2009-09

Planning the optimal transit for a ship through a mapped minefield

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Monterey, California: Naval Postgraduate School

http://hdl.handle.net/10945/4548

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

PLANNING THE OPTIMAL TRANSIT FOR A SHIP THROUGH A MAPPED MINEFIELD

by

Pei-Chieh Li

September 2009

Thesis Advisor:	R. Kevin Wood
Second Reader:	James N. Eagle

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REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** September 2009	**3. REPORT TYPE AND DATES COVERED** Master's Thesis

4. TITLE AND SUBTITLE
Planning the Optimal Transit for a Ship Through a Mapped Minefield

5. FUNDING NUMBERS

6. AUTHOR(S) Pei-Chieh Li

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES)
N/A

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (maximum 200 words)

This thesis develops (a) a mission-planning tool for a Navy Mine Counter Measure (MCM) force to find a minimum-risk route for a surface ship through a mapped minefield, and (b) a heuristic to identify a sequence of mines whose clearance (removal and/or deactivation) leads to a rapid reduction of the risk of a minimum-risk path. All modeling concepts reflect the requirements of the Republic of China Navy's MCM operations.

The problem is formulated and solved as a shortest-path problem in a network. A grid of nodes, representing waypoints, is embedded in a representation of the operating area, while arcs are created to link waypoints. The risk function is defined in terms of the closest point of approach distance between each mine that falls within a maximum danger radius along a route.

A complete planning tool is implemented using Excel and Visual Basic for Applications. A basic test scenario describes an operational area of 1,000 by 3,000 yards containing 30 mines; node spacing is 100 yards. The minimum-risk path is found in few seconds on a laptop computer, while a greedy "mine clearance list" is found in a few minutes.

14. SUBJECT TERMS

Shortest-path algorithm, Mine countermeasure warfare, optimal routing

15. NUMBER OF PAGES
67

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**19. SECURITY CLASSIFICATION OF ABSTRACT**	**20. LIMITATION OF ABSTRACT**
Unclassified	Unclassified	Unclassified	UU

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PLANNING THE OPTIMAL TRANSIT FOR A SHIP THROUGH A MAPPED MINEFIELD

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

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EXECUTIVE SUMMARY

This thesis develops (a) a mission-planning tool for a Navy Mine Counter Measure (MCM) force to find a minimum-risk route for a surface ship through a mapped minefield, and (b) a heuristic to identify a sequence of mines whose removal ("clearance") leads to a rapid reduction of the risk of a minimum-risk path. The problem is formulated and solved as a shortest-path problem in a network. A grid of nodes, representing waypoints, is embedded in a representation of the operating area, while arcs are created to link waypoints. For a specified set of mapped mine locations, the risk function for a candidate route is a sum of the risks for each arc along the route. The risk to an individual arc is the sum of the risks generated by mines in the vicinity of that arc. And the risk a specific mine generates for a specified arc increases as the minimum distance from the arc to the mine decreases. Additional arcs are added to the usual network formulation to encourage each mine to interact with at most one "long arc" in the optimal solution. This would allow the user to incorporate actuation-curve data, which is familiar to mine warfare planners and would provide a stronger probabilistic foundation for the optimization modeling. We do not test a model that uses actuation-curve data, but describe how such a model would be constructed in a separate chapter.

The test scenario for this thesis defines an operational area of 1000 by 3000 yards containing 30 mines; node spacing in the grid network is 100 yards. Multiple runs of the model are made to test the effects of (a) "long arcs," in addition to arcs that connect nearest-neighbor nodes, (b) the inclusion or exclusion of a "head-node penalty," and (c) the effectiveness of a greedy mine-clearance heuristic compared to an optimal integer-programming model. Using long arcs and head-node penalties encourages each mine along the optimal path to interact with only one arc. This allows approximate network arc costs (or "lengths," to maintain the analogy with shortest paths) to be computed from lateral range curve data for actuation and damage probabilities of mines against ships, and provides a probabilistic interpretation of the optimization objective function.

Test results show that (1) models with "long arcs" allow greater flexibility in routing and can provide lower-risk routing solutions, (2) removing a "head-node penalty"

that may double count the risk contribution of certain mines to an optimal path does not significantly improve routing solutions, and (3) the greedy mine-clearance heuristic can identify a sequence of mines whose clearance leads to a rapid reduction of the risk of a minimum-risk path and, furthermore, if mines are “nearly uniformly distributed” across the operational area, then the greedy solution will be optimal or near-optimal.

The problem we study in this thesis assumes that a “Q-route” has been established in a particular area, nominally the entrance to a harbor. A Q-route is a preplanned system of shipping lanes in mined or potentially mined waters designed to reduce the size of the area in which a mine warfare commander must manage the risk from mines. Countermine patrols have been carried out along the Q-route (using manned and/or unmanned vessels), the location of each relevant mine has been mapped and its type established. Several assumptions are made to simplify the problem in order to develop a practical model: we assume each mine position is known exactly, and each has known characteristics (e.g., activation method, explosive force), and no own-ship navigation errors occur. We further assume that the enemy does not “re-seed” the minefield during the period of interest.

A complete planning tool is implemented using Excel and Visual Basic for Applications. For the test scenario, the minimum-risk path is found in few seconds on a laptop computer, while a greedy “mine-clearance list” is found in a few minutes. This prototype should provide the framework for a usable mission-planning tool for the ROC Navy MCM force.

ACKNOWLEDGMENTS

I would like to dedicate this work to my wife, and to express my appreciation for her loving support throughout this course of study. I could not have done it without you. I also wish to thank my advisor, Professor R. Kevin Wood, for his guidance, endless support and patience. It is largely because of him that this thesis is completed. To my second reader, Professor James N. Eagle, I would like to thank him for his expertise and guidance. Finally, I would also like to thank Admiral Chihlung Tan, for his encouragement.

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I. INTRODUCTION

In the event of a conflict with the People's Republic of China (PRC), the navy of the Republic of China (ROC, also known as Taiwan) must be prepared to deal with its ports being blockaded by naval mines: the ROC Navy must be able to maintain safe maritime passage into and out of designated ports at all times. To help with this effort, this thesis (a) describes the problem of finding a minimum-risk path for a surface ship through a mapped minefield, (b) implements a solution as a variant of a shortest-path problem in a network, and (c) develops a heuristic to identify a sequence of mines to remove ("clear") that quickly reduces the risk of the minimum-risk path to zero, and which should yield a good solution if exigencies cut short the sequence of removals. The thesis also develops a prototypic decision-support tool for the ROC Navy MCM force. The tool will run on most personal computers and is available for immediate use.

A. THE PROBLEMS: AVOIDING MINES AND CLEARING MINES

This thesis studies the problem of how to reduce the effectiveness of an enemy's sea mines through optimized avoidance. In particular, we develop an automated method for finding a minimum-risk path for naval or commercial vessels to transit through a mapped minefield. (Because of possible confusion with a related term "Q-route," we use "path" rather than "route" to refer to a ship's movement through a minefield.) "Risk" can have several meanings, but for our purposes the risk a specified minefield generates against a specified track through that minefield is the sum of the risks on the component arcs of the route. And the risk to any arc is the sum of the risks generated by the mines near that arc. And the risk a mine generates for an arc is a deterministic measure that increases as the minimum distance between the mine and arc decreases. This is a deterministic approach for designing paths through a mapped minefield, which stays furthest away from the most dangerous mines. The formulation in this study adds additional "long arcs" to the standard network formulation to encourage in the optimal solution each mine to engage with at most one arc. This allows the model to use actuation-curve data, familiar to mine warfare planners, in the cost function (see Washburn and Kress 2009, Chap. 8).

For simplicity, we assume all mine positions are known exactly, and all have the same characteristics (e.g., activation method, explosive force). Furthermore, all mines have a known damage radius and no own-ship navigation errors occur. The model easily generalizes to allow mines of different types as long as the characteristics of each mine are known. For practicality, i.e., speed of computation, we use a shortest-path based methodology much like that described by Bekker and Schmid (2006) for finding mine-avoiding paths; see also Boerman (1994). This is essentially the same model type that others use for the routing of manned and unmanned aircraft (e.g., Carlyle et al. 2007; Reber 2007). In particular, the area of operations (AO) is discretized into a grid of nodes representing waypoints, and arcs connecting nodes to represent potential transitions between waypoints. We will call this network the *AO network*.

Because of the great uncertainty about if and when a mine will detonate and how much damage it might cause, identifying a true "minimum-risk path" is probably impossible in our context. In this study, we will (a) compute a "risk measure" for each arc in the AO network as the probability of mine actuation that a ship will incur if it travels an infinite straight-line path containing this arc, and (b) identify a path through the network that minimizes the sum of arc risk-measure values along that path. (To avoid identifying paths that may be too long and circuitous, we may also add an arc-length penalty to the "risk-measure value" assigned to each arc.)

Two different risk measures for paths in minefield have been used in the literature: (a) a measure based on the closest point of approach (CPA) to the closest single mine along a path (Boerman 1994), and (b) a measure based on the CPA to each mine that might conceivably endanger transit along the path (Bekker and Schmid 2006). Case (a) computes risk with respect to a single mine; case (b) could involve all mines in a minefield, but would normally involve only a small number of mines that are within some "maximum danger radius" of the arc.

If “true risk” corresponds to the maximum explosive force that a ship can withstand and maintain some specified level of operational effectiveness, the “single closest CPA” might be the right measure to use. If “true risk” involves a set of explosions that a ship can experience with an acceptable level of resulting damage, then considering the “CPA to each mine within a maximum danger radius” is a reasonable approach. Because we focus on naval operations, and because warships are designed for flotation that is robust against damage, we will investigate arc risk measures based on case (b). In the simplest case, we will compute an additive risk-measure for each arc in the network, identify a “minimum-risk-measure path” and label that as an approximation to the true “minimum-risk path.”

In this thesis, we first implement a model similar to that of Bekker and Schmid in Excel and VBA (Visual Basic for Applications), and then extend the model by adding additional arcs to encourage the optimal solution to use longer arcs. The reason for this is two-fold: (1) paths with fewer turns are easier to execute accurately, and navigational accuracy is critical when crossing a minefield; and (2) when a mine interacts with a single, straight-line arc, the probability of mine actuation can be approximated with lateral range curve data, generally understood and available to mine warfare planners (Washburn and Kress 2009, Chap. 8). Our testing does not actually use the actuation-curve model—it uses an intuitively appealing model of risk that varies inversely with the closest point of approach to each mine—but our procedures are ready to accept the former model.

In addition, we implement a greedy heuristic for mine clearance that may have useful properties when trying to quickly reduce mine risk. Motivation for this result is that a ship may need to transit the minefield before all the mines on the list have been cleared and a zero-risk path established. To illustrate, suppose that a commander identifies the smallest set of mines whose clearance (removal) results in a zero-risk path. Further, suppose that the smallest set has cardinality five. MCM forces begin removing these mines, but only have time to clear three before operations must halt, and a ship must transit the minefield. There is no reason to believe that the three mines that were cleared were the best three to clear. Naturally, identifying a minimum set of mine removals that gives good intermediate solutions might be impossible, but we provide a greedy heuristic that identifies a sequence of mines, always choosing the next best mine to clear. This greedy approach might lead to a poor overall solution, but the sequence of removals that it generates has the chance of being a good sequence that can be interrupted at any point with reasonable results. This issue is investigated computationally in Chapter V.

An important aspect of our solution approach is that it can be, and has been, fully implemented in Excel and VBA, so that it can run on most personal computers. Although we can find a path that minimizes our definition of risk, we have not validated the model with empirical data. This must wait for further research.

B. MOTIVATION

According to the Quadrennial Defense Review (QDR) of the Ministry of National Defense (MND), ROC, partial blockade of Taiwan's important ports is one of the five possible military actions that the PRC might apply against Taiwan in the case of hostilities (ROC MND QDR 2009, pp. 41-42). Also, a US Congressional Research Service Report points out that a maritime quarantine or blockade by naval mines of the Taiwan's ports is one of the PRC's options in a military conflict with the ROC (O'Rourke 2008, pp. 47-48).

It is critical for the ROC Navy to maintain the safe access into and out of certain harbors at all times. Establishing a "Q-route" system is a standard method to help with this access problem, if harbors or nearby waters may be mined (Vego 2008). A Q-route is a preplanned system of shipping lanes in mined or potentially mined waters. By making use of extensive route surveys conducted prior to hostilities, the MCM force can rapidly verify the presence (or absence) of mines in the designated routes and take appropriate clearance (mine-removal) actions. Q-routes are used to minimize the area that an MCM commander must patrol and clear and yet still be able to provide safe passage for friendly ship movements (Holden 1994).

The problem we study in this thesis assumes that a Q-route has been established in a particular area, nominally the entrance to a harbor. Countermine patrols have been carried out along the Q-route (using manned and/or unmanned vessels), the depth and position of each relevant mine has been mapped and its type established. We further assume that the enemy does not "re-seed" the minefield during the period of interest. The basic problems that remain are (a) to find a minimum-risk path through the Q-route, and/or (b) to find a prioritized list of mines whose removal will result in a rapid reduction in the risk of a minimum-risk path, and ultimately lead to a zero-risk path.

The main reason we want to find a prioritized list of mines for disposal, instead of the best set of n mines to clear, is to reflect how MCM operations might actually evolve. For instance, suppose a Q-route has been established to enable the egress of a naval combat force from a harbor. Ideally, the MCM force would clear all mines required to create a zero-risk path for all of the force's ships to exit the harbor. However, suppose that an attack on the harbor by aircraft-borne missiles is anticipated and the force must evacuate its ships from the harbor immediately. Furthermore, suppose that only half of the n mines necessary to achieve a zero-risk route have been cleared. Has risk been reduced to an acceptable level? It is possible that risk has hardly been reduced at all, actually.

This kind of scenario illustrates a type "optimal priority list problem" (Koc et al. 2008). Koc et al. solve an optimal prioritized-list problem (under uncertainty) for creating a project portfolio, and that is the kind of model that we would like to solve, to create a prioritized mine-removal list. But since that is too complicated for our computational platform, we are going to use a heuristic to solve this problem approximately, and hope to get good intermediate solutions. We note that Pfarrer (2000) uses a greedy heuristic in his thesis to approximately solve a acquisition-prioritization model that is much like the prioritized-list model that Koc et al. (2008) solve (for project prioritization). Pfarrer reports near-optimal results with his greedy heuristic, making it seem more likely that the greedy mine-removal heuristic will also work well. Computational experiments will investigate this expectation.

There are many combat-related instances of optimal prioritized-list problems, with one example being the Allied amphibious operations at Guadalcanal during World War II (Miller 1995). In the Guadalcanal Islands, the Allies wished to complete a landing mission encompassing approximately 150 landing craft loaded with troops, ammunition, fuel, etc. However, the Japanese began an air attack and the landing operation broke down completely after about 100 craft had landed. Clearly, mission planning should have tried to achieve a good sequence of landing-craft landings, i.e., a prioritized-list of landings, so that if the full operation were cut short, the successfully landed forces would be equipped as best possible. There is no evidence that a good sequence of landing-craft landings was investigated, however.

C. THESIS OUTLINE

This thesis is structured as follows. This chapter has defined, in general terms, the problem of finding a minimum-risk route through a mapped minefield, and the problem of finding a good mine-clearance sequence. It has also motivated the need for solving both of these problems. Chapter II presents several models for a minimum-risk routing through a mapped minefield and describes solution techniques. Chapter III develops a greedy heuristic for mine clearance. Chapter IV describes how actuation-curve data could be incorporated into our basic model at a later date. Chapter V develops test scenarios and provides computational results for finding a minimum-risk path through the Q-route, also comparing the effects for mine removal using greedy heuristic and integer-programming model. Finally, Chapter VI presents a summary and conclusion, also suggests areas of further research.

II. MODELING A MINIMUM-RISK ROUTE THROUGH A MAPPED MINEFIELD

This chapter starts by defining a network model that can represent the transit of ship through a Q-route. Then, we define a simple, additive risk function for each arc in the network that should reflect, approximately, the true risk that a ship would experience from mines that would be approached during a transit of that arc. Once the risk function and network structure are defined, any standard shortest-path algorithm can be used to find an "approximate minimum-risk path."

The risk function we use may imply risk from a single mine across multiple arcs, and traversing those arcs could amount "double counting" of risk. To reduce this effect, we add "long arcs" to the model, which will tend to be preferred in the optimal solution. This will encourage each mine to engage with at most one arc in the optimal solution. Furthermore, the use of long arcs should simplify the incorporation of actuation-curve data at a later date.

A. NETWORK STRUCTURE AND SHORTEST-PATH MODEL

This thesis models two types of Navy MCM operations in an established Q-route, (a) mine-avoidance, known as Passive Defensive MCM, and (b) mine-clearance, known as Active Defensive MCM (Holden 1994). We assume that all mines in the Q-route of interest have been mapped. For simplicity, we also assume that a rectangular area defines the Q-route, as might be the situation with a potentially mined harbor entrance. As in Bekker and Schmid (2006), we define a two-dimensional grid of nodes $i \in N$ in that area to represent waypoints for a transiting ship, and connect those nodes with a set of directed arcs $(i, j) \in A$, $i, j \in N$ and $i \neq j$. The arcs represent potential transitions between waypoints. Together, the nodes and arcs define a directed graph $G = (N, A)$. A set of nodes S at one end of the network represents potential starting points for a transit, and a set T at the other end represents potential endpoints, beyond which the transiting ship is assumed free from the danger of mines. See Figure 1.

We wish to find a route with the lowest total risk for a ship to transit through the area. The simplest way to solve this problem approximately is to assign an additive risk measure to each arc as a length, and find a shortest path from some node in *S* to some node in *T*, using any reasonably efficient shortest-path algorithm. (We use a label-correcting algorithm implemented with a deque; see Ahuja et al. 1993, pp. 136–143.) The risk on each arc is computed as some function of the mines that fall within a prescribed "maximum-danger radius" if a ship were to transit the arc.

There is a problem with that simple approximation, however, because the maximum-danger radius of any single mine may cover more than one arc that might be traversed along a ship's path, and risk may not truly be additive. Suppose, for instance, that we define a risk measure to be $1/CPA$, and one path crosses a single arc with a CPA to mine m of 50 yards, and suppose another path crosses two arcs each with a CPA of 100 yards to mine m. (No other mines come into play, and the maximum-danger radius exceeds 100 yards.) The specified additive risk measure would evaluate the two paths identically with respect to risk, but a ship's captain might find the former situation "riskier." For simplicity, we will implement a simple additive risk function first, and then look at the alternatives that may better represent the true risk for a transiting ship.

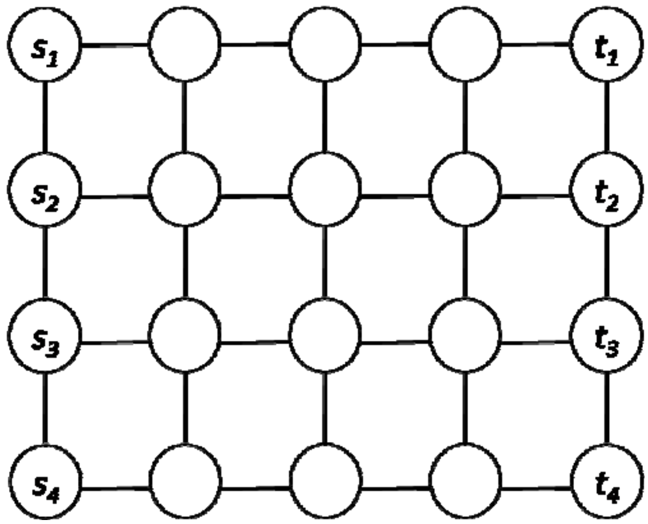

Figure 1. Network structure for the minimum-risk routing model without long arcs. A set of nodes $S = \{s_1, s_2, s_3, s_4\}$ represents potential starting points, and a set of nodes $T = \{t_1, t_2, t_3, t_4\}$ represents potential endpoints. Between *S* and *T*, nodes represent potential waypoints for a transit; beyond *T*, the transiting ship is assumed free from the danger of mines. For simplicity, the figure displays an undirected version of a portion of an AO-network, with only nearest-neighbor nodes connected. In reality, the network will be directed, and the connectivity of the arcs will depend on modeling assumptions.

B. RISK FUNCTION

Two different additive risk measures for arcs have been used in the literature on mine-avoiding paths. Roughly, they are (a) the CPA to the closest single mine along a path (Boerman 1994), and (b) CPA to each mine that might conceivably endanger transit along a path (Bekker and Schmid 2006). Whatever we use to define the risk measure, it will relate to the CPA distance from an arc to a mine. There will likely be some minimum ship-to-mine distance at which unacceptable damage must occur to the ship if the mine explodes. This will be called the *mine-damage radius*, and any arc that passes within such a distance will be heavily penalized: unless there is no other way, a ship should never traverse such an arc. This distance can be calculated, and depends on the mine's depth, explosive charge weight, and the relevant type of vessel (TM Navord Op 3696 1996). For computational efficiency, we also define a *maximum danger radius* beyond which a ship cannot possibly be damaged by a mine. Thus, we need not compute any contribution to risk from mine m to arc (i, j) if all points on that arc fall outside the maximum danger radius. The following describes our basic shortest-path model with all risk-measure calculations.

Indices:

$i, j \in N$	nodes in AO network $G = (N, A)$
$(i, j) \in A$	arcs in AO network $G = (N, A)$
$m \in M$	mines in the AO
$s \in S \subset N$	start nodes in AO network
$t \in T \subset N$	end nodes in AO network
A_{path}	arcs in a simple directed path in G from some node $s \in S$ to some node $t \in T$ (all paths assumed to be simple, i.e., with no nodes repeated)
$\mathcal{A}(S,T)$	the set of all simple paths in G from some node $s \in S$ to some node $t \in T$

Data [units]:

$\underline{D}$	mine-damage radius [yards]
$\bar{D}$	maximum danger radius, with $\bar{D} = \eta \times \underline{D}$ [yards], where $\eta \geq 1$ will be set by the decision maker ($\eta = 2$ in this thesis)
D_{ijm}	CPA distance from mine m to arc (i, j) [yards]
κ	risk-measure exponent, where $\kappa \geq 1$ and can be decided by decision maker's risk preference ($\kappa = 2$ in this thesis)
r_{ijm}	risk measure for arc (i, j) from mine m
$r_{\max}$	$\max_{(i,j) \in A,\, m \in M} r_{ijm}$
μ	mine-damage penalty constant, where $\mu \geq \lvert N \rvert r_{\max}$ ($\mu = 1$ in this thesis)

Single-mine Arc Risk Measure:

$$r_{ijm} = \begin{cases} \mu & \text{if } D_{ijm} < \underline{D} \\ 1/D_{ijm}^{\kappa} & \text{if } \underline{D} \le D_{ijm} < \overline{D} \\ 0 & \text{if } D_{ijm} \ge \overline{D} \end{cases} \tag{1}$$

Multiple-mine Arc Risk Measure:

$$r_{ij} = \sum_{m \in M} r_{ijm} \tag{2}$$

Formulation of Basic Minimum-Risk Routing Model:

$$\min_{A_{path} \in \mathcal{A}(S,T)} \sum_{(i,j) \in A_{path}} r_{ij} \tag{3}$$

We define the AO network only for a Q-route, since only the region in the Q-route has been surveyed. Thus, no ship will be allowed outside the Q-route. Furthermore, for safety's sake, we also assume that fictitious mines m' exist just a small distance $\varepsilon > 0$ beyond the borders of Q-route (only along the two side borders of the Q-route that do not include *S* and *T*). We apply the same risk function to those regions along the side borders; consequently we include a risk contribution from a fictitious mine m' for each arc (i, j) in the upper and lower row of arcs in the grid when CPA distance from the borders $(D_{ijm'})$ is within maximum danger radius ($r_{ijm'} = \mu$ if $D_{ijm'} < \underline{D}$; $r_{ijm'} = 1/D_{ijm'}^{k}$ if $\underline{D} \le D_{ijm'} < \overline{D}$).

C. GRID SPACING, ARC STRUCTURE, AND LONG ARCS

It is important to set the proper grid spacing for the AO network. Although we might obtain better resolution with a finer grid spacing (and with more arcs to give a greater variety of angles), that finer grid comes with higher computational cost. But, if we set the grid spacing too large, then the approximation of risk and a ship's ability to maneuver will be poor. Since this model is designed as a decision-support tool for navy MCM operations, and want to provide waypoint coordinates that a vessel can navigate by, a reasonable grid spacing is the minimum turn radius that a vessel can achieve (known as the "90-degree turn radius"). Thus, this spacing will depend on the relevant vessel's maneuverability: small for small and more-maneuverable vessels, and large for large and less-maneuverable vessels. For simplicity in this thesis, we fix the grid spacing to 100 yards, which roughly corresponds to an 800-tons Aggressive-class ocean minesweeper when guiding a convoy through the Q-route for port ingress or egress (known as a "lead-through operation," see Holden 1994). (Note that the 90-degree turn radius for a naval ship would likely constitute classified data.)

The topological structure chosen will directly affect the quality of the path found through the Q-route. Bekker and Schmid (2006) use a topology that consists of a square grid, with each node i connected to all its nearest neighbor nodes j, including those on the diagonal; see Figure 2. Our model disallows certain turns sharper than an acceptable 90 degrees by removing backward arcs and by adding long arcs (Figure 3). Unfortunately, our model does not eliminate all sharper-than-90-degree turns, and might disallow an optimal path that takes no sharp turns but which does move back toward S at some point. Further development will require the implementation of a turn-restricted shortest-path model (e.g., Caldwell 1961, Carlyle et al. 2007).

Long arcs are critical to the model because they tend to be preferred to short arcs in an optimal solution. This occurs because long arcs are "cheaper" in that they move further across the minefield with the same cost (risk) as shorter arcs. In so doing, they encourage optimal paths to avoid counting risk from the same mine on multiple arcs, which would lead to an overestimation of risk. As discussed in Chapter 0, long arcs also more accurately approximate lateral range curve and actuation curve geometry, and would be important in a future implementation that used such constructs.

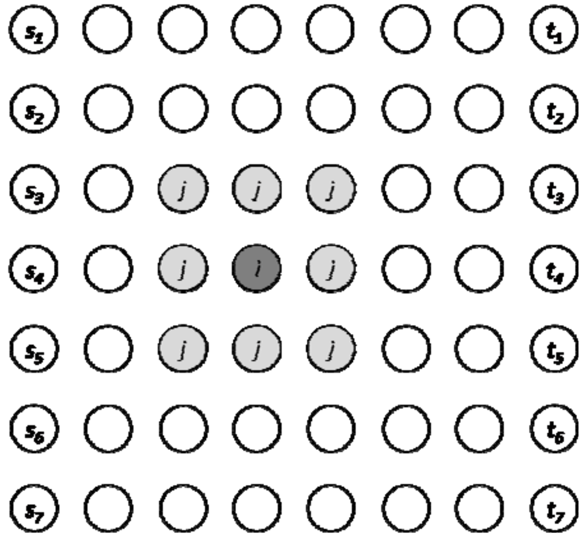

Figure 2. Network topology used by Bekker and Schmid (2006). They use a regular, square grid of nodes and connect each node i to each of its nearest neighbors j, using a directed arc (i, j). Note that "nearest neighbors" includes nodes that are diagonally adjacent. For simplicity, no arcs are shown.

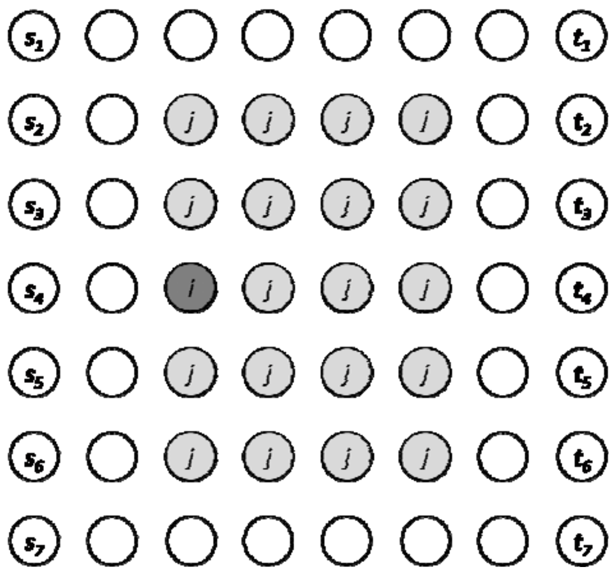

Figure 3. Network topology used in this thesis (we call this the "long-arc topology"). Nodes are laid out in a regular, square grid, with each node i connected by directed arcs (i, j) to neighbor nodes j, as shown. For simplicity, no arcs are shown.

D. MODIFIED MINIMUM RISK PATH

In the basic minimum-risk routing model defined in Chapter II, an approximate minimum-risk path is found by assigning arcs lengths $c_{ij} = r_{ij}$ to each arc $(i, j) \in A$, and finding a shortest path from any node $s \in S$ to any node $t \in T$. However, we may wish to keep the transiting ship's route reasonably short to reduce the effects of technical issues such as mine-position errors, own-ship navigation errors, etc. (It is also possible that a long path would require too much time, and might subject the ship to other risks such as missile strikes.) This can be accomplished effectively by adding a small distance penalty to any arc length and then finding a shortest path (see Bekker and Schmid 2006). Thus, if an arc's physical length is denoted d_{ij}, we find a "shortest path" using arc lengths

$$c_{ij} = r_{ij} + \alpha d_{ij} \tag{4}$$

where $\alpha \geq 0$ is a user-specified value ($\alpha = 10^{-7}$ in this thesis). We end up with the following routing model:

Formulation of Modified Minimum-Risk Routing Model:

$$\min_{A_{path} \in \mathcal{A}(S,T)} \sum_{(i,j) \in A_{path}} c_{ij} \tag{5}$$

E. REDUCING "DOUBLE COUNTING"

We intend to use the minimum-risk routing model that culminates in Equation (5). That model is susceptible to the "double counting" of risk as described in Chapter II.B, but it may be possible to reduce that using the long arcs also described in that section. Further reductions in double counting may be possible by ignoring the risk associated with certain mine-arc combinations. In particular, suppose that mine m has a CPA to arc (i, j) at the head node j of the arc, and CPA is within the maximum danger radius; see Figure 4. In this case, r_{ijm} will not be included in the computation of r_{ij} and we call this modified risk penalty calculation as "without head-node penalty." This modified calculation is correct because, if (i, j) appears on a ship's path, some arc (j, k) must also appear on that path, and r_{jk} will include a contribution from mine m. (This assumes that all mines m lie strictly within the region of the Q-route.) Both techniques, "long arcs" and "without head-node penalty," may reduce double counting, and this issue is investigated computationally in Chapter V.

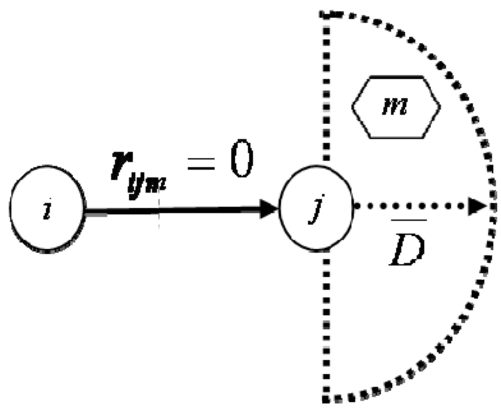

Figure 4. Modified risk penalty calculation to reduce the effect of double counting. If a mine m has its CPA to arc (i, j) at the head node j (within the maximum-danger radius so that, nominally, $r_{ijm} > 0$), the model will ignore the risk measure from mine m to arc (i, j), i.e., it will use $r_{ijm} = 0$. This modification is correct because a ship transiting (i, j) must use some other arc (j, k), and r_{jk} will include a contribution from mine m as r_{jkm}.

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III. REMOVING MINES TO REDUCE RISK

Depending on assumptions, different algorithms and/or models may apply to finding an optimal or good set of mines to clear. For example, the heuristic used by Bekker and Schmid (2006) tries to find a smallest set of mines whose clearance creates a "sufficiently safe passage." We will take "sufficiently safe" to mean zero-risk. A solution from their heuristic will be good if we assume that all the required mines can be cleared before a ship must transit the minefield. But, in a naval MCM operation, the MCM force may need to halt the mine-clearance operation before the full "optimal" set of mines has been cleared. This may happen because a higher-risk situation presents itself: for instance, ships in port anticipate a guided-missiles attack, and they must exit the port before those missiles arrive. This scenario leads us to another approach for mine clearance.

As discussed in the Introduction, the naval MCM operational scenario has the characteristics of an "optimal priority list problem." In lieu of formally modeling this problem, we have argued that applying a sequential heuristic to identify a sequence of mines to clear—the heuristic will always choose the next-best mine to clear—may be a good way to solve this problem. Thus, our algorithm will not only provide a list of mines to clear, but also the sequence in which they should be cleared. This additional information should be useful for an MCM force. (For simplicity, this thesis must ignore the issue of the safety of MCM forces. That is, we assume that a minesweeper can reach and clear any mine in any sequence, safely.) Computational tests in Chapter 0 will verify whether this reasoning holds.

Pseudo code follows for the heuristic greedy algorithm that has been implemented. The code is self-explanatory, and we add no discussion.

Algorithm Greedy Mine-clearance Heuristic

Description: A heuristic greedy algorithm to approximately solve the optimal mine-clearance problem.

Input: All data for the minimum risk path transit on $G = (N, A)$, with mine set M .

Output: A prioritized list of mines whose removal will greedily reduce the risk of a minimum-risk path until a zero-risk path is found.

{
$List \leftarrow \varnothing$; /* Create an empty prioritized list */
$z \leftarrow \min\limits_{A_{path} \in \mathcal{A}(S,T)} \sum\limits_{(i,j) \in A_{path}} c_{ij}$; /* Find min-risk path with original mine set M */
Let A^*_{path} denote the optimal path found above;
$z' \leftarrow \sum\limits_{(i,j) \in A^*_{path}} r_{ij}$; /* Compute "true risk" for optimal path, i.e., ignore arc lengths */
while ($z' > 0$) {
  for (each $m \in M$) {
    $M \leftarrow M - m$; /* Remove m from M */
    Recompute r_{ij} and c_{ij} with respect to M ;
    $z_m \leftarrow \min\limits_{A_{path} \in \mathcal{A}(S,T)} \sum\limits_{(i,j) \in A_{path}} c_{ij}$; /* Find min-risk path with new mine set M */
    $M \leftarrow M + m$; /* Put m back into M */
  } /* end for */
  $m^* \leftarrow \operatorname{argmin}_{m \in M} z_m$; /* best single mine to clear next */
  $M \leftarrow M - m^*$;
  Add m^* to back of $List$;
  Recompute r_{ij} and c_{ij} with respect to new M ;
  $z \leftarrow \min\limits_{A_{path} \in \mathcal{A}(S,T)} \sum\limits_{(i,j) \in A_{path}} c_{ij}$; /* Find min-risk path with new mine set M */
  Let A^*_{path} denote new optimal path found above;
  $z' \leftarrow \sum\limits_{(i,j) \in A^*_{path}} r_{ij}$; /* Compute "true risk" for A^*_{path}, i.e., ignore arc lengths */
} /* end while */
Print ("Clear mines in this order:", *List*)} /* "*List*" referred to as M^* elsewhere */
}

IV. A PROBABILISTIC OBJECTIVE FUNCTION, AND ACTUATION CURVES

This chapter gives a probabilistic interpretation to the additive risk function for mine avoidance described in Chapter II. It also describes how standard mine-actuation data could be used to compute risk measures for individual arcs that would be consistent with that probabilistic interpretation.

This thesis does not attempt to implement the models described in this chapter, but we hope that this provides useful information for future research.

A. A PROBABILISTIC OBJECTIVE FUNCTION

We return to the basic model for minimizing risk when transiting a minefield, ignoring the physical lengths of arcs for simplicity:

Basic Minimum-Risk Routing Model:

$$\min_{A_{path} \in \mathcal{A}(S,T)} \sum_{(i,j)\in A_{path}} r_{ij} \qquad (6)$$

Written in terms of the risk from individual mines, r_{ijm}, and letting $M_{ij} = \left\{ m \in M \mid r_{ijm} > 0 \right\}$, Equation (6) becomes

$$\min_{A_{path} \in \mathcal{A}(S,T)} \sum_{(i,j)\in A_{path}} \sum_{m \in M_{ij}} r_{ijm} \,. \qquad (7)$$

A reasonable model for the safe transit of the ship through the minefield is one that maximizes the probability that the ship experiences <u>no</u> mine actuations that damage the ship during its transit. Let p_{ijm} denote the probability that mine m actuates and damages the ship in question while it transits arc (i, j), and let $q_{ijm} = 1 - p_{ijm}$. Assuming independence of actuation and damage events leads to the following model:

$$\max_{A_{path} \in \mathcal{A}(S,T)} \prod_{(i,j)\in A_{path}} \prod_{m \in M_{ij}} q_{ijm} \qquad (8)$$

Independence is a strong assumption, to be discussed momentarily, but is made more plausible if each mine in the optimal solution endangers at most one arc. The network topology and "long-arc" model used here encourage this to happen.

Now, using a standard transformation (e.g., Ahuja et al. 1993, p. 130), if we let $r_{ijm} \equiv -\ln q_{ijm}$, it is easy to see that the models of Equations (7) and (8) are essentially identical. Thus, our basic model has a straightforward probabilistic interpretation if the q_{ijm} can be computed, and if independence holds. We discuss one standard method of computing the q_{ijm} in the following section, but take up the issue of independence here.

If a particular mine implies risk on two separate arcs of a transit, then independence could be lost, depending on how risk is interpreted. To see this, imagine that a mine that has a probability of .75 of being operational and .25 of being inert. Then, one close pass by this mine results in a probability of actuation of .75, and ten close passes produce the same value. The pass-to-pass actuation events are, in this case, completely correlated. If, on the other hand, we assume that the mine is always operational, and on each pass the mine receives an independent look at the target with a probability of actuation of .75, then the cumulative probability of actuation accumulates very quickly with each pass.

It is reasonable to assume that actual mines have both reliability and actuation probabilities, which could complicate the modeling. However, if we can allow that each mine interacts with only one arc in the optimal solution, we need only assume mine-to-mine independence along the path, which is more reasonable than is pass-to-pass independence for a single mine.

The restriction that paths move generally forward will encourage mines to interact with a single arc, as will the "long-arc topology." In particular, (a) the restriction of "generally forward" disallows a path that passes close to a mine in one direction, moves some distance away, and then returns to pass by the mine in a different direction; and (b) the long-arc topology tends to have fewer arcs and thus fewer instances where adjacent arcs have risk measures associated with the same mine, which would imply dependence along the path. Computational results in Chapter 0 will indicate whether or not we have successfully dealt with this type of dependence, and will thus indicate whether or not our basic model is a good candidate for a probabilistic interpretation.

Mine-to-mine dependence which could cause difficulties in the use of the basic model include, for instance, the minefield-wide environmental effects of temperature and salinity, or randomness in the blast hardening as actuation signature of the transiting ship. Those issues need further study, and are beyond the scope of this thesis.

B. ESTIMATING PROBABILITIES OF DAMAGE: ACTUATION CURVES

If a ship transits along a straight path, infinite in both directions, and with CPA x_m to mine *m*, then the probability of mine actuation during the transit has been called "the actuation probability" and can be denoted $A(x_m)$. This function is called the actuation curve for the mine/ship pair (Washburn and Kress 2009, p. 165). Actuation curves are familiar to mine warfare planners and have been measured or modeled for some mine types and ships; see Figure 5.

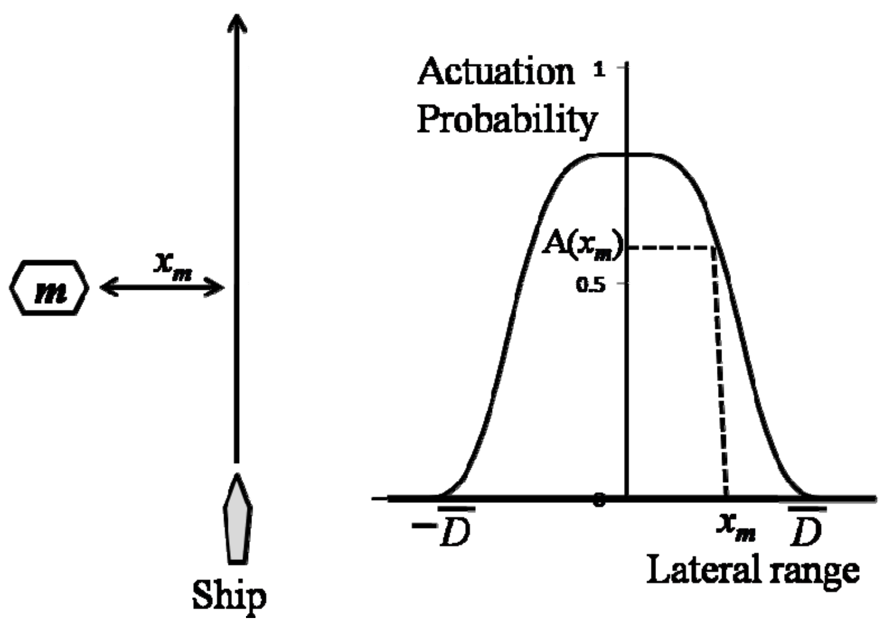

Figure 5. Actuation curves. When a ship transits along a straight path, infinite in both directions, and with CPA x_m (lateral range) to a mine *m*, then the probability of mine actuation during the transit is $A(x_m)$.

One conservative assumption is that a ship will be damaged by mine m if that mine actuates. Thus, if arc (i, j) passes within the maximum danger radius of mine m and if that arc is sufficiently long, then it is reasonable to define

$$q_{ijm} = 1 - A(L_{ijm}), \tag{9}$$

where L_{ijm} denotes the CPA of mine m to the infinite extension of arc (i, j); see Figure 6.

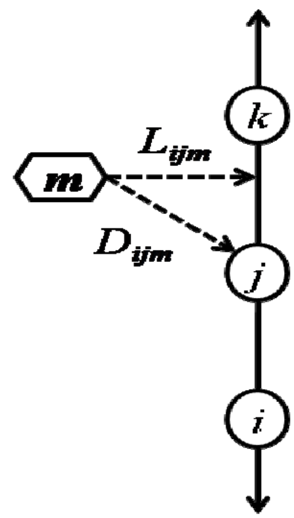

Figure 6. Definition of D_{ijm} and L_{ijm}. D_{ijm} is the CPA distance from mine m to arc (i, j), while L_{ijm} is the CPA distance from mine m to arc (i, j) extended in both directions to edge of Q route.

Thus, an approximate risk-measure calculation for the basic model, when using actuation-curve data is:

$$r_{ijm} = \begin{cases} -\ln(1 - A(L_{ijm})) & \text{if} \quad D_{ijm} < \overline{D} \\ 0 & \text{if} \quad D_{ijm} \geq \overline{D} \end{cases}$$

Again, the long-arc topology should help to make this risk measure more accurate, because a long arc more closely resembles the hypothetical, infinitely long straight path than a short arc does. Typically, we also expect that $L_{ijm} = D_{ijm}$ with long arcs.

It should also be noted that when the actuation probabilities $A(L_{ijm})$ are small, $-\ln(1 - A(L_{ijm})) \approx A(L_{ijm})$. Thus, minimizing the sum of small actuation probabilities approximately maximizes the product of the non-actuation probabilities. And, from the standpoint of the transiting ship, we would hope that the actuation probabilities in the optimal solution would, in fact, be small.

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V. COMPUTATIONAL RESULTS

This chapter starts by introducing a standard test scenario and the three tests to be conducted. We apply the minimum-risk routing model on the test scenario and compare results (a) with and without long arcs, and (b) with and without head-node penalties. Also, (c) compare the greedy mine-clearance heuristic to the solution of an optimal integer-programming model. All tests are carried out on a laptop computer with a 1.79 GHz AMD Turion processor, 384MB of RAM, and the Microsoft Windows XP Professional operating system. Programs are written in Excel 2007 and Visual Basic for Applications (VBA). A minimum-risk path is found in a few seconds, and greedy "mine-clearance list" is found in a few minutes.

A. TEST SCENARIO

The test scenario for this thesis models a Navy MCM force performing mine-avoidance (for example, a lead-through operation) and mine-clearance operations in an established Q-route. There are three tests to be conducted:

> (a) Evaluate the effects of using long arcs to reduce the issue of double counting, and to achieve lower-risk paths.
>
> − We claim that the long-arc structure can give our model more flexibility and result in a lower-risk path for the minimum-risk path problem. First, we run the modified minimum-risk routing model on the same test scenario with and without long arcs, and then compare the two solution values using the "approximate path-risk measure" and the "true path-risk measure. The approximate path-risk measure is the model objective value, while the true path-risk measure is computed after the (approximate) minimum-risk path is found, by tracing the arcs in the path and the mines that interact with those arcs, and adding each mine's risk contribution to the overall path-risk measure only once. That contribution is computed for the CPA of the mine to the path, rather than the CPAs of the mine to each arc in the path. Finally, we check the result statistically to see if long arcs can achieve an advantage.

(b) Determine if the modified risk-penalty calculation that ignores head-node penalties can reduce the issue of double counting, and can lead to lower-risk paths.

– Since we cannot avoid double counting when using a simple additive model, we try to use this modified risk-penalty calculation to reduce the issue. We also run the modified minimum-risk routing model on the same test scenario with and without head-node penalties, and then compare approximate path-risk measure and true path-risk measure, as described above. We check the result statistically to see if this adjusted risk measure can reduce the issue of double counting.

(c) Evaluate the effect of reducing risk for a minimum-risk path by our greedy heuristic.

– We hope that our greedy heuristic will find a prioritized list of mines whose sequential clearance will quickly reduce the risk of a minimum-risk path to zero. We compare our heuristic solution to an "optimal" solution computed through an integer-programming model (see Appendix A). We compare the greedy and optimal solutions in terms of (1) the total number of mines (m^*) required to be cleared, and (2) the amount of risk reduction when a subset of mines of size m' is to be cleared, where $m' < m^*$.

Specific input values for the test scenarios follow:

- Area of AO network: 1000 yards wide (y-axis) by 3000 yards long (x-axis).
- Grid spacing: 100 yards (11 nodes on y-axis, 31 nodes on x-axis, total nodes $N = 341$).
- Naval mines in the AO network: $M = 30$ with uniformly distributed x-axis coordinates and uniformly distributed y-axis coordinates).
- Mine-damage radius: $\underline{D} = 100$ yards.
- Maximum danger radius: $\overline{D} = 200$ yards ($\overline{D} = \eta \times \underline{D}$, where $\eta = 2$).

- Single mine risk measure: $r_{ijm} = \begin{cases} \mu \text{, if } D_{ijm} < \underline{D} \\ 1/D_{ijm}^{\kappa}\text{, if } \underline{D} \le D_{ijm} < \overline{D} \\ 0\text{, if } D_{ijm} \ge \overline{D} \end{cases}$, where $\mu = 1$, $\kappa = 2$.

- Arc "length": $c_{ij} = r_{ij} + \alpha d_{ij}$, where $\alpha = 10^{-7}$.

Mathematical symbols used for the tests follow:

z_1 random approximate risk measure computed without long arc

z_2 random approximate risk measure computed with long arc

z_3 random true risk measure computed without long arc

z_4 random true risk measure computed with long arc

z_5 random approximate risk measure computed with head-node penalty

z_6 random approximate risk measure computed without head-node penalty

z_7 random true risk measure computed with head-node penalty

z_8 random true risk measure computed without head-node penalty

μ_1 mean approximate risk measure without long arc ($\mu_1 \equiv E[z_1]$)

μ_2 mean approximate risk measure with long arc ($\mu_2 \equiv E[z_2]$)

μ_3 mean true risk measure without long arc ($\mu_3 \equiv E[z_3]$)

μ_4 mean true risk measure with long arc ($\mu_4 \equiv E[z_4]$)

μ_5 mean approximate risk measure with head-node penalty ($\mu_5 \equiv E[z_5]$)

μ_6 mean approximate risk measure without head-node penalty ($\mu_6 \equiv E[z_6]$)

μ_7 mean true risk measure with head-node penalty ($\mu_7 \equiv E[z_7]$)

μ_8 mean true risk measure without head-node penalty $(\mu_8 \equiv E[z_8])$

H_0 null hypothesis

H_a alternative hypothesis

B. RESULTS FOR TEST 1 (LONG ARCS)

To evaluate the effects of using long arcs, we first run the modified minimum-risk routing model with and without long arcs for the same test scenario, and compare resulting approximate risk measures (objective values). Then, we calculate the true risk measure for each path, and see if the long-arc structure can actually reduce double counting and provide a lower-risk solution. Test results with and without long arcs, for 30 trials on different minefields, are summarized in Table 1. Figure 7 shows an example test result of the approximate minimum-risk path with and without long arcs. Appendix B contains the raw solution data for Test 1.

The test results show with long arcs, both mean approximate risk measure and mean true risk measure are smaller (better) than the ones without long arcs $(\mu_2 < \mu_1, \mu_4 < \mu_3)$. To check if the mean of different with and without long arcs is significant, we conduct paired t hypothesis test on mean of difference for approximate risk measure $(H_0 : \mu_1 - \mu_2 \le 0; H_a : \mu_1 - \mu_2 > 0)$ and true risk measure $(H_0 : \mu_3 - \mu_4 \le 0; H_a : \mu_3 - \mu_4 > 0)$. The result of the hypothesis test shows that with long arcs, both mean of difference for approximate risk measure and true risk measure are greater than 0 (significant enough to reject H_0) under a 90% confidence interval. So we conclude that using long arcs in the routing structure does give our model more flexibility and able to reduce the issue of double counting; furthermore, long arcs can result in lower-risk paths.

Path-risk measure	Long-arc topology	Mean risk measure	Standard error	Paired t-test result (90% CI)
Approximate	Without	0.135798 (μ_1)	$\sigma_{(z_1-z_2)}$ =0.073459	$H_a : \mu_1 - \mu_2 > 0$
	With	0.034454 (μ_2)		p-value=0.0891
True	Without	0.100907 (μ_3)	$\sigma_{(z_3-z_4)}$ =0.046311	$H_a : \mu_3 - \mu_4 > 0$
	With	0.034008 (μ_4)		p-value=0.0796

Table 1. Summary for Test 1. The mean objective value is better (smaller) when routing structure in the network with long arcs (0.034454) than without (0.135798); and mean true risk measures for the found path is also better (smaller) with long arcs (0.034008) than without (0.100907). The Paired t hypothesis test shows that both data for approximate and true risk measure are significant enough to reject the null hypothesis (accept the alternative hypothesis H_a) under a 90% confidence interval. This means that it is likely that a lower-risk solution can be achieved using long arcs than without using them.

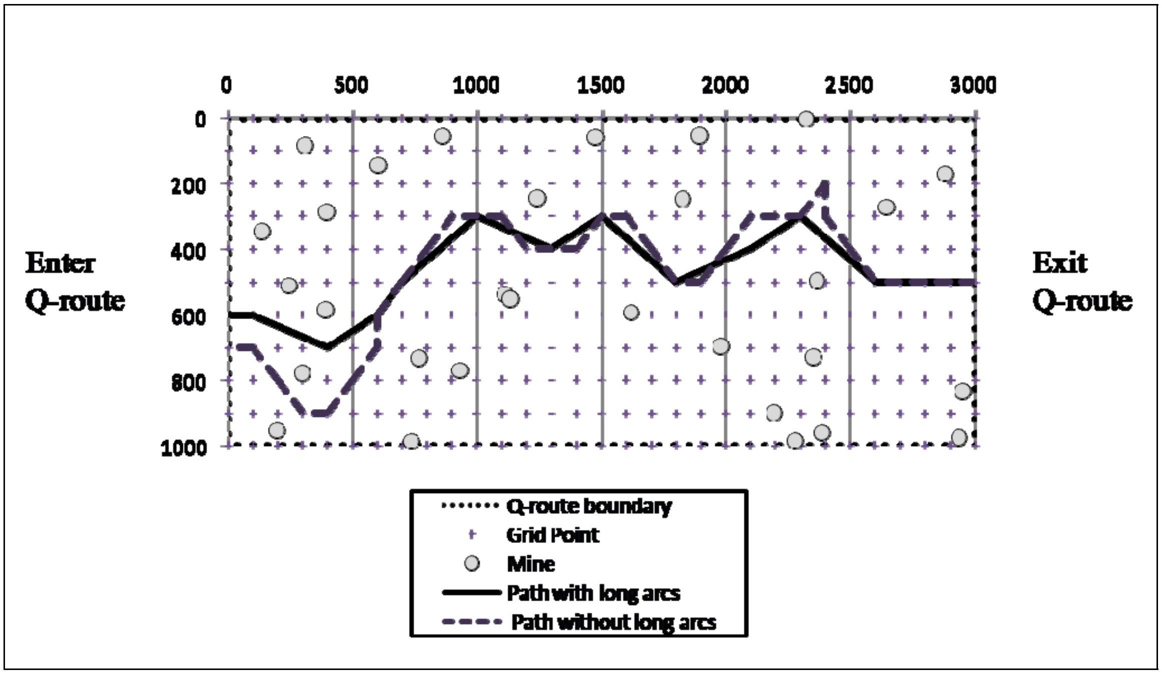

Figure 7. Examples of the approximate minimum-risk path with and without long arcs. All data correspond to the basic scenario described in section V.A. The approximate risk measure of the paths with and without long arcs are 0.000974035 and 1.007169169 respectively; and true risk measure with and without long arcs are 0.000753084 and 1.00094265 respectively. The path with long arcs results in a lower-risk solution. (The fact that the "short-arc risk measures" are slightly larger than 1.0, means that the lack of flexibility with short arcs forces the best path to pass within the mine-damage radius for one mine: recall that the penalty for this occurring is $\mu = 1.0$.)

C. RESULTS FOR TEST 2 (IGNORING HEAD-NODE PENALTIES)

To further investigate the issue of double counting, we test a modified penalty calculation, which ignores head-node penalties. We run the modified minimum-risk routing model with and without head-node penalties for the same test minefield first, and then compare resulting approximate risk measures and true risk measures for each path to see if this modified penalty calculation can reduce the issue of double counting and provide a better solution. Table 2 summarizes test results, with and without head-node penalties, for 30 trials on different minefields. Figure 8 shows an example test result of the approximate minimum-risk path with and without head-node penalties. Appendix C contains the raw solution data for Test 2.

The test results show the mean approximate risk measure without head-node penalties is smaller than with head-node penalties $(\mu_6 < \mu_5)$, but the mean true risk measure does not get better when using this modified penalty calculation $(\mu_8 > \mu_7)$. We also conduct paired t hypothesis test on mean of difference for approximate risk measure $(H_0 : \mu_5 - \mu_6 \leq 0; H_a : \mu_5 - \mu_6 > 0)$ and true risk measure $(H_0 : \mu_7 - \mu_8 \leq 0; H_a : \mu_7 - \mu_8 > 0)$. These results show that without head-node penalties, the mean of difference for approximate risk measure is greater than 0 (significant enough to reject H_0) under a 90% confidence interval; but for true risk measure, the mean of difference is not greater than 0 (not enough evidence to reject H_0). So we conclude that using this modified penalty calculation can not actually reduce the issue of double counting, and it will not guarantee to provide a better minimum-risk path solution.

Path-risk measure	Head-node penalties	Mean risk measure	Standard error of	Paired t-test result (90% CI)
Approximate	With	0.034454 (μ_5)	$\sigma_{(z_5 - z_6)}$ =0.000804	$H_a : \mu_5 - \mu_6 > 0$
	Without	0.034182 (μ_6)		p-value=0.0372
True	With	0.034008 (μ_7)	$\sigma_{(z_7 - z_8)}$ =0.000204	$H_0 : \mu_7 - \mu_8 \leq 0$
	Without	0.034083 (μ_8)		p-value=0.9725

Table 2. Summary for test 2. The mean approximate risk measure for the modified penalty calculation (without head-node penalty, 0.034182) is better (smaller) than with head node penalty (0.034454) as expected, since it ignores the head-penalties. But mean true risk measures for the found minimum-risk path, without head-node penalty (0.034083) is not better than with (0.034008). The Paired t hypothesis test also shows that for approximate measure is significant enough to reject null hypothesis (accept alternative hypothesis (H_a)) under 90% confidence interval, but for true risk measure there is not enough evidence to reject null hypothesis (accept null hypothesis (H_0)). That implies this modified penalty calculation (without head-node penalties) does not provide a lower-risk solution.

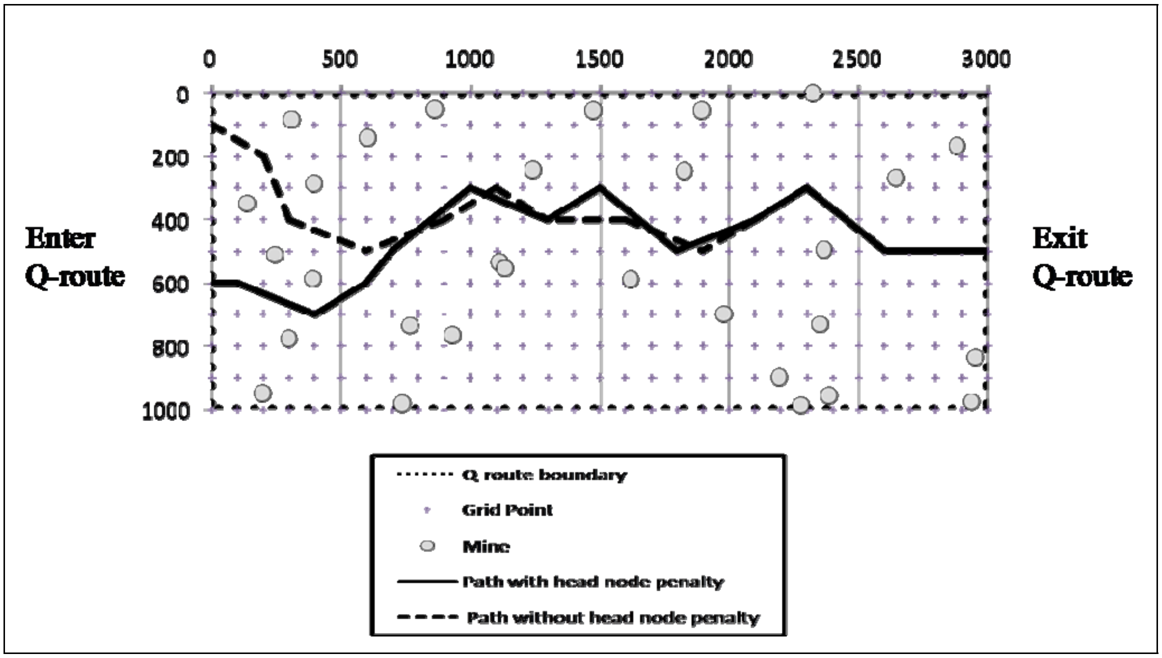

Figure 8. Examples of the approximate minimum-risk path with and without head-node penalties. All data correspond to the basic scenario described in section V.A. The approximate risk measure of the paths with and without head-node penalties are 0.000974035 and 0.000904875 respectively; and true risk measure with and without head-node penalties are 0.000753084 and 0.000867963 respectively. The modified penalty calculation (without head-node penalties) does not result in a lower-risk solution.

D. RESULTS FOR TEST 3 (MINE REMOVAL)

To evaluate the greedy heuristic for mine removal, we first we run both the greedy heuristic and an optimal integer-programming model to obtain a zero-risk path from each. Then, we compare the number of mines, m^*, required to be cleared. Also, since we expect that the greedy heuristic can find a prioritized list of mines whose removal will result in a rapid reduction in the risk of a minimum-risk path, we will clear each subset of mines of size $m' < m^*$, following the prioritized list, and compare its risk-reduction curve to the curve achieved by a sequence of optimal subsets of cleared mines. The latter curve provides a lower bound on the risk reduction for any prioritized list of mine removals. Test results from both models, for five trials on different minefields, are summarized in Table 3. Figure 9 shows an example test result of required mines to clear for both models to achieve a zero-risk path.

The test results show that for both models to achieve a zero risk path, not only the number of mines required to clear are the same, but also those solutions are the same set of mines. Furthermore, when we follow the prioritized list from greedy heuristic and start to clear one mine at a time until zero-risk path achieved, all subsets of removal mines are exactly the same as those from the integer-programming model which clears optimal subsets. From this, we conclude that when naval mines are "near uniformly distributed" in the area of operation, the greedy heuristic solution will be close to (or the same as) the optimal solution.

To contrast an optimal solution with the greedy-heuristic solution when mines are not "near uniformly distributed," we illustrate a scenario with 33 mines, with the mines distributed in the pattern shown in Figure 10. In that case, suppose that a commander uses both methods to try to find the smallest set of mines to clear, and the minimum number found from optimal and greedy solutions are three and five respectively. MCM forces begin removing the three optimal mines, but only have time to clear two (the actual order of removal is immaterial) when operations must halt, and a ship must transit the minefield. With the optimal solution, after two mines being cleared, the risk measure of the minimum-risk path will be higher than after removing two mines but following the prioritized list obtained from greedy heuristic (risk reduction curve as Figure 11). Since our heuristic identifies a sequence of mines to clear, always choosing the next best mine to clear, this case shows it generates a good sequence that can be interrupted at any point but still with reasonable results.

Test	Model	Number of mines cleared to achieve a zero-risk path (m^*)	Set of total mines to clear (M^*)	Subset of mines to clear (M')
1	Greedy	5	Same set $(M^*_{Greedy} = M^*_{Optimal})$	Same all subsets $(M'_{Greedy} = M'_{Optimal})$
	Optimal	5		
2	Greedy	4	Same set $(M^*_{Greedy} = M^*_{Optimal})$	Same all subsets $(M'_{Greedy} = M'_{Optimal})$
	Optimal	4		
3	Greedy	5	Same set $(M^*_{Greedy} = M^*_{Optimal})$	Same all subsets $(M'_{Greedy} = M'_{Optimal})$
	Optimal	5		
4	Greedy	5	Same set $(M^*_{Greedy} = M^*_{Optimal})$	Same all subsets $(M'_{Greedy} = M'_{Optimal})$
	Optimal	5		
5	Greedy	4	Same set $(M^*_{Greedy} = M^*_{Optimal})$	Same all subsets $(M'_{Greedy} = M'_{Optimal})$
	Optimal	4		

Table 3. Summary for Test 3. In all cases, both solution methods (greedy heuristic and integer programming) clear the same (optimal) set of mines in order to achieve a zero-risk path. Furthermore, in all cases, both methods result in the same subsets of mines to clear. This means that the greedy heuristic in this test scenario will have the same optimal risk-reduction curve as integer-programming model.

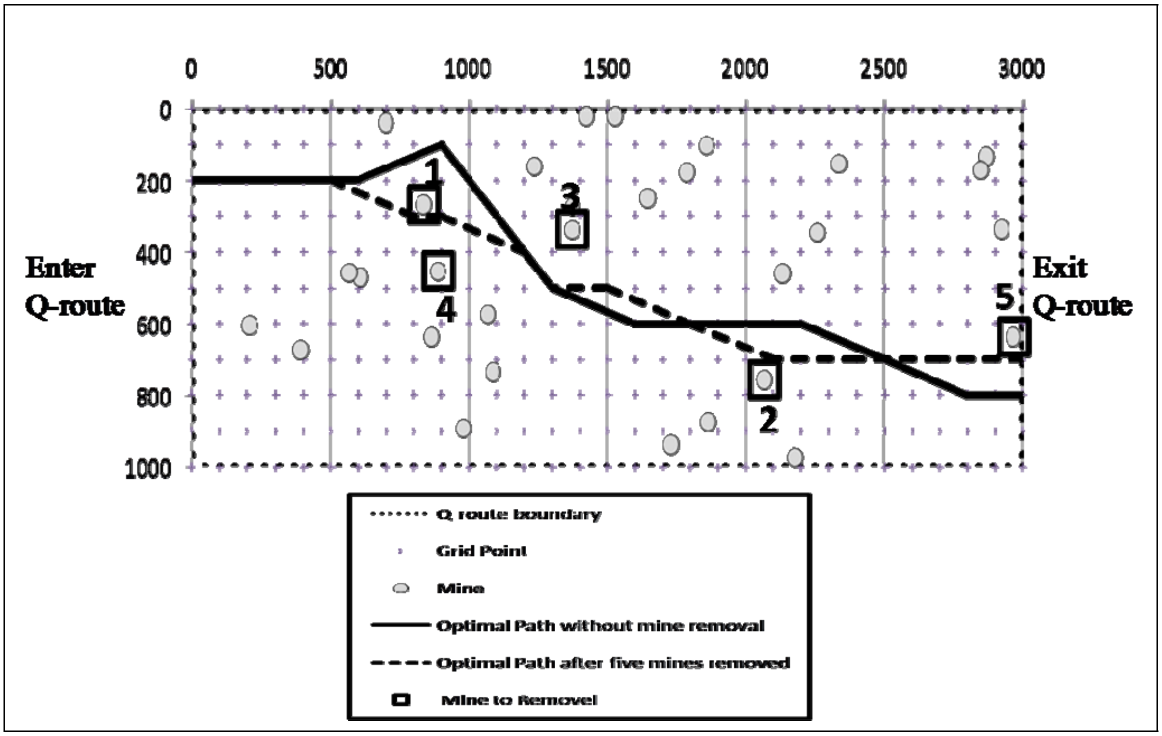

Figure 9. Examples in which five mines, in a "nearly uniformly distributed" minefield, must be cleared to achieve a zero-risk path. Both optimal and greedy-heuristic solutions clear the same set of five mines. Furthermore, both solutions clear the same subsets of mines, prioritized from one to five.

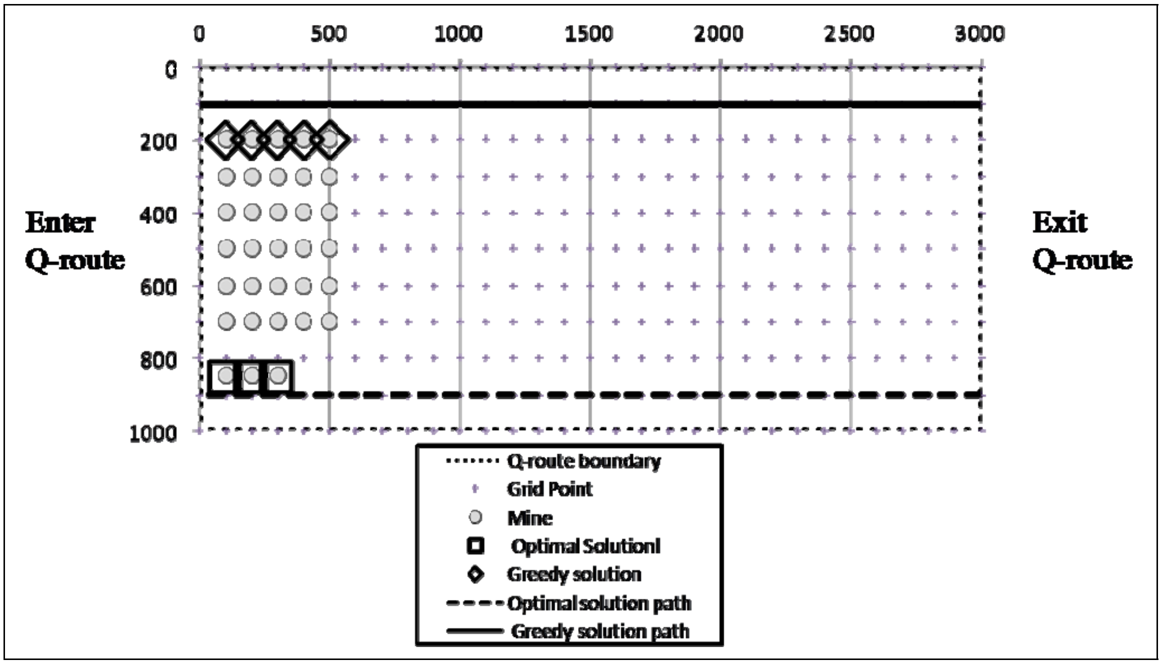

Figure 10. Examples of optimal and greedy heuristic solutions in an AO network in which mines are not "near uniformly distributed." The greedy heuristic needs to clear five mines to achieve a zero-risk path, but optimal solution only needs to clear three.

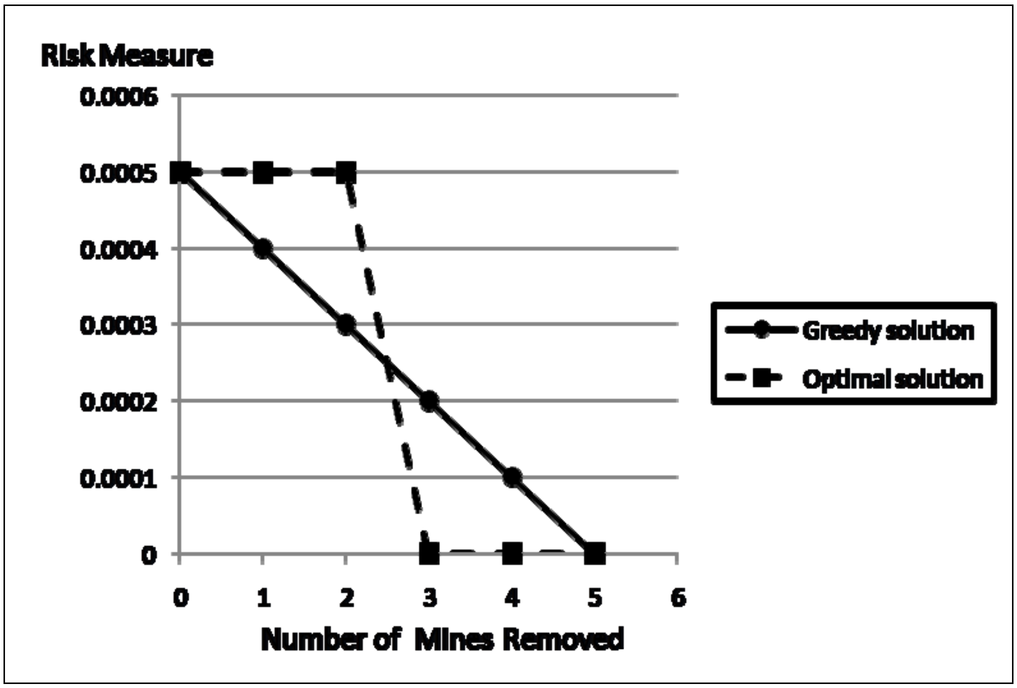

Figure 11. Examples of risk-reduction curves. All data correspond to the scenario shown in Figure 10. The greedy heuristic needs to clear five mines to achieve a zero-risk path, but the optimal solution only needs to clear three. But, suppose that we begin clearing the three "optimal mines," and only have time to clear two before operations must halt, and a ship must transit the minefield. (The actual order of clearance is immaterial.) The risk measure for the minimum risk path will be the same (0.0005) as before clearing any mines, since one mine with penalty μ remains, and the minimum-risk path does not change. (That path avoids all three "optimal mines" entirely.) If we clear the two mines specified by the greedy heuristic, the risk measure drops to 0.0003, however.

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VI. SUMMARY AND CONCLUSIONS

This thesis develops a mission-planning tool for a Navy Mine Counter Measure (MCM) force to find a minimum-risk route for a surface ship through a mapped minefield ("mine avoidance"). The thesis also develops a heuristic to identify a sequence of mines whose clearance leads to a rapid reduction of the risk of a minimum-risk path ("mine clearance"). The routing problem is formulated and solved as a shortest-path problem in a network. A grid of nodes, representing waypoints, is embedded in a representation of the operating area, while arcs are created to link waypoints. The risk assigned to a path is the sum of the risks assigned to the component arcs of the path. And the risk to an arc is a function of the distance from the arc to nearby mines.

The model investigates computationally potential improvements to the risk-measure approximations in a highly dense minefield scenario. Test results shows that for the mine-avoidance problem, "long arcs" can provide lower-risk solutions and allow the use of activation curve data. ("Long arcs" mean that not only are nearest-neighbor waypoints connected to each other, but so are those that are several echelons away in the grid of nodes.) On the other hand, removing "head-node penalties" does not produce lower-risk solutions. For the mine-clearance problem, if mines are "nearly uniformly distributed" in the minefield, the greedy heuristic solution will be close to the optimal solution.

The problem we study in this thesis assumes that a Q-route has been established in a particular area, nominally the entrance to a harbor. Countermine patrols have been carried out along the Q-route (using manned and/or unmanned vessels), the location of each relevant mine has been mapped and its type established. Several assumptions are made to simplify the problem in order to develop a practical model: we assume all mine positions are known exactly, and all mines have known characteristics (e.g., activation method, explosive force). Also we assume all mines have known mine-damage radius, and no own-ship navigation errors occur. Furthermore, we assume that the enemy does not "re-seed" the minefield during the period of interest.

Those assumptions may not be entirely true in reality, but they are reasonable for a prototype. An approximate, minimum-risk path can be found in few seconds on a laptop computer, and a greedy "mine-clearance list" can be found in a few minutes. This prototype should provide the framework for a usable mission-planning tool for ROC Navy MCM force.

A. RECOMMENDATIONS FOR FUTURE RESEARCH

Additional work is needed to solidify and generalize the probabilistic underpinnings of optimization approaches to modeling mine-avoidance and min-clearance. This thesis does describe (but does not test) a mine-avoidance model with a risk-function sub model that has a probabilistic interpretation. Furthermore, we describe how standard lateral range curve data can be used to calculate risk-function values. But this modeling requires a number of assumptions and approximations that may not be valid in practice, or which do not fit well into the computationally attractive paradigm of shortest paths. Further research is needed.

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APPENDIX A. FORMULATION FOR INTEGER-PROGRAMMING MODEL

Integer-programming model that identifies the set of m^* mines whose removal results in the least-risk route being minimized.

Indices and index Sets:

$i \in N$	nodes
$s \in S \subset N$	source nodes
$t \in T \subset N$	sink nodes
$(i, j) \in A$	directed arcs with tail node i and head node j
$m \in M$	mines

Data:

d_{ij}	physical length of arc (i, j)
r_{ijm}	risk for arc (i, j) contributed by mine m
α	arc length penalty ($\alpha = 10^{-7}$ in this thesis)
m^*	total number of mines to clear

Variable:

z	variable representing the approximate risk measure for the identified path
y_{ij}	1 if arc (i, j) is on the optimal path, 0 otherwise
v_{ijm}	1 if arc (i, j) on the optimal path and mine m is not cleared, 0 otherwise
x_m	1 if mine m is cleared, 0 otherwise

Formulation:

$$z = \min \sum_{(i,j)\in A} \sum_{m\in M} r_{ijm} v_{ijm} + \sum_{(i,j)\in A} \alpha d_{ij} y_{ij}$$

s.t. $$\sum_{(i,j)\in A} d_{ij} y_{ij} - \sum_{(j,i)\in A} d_{ji} y_{ji} = 0 \quad \forall i \in N - S - T$$

$$\sum_{(i,j)\in A} y_{ij} = 1 \quad \forall i \in S$$

$$\sum_{(j,i)\in A} -y_{ji} = -1 \quad \forall i \in T$$

$$v_{ijm} \geq y_{ij} - x_m \quad \forall (i,j) \in A, m \in M$$

$$\sum_{m\in M} x_m = m^*$$

All variables binary

APPENDIX B. OUTPUT DATA FOR TEST 1

	Long arcs with head node penalty		Short arcs with head node penalty	
Trial #	Approximate Risk	True Risk	Approximate Risk	True Risk
1	0.000577562	0.000546705	0.000966341	0.000603858
2	0.001123551	0.000845455	0.004192647	0.003206728
3	0.000672681	0.000579381	0.001173679	0.000645828
4	0.000925368	0.000732076	0.001513938	0.000746541
5	0.001001279	0.000766743	0.001852396	0.000903726
6	0.000480648	0.000417294	0.000687652	0.000448816
7	0.000807330	0.000670834	0.007279079	0.000934303
8	0.000522961	0.000458028	0.000895349	0.000496554
9	0.001190035	0.000890471	0.001880890	0.000901087
10	0.000974035	0.000753084	1.007169169	1.000942650
11	0.000476191	0.000436464	0.000814873	0.000518526
12	0.000733513	0.000604009	0.001239326	0.000696952
13	0.000680443	0.000571147	0.001186336	0.000671436
14	0.001250837	0.000952037	0.002061613	0.001002332
15	0.001410857	0.000957398	0.008017813	0.001536707
16	0.000614547	0.000584754	0.001099147	0.000642474
17	0.000687018	0.000586362	0.001014498	0.000607021
18	0.000846754	0.000641033	0.001417664	0.000706301
19	0.000962261	0.000719946	0.001481260	0.000798301
20	0.000747701	0.000645377	0.001023361	0.000618937
21	1.001051698	1.000768747	1.003772574	1.002920264
22	0.001255361	0.000950139	0.002141887	0.001064435
23	0.000530272	0.000498306	0.000737661	0.000475478
24	0.001018953	0.000730815	0.001649367	0.000883335
25	0.000788122	0.000649133	0.001309409	0.000685438
26	0.000637582	0.000579532	0.001019192	0.000626495
27	0.006617582	0.000736728	0.007093330	0.000856236
28	0.000571352	0.000533636	0.000945936	0.000593624
29	0.000992145	0.000772604	0.007284760	0.000938725
30	0.003468737	0.000672653	2.001018043	1.000538656

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APPENDIX C. OUTPUT DATA FOR TEST 2

	Long arcs with head node penalty		Long arcs without head node penalty	
Trial #	Approximate Risk	True Risk	Approximate Risk	True Risk
1	0.000577562	0.000546705	0.000560755	0.000560749
2	0.001123551	0.000845455	0.000747592	0.001830270
3	0.000672681	0.000579381	0.000641069	0.000637741
4	0.000925368	0.000732076	0.000866159	0.000783442
5	0.001001279	0.000766743	0.000864260	0.000766743
6	0.000480648	0.000417294	0.000438567	0.000438072
7	0.000807330	0.000670834	0.000712321	0.000682444
8	0.000522961	0.000458028	0.000484516	0.000484516
9	0.001190035	0.000890471	0.000949839	0.000913389
10	0.000974035	0.000753084	0.000904875	0.000867964
11	0.000476191	0.000436464	0.000448316	0.000448316
12	0.000733513	0.000604009	0.000627596	0.000599062
13	0.000680443	0.000571147	0.000634697	0.000558852
14	0.001250837	0.000952037	0.001071356	0.000943874
15	0.001410857	0.000957398	0.001018941	0.001013009
16	0.000614547	0.000584754	0.000562649	0.000561037
17	0.000687018	0.000586362	0.000642231	0.000602060
18	0.000846754	0.000641033	0.000697065	0.000667054
19	0.000962261	0.000719946	0.000700646	0.001300646
20	0.000747701	0.000645377	0.000655630	0.000618051
21	1.001051698	1.000768747	1.000641965	1.000841965
22	0.001255361	0.000950139	0.001017353	0.000967557
23	0.000530272	0.000498306	0.000479842	0.000478573
24	0.001018953	0.000730815	0.000856403	0.000758809
25	0.000788122	0.000649133	0.000689767	0.000684106
26	0.000637582	0.000579532	0.000565831	0.000565371
27	0.006617582	0.000736728	0.002126748	0.000717334
28	0.000571352	0.000533636	0.000529262	0.000528447
29	0.000992145	0.000772604	0.000935396	0.000808023
30	0.003468737	0.000672653	0.003388454	0.000863583

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