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Gait and Foot Trajectory Planning for Versatile Motions of a Six-Legged Robot

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This article deals with the problem of planning and controlling a radially symmetric six-legged walker on an uneven terrain when a smooth time-varying body motion is required. The main difficulties lie on the planning of gaits and foot trajectories. As for the gaits, this article discusses the forward wave gait of a variable duty factor and a variable wave direction. With the commanded body motion, the maximum possibly duty factor is computed using the speed limit of the leg swing motion. Guaranteeing this maximum duty factor contributes to obtain higher stability. We prove the “continuity” of this forward wave gait planning algorithm adds the versatility to gaits planned. The
foot trajectory planning algorithm dynamically generates a smooth foot trajectory as a function of the instantaneous body motions by modifying standard leg motion templates. The robot can negotiate an uneven terrain by modifying a vertical leg motion by a signal of tactile sensors on the foot. The experiments prove that the robot can successfully track smooth curves with body rotations on an uneven terrain, and thus prove the robustness of the algorithms. © 1997 John Wiley & Sons, Inc.

1. INTRODUCTION

A practical walking machine should be able to walk versatile following a continuous curvature of the commanded locomotion trajectory. Especially, walking omnidirectionally on uneven terrain is an indispensable feature of legged machines. This research is related to a radially symmetric six-legged robot, which has an advantage of having a uniform performance in an arbitrary crab angle.

A number of periodic gait planning methods for six-legged walkers have already been proposed. For six-legged robots, the tripod gait is the simplest periodic gait, in which two set of tripods move alternately. Although the control method of this gait is simple, it is appropriate only at a specific speed as seen later. We want a more general gait planning algorithm that gives larger stability for a slower speed.

The (forward) wave gait for a longitudinally symmetric multi-legged robot is a gait in which a wave of lift-off events propagates forward along each side. Song and Waldron demonstrated that the wave gait provides the largest stability margin at any given duty factor on a longitudinally symmetric six legged walker. It seems to have largest stability margin for a radially symmetric walker also. This research issue was systematically investigated by Zhang and Song. However, the conventional wave gait had limitations in at least two aspects. A larger duty factor means a larger stability margin. Therefore, we should use the maximum duty factor determined by the robot’s commanded speed and the leg’s maximum swing speed, which are generally variable over time. In the conventional wave gait, a duty factor is kept constant and cannot be changed when walking. This is the first of the two limitations.

To obtain larger stability, it is essential for a walker to have its wave propagating direction equal to the desired crab angle when walking (crab angle is the motion direction in the robot’s coordinate system). However, it is not possible in the conventional wave gait to continuously change the wave propagating direction when walking. This is the second limitation. This capability of changing the wave propagation is needed for a walking robot to track a curved trajectory. Lee and Orin proposed a method to change the crab angle by using only two wave propagation directions: backward or forward. However, in this gait, the wave propagation direction and crab angle are not the same and some hysteresis is unavoidable in the direction switching. A gait is said to be semi-periodic if its duty factor or relative phases are variable over time.

In this article, we will show a variable parameter gait planning algorithm that generates semi-periodic gaits with a variable speed, with a variable motion direc-
tion, and with a variable body rotation, answering the aforementioned research questions. More specifically, this algorithm embodies the best duty factor and the best combination of relative phases for each leg under a given motion command. The main theoretical results in this article are explicitly stated in three Propositions. The computation time for this algorithm is much less than that for fully intelligent free gait planning algorithms.\textsuperscript{11,12} This algorithm generates not only the conventional wave gaits (such as tripod gaits), but other useful gaits as well.

As for the trajectory of a transferring foot, Lee and Orin\textsuperscript{13,14} described an algorithm for a foot to follow the body motion. They used a rectangle trajectory, which is not suitable for smooth foot motion. Sakakibara and Hirose proposed to use a smooth foot trajectory, in which the next foothold must be determined by the time of its lift-off. Hence, their algorithm cannot execute a curved body trajectory.\textsuperscript{13,14} The features of the foot trajectory planning algorithm investigated in this article are (1) a smooth foot trajectory template is adopted, (2) omni-directional motion commands are allowed, (3) footholds are dynamically determined to deal with the omni-directional motions, and (4) each foot velocity is dynamically evaluated because of the foothold point’s possible motion. The algorithm stated here is the most general one among others reported so far, because the method given by ref. 5 satisfies (2) and (3), but not (1), and because the method given by refs. 13 and 14 satisfies (1), but neither (2) nor (3).

The activities described in this article are related to the robot AQUAROBOT, which was constructed by the Port and Harbour Research Institute (PHRI), Ministry of Transport of Japan. PHRI has constructed three experimental six-legged underwater walking robots, AQUAROBOT, for underwater inspection tasks. The second version was tested in a real environment. The task was to inspect the unevenness of a rubble mound of size $42 \times 77$ m at an average depth of 24 m in the caisson yard for Kamaishi Bay Breakwaters.\textsuperscript{15,16}

This research was a part of the international cooperated research project administered mainly by the Naval Postgraduate School (NPS) in the US and PHRI.\textsuperscript{17} The research activities in the U.S. have been mostly supported by the National Science Foundation for three years beginning in February 1992. Activities in Japan have been partially supported by the Science and Technology Agency for the Japanese fiscal year of 1993.\textsuperscript{18,19}

One of the purposes of this research project was to improve the performance of these underwater walking robots through international collaboration, and, more specifically, to make its walking speed faster. An implementation of these algorithms was done on the AQUAROBOT system for evaluating their correctness and efficiency. The successful results proved not only the correctness, but the fact that the algorithms require only modest computation time to make real-time control of the robot hardware possible. The research goals of this whole project also included linkage dynamics, a virtual reality simulation model, and task and mission level control architecture.\textsuperscript{20–22}

2. Problem Statements and Overview of the Control System

The objective of this research is to find a leg motion planning algorithm for a radially symmetric walking robot to embody smooth continuous body motion commands on an uneven terrain with optimal stability. The input to this algorithm is a rigid body motion specification to the robot, and the output is a description of foot motions. The difficulties of this problem lie in the fact that the gait must have larger stability for any situation, that the leg motion must be smooth and must negotiate a terrain roughness, that the body rotation may be superimposed on translational motion, and that the body speed may change with a realtime command.

To solve the problem of realizing a versatile control algorithm as stated above, we propose a global hierarchical architecture of the control system as shown in Figure 1. The body motion description block provides a smooth body motion command to follow a path that is specified for global robot tasks. Once the body motion command is given, the gait planning block generates a sequence of the legs’ lift-off and touch-down events to produce a larger stability margin. A static stability margin on an uneven terrain is defined the same as that on a flat terrain. Thus, this gait planning algorithm is available also on an uneven terrain. The gait is specified by a duty factor and relative phases. Hence, the gait planning block has the functions to decide a duty factor and relative phases based on the body motion command. The foot trajectory planning block generates a foot motion that allows the body to follow the commanded motion. It includes an uneven terrain negotiation algorithm. The foot motion is planned in the world coordinate system, and translated to the body coordinates to calculate joint angular rates.
first find the maximum duty factor $\beta_{\text{max}}$ for a given motion command $\mathcal{A}(t)$. Next, we embody this maximum duty factor $\beta_{\text{max}}$ by continuously changing $\beta(t)$. This method works to obtain larger stability.

A crab angle $\alpha(t)$ is the difference between the robot’s heading (direction of leg 1) and body motion direction $\theta(t)$. The relative phase $\phi_i$ of leg $i$ is the touchdown instance normalized by one cycle time. The legs are numbered as shown in Figure 2. A crab angle $\alpha$ divides the six legs into two groups: left and right. (In a special case where the body motion direction equals the direction of one leg, assume that both left and right leg groups include the forefront and rearmost legs.)

A forward wave gait for a radially symmetric walking robot is defined as a gait in which the wave of touchdown events propagates forward in each group, and the phase difference between the neighbors in each group is $1/2$. That means when one leg touches down, the next one is lifted off. In the conventional forward wave gait, the phase difference between two groups is $1/2$ so that the two legs in a longitudinally symmetric positions move alternately. However, in a radially symmetric walker, the phase difference between two groups may not necessarily be equal to $1/2$.

The gait planning algorithm developed here generates the conventional forward wave gait as a special case where $\alpha(t) = (2n - 1)\pi / 6$. No jerky motion occurs in any case, because the gait generator function is continuous.

When the body motion direction changes over time, we continuously change the wave propagating direction so that both directions become equal. This method also contributes to obtaining larger stability.

A regular / periodic gait is normally described by a duty factor $\beta$ and a set $\{\psi_i, \ldots, \psi_6\}$ of relative phases, which are constant over time. However, in this article, the duty factor $\beta(t)$ and the relative phases $\psi_i(t)$ are functions of time and defined more generally as follows. A variable parameter forward wave gait is a collection of regular and periodic reference gaits $\mathcal{G}(t)$. That
means, at each given time $t_i$, the gait $\phi(t_i)$ is a regular / periodic gait. Its duty factor $\beta(t_i)$ and relative phases $\psi(t_i)$ are defined as being of the instantaneous reference regular / periodic gait $\phi(t_i)$. Other instanta- neously definable values, such as positions and velocities of the legs, are also those of the reference gain.

Under this variable parameter forward wave gait planning method, we define a gait generator $\Psi_i(t_i = 1, \ldots, 6)$ of a crab angle $\alpha(t_i)$ and a duty factor $\beta(t_i)$ to obtain a relative phase $\psi(t_i)$ of leg $i$:

$$ \psi_i(t) = \Psi_i(\alpha(t), \beta(t)) \quad i = 1, \ldots, 6 \quad (2) $$

A gait generator $\Psi_i$ must be a continuous function of $\alpha(t_i)$ and $\beta(t_i)$ to obtain dynamically changing gaits when the parameters $\alpha(t_i)$ and $\beta(t_i)$ change. A discontinuity in a relative phase causes jerky leg motion. For instance, if $\Psi_i$ is discontinuous, a free leg may need to become a support leg instantaneously, even when the leg is still horizontally moving.

### 3.2. Maximum Duty Factor

In a regular gait without rotation, an average velocity of

$$ u(t) = \frac{\beta(t)}{1 - \beta(t)} v(t) \quad (3) $$

is required for a swing leg, where $\beta(t)$ is a duty factor and $v(t)$ is body motion speed. The duty factor defined in section 3.1 is still valid for a variable parameter gait. In a motion with a translational component $v(t)$ and a rotational component $\omega(t)$, the average velocity $u(t)$ at the outermost leg is

$$ u(t) = \frac{\beta(t)}{1 - \beta(t)} (v(t) + |\omega(t)| R), \quad (4) $$

where $R$ is the distance from the body center to the outermost point of a leg’s constrained working area. If $U_{\text{max}}$ denotes the maximum swing velocity,

$$ \beta(t) = \frac{u(t)}{v(t) + |\omega(t)| R + u(t)} \leq \frac{U_{\text{max}}}{v(t) + |\omega(t)| R + U_{\text{max}}} = \beta_{\text{max}}(t), \quad (5) $$

where $\beta_{\text{max}}(t)$ denotes the maximum duty factor. This value $\beta_{\text{max}}(t)$ is the upper limit of the duty factor for any foot position. From now on, we try to realize this velocity-dependent maximum duty factor $\beta_{\text{max}}(t)$ all the time to obtain the maximum stability.

### 3.3. Varying the Duty Factor

Now we want to find a set of relative phases of all six legs satisfying the maximum duty factor $\beta_{\text{max}}(t)$ given by Equation (5) so as to optimize the stability margin. We first consider the case in which the crab angle $\alpha = \pi / 6$ and the commanded velocity is changing. Namely, we will find the function value $\Psi_i(\pi / 6, \beta)$ in this section.

To find a continuous function $\Psi_i(\pi / 6, \beta)$, we design a gait pattern of some specified duty factor shown in Figure 3. The solid segments of each leg show a support phase at each duty factor. The segments were horizontally shifted from those in a conventional gait diagram to minimize the total amount of the phase variations. By linearly interpolating these
Figure 4 shows supporting leg combinations at a kinematic phase \( \phi \) (a time normalized by one cycle time)\(^1\) for a given duty factor \( \beta \). During its motion, a robot's state corresponds to a point with the current \( \beta \) and \( \phi \) in the chart. If a robot is walking at a constant translational speed \( v \) without rotation, the robot’s state moves along the horizontal line at the duty factor \( \beta \) given by Equation (5). In a special case of \( \beta = 1/2 \), this extended wave gait becomes the well-known tripod gait. When the robot increases its current translational speed \( v(t) \), the current duty factor \( \beta(t) \) decreases according to Equation (5), and the robot’s state moves downward in this chart as its kinematic phase proceeds.

A gait with a duty factor \( \beta < 1/2 \) is statically unstable for a six-legged walker. However, some of these gaits are still stable for a walker with more than six legs.

### 3.4. Varying the Crab Angle

In this section, we are finding a wave gait planning method when its crab angle \( \alpha(t) \) is variable. We first plan gaits for a variable crab angle \( \alpha(t) \) with a fixed duty factor \( \beta = 2/3 \). In Figure 5, for each leg, the upper line segment shows its supporting interval with \( \pi / 6 \) and \( 2/3 \), and the lower line segment with \( \pi / 6 \) (\( \beta = 2/3 \) in both cases). By interpolating these two segments, we obtain a gait for an arbitrary intermediate crab angle. This intermediate gait should also be classified into a forward wave gait, because a wave of lift-off events propagates forward.

For this fixed duty factor, it is straightforward to obtain a gait with a crab angle in the expanded range

\[ \Psi_i(\beta, \phi) = \left[ 6 \left( \left( \beta - \frac{1}{2} \right) \Psi_i\left( \frac{\pi}{6} \frac{2}{3} \right) \right) - \left( \beta - \frac{2}{3} \right) \Psi_i\left( \frac{\pi}{6} \frac{1}{2} \right) \right] \mod 1 \quad (i = 1 \text{ to } 6) \]

for the full duty factor range of (0, 1]. Thus, the relative phase \( \Psi_i \) for \( \alpha = \pi / 6 \) is found as a linear continuous function of \( \beta \).

**Proposition 1.** There exists a continuous gait generator \( \Psi_i \) of \( \beta \) such that \( \Psi(\pi / 6, \beta) \) is a forward wave gait for any duty factor \( \beta \in (0, 1] \).

**Proof.** Since \( \alpha = \pi / 6 \), the left leg group is (5, 6, 1) and the right leg group is (4, 3, 2) from the rear to the front, respectively. Consider the gait class shown in Figure 3 and Equation (6). The touch-down timing of leg 5 is equal to the lift-off timing of leg 6, and the touch-down timing of leg 6 is equal to the lift-off timing of leg 1 for any duty factor. The same relation holds on the other side. The phase differences of the touch-down timings of leg pairs (1, 2), (6, 3), and (5, 4) are 1/2, respectively. Thus, the gaits generated by Equation (6) are forward wave gaits.

![Figure 4. Supporting leg chart of variable duty factor wave gait (\( \alpha = \pi / 6 \)).](image)

![Figure 5. Gait diagram obtained by interpolation on \( \alpha(\beta = 2/3) \).](image)
of $[0, 2\pi)$ taking into account a hexagonal symmetry. Figure 6 shows the relative phase function $\Psi_i(\alpha, \beta)$ for $\beta = 2/3$ and $\beta = 1/2$. These functions are explicitly described as follows:

For $i = 1, 3, 5$,

$$\Psi_i(\alpha, \frac{2}{3}) = \begin{cases} \frac{\alpha'}{2\pi} & (0 \leq \alpha' < 2\pi/3), \\ 1/3 & (2\pi/3 \leq \alpha' < \pi), \\ -\alpha'/(2\pi) + 5/6 & (\pi \leq \alpha' < 5\pi/3), \\ 0 & (5\pi/3 \leq \alpha' < 2\pi), \\ \Psi_i(\alpha, \frac{1}{2}) = 1/4. \end{cases}$$

For $i = 2, 4, 6$,

$$\Psi_i(\alpha, \frac{2}{3}) = \begin{cases} \frac{\alpha'}{2\pi} + 1/2 & (0 \leq \alpha' < 2\pi/3), \\ 5/6 & (2\pi/3 \leq \alpha' < \pi), \\ -\alpha'/2\pi + 1/3 & (\pi \leq \alpha' < 5\pi/3), \\ 1/2 & (5\pi/3 \leq \alpha' < 2\pi), \\ \Psi_i(\alpha, \frac{1}{2}) = 3/4, \end{cases}$$

where

$$\alpha' = \left[ \alpha - \frac{(2i - 1)\pi}{6} \right]_{\text{mod} 2\pi}$$

Proposition 2. There exists a continuous gait generator $\Psi_i$ of $\alpha$ such that $\Psi_i(\alpha, 2/3)$ is a forward wave gait for any crab angle $\alpha \in [0, 2\pi)$.

Proof. Consider the class of gaits shown in Figure 6 and Equation (7). The touch-down events propagate from the rear to the front for any crab angle, and the phase lag of each leg to the neighboring one is 1/3. The phase differences of left and right legs are 1/2 for $\alpha(t) = (2n - 1)\pi/6$. Thus, the gaits generated by Equation (7) are forward wave gaits.

Now we consider the cases in which the duty factor is also variable. To plan a gait for an arbitrary combination of $\alpha$ and $\beta$, we combine the algorithm described in section 3.3 and the previous result (7). By substituting each occurrence of $\pi/6$ in Equation (6) by $\alpha$, we obtain

$$\Psi_i(\alpha, \beta) = [6(\beta - \frac{2}{3})\Psi_i(\alpha, \frac{2}{3}) - (\beta - \frac{1}{2})\Psi_i(\alpha, \frac{1}{2})]_{\text{mod} 1} \quad (i = 1 \text{ to } 6),$$

where $\Psi_i(\alpha, \frac{2}{3})$ and $\Psi_i(\alpha, \frac{1}{2})$ are obtained by Equation (7). That is, this algorithm generates a linear inter /

extrapolation for both parameters of a crab angle and a duty factor.

Proposition 3. There exists a continuous gait generator $\Psi_i$ of $\alpha$ and $\beta$ such that $\Psi_i(\alpha, \beta)$ is a forward wave gait for any crab angle $\alpha \in [0, 2\pi)$ and for any duty factor $\beta \in (0, 1]$.

Proof. Consider the class of gaits given by Equation (9). The phase lag of each leg to the neighboring one is 1/3 in $\Psi_i(\alpha, \frac{2}{3})$, and is 1/2 in $\Psi_i(\alpha, \frac{1}{2})$ as demonstrated in Proposition 2. By substituting these values for $\Psi_i(\alpha, \frac{2}{3})$ and $\Psi_i(\alpha, \frac{1}{2})$, we find the total phase lag to be $1 - \beta$. That means the gaits generated by Equation (9) are forward wave gaits.

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### 3.5 Stability Margin

The stability margin is defined as the ratio $C/D$, where $C$ is the horizontal distance between the body center and the nearest edge of the convex hull formed by the supporting feet, and $D$ is the horizontal distance from the body reference point to a foot in the standard posture of the robot. Figure 7 shows the stability margins obtained by simulation for three distinct cases. The first one, shown in a solid line, is the result for a variable parameter forward wave gait along a circular path with a constant body orientation in the world coordinate system (a variable crab angle gait). The second one, shown in a broken line, is the result for
Figure 7. Comparison of stability margins. The minimum values of the proposed omnidirectional gait and forward wave gait are significantly larger than that of backward wave gait.

a classical forward wave gait with $\alpha = \pi/6$ along a straight path. The third one, shown in a dotted line, is the result for a backward wave gait along a straight path. Notice that the stability margin of variable parameter forward wave gaits and that of fixed crab angle forward wave gaits are not much different, and they are significantly better than that of backward wave gaits. This observation is consistent with what was proved on a longitudinally symmetric walker.\textsuperscript{2}

4. FOOT TRAJECTORY PLANNING

4.1. Objectives

The problem of foot motion planning for a walking robot is, given its gait, determining appropriate trajectories of the feet in the space to embody the gait.

The inputs to the algorithm are the body motion command, gait parameters such as $\beta$ and $\phi_i$, and the robot's state variables such as the phase variables, foot positions, and foot speeds. Notice that these inputs are dynamic. The outputs are commanded speeds to the joints. Therefore, this algorithm obtains optimal foot motions at every sampling time in a time-varying body commanded motion. After defining fundamental concepts in foot motion planning, we describe desirable properties for foot motions, and next we design an algorithm to plan these kinds of foot trajectories.

4.2. Computing Phase Variables

Evaluating phase variables is essential to the periodic behavior of a walking robot. The method described in this section is based on the work by Lee and Orin.\textsuperscript{5}

The concept of the constrained working volume (CWV) of a leg is essential in foot trajectory design.\textsuperscript{5} This concept was introduced to analyze interference among legs, and to implement the spatial aspect of the semi-periodic gait.\textsuperscript{5} Generally the reachable vol-
umes of all the feet of a robot overlap each other. In this article, a cylindrical subset of the reachable volume of a leg is defined as the CWV of the leg in such a way that the CWVs of any two legs do not have intersections (the radius of the cylinder is \( r_{cwv} \)). This decision makes the foot trajectory computation task simpler.

The temporal kinematic margin \( t_{si} \) of leg \( i \) is a predicted time-to-go until the foot-tip of leg \( i \) in the support phase reaches the surface of the CWV with the present foot velocity. By this definition,

\[
t_{si} = \frac{d}{|v_{fi}|},
\]

where \( v_{fi} \) is the present foot velocity of leg \( i \) relative to the body, and \( d \) is the distance from the support point to the surface of the CWV in the direction of \( v_{fi} \). The kinematic period \( \tau \) is given by:

\[
\tau = \frac{1}{\beta} \min_i \left\{ \frac{t_{si}}{1 - \phi_{bi}} \right\}.
\]

The leg phase variable \( \phi_{bi} \), which determines the phase of the leg \( i \), is defined as:

\[
\phi_{bi} = [\phi - \psi_i]_{mod 1},
\]

where,

\[
0 \leq \phi_{bi} < \beta \quad \text{support phase},
\]

\[
\beta \leq \phi_{bi} \leq 1 \quad \text{transfer phase}.
\]

The support phase variable \( \phi_{bi} \in [0, 1] \) of leg \( i \), which represents the phase during its support phase, being 0 at the touchdown and 1 at the lift-off, is computed as follows:

\[
\phi_{bi} = \frac{\phi_{bi}}{\beta}.
\]

The transfer phase variable \( \phi_{ti} \in [0, 1] \) of leg \( i \), which represents the phase during its transfer phase, being 0 at the lift-off and 1 at the touchdown, is computed as follows:

\[
\phi_{ti} = \frac{\phi_{ti} - \beta}{1 - \beta}.
\]

### 4.3. Foot Trajectory Design

#### 4.3.1. Trajectory Template

This section discusses a foot motion planning algorithm in the period while a foot is moving (or transferring). A foot trajectory is a curve in the three-dimensional space, and there exists a tremendous number of distinct curve classes for our purpose. We specifically selected a trajectory class for foot motions that satisfies the following requirements.

1. Trajectories are smooth so that acceleration of legs is easier for the motor controllers and foot motions would not give bad reactions to the body.
2. Trajectories have vertical portions at landing and leaving periods in order not to cause horizontal slippage of the feet.
3. Vertical foot motions at landing must be decelerated to reduce the landing impacts.
4. Omni-directional body motions must be embodied.
5. Each foot trajectory is constrained in a vertical flat plane named a foot trajectory plane to simplify the trajectory design task.
6. Each trajectory (in a trajectory plane) is generated by a foot trajectory template to make the generation task easier. Because of Requirement (1), the trajectory template must also be smooth.

As shown later, we found these requirements were effective for controlling the robot in real time and to make the resultant motions smooth and natural.

We adopted a trajectory that satisfies the previous requirements in Figure 8(a). Figure 8(b) shows the vertical and horizontal velocities to generate the trajectory. A foot trajectory instance is generated depending on each dynamic situation on the current trajectory plane based on the trajectory template.

Let \( \phi_{ni} \in [0, 1] \) denote the transfer phase variable of leg \( i \), which represents the phase during its transfer phase, 0 at the lift-off time, and 1 at the touchdown time. This variable \( \phi_{ni} \) describes each foot motion as shown in Figure 8(b). The horizontal velocity \( \xi_{si} \) and vertical velocity \( \eta_{si} \) are normalized.

As a part of foot trajectory planning task, each leg’s next precise foothold position should be evaluated at each sampling time. The foothold position determines the foot trajectory instance. The next foothold position should be determined in relation to the motion of CWV.
First, we find the horizontal velocity of foothold position on \(X_W-Y_W\) plane. Let \(\mathbf{p}_{li} = (p_{lix}, p_{liy}, p_{liz})^T\) be the CWV center of leg \(i\) in the world coordinate system, and \(\mathbf{v}_{li} = (v_{lix}, v_{liy}, v_{liz})^T\) be the velocity of the CWV center in the world coordinate system. The predicted foothold \(\mathbf{p}_{\text{Hi}} = (p_{\text{Hi}x}, p_{\text{Hi}y}, p_{\text{Hi}z})^T\) in the world coordinate system is computed as the intersection of the edge of the CWV and the ray drawn from the CWV center in the \(\mathbf{v}_{li}\) direction. Therefore, at the end of the transfer phase (\(\phi_{ti} = 1\)), the predicted foothold position will be

\[
\begin{align*}
p_{\text{Hi}x} &= p_{lx} + (1 - \phi_{t1})r_{cvi}v_{lx} + \frac{r_{cvi}v_{lx}}{v_{lx}^2 + v_{ly}^2}, \\
p_{\text{Hi}y} &= p_{ly} + (1 - \phi_{t1})r_{cvi}v_{ly} + \frac{r_{cvi}v_{ly}}{v_{lx}^2 + v_{ly}^2},
\end{align*}
\]

where \(r_{cvi}\) is the transfer period.

### 4.3.2. Negotiation on Uneven Terrain

Next, let us discuss the issue of how to negotiate on uneven terrains. We first assume that the robot’s terrain is globally flat and there exists a local variance in the vertical unevenness of \(\pm h\). The average height of the terrain is called the ground level, where \(z = z_0\).

To adapt this much terrain roughness, the vertical component of a foothold position is initially planned at a height below the ground level by \(h\). In a real robot’s operation, the phase of a foot is switched from a transfer to a support phase when its tactile sensor elastically, and this is possible through changing the coefficients \(\xi_{li}, \xi_v\) of the trajectory templates. Hence the foot velocity \(\mathbf{v}_{li}\) of leg \(i\) at the foot position of

\[
p_{\text{Hi}z} = \begin{cases} 
  z_0 - h, & \text{for } 0 \leq \phi_{ti} < \phi_1, \\
  z_0 - H_L, & \text{for } \phi_1 \leq \phi_{ti} < \phi_2, \\
  z_0 + h, & \text{for } \phi_2 \leq \phi_{ti} < 1,
\end{cases}
\]

where \(\phi_1, \phi_2,\) and \(\phi_3\) are the phases when a foot reaches a height of \(h\), a clearance height of \(H_L\), and a height of \(-h\), respectively (Fig. 8).

So far we have considered environments in which the ground is globally horizontal with the ground level of \(z_0\) even though there is a small disturbance of \(\pm h\). If a terrain is globally not horizontal, we propose the following method to negotiate it.

A local terrain is approximated by a plane

\[
z = Ax + By + C,
\]

where \(A, B,\) and \(C\) are constant. These are computed by the latest three supporting foot positions. Then the current terrain’s height \(z_0\) is dynamically evaluated as

\[
z_0 = Ap_{Hi} + Bp_{Hiy} + C.
\]

Since the robot can negotiate the unevenness of \(\pm h\) from this variable \(z_0\), it can walk on a slope as well with a semi-periodic gait.

### 4.3.3. Dynamic Trajectory Planning

If the robot is steadily walking at a constant speed and a direction, the trajectory plane of each foot has the same direction as the body translation direction, and the stride and trajectory of each foot are the same. However, if the motion commands are changing, the trajectory plane may be bent and may be shrunk or stretched to coordinate with the commanded motions. The directions of the trajectory planes are not equal. The strides and speeds of all legs may not be equal anymore. Such an algorithm is needed to embody omni-directional body motions. This capacity is implemented by calculating an appropriate foot speed as described below.

When a body motion command is a variable over time, a foot trajectory needs to be expanded or shrunk elastically, and this is possible through changing the coefficients \(\xi_{li}, \xi_v\) of the trajectory templates. Hence the foot velocity \(\mathbf{v}_{li}\) of leg \(i\) at the foot position of
\[ w_{Fi} = (p_{Fix}, p_{Fiy}, p_{Fiz}) \] in the world coordinate system becomes

\[
\begin{bmatrix}
\frac{p_{Fix} - p_{Fix}}{\tau_i \int_{\phi_i}^\phi \xi_i(\phi') \, d\phi'} \\
\frac{p_{Fiy} - p_{Fiy}}{\tau_i \int_{\phi_i}^\phi \xi_i(\phi') \, d\phi'} \\
\frac{p_{Fiz} - p_{Fiz}}{\tau_i \int_{\phi_i}^\phi \xi_i(\phi') \, d\phi'}
\end{bmatrix} \xi_i(\phi_i),
\] (20)

where

\[
\phi_i = \begin{cases} 
\phi_2, & \text{for } 0 \leq \phi_i < \phi_2 \\
1, & \text{for } \phi_2 \leq \phi_i < 1
\end{cases}.
\] (21)

The foot speed represented in the world coordinate system is converted into the body coordinate system and next translated into robot-dependent joint velocities using an inverse Jacobian matrix.

\[
\begin{aligned}
&\dot{\theta}_i = \begin{bmatrix} \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 \end{bmatrix} \\
&\begin{bmatrix} p_{Fix} - p_{Fix} \\
\tau_i \int_{\phi_i}^\phi \xi_i(\phi') \, d\phi' \\
\tau_i \int_{\phi_i}^\phi \xi_i(\phi') \, d\phi'
\end{bmatrix} \xi(\phi_i),
\end{aligned}
\] (22)

where \( R \) is the rotation matrix from the world to the body coordinate system.

\[
\begin{aligned}
&\dot{\theta}_i = J^{-1}(\Theta) \dot{v}_{Bi},
\end{aligned}
\] (23)

where \( \Theta = [\theta_1, \theta_2, \theta_3] \) is the joint angles of each leg, and its derivative \( \dot{\Theta} \) is the joint angular velocities. These final joint angular velocities are applied to the actuators.

## 5. EXPERIMENTAL RESULTS

This section describes the implementation results of the body motion, gait, and foot motion planning algorithms (sections 3 and 4) on the robot system.

### 5.1. Variable Duty Factor

Figure 9 shows the snapshots of the walking experiment in which locomotion speed increases from Figure 9 parts (a) to (c). Thus, the duty factor decreases. The line on the floor shows that the robot walks from right to left. In Figure 9(a), the duty factor is about \( 1/6 \), and only leg #2 (left behind) is in the air. In Figure 9(b), the duty factor is about \( 2/6 \), and legs #1 and #3 are in the air. In Figure 9(c), the duty factor

Figure 9. Experiment of variable duty factor walk. The number of the swing leg is changing.
is about 3/6, and legs #2, #4, and #6 are in the air. The number of legs in the air is increased to produce a higher speed locomotion.

5.2. Variable Crab Angle

Figure 10 shows the snapshots of the walking experiment in which the crab angle increases from (a) to (b). The line on the floor shows that the robot walks from right to left, and the direction of leg #1 (white shank) shows that the robot rotates counterclockwise in the top view. In Figure 10(a), legs #3 and #5 are in the air. In Figure 10(b), legs #1 and #3 are in the air. Thus, leg #3 is paired with #5 in (a) and with #1 in (b). The variation of the leg combination is planned to make a highly stable gait.

5.3. Walking on an Uneven Terrain

Figure 11 shows that the AQUAROBOT is stably walking on an unknown uneven terrain. This motion was generated by the method described in Equation (20). The foot is able to step on and off an obstacle with a height of 10 cm. Foot trajectories in Figure 11 demonstrate that the foot motion algorithm absorbs differences in foot holding height. Although the current control system does not include a body posture stabilizing feedback using an inclinometer, experiments of walking over an uneven terrain worked perfectly as long as the walking path was not very long.

Through our prior experiments, we understand that the stability of a walking robot often fails due to an elevation error of a landing point or a time lag of touchdown, because it causes an inclination of the body, especially when fewer legs are supporting the body. A larger stability margin and more supporting legs obtained by the new algorithm make the robot’s performance better even under disadvantageous conditions. The correctness and effectiveness of the algorithms were thus demonstrated through these experiments.

6. CONCLUSIONS

The body and foot motion planning algorithms investigated here handle variable speeds, variable crab
angles, and motion commands with superimposed rotation cases perfectly. This algorithm has a striking feature in that it always maximizes the number of supporting legs in each situation. Thus, a large stability margin is obtained. The foot trajectory planning algorithm can deal with dynamic situations through versatile commanded motions.

Correctness and effectiveness of the body/foot motion planning algorithms were proven by implementing them on AQUAROBOT. The robot walked smoothly with a variable speed and crab angle on a smooth curved commanded path. The foot motion planning algorithm for an uneven terrain was also proven successful.

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