Linear Matrix Inequalities (LMIs) [1].

Let $\mathcal{G}$ admit the realization

$$\mathcal{G} = \begin{cases} \dot{x} &= Ax + B_w w + B_u u \\ z &= Cx + Du \\ y &= x \end{cases}$$

where $x \in \Re^n$, $w \in \Re^m$, $u \in \Re^q$, and $z \in \Re^p$. Assume that all the states are available for feedback, the pair (A, B_u) is stabilizable and that matrix D has full rank. Then, there exits a state feedback controller K such that $\|\mathcal{T}_{\mathbf{zw}}\|_\infty < \gamma$ if and only if there exist a symmetric positive definite matrix $X \in \Re^{n\times n}$ and a matrix $W \in \Re^{q\times n}$ such that the linear matrix inequality (LMI) $R_\infty(X, W, \gamma) < 0$ holds, where $R_\infty(X, W, \gamma)$ is defined by

$$\begin{bmatrix} AX_\infty + B_u W + X_\infty A^T + W^T B_u^T & B_w & X_\infty C^T + W^T D^T \\ B_w^T & -\gamma I & D^T \\ CX_\infty + DW & D & -\gamma I \end{bmatrix}. \tag{4}$$

If the LMI is feasible, a state feedback gain K is obtained as $K = WY^{-1}$. In view of of the discussion in the previous sections, the variables above should be viewed as functions of the vehicle's trimming point and the size of the control surfaces. However, the approximations introduced before guarantee that they depend linearly on the variables ζ_b and ζ_s.

Using the LMI framework for control system design, it is possible to set-up a finite number of simultaneous H_∞ constraints to be satisfied at each operating point. This is simply done by stacking together the LMIs for the different operating points and viewing the inequality that results as a larger LMI to be satisfied.

In some applications, the AUV maneuverability requirements are only important when the vehicle is inspecting the seabed in straight, level flight since in this case the vehicle must be able to change depth quickly in order to avoid unforeseen obstacles. In this case, it is interesting to incorporate the additional constraint that the controller developed for the straight level maneuver stabilize the vehicle about the other operating points. The additional constraints can be easily cast as an LMI denoted R_s where, for the sake of simplicity, the LMI arguments are omitted. Again, this LMI will be linear in the variables ζ_b and ζ_s.

Open loop constraints can also be easily incorporated in the design process by using the concept of LMI regions in the complex plane, as introduced by Chilali and Gahinet [2]. A simple example of an LMI region arises in the case where the eigenvalues of the matrix A are required to lie in the region defined by $Re(z) < -a$. It is well known that this property is satisfied if and only if there exists a symmetric positive matrix $X_o l > 0$ that verifies the generalized stability Lyapunov inequality

$$R_{ol}(A, X_{ol}, a) := (A + aI)X_{ol} + X_{ol}(A + aI)^T < 0. \tag{5}$$

is satisfied. Again, open loop constraints can be easily cast in the LMI framework to force the AUV to display adequate open loop stability over a finite set of operating points.

5 Plant / Controller Optimization. A design example.

Equipped with the mathematical tools described, it is now possible to set-up a combined plant controller optimization problem where the objective is to minimize the functional $J(\zeta_b, \zeta_s)$ subject to the conditions that a finite set of LMIs R_∞, R_s, and R_{ol} as well as the linear constraints R^+_{trim} and R^-_{trim} are met. Notice that the cost $J(\zeta_b, \zeta_s)$ depends linearly on the variables $\zeta_b, \zeta_s, \zeta_b^{-1}$ and ζ_s^{-1} and therefore it is not a linear function of ζ_b and ζ_s only. However, the cost can be made linear by introducing two extra variables η_b and η_s and enforcing the equalities $\eta_b = \zeta_b^{-1}$ and $\eta_s = \zeta_s^{-1}$ through a slack variable that is forced to zero in the minimization process. The structure of the particular problem studied here shows that this can be achieved by introducing an extra LMI constraint relating the above variables; see [7]. At this stage, one can resort to efficient interior-point optimization algorithms - that are available under the Matlab LMI Toolbox - to minimize a linear cost subject to LMI constraints. It is important to point out that the LMIs considered are separately linear in ζ_b and ζ_s and in the remaining arguments, but not in both simultaneously. This difficulty can be overcome by introducing a loop that alternatively holds constant the variables ζ_b, ζ_s and

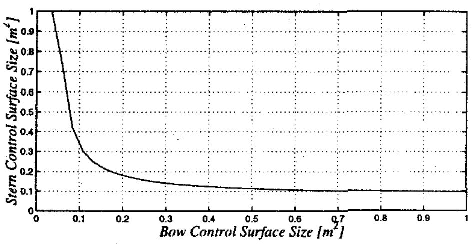

Figure 4: H_∞ constraint $\gamma \leq 0.8$: boundary of allowable surface sizes

the remaining LMI parameters. See [7] for a complete discussion of the resulting algorithm and its implementation using the LMI Toolbox.

We now describe a design exercise in which the tools developed in the previous sections were applied to the resizing of the control surfaces for a prototype AUV to execute the following three phase mission: i) Vehicle diving: $v_t = 1.5\,\mathrm{m/s}$, $\gamma = -15$ deg. Duration: 30% of total mission time (TMT); ii) Vehicle on level flight: $v_t = 1.5\,\mathrm{m/s}$. Duration: 40% TMT; iii) Vehicle climbing: $v_t = 1.5\,\mathrm{m/s}$, $\gamma = +15$ deg. Duration: 30% TMT.

The objective of the design was to find the optimal size of the bow and stern planes so as to minimize the average mission power, subject to the following conditions: i) the open loop AUV system should exhibit an adequate degree of stability as specified by the requirement that the real part of the eigenvalues be less than or equal to -0.1 rad/s, ii) the maximum deflection of the bow and stern planes at trimming should not exceed 15 deg, iii) there should exist a single state feedback controller that simultaneously stabilizes the AUV about all trimming conditions, iv) the resulting controller should exhibit satisfactory dynamic behaviour about the level flight condition, as measured by the requirements of zero steady state in response to depth commands, minimum depth command bandwidth of 0.5 rad/s, maximum bow and stern plane control bandwidths of 2 rad/s, and gain and phase margins of 8 db and 35 deg in the bow and stern plane channels, respectively. As usual, the crucial step in the design process was to convert the dynamic requirements into an H_∞ constraint, as expressed in the design weights and the constant γ. This was done by selecting a combination of bow and stern planes and carrying out a separated controller design exercise until all dynamic specifications were met, after which the design weights and the value of γ were frozen in the optimization process.

The results of the constrained optimization procedure are summarized in figure 5, which shows the open loop and closed loop constraints as well as the evolution of the optimal search procedure. This figure should be examined together with figure 4, which presents the boundary curve Γ_∞ above which the surface sizes must lie in order to guarantee that an $\mathcal{H}_\infty$ controller exists that meets the design specifications. In this design example the minimum of the cost J is attained in Γ_∞. The strip region inside the two horizontal lines denoted Γ^+ and Γ^- corresponds to the open loop degree of stability requirement. Different scenarios will of course lead to other

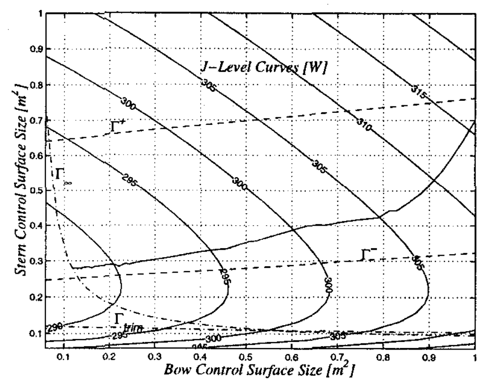

Figure 5: Evolution of the cost J.

optimal surface sizes, which are therefore mission dependent. Figures 4 and 5 illustrates very clearly that in some circumstances it is advantageous to use both bow and stern planes, instead of simply stern planes.

6 Conclusions

This paper introduced a new methodology for the integrated design of AUV plant parameters and feedback controllers to meet mission performance requirements with minimum energy expenditure. The results obtained indicate that the methodology developed holds great promise as a powerful tool to study tradeoffs among possibly conflicting AUV performance requirements as a function of plant parameters.

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