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Menichini, Amilcar A.

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Amilcar A. Menichini*

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Abstract

We use a dynamic model of the firm to ascertain both the value and the determinants of the tax benefits of debt. A standard parameterization suggests that the value of the interest tax shields represents around 3% of firm value, and it varies considerably across industries. In addition, our results show that this component of the stock price behaves countercyclically over the business cycle. Finally, besides the interest rate on debt and the corporate income tax rate, we find that the curvature of the production function is one of the most important determinants of the tax advantage of debt.

JEL classification: G31, G32

Keywords: Interest Tax Shields; Dividend Discount Model; Gordon Growth Model; Dynamic Programming

*Amilcar A. Menichini is from the Graduate School of Business and Public Policy, Naval Postgraduate School, Monterey, CA 93943 (e-mail: aamenich@nps.edu). We thank Natalia Lazzati for her helpful comments.

1 Introduction

The capital structure choice is one of the most important decisions made by corporate financial managers. Modigliani and Miller (1963) show that, under certain assumptions, the value of the firm increases with debt in an amount equal to the present value of the interest tax shields. However, from an applied perspective, the question of how important debt tax shields are in practice is still under great debate. We use a dynamic model of the firm to investigate the proportion of firm value that is explained by the tax advantage of debt as well as the determinants of the tax benefit.

In an influential paper, Graham (2000) finds that the mean value of the interest tax shields equals approximately 10% of the value of the firm.[1] However, in subsequent work, Blouin, Core, and Guay (2010) argue that Graham's value might be overestimated due to his underlying random walk assumption for earnings, as opposed to the usual mean-reverting process.[2] Thus, by substituting the former with the latter in the dynamic model, we are able to quantify the overestimation generated by the random walk restriction. Under a standard parameterization in the corporate finance literature, we find that, in our model, the debt tax shields represent only about 3% of firm value, which is in line with the conjecture of Blouin, Core, and Guay.

Consistently with the findings of Korajczyk and Levy (2003) and Bhamra, Kuehn, and Strebulaev (2010) regarding observed leverage ratios, we also find that the value of the tax benefits of debt as a fraction of firm value is countercyclical over the economic cycle.

In addition, we do a cross-sectional comparison of the interest tax shields from firms in different industries (e.g., Oil and Gas Extraction, Chemicals, and Printing and Publishing), and find that the tax benefits of debt vary substantially across them, which agrees with the evidence reported by Cordes and Sheffrin (1983). This variability arises as these firms choose considerably different levels of optimal debt due to their dissimilar fundamental characteristics. In order to shed further light on this finding, we study the importance of the different primitive features of

[1] In related papers, van Binsbergen, Graham, and Yang (2010) and Korteweg (2010) report that the net benefits of debt account for 3.5% and 5.5% of firm value, respectively.

[2] A large body of accounting literature on earnings suggests that they are mean-reverting (see, e.g., Hayn (1995) and references therein).

the firm through a comparative statics analysis of the fraction of the stock price explained by the interest tax shields. Jointly with the interest rate on debt and the income tax rate, we find that the curvature of the production function is a fundamental determinant of the tax benefits of debt. This novel result is significant because, while this primitive firm characteristic has been studied substantially by the industrial organization literature in economics, it has received little attention in finance as a determinant of corporate borrowing.[3]

By highlighting the importance of the curvature of the production function, our work complements the extensive finance literature studying the determinants of the interest tax shields and leverage. For instance, in line with our results, DeAngelo and Masulis (1980) find that the non-debt tax deductions (e.g., the operating costs and capital depreciation) are important determinants of the tax benefits of debt. Furthermore, Bradley, Jarrell, and Kim (1984) and Long and Malitz (1985) also show that the benefits of debt are strongly related to the volatility of earnings and growth opportunities, while Long and Malitz (1985), Titman and Wessels (1988), and Rajan and Zingales (1995) find a negative association between profitability and leverage.[4]

The paper is organized as follows. In Section 2, we derive the analytic solution of the dynamic model of the firm and define the interest tax shields. In Section 3, we study the economic importance of the tax advantage of debt. The comparative statics analysis of the proportion of the stock price explained by the tax benefits of leverage is in Section 4. Section 5 concludes. Appendix 1 provides two robustness checks; it shows our results remain valid even if we allow for risky debt and costly external finance. Appendix 2 describes the calibration of model parameters.

2 A Dynamic Model of the Firm

This section solves the dynamic model of the firm in closed-form and defines the value of the interest tax shields.

In every period (e.g., quarter, year, etc.), the firm chooses capital and debt in order to

[3]See, e.g., Ackerberg, Benkard, Berry, and Pakes (2007), and the references therein.

[4]The list of important papers in this literature is vast, including, but not limited to, Harris and Raviv (1991), Shyam-Sunder and Myers (1999), Hovakimian, Opler, and Titman (2001), Graham and Harvey (2001), Fama and French (2002), Flannery and Rangan (2006), Lemmon, Roberts, and Zender (2008), and Frank and Goyal (2009).

maximize its current stock price. We write a tilde on variable X (i.e., $\widetilde{X}$) to indicate that it is growing over time. The book value of assets in period t is denoted by variable $\widetilde{K}_t$ and depreciates at rate $\delta \in [0,1]$ in each period. The firm uses debt $\widetilde{D}_t$ in period t, which we assume expires in one period. Without any loss of generality, we let the market cost of debt r_B be equal to the coupon rate c_B, which makes the market value of debt $\widetilde{B}_t$ equal the book value of debt $\widetilde{D}_t$. We initially follow DeAngelo, DeAngelo, and Whited (2011) and assume the firm issues risk-free debt. This feature allows us to both capture the apparent conservatism in debt usage reported by Graham (2000) and obtain a closed-form solution for our model, which improves the description of the results.[5] In this context, the market cost of debt, r_B, equals the risk-free interest rate, r_f, in the economy. Then, in Appendix 1, we show that our findings do not change when we consider risky debt.

The profit shock, z_t, is a random variable that follows a logarithmic AR(1) process

$$\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \varepsilon_t \tag{1}$$

where $c > 0$ is a constant and parameter $\rho \in (0,1)$ defines the degree of mean-reversion of profits. Finally, we assume ε_t is an *iid* normal random term with mean 0 and variance σ^2.

The firm obtains net profits in period t according to the following function

$$\widetilde{N}_t = \left[(1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta\widetilde{K}_t - r_B\widetilde{B}_t\right](1-\tau) \tag{2}$$

where $(1+g)^{t(1-\alpha)}$ represents the level of technology in period t and allows the firm to grow at rate $g \geq 0$. Parameter $\alpha \in (0,1)$ denotes the curvature of the production function with respect to capital, $f > 0$ indicates the operating costs, and parameter $\tau \in [0,1)$ represents the income tax rate.[6] Then, the dividend that shareholders receive from the firm in period t is

$$\widetilde{L}_t = \widetilde{N}_t - \left[\left(\widetilde{K}_{t+1} - \widetilde{K}_t\right) - \left(\widetilde{B}_{t+1} - \widetilde{B}_t\right)\right]. \tag{3}$$

The market cost of equity is denoted by r_S, the market cost of capital is indicated by r_A, and we impose the restriction that the market cost of capital exceeds the growth rate (i.e., $r_A > g$) to guarantee the existence of share price.

[5] Lazzati and Menichini (2014a) show that these assumptions about debt produce leverage predictions that are in line with several important findings reported by the literature on capital structure.

[6] We also assume $(f+\delta)(1-\tau) \leq 1$ to guarantee that the stock price is weakly positive.

At the beginning of its life (i.e., at $t = 0$), the firm selects the future levels of capital and debt, $\left\{\widetilde{K}_{t+1}, \widetilde{B}_{t+1}\right\}_{t=0}^{\infty}$, in order to maximize the stock price. Accordingly, the market value of equity is given by

$$\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right) = \max_{\left\{\widetilde{K}_{t+1}, \widetilde{B}_{t+1}\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{\prod_{j=0}^{t}\left(1 + r_{S_j}\right)} \widetilde{L}_t \tag{4}$$

subject to the restriction of risk-free debt and where E_0 denotes the expectation over future profit shocks given current information (i.e., $\widetilde{K}_0, \widetilde{B}_0, z_0$). We say debt is risk-free if, at the end of each period, net profits plus assets, $\widetilde{N}_t + \widetilde{K}_t$, are sufficient to pay off the debt, $\widetilde{B}_t$. Leland (1994) uses a similar type of debt covenant. We extend this model in Appendix 1 to include risky debt as well as costly external finance.

We solve Equation (4) in closed-form and find the following expression for the stock price

$$\widetilde{S}_t\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right) = \left[(1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta\widetilde{K}_t - r_B\widetilde{B}_t\right](1-\tau) + \widetilde{K}_t - \widetilde{B}_t + \widetilde{G}_t(z_t) \tag{5}$$

where the going-concern value is $\widetilde{G}_t(z_t) = \widetilde{M}_t(z_t) P^*$. Variable $\widetilde{M}_t(z_t)$ is given by

$$\widetilde{M}_t(z_t) = (1+g)^t e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} \sum\nolimits_{n=1}^{\infty} \left(\frac{1+g}{1+r_A}\right)^n E\left[z_{t+n}^{1/(1-\alpha)} | z_t\right] \tag{6}$$

with the general term

$$E\left[z_{t+n}^{1/(1-\alpha)} | z_t\right] = \left(c^{\frac{1-\rho^n}{1-\rho}} z_t^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{\left(1-\rho^{2n}\right)}{\left(1-\rho^2\right)} \frac{1}{(1-\alpha)}}\right)^{\frac{1}{1-\alpha}}, \quad n = 1, 2, ... \tag{7}$$

and variable P^* takes the form

$$P^* = \left(W^{*^{\alpha}} - fW^* - \delta W^*\right)(1-\tau) - r_A W^* + \left(\frac{1+r_A}{1+r_B}\right) r_B \tau \ell^* W^* \tag{8}$$

with

$$W^* = \left(\frac{\alpha}{\frac{r_A}{1-\tau} + f + \delta}\right)^{\frac{1}{1-\alpha}} \text{ and } \ell^* = \frac{1 - (f+\delta)(1-\tau)}{1 + r_B(1-\tau)}. \tag{9}$$

Equation (5) exhibits the stock price, which represents an analytic solution of the Gordon Growth Model in the dynamic and stochastic setting.[7]

[7] Lazzati and Menichini (2014b) include the proof of Equation (5) and describe each of its parts in detail.

The tax advantage of debt is defined as the present value of the reduction in current and future corporate income tax bills due to interest payments on debt. Given our previous results, the value of the interest tax shields takes the following form.

The Value of the Interest Tax Shields

$$\widetilde{T}_t\left(\widetilde{B}_t, z_t\right) = r_B \tau \widetilde{B}_t + r_B \tau \left[\widetilde{M}_t\left(z_t\right)\left(\frac{1+r_A}{1+r_B}\right)\ell^* W^*\right]. \tag{10}$$

The first term in equation (10) denotes the interest tax shield from the interest payment on current-period debt. The second term indicates the present value of the interest tax shields stemming from the expected future interest payments. The expression in square brackets represents the discounted value of the future stream of optimal debt decisions. From equations (6) through (9), we can observe that the second term in equation (10) depends explicitly on all the primitive characteristics of the firm (i.e., the elasticity of capital, the capital depreciation rate, etc.) In particular, it depends on the kind of process followed by firm profits, which is defined by parameter ρ. In our baseline analysis, we let profits follow a mean-reverting process (i.e., $0 < \rho < 1$). We then show that if we assume instead a random walk process, results change substantially. This finding sheds light on the current debate regarding the actual value of the tax advantage of debt, as we discuss below.

3 The Value of the Interest Tax Shields

In Subsection 3.1, we ascertain the economic importance of the interest tax shields as a component of the stock price, and investigate some of their relevant time-series properties. We then study the significance of the tax benefits of debt for different SIC industries in Subsection 3.2.

3.1 Analysis of a Representative Firm

We start parameterizing the model for a representative firm. The parameterization follows Lazzati and Menichini (2014b) and is standard in the corporate finance literature (see, e.g., Hennessy and Whited (2007) and DeAngelo, DeAngelo, and Whited (2011)). The elasticity of capital (α) is set at 0.65, the volatility of the innovations (σ) equals 0.20, and the persistence of profit shocks (ρ)

is 0.75. We set operating costs (f) and the capital depreciation rate (δ) equal to 0.20 and 0.10, respectively. Furthermore, we use standard values for the corporate income tax rate, $\tau = 0.35$, the market cost of debt, $r_B = 0.02$, the market cost of capital, $r_A = 0.08$, and the long-run growth rate, $g = 0.01$. Finally, we set parameter c equal to 1 because we are studying a representative firm. The parameter values we just described refer to a period of a year.

For simplicity, we assume that the firm is at the beginning of its life (i.e., $t = 0$) and the current state $\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$ is at the mean of the stationary distribution of profit shocks

$$z_0 = c^{\frac{1}{1-\rho}} e^{\frac{1}{2}\sigma^2 \frac{1}{(1-\rho^2)}}, \quad \widetilde{K}_0 = \left[c^{\frac{1}{1-\rho}} e^{\frac{1}{2}\sigma^2 \frac{1}{(1-\rho^2)}}\right]^{\frac{1}{1-\alpha}} W^*, \quad \text{and} \quad \widetilde{B}_0 = \ell^* \widetilde{K}_0. \tag{11}$$

Using the previous parameter values, we obtain $z_0 = 1.05$, $\widetilde{K}_0 = 3.89$, and $\widetilde{B}_0 = 3.09$.

Table 1 exhibits our main results. **In the base case parameterization, the interest tax shields represent 3.47% of the stock price and 2.73% of firm value.** This value of the tax benefits of debt is lower than the 10% suggested by Graham (2000). As we explained before, a difference between his work and ours is the distinct assumption made about the evolution of profits over the business cycle. We assume that income follows a mean-reverting process while Graham assumes it follows a random walk. Blouin, Core, and Guay (2010) argue that the latter overestimates the benefits from interest tax deductions as a consequence of the random walk restriction. Our model confirms this prediction and allows us to quantify the degree of overvaluation.

[Insert Table 1 here]

In order to connect our results more closely with those of Graham (2000), we explore the effect of changing parameter ρ on the fraction of firm value explained by the tax benefits of debt, $\widetilde{T}_t\left(\widetilde{B}_t, z_t\right) / \left[\widetilde{B}_t + \widetilde{S}_t\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right)\right]$. In Section 2, we described that parameter ρ defines the type of process followed by firm profits. In our model, the restriction $0 < \rho < 1$ makes them mean-reverting. However, as we increase ρ and make it closer to 1, the behavior of profits approximates a random walk. Accordingly, Panel A in Figure 1 shows the fraction of firm value explained by the tax advantage of debt as we increase ρ (while we keep firm value constant)[8]. It is clear

[8]Graham (2000) computes the interest tax shields assuming profits follow a random walk, and divides them by

that, as ρ approximates 1, that proportion increases steadily and exceeds 10%, suggesting that a random walk assumption for firm profits would overestimate the tax benefits of debt.

[Insert Figure 1 here]

We close this subsection describing some interesting time-series properties of the interest tax shields. Panel B in Figure 1 displays the stochastic evolution over time of the proportion of the value of the firm explained by the tax benefits of debt as well as the profit shock, z_t. We simulate the model over 100 periods with the base case parameterization described above. We can observe that the proportion of firm value explained by the interest tax shields (solid line) is negatively related to the profit shocks (dashed line). **This phenomenon implies that the fraction of firm value explained by the interest tax shields is countercyclical over the business cycle. That is, when profit shocks are high, the proportion of firm value explained by this component falls and vice versa.** This pattern occurs because market equity is more sensitive to profit shocks than are the interest tax shields, and suggests that the economic importance of the latter increases during economic recessions as opposed to expansions. Korajczyk and Levy (2003) and Bhamra, Kuehn, and Strebulaev (2010) report evidence suggesting that observed leverage ratios move countercyclically, which is consistent with our results.

3.2 Cross-Industry Analysis

In the previous subsection, we analyzed the value of the interest tax shields for a representative firm. We now extend those results to compare the importance of the tax advantage of debt for firms in different SIC industries. We consider the following industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). These firms differ from each other with respect to all of the fundamental characteristics, specially the curvature of the production function and the non-debt tax deductions. These differences make those firms select considerably dissimilar optimal debt ratios and, as consequence, they exhibit quite different levels of interest

the market value of the firm (which we suppose is determined by the market assuming mean-reverting profits). For that reason, we vary the interest tax shields while we keep firm value constant.

tax shields. We use Compustat data to parameterize the model for each industry and show their values in Panel A of Table 2. Appendix 2 describes the procedure used to obtain those parameters for each industry.

[Insert Table 2 here]

Panel B of Table 2 shows the results of the cross-sectional comparison. The low non-debt tax deductions (i.e., operating costs, f, plus depreciation, δ) in the OGE industry make those firms have high book leverage ratio ($\ell^* = 0.76$) while the opposite is true for firms in the C industry ($\ell^* = 0.36$). Optimal leverage for firms in the PP industry is between these two extreme values ($\ell^* = 0.69$). Accordingly, the fraction of firm value explained by the tax advantage of debt is 2.07% for OGE firms, 1.38% for PP firms, and just 0.34% for C firms. As we show in the following section, the fact that OGE firms have higher elasticity of capital (α) than the other two industries also contributes to the larger proportion of interest tax shields.

In summary, there is substantial variation in the value of the tax advantage of debt across industries. We find that this variability stems from the fact that these industries differ in all their primitive features, specially the elasticity of capital and the non-debt tax deductions which, as we describe in the next section, are among the most important determinants of the tax benefits of debt. This result is in line with the evidence reported by Cordes and Sheffrin (1983).

4 The Determinants of the Interest Tax Shields

In order to study the determinants of the interest tax shields, we perform a sensitivity analysis that allows us to ascertain how the fraction of the stock price explained by the tax advantage of debt, $\widetilde{T}_0\left(\widetilde{B}_0, z_0\right)/\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, varies when we change the primitive firm characteristics. In addition to understanding the directional impact of those fundamental features, we identify the ones with the largest influence on the interest tax shields. We do this study using the base case parameterization described in Subsection 3.1 and changing those values by up to $\pm$ 20%.

Table 3 presents the results of this comparative statics analysis. **The table shows clearly that the curvature of the production function (α) is the main determinant of that proportion.** For example, for the base case parameter value of $\alpha = 0.65$, the interest tax shields represent 3.47% of the stock price, percentage that goes up to 6.74% when α increases to 0.78. It turns out that the interest tax shields, $\widetilde{T}_0\left(\widetilde{B}_0, z_0\right)$, are considerably more sensitive to α than is the stock price, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, producing the result just described.

As expected, the income tax rate (τ) **and the market cost of debt** (r_B) **are also important parameters.** A 20% increment in τ from 0.35 to 0.42 increases the proportion of interest tax shields from 3.47% to 4.56%, while a similar proportional increment in r_B (i.e., from 0.02 to 0.024) augments the fraction of the tax advantage of debt from 3.47% to 4.11%. **Finally, the joint effect of the non-debt tax deductions** $(f + \delta)$ **also plays a significant role.** The other parameters affect this fraction to a lesser extent.

[Insert Table 3 here]

5 Conclusion

We use a dynamic model of the firm to investigate both the economic importance of the interest tax shields and their main determinants. Assuming a random walk process for profits, Graham (2000) finds that the value of the tax benefit of leverage is around 10% of firm value. By substituting that assumption with a mean-reverting process, we find that the tax advantage of debt represents about 3% of the value of the firm. Our results are consistent with Blouin, Core, and Guay (2010), who suggest that the random walk assumption for earnings overestimates the value of the interest tax shields. In addition, we find that the value of the debt tax shields is countercyclical over the business cycle.

Regarding the underlying factors, we find that the curvature of the production function is among the main determinants of the proportion of firm value explained by the interest tax shields. This new result is significant because it highlights an aspect of the problem that so far has not received great attention from the corporate finance literature. As expected, we also find that the corporate income tax rate and the interest rate on debt are important determinants of that proportion.

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6 Appendix 1: Robustness Checks

6.1 Risky Debt

In this subsection, we extend the model presented in Section 2 to include the possibility of the firm to issue risky debt. This robustness check is necessary since the risk-free debt assumption might undervalue the tax benefits of debt. As we show next, our results regarding the economic importance of the interest tax shields remain the same in this more general setting.

The risky debt model only differs from the risk-free debt one in that it includes bankruptcy costs and a bankruptcy triggering event (i.e., all the other features of the firm are the same). We follow Hennessy and Whited (2007) and assume the bankruptcy costs in period t are given by $\xi\widetilde{K}_t$, where parameter $\xi > 0$ represents the proportion of assets that is lost in bankruptcy. In this model, the event of bankruptcy happens whenever

$$\left(z_t\widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta\widetilde{K}_t - r_{B_t}\ell\widetilde{K}_t\right)(1-\tau) + \widetilde{K}_t - \ell\widetilde{K}_t < 0. \tag{12}$$

That is, bankruptcy is triggered when the profit shock, z_t, is such that the sum of net profits, $\left(z_t\widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta\widetilde{K}_t - r_{B_t}\ell\widetilde{K}_t\right)(1-\tau)$, and the value of assets, $\widetilde{K}_t$, is insufficient to cover current debt, $\ell\widetilde{K}_t$. In this case, we assume the firm pays the bankruptcy costs and shuts down.

Another important feature of the risky debt model is that the interest rate charged by debt-holders, r_{B_t}, becomes endogenous. The latter is determined by the following equation

$$\widetilde{D}_{t+1} = \frac{1}{1+r_f}E\left[(1-\Phi_{t+1})\,\widetilde{D}_{t+1}\left(1+r_{B_{t+1}}\right) + \Phi_{t+1}\widetilde{R}_{t+1}|z_t\right] \tag{13}$$

where the indicator function Φ_t equals 1 if the firm goes into bankruptcy in period t, and 0 otherwise. Variable $\widetilde{R}_{t+1}$ is the amount of money received by the debt claimants in the case of bankruptcy. Specifically,

$$\widetilde{R}_t = \min\left\{\widetilde{D}_t, \widetilde{K}_t + \widetilde{N}_t - \Phi_t\xi\widetilde{K}_t\right\} \tag{14}$$

which suggests that creditors receive the minimum between the nominal value of the debt and the value of the assets in bankruptcy. Equation (13) means that debt claimants require an interest rate that equates the nominal value of the debt to the expected discounted payoff of debt in the next period.[9]

[9] Similar to Moyen (2004) and Hennessy and Whited (2007), we assume that bond-holders are risk-neutral.

With the previous assumptions, the stock price is finally given by

$$\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right) = \max_{\left\{\widetilde{K}_{t+1}, \widetilde{B}_{t+1}\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{\prod_{j=0}^{t}\left(1+r_{S_j}\right)} \widetilde{L}_t \tag{15}$$

where $\widetilde{L}_t = \widetilde{N}_t - \left[\left(\widetilde{K}_{t+1} - \widetilde{K}_t\right) - \left(\widetilde{B}_{t+1} - \widetilde{B}_t\right)\right] - \Phi_t \xi \widetilde{K}_t$. Unfortunately, the expression above does not have an analytic solution, so we proceed to solve it numerically by backward induction.

We calibrate all previous model parameters as we explained in Subsection 3.1. Following Andrade and Kaplan (1998), we calibrate the new parameter ξ such that it represents 20% of firm value. We then reproduce the information in Table 1 using the risky-debt model and show the results in Panel A of Table 4. **We find that the interest tax shields represent 4.15% of the stock price and 3.27% of firm value.** These results imply that the possibility of the firm to issue risky debt does not affect our main conclusions. That is, the assumption about the stochastic process followed by income (i.e., mean-reversion vs. random walk) plays a fundamental role in the valuation of the benefits from interest tax deductions, with the random walk assumption producing an overvaluation of those benefits. **Overall, we find that the predictions of the risky debt model confirm those of the risk-free debt model.**

[Insert Table 4 here]

6.2 Costly External Finance

In Subsection 3.1, we investigated the value of the interest tax shields in a context of no issuance costs of debt or equity. In this subsection, we study the effects of adding those costs. In order to do this analysis, we extend the risky debt model described in the previous subsection by adding a cost function of external finance.

Altinkilic and Hansen (2000) suggest that the costs of issuance are convex, both for debt and equity. Accordingly, we use the following linear-quadratic cost function of external finance

$$\widetilde{C}_t = \phi_d \left[\lambda_1^d \left(\widetilde{B}_{t+1} - \widetilde{B}_t\right) + \lambda_2^d \frac{\left(\widetilde{B}_{t+1} - \widetilde{B}_t\right)^2}{\widetilde{B}_t}\right] + \phi_e \left[\lambda_1^e \widetilde{X}_{t+1} + \lambda_2^e \frac{\left(\widetilde{X}_{t+1}\right)^2}{\widetilde{K}_t - \widetilde{B}_t}\right] \tag{16}$$

where $\widetilde{X}_{t+1} = \left(\widetilde{K}_{t+1} - \widetilde{K}_t\right) - \left(\widetilde{B}_{t+1} - \widetilde{B}_t\right) - \widetilde{N}_t$ represents the equity issuance in period t. The indicator function ϕ_d equals 1 if $\widetilde{B}_{t+1} - \widetilde{B}_t > 0$, and 0 otherwise. Likewise, the indicator function

ϕ_e equals 1 if $\widetilde{X}_{t+1} > 0$, and 0 otherwise. This feature implies that issuing debt and/or equity is costly, while reducing them is not. Parameters λ_1^d and λ_2^d denote the linear and quadratic costs of issuing debt, respectively, while parameters λ_1^e and λ_2^e reflect the analogous costs for equity. Finally, we assume that the costs of external finance are tax deductible.

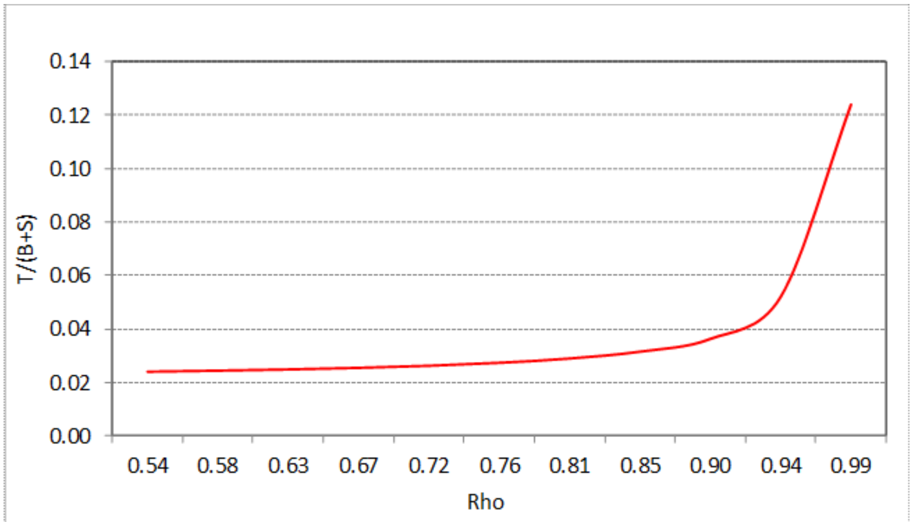
We proceed to calibrate the parameters in equation (16) following the evidence reported by Altinkilic and Hansen (2000) and Hennessy and Whited (2007). Accordingly, we let $\lambda_1^d = 0.01$, $\lambda_2^d = 0.0002$, $\lambda_1^e = 0.1$, and $\lambda_2^e = 0.0004$, which reflect the empirical observation that issuing equity is more expensive than issuing debt. All other model parameters are calibrated as before. We then repeat the analysis in Table 1 and present our findings in Panel B of Table 4. **The fraction of share price and firm value explained by the interest tax shields is 4.04% and 3.19%, respectively.** These results are similar, though slightly lower, than those of the previous subsection and suggest that costly external finance does not play an important role regarding the value of the benefits from interest tax deductions.

7 Appendix 2: Calibration of Model Parameters

We need to find parameter values for $c, \rho, \sigma, \alpha, f, \delta, \tau, r_B, r_A$, and g for each of the three industries. We calibrate the model using Compustat annual data for all firms in each of the three SIC codes (i.e., Oil and Gas Extraction (OGE) is SIC 13, Printing, Publishing, and Allied Products (PP) is SIC 27, and Chemicals and Allied Products (C) is SIC 28). The sample covers the period 1990-2013 and includes 9,476 firm-years for the OGE industry, 1,859 firm-years for the PP industry, and 11,162 firm-years for the C industry.

In order to obtain parameter f, we average the ratio Selling, General, and Administrative Expense (XSGA)/Assets - Total (AT) for all firm-years in each industry. We follow the same procedure to get δ as the ratio of Depreciation and Amortization (DP) over Assets - Total (AT), and τ as the fraction Income Taxes - Total (TXT)/Pretax Income (PI). We trim these ratios at the lower and upper one-percentiles to reduce the effect of outliers and errors in the data. Following Moyen (2004), we obtain parameters ρ, σ, and α for each industry using the firm's autoregressive profit shock process of equation (1) and the gross profits function in equation (2), $(1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha}$. The data we use with these equations are Gross Profit (GP) and Assets - Total (AT). Given that we are working with representative firms, we set $c = 1$ for the three industries. We keep the assumption that the risk-free interest rate ($r_f = r_B$) is 0.02. We derive r_A using CAPM with the corresponding (unlevered) industry betas estimated by Fama and French (1997) and assuming an expected market return (r_M) of 0.08. Finally, we obtain g for each industry from Jorgenson and Stiroh (2000).

Panel A



Panel B

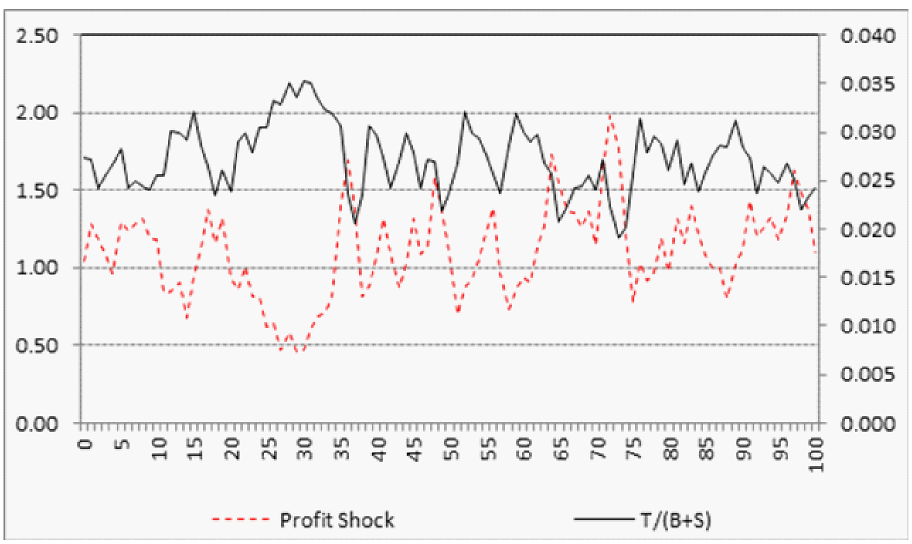


Figure 1. Interest Tax Shields as a Fraction of Firm Value. Panel A: The figure exhibits the fraction of firm value explained by the tax benefits of debt (solid line) for different values of parameter ρ. Panel B: The figure exhibits the evolution over time of the proportion of firm value explained by the interest tax shields (solid line, right Y-axis) as well as the profit shocks (dashed line, left Y-axis). The model is simulated over 100 periods with the parameterization described in Subsection 3.1.

Table 1

Value of Market Equity and Interest Tax Shields

The table exhibits the base case results for the dynamic dividend discount model. The variables are the market value of equity, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; the market value of the firm, $\widetilde{B}_0 + \widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; and the interest tax shields, $\widetilde{T}_0\left(\widetilde{B}_0, z_0\right)$.

Variable	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0 + \tilde{S}_0$*
$\tilde{S}_0$	11.47	100.00	78.79
$\tilde{B}_0 + \tilde{S}_0$	14.56	126.92	100.00
$\tilde{T}_0$	0.40	3.47	2.73

Table 2

Cross-Sectional Value of Market Equity and Interest Tax Shields

Panel A: The table presents the values used to parameterize the dynamic dividend discount model for three different SIC industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). Panel B: The table exhibits the results of the dynamic dividend discount model for the three SIC industries. The variables are the market value of equity, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; the market value of the firm, $\widetilde{B}_0 + \widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; the interest tax shields, $\widetilde{T}_0\left(\widetilde{B}_0, z_0\right)$; and the optimal book leverage ratio (ℓ^*).

Panel A

Parameter	*Value*		
	OGE Firms	*C Firms*	*PP Firms*
c	1.0000	1.0000	1.0000
ρ	0.4748	0.5483	0.5603
σ	0.3633	0.2857	0.1787
α	0.6905	0.6146	0.5823
f	0.2079	0.7818	0.3746
δ	0.0925	0.0452	0.0617
τ	0.2519	0.2281	0.3210
r_B	0.0200	0.0200	0.0200
r_A	0.0710	0.0854	0.0902
g	0.0043	0.0347	0.0251

Panel B

Variable	*OGE Firms*			*C Firms*			*PP Firms*		
	Value	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0 + \tilde{S}_0$*	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0 + \tilde{S}_0$*	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0 + \tilde{S}_0$*
$\tilde{S}_0$	22.25	100.00	78.50	4.35	100.00	96.91	5.83	100.00	88.26
$\tilde{B}_0 + \tilde{S}_0$	28.34	127.39	100.00	4.49	103.18	100.00	6.60	113.30	100.00
$\tilde{T}_0$	0.59	2.64	2.07	0.02	0.35	0.34	0.09	1.56	1.38
l^*	0.76			0.36			0.69		

Table 3

Comparative Statics Analysis of Interest Tax Shields as a Fraction of Share Price

The table shows the proportion of share price, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, that is explained by the value of the interest tax shields, $\widetilde{T}_0\left(\widetilde{B}_0, z_0\right)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Subsection 3.1 while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of capital (r_A), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$\widetilde{T}_0/\widetilde{S}_0$	2.47%	2.71%	2.95%	3.15%	3.33%	3.47%	3.57%	3.64%	3.70%	3.73%	3.76%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$\widetilde{T}_0/\widetilde{S}_0$	3.43%	3.44%	3.44%	3.45%	3.46%	3.47%	3.48%	3.49%	3.51%	3.53%	3.57%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$\widetilde{T}_0/\widetilde{S}_0$	3.44%	3.44%	3.45%	3.45%	3.46%	3.47%	3.47%	3.48%	3.48%	3.49%	3.50%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$\widetilde{T}_0/\widetilde{S}_0$	2.15%	2.34%	2.56%	2.82%	3.11%	3.47%	3.89%	4.39%	5.02%	5.78%	6.74%
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$\widetilde{T}_0/\widetilde{S}_0$	3.98%	3.87%	3.76%	3.66%	3.56%	3.47%	3.38%	3.29%	3.20%	3.12%	3.04%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\widetilde{T}_0/\widetilde{S}_0$	3.71%	3.66%	3.61%	3.56%	3.51%	3.47%	3.42%	3.38%	3.33%	3.29%	3.25%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$\widetilde{T}_0/\widetilde{S}_0$	2.54%	2.71%	2.89%	3.08%	3.27%	3.47%	3.67%	3.88%	4.10%	4.33%	4.56%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$\widetilde{T}_0/\widetilde{S}_0$	2.81%	2.94%	3.07%	3.20%	3.34%	3.47%	3.60%	3.72%	3.85%	3.98%	4.11%
r_A	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$\widetilde{T}_0/\widetilde{S}_0$	3.72%	3.67%	3.61%	3.56%	3.51%	3.47%	3.42%	3.37%	3.32%	3.28%	3.23%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$\widetilde{T}_0/\widetilde{S}_0$	3.46%	3.46%	3.46%	3.46%	3.46%	3.47%	3.47%	3.47%	3.47%	3.47%	3.48%

Table 4

Value of Market Equity and Interest Tax Shields: Risky Debt and Costly Issuance of Debt and Equity

Panel A: The table exhibits the base case results for the dynamic dividend discount model with risky debt.

Panel B: The table presents the base case results for the dynamic dividend discount model with both risky debt and costly issuance of debt and equity. The variables are the market value of equity, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; the market value of the firm, $\widetilde{B}_0 + \widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; and the interest tax shields, $\widetilde{T}_0\left(\widetilde{B}_0, z_0\right)$.

Panel A

Variable	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0 + \tilde{S}_0$*
$\tilde{S}_0$	11.55	100.00	78.91
$\tilde{B}_0 + \tilde{S}_0$	14.64	126.73	100.00
$\tilde{T}_0$	0.48	4.15	3.27

Panel B

Variable	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0 + \tilde{S}_0$*
$\tilde{S}_0$	11.54	100.00	78.89
$\tilde{B}_0 + \tilde{S}_0$	14.63	126.76	100.00
$\tilde{T}_0$	0.47	4.04	3.19