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http://hdl.handle.net/10945/48188

[8] Ender, J.
Detection and estimation of moving target signals by multi-channel SAR.
In *Proceedings of EUSAR'96 Conference*, 1996, 411–417.

[9] Friedlander, B., and Porat, B.
VSAR: A high resolution radar system for detection of moving targets.
IEE Proceedings—Radar, Sonar & Navigation, **144**, 4 (1997), 205–218.

[10] Wang, G., Xia, X.-G., Chen, V. C., and Fiedler, R. L.
Detection, location, and imaging of fast moving targets using multifrequency antenna array SAR.
IEEE Transactions on Aerospace and Electronic Systems, **40**, 1 (2004), 345–355.

[11] Li, G., Xu, J., Peng, Y.-N., and Xia, X.-G.
Moving target location and imaging using dual-speed velocity SAR.
IET Radar Sonar & Navigation, **1**, 2 (2007), 158–163.

[12] McClellan, J. H., and Rader, C. M.
Number Theory in Digital Signal Processing.
Englewood Cliffs, NJ: Prentice-Hall, 1979.

[13] Wood, J., and Barry, D.
Linear signal synthesis using the Radon-Wigner transform.
IEEE Transaction on Signal Processing, **42**, 8 (1994), 2105–2111.

Weather Radar Equation Correction for Frequency Agile and Phased Array Radars

This paper presents the derivation of a correction to the Probert-Jones weather radar equation for use with advanced frequency agile, phased array radars. It is shown that two additional terms are required to account for frequency hopping and electronic beam pointing. The corrected weather radar equation provides a basis for accurate and efficient computation of a reflectivity estimate from the weather signal data samples. Lastly, an understanding of calibration requirements for these advanced weather radars is shown to follow naturally from the theoretical framework.

I. INTRODUCTION

The classical weather radar equation was introduced by Probert-Jones in 1962 [1] and has since been universally used to implement reflectivity algorithms. These algorithms are used to compute reflectivity estimates from the signal sample data collected by weather radars. Existing weather radars typically operate at a fixed frequency and employ antennas that are mechanically scanned. Recently however, there has been interest in adding a weather processing capability to advanced radars originally developed for other purposes. Some of these radars are frequency agile and use phased array antennas. Adaptive waveforms and phased array technology for agile beam scanning strategies have also been identified as technologies that should be investigated for the next generation of U.S. national weather radars [2].

The cross section density of precipitation in the Rayleigh region varies with frequency, as does antenna gain and beamwidth. Further, antenna gain and beam solid angle also vary when the beam of a planar phased array is electronically pointed off broadside. These inter-related effects impact the radar effective radiated power, the size of the radar resolution cell and ultimately the observed average power return and reflectivity estimate. These effects raise doubts about the direct applicability of the Probert-Jones equation to these radars. If used, the classical Probert-Jones weather radar equation would lead to reflectivity errors because frequency hopping and electronic beam pointing effects are not inculded. These errors would exceed the reflectivity accuracy objectives of most modern weather radars. The objective of the work described here is to analytically account for the effects of weather radar frequency agility and electronic beam pointing in the weather radar equation. Analytically accounting for these effects leads to a theoretical result that permits a reflectivity estimate to be computed simply, accurately, and efficiently. The theoretical framework also leads to a clear understanding of the calibration requirements for frequency agile, phased array weather radars.

II. PHASED ARRAY FUNDAMENTALS

A. Scanned Array Gain

Consider a rectangular planar array with separable aperture distribution

$$E_a(x,y) = E_{a1}(x)E_{a2}(y) \tag{1}$$

where $E_{a1}(x)$ and $E_{a2}(y)$ are the aperture field distributions in the x and y directions, respectively. It can be shown [3] that when the aperture distribution is separable, the directivity D is also separable and is given by

$$D = \pi D_x D_y \cos\theta \tag{2}$$

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where θ is the angle of the antenna beam with respect to boresight or the normal to the array face. The principal plane directivities of the array are given by

$$D_x = c_x \frac{L_x}{\lambda}, \qquad D_y = c_y \frac{L_y}{\lambda} \tag{3}$$

where L_x and L_y are the x and y dimensions of the array, λ is the wavelength and the constants c_x and c_y are determined by the tapers of the aperture distribution. It is assumed here that the taper does not change as the antenna beam is electronically pointed off boresight. Thus the array directivity is

$$D = \pi c_x c_y \frac{L_x L_y}{\lambda^2} \cos\theta \tag{4}$$

where $L_x L_y = A_p$ is the physical aperture area. Gain G is related to directivity by the array radiation efficiency e_r, so

$$G = e_r D = e_r \pi c_x c_y \frac{A_p}{\lambda^2} \cos\theta. \tag{5}$$

If we choose an arbitrary reference frequency f_0 such that

$$G_0 = e_r c_x c_y \pi \frac{A_p}{\lambda_0^2}, \qquad \theta = 0 \tag{6}$$

then gain can be written as

$$G = G_0 (\lambda_0/\lambda)^2 \cos\theta \tag{7}$$

where G_0 is the gain at the reference frequency f_0 when the beam is pointed in the broadside direction. Note that for a uniformly illuminated aperture, $c_x = c_y = 2$ ($c_x c_y = 4$) and for a tapered aperture, $c_x c_y < 4$.

B. Scanned Array Beam Solid Angle

An alternative but fundamental expression for the directivity of any antenna, including a phased array is

$$D = \frac{4\pi}{\Omega_B} \tag{8}$$

where Ω_B is the beam solid angle. In any arbitrary direction, the beam of a high gain array forms an ellipse on the surface of a sphere as illustrated in Fig. 1. As the beam of a phased array is scanned, the orientation of this ellipse varies, as does the beam solid angle. Let θ_u and φ_v be the beamwidths in the orthogonal planes defined by the major and minor axes of the ellipse. Then

$$\Omega_B = \text{const} \times (\theta_u \varphi_v). \tag{9}$$

Since $\Omega_B D = 4\pi$ is a constant, we can also write that

$$\theta_u \varphi_v = \frac{\theta_{B0}^{br} \varphi_{B0}^{br} (\lambda/\lambda_0)^2}{\cos\theta} \tag{10}$$

where θ_{B0}^{br} and φ_{B0}^{br} are the broadside 3 dB principal plane beamwidths at the reference frequency f_0 [3]. It should be noted that the half-power beamwidths

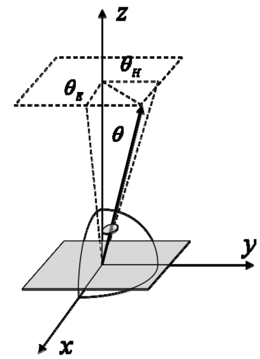

Fig. 1. Planar array with z axis normal to array face. α and β are beam angles measured in x-z and y-z planes.

required in the classical Probert-Jones equation are θ_u and θ_v. Equation (10) gives these beamwidths in terms of other known quantities.

C. Beam Pointing Angle

Assume the planar array lies in the x-y plane as illustrated in Fig. 1. In general the beam of the array can be steered by changing frequency, changing phase using phase shifters, or some combination of the two. The objective is to point the beam by changing the phase shift across elements in the x and y directions. Further assume that α is the beam angle measured in the x-z plane and β is the beam angle measured in the y-z plane. For a linearly polarized antenna, one of these planes would typically be referred to as the E-plane and the other as the H-plane. Simple geometry relates θ, the angle between the z axis and the beam axis, to α and β. The relationship is

$$\theta = \arctan\{[(\tan\alpha)^2 + (\tan\beta)^2]^{1/2}\}. \tag{11}$$

Planar phased arrays for surface-based radars are typically tilted in elevation. The elevation plane is the H-plane for phi polarized antennas and the E-plane for theta polarized antennas. The z axis remains normal to the face of the tilted array but an expression for elevation angle with respect to the horizon is desired. The angle in the elevation plane must be corrected for the tilt. If we define θ_e as the elevation angle of the beam with respect to the horizon, θ_{tilt} as the tilt angle and assume the x-z plane is the elevation plane, then

$$\theta_e = \alpha + \theta_{\text{tilt}}. \tag{12}$$

III. POINT TARGET RETURN

The return from a point target is of interest because a weather radar must be calibrated to accurately determine radar cross section from the measured receiver output power. One way of calibrating a radar is to measure the power returned from a reference target such as a metal sphere.

For a point target, the echo power at the receiver input [4] is given by

$$P_{\text{in}} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_s L_a} \tag{13}$$

where

P_{in} = receiver input power

P_t = transmit power

G = antenna gain (peak)

λ = wavelength

σ = point reference target radar cross section

R = range to target

L_s = radar system losses

L_a = atmospheric propagation loss.

Using (7), P_{out}, the power at the coherent receiver output is given by

$$P_{\text{out}} = \frac{P_t G_{rx}\{G_0^2(\lambda_0/\lambda)^4 \cos^2\theta\}\lambda^2\sigma}{(4\pi)^3 R^4 L_s L_a} \tag{14}$$

where G_{rx} is the receiver gain. Rearranging terms, (14) can be written as

$$P_{\text{out}} = \left[\frac{P_t G_{rx} G_0^2}{L_s}\right]\left[\frac{(\lambda_0/\lambda)^4(\cos^2\theta)\lambda^2\sigma}{(4\pi)^3 R^4 L_a}\right]. \tag{15}$$

The first term on the right side of (15) contains radar system parameters which may not be precisely known. The objective of a calibration is to determine the composite of these factors. For the case where the reference target is on boresight $\theta = 0$, the measurement is made at the reference frequency $\lambda = \lambda_0$, and the range is short ($L_a = 1$), (15) simplifies to

$$\left[\frac{P_t G_{rx} G_0^2}{L_s}\right] = \left[\frac{(4\pi)^3 R^4 P_{\text{out}}}{\lambda_0^2 \sigma}\right]. \tag{16}$$

From (16), it is clear that a measurement of receiver output power and range for a reference target of known cross section ($\theta = 0$, $\lambda = \lambda_0$, $L_a = 1$) permits the radar system constant on the left side of (16) to be determined. For cases where the reference target return cannot be measured on boresight, (15) can easily be used to derive an alternative form of (16) for another beam pointing angle.

IV. VOLUME TARGET RETURN

A. Classical Weather Radar Equation

Next consider the Probert-Jones weather radar equation [1],

$$Z = \frac{1024 \ln 2 L_s L_a R^2 \lambda^4 \overline{P_{\text{in}}}}{P_t G^2 \lambda^2 (\theta_B \varphi_B)(c\tau)\pi^3 |K|^2} \tag{17}$$

or

$$Z = \frac{1024 \ln 2 L_s L_a R^2 \lambda^4 \overline{P_{\text{out}}}}{P_t G_{rx} G^2 \lambda^2 (\theta_B \varphi_B)(c\tau)\pi^3 |K|^2} \tag{18}$$

where

Z = reflectivity (mm^6/m^3)

$\overline{P_{\text{out}}}$ = average output power (W)

$c = 3 \times 10^8$ m/s

τ = pulsewidth (s)

$|K|^2 = |(\varepsilon_r - 1)/(\varepsilon_r + 2)|^2 \approx 0.93$ (H_2O)

ε_r = hydrometeor relative permitivitty.

Note that $\theta_B \varphi_B = \theta_u \varphi_v$ are the beamwidths required in (17) and (18) as these are the beamwidths that define the beam solid angle for an arbitrary beam pointing direction.

B. Weather Radar Equation Correction

Equations (7) and (10) can be used in (18) to correct the classical weather radar equation. Substituting and rearranging terms, we obtain

$$Z = \left[\frac{1024 \ln 2 \lambda_0^4 (\lambda/\lambda_0)^4}{\pi^3 (c\tau)|K|^2 \lambda_0^2 (\lambda/\lambda_0)^2}\right]\left[\frac{\cos\theta}{(\lambda/\lambda_0)^2(\theta_{B0}^{br}\varphi_{B0}^{br})}\right] \times \left[\frac{L_s L_a}{P_t G_{rx} G_0^2 (\lambda_0/\lambda)^4 \cos^2\theta}\right](R^2)\overline{P_{\text{out}}}. \tag{19}$$

Simplifying, for frequency agile, phased array weather radars, the equation for reflectivity takes the form

$$Z = \left[\frac{1024 \ln 2 \lambda_0^4}{\pi^3 (c\tau)|K|^2 \lambda_0^2}\right]\left[\frac{1}{(\theta_{B0}^{br}\varphi_{B0}^{br})}\right] \times \left[\frac{L_s}{P_t G_{rx} G_0^2}\right]\left[\frac{L_a \overline{P_{\text{out}}}}{(f/f_0)^4 \cos\theta}\right](R^2). \tag{20}$$

Note that the units of Z are given correctly (mm^6/m^3) by the first two terms of (20) if range is in millimeters, λ_0 in the numerator of the first term is in millimeters and the quantities in the denominator of the first term are in meters.

All factors in (20) appear in the classical Probert-Jones weather radar equation [1] except the terms $(f/f_0)^4 \cos\theta$. These terms analytically account for the Rayleigh region frequency dependence of scattering from precipitation, the frequency dependence of antenna gain and the change in antenna gain, and beamwidth as the antenna beam is pointed off boresight.

To illustrate the application of (20), consider the MWR-05XP radar [5] shown in Fig. 2. This is a mobile, X-band, frequency agile, phased array weather

Fig. 2. Naval Postgraduate School MWR-05XP mobile, frequency agile, phased array weather radar [5].

radar with operating parameters as follows:

$$f_0 = 9370 \text{ MHz}$$
$$\tau = 1\ \mu\text{s}$$
$$\theta_{B0}^{br}\varphi_{B0}^{br} = 1.096\times 10^{-3} \text{ rad}$$
$$P_t G_0 = 111.1 \text{ dBm}$$
$$G_{rx} = 42.1 \text{ dB}.$$

The receiver gain G_{rx} can be adjusted using an IF attenuator and the value given above is measured with the attenuator set at $A_{\text{dB}} = 28$ dB. The attenuator is set to achieve an average radar system noise level about 10 dB above the digital receiver noise floor. This is necessary if postdetection integration is to be implemented.

For the MWR-05XP, the terms in (20) have the following values:

$$\left[\frac{1024\ln 2\lambda_{0,mm}^4}{\pi^3(c\tau)|K|^2\lambda_{0,m}^2}\right](R_{mm}^2) = 139.3 \text{ dB} + 20\log(R_{\text{meters}})$$
$$\left[\frac{1}{(\theta_{B0}^{br}\varphi_{B0}^{br})}\right] = 29.6 \text{ dB}$$
$$\left[\frac{L_s}{P_t G_{rx} G_0^2}\right] = -191.7 \text{ dBm}$$
$$\left[\frac{\overline{P_{\text{out}}}}{(\lambda_0/\lambda)^4\cos\theta}\right] = -40\log(f/f_0) - 10\log[\cos\theta] - \overline{P_{\text{out(dBm)}}}.$$

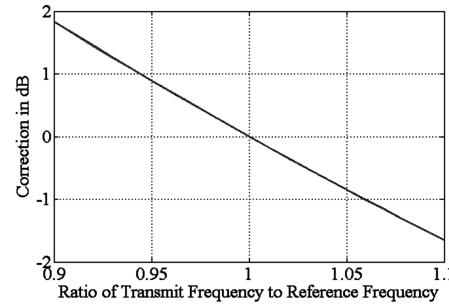

Fig. 3. Reflectivity correction to Probert-Jones equation due to frequency hopping.

Substituting the above values of the various terms for the MWR-05XP weather radar in (20), we obtain

$$Z_{\text{dBZ}} = -22.8 \text{ dBm} + L_{a(\text{dB})} + 20\log(R_{\text{meters}}) + \overline{P_{\text{outdBm}}} - 40\log(f/f_0) - 10\log[\cos\theta]. \quad (21)$$

The last two terms in (21) account for the effects of frequency agility and beam pointing.

C. Correction Associated with New Terms

It is of interest to investigate and quantify the correction that might typically accrue from the two new terms that appear in (21). Consider a radar with a fractional bandwidth of about 6% and assume the beam is pointed 45 deg off boresight. The frequency hopping and beam pointing terms (last two terms) in (20) yield

$$-40\log(f/f_0)|_{f/f_0=1.06} \approx -1 \text{ dB} \quad (22a)$$

and

$$-10\log(\cos\theta)|_{\theta=45^\circ} \approx +1.5 \text{ dB}. \quad (22b)$$

It should be noted that the sign of the result in (22a) would change if the reference frequency f_0 were chosen to be the highest frequency in the band rather than the lowest, as assumed in (22a). Thus, the correction associated with the two terms could be as high as 2.5 dB for the conditions assumed. This error exceeds the reflectivity accuracy objective for most modern weather radars.

Fig. 3 shows the correction to reflectivity due to frequency hopping. The dB value of the correction may be positive or negative depending on the instantaneous transmit frequency relative to the reference frequency. The choice of reference frequency within the operating band of the radar is arbitrary so the correction may be positive or negative.

Fig. 4 shows the correction to reflectivity due to beam pointing. Phased array beams are usually pointed up to 45 deg off boresight and the correction for this pointing angle is 1.5 dB as noted above.

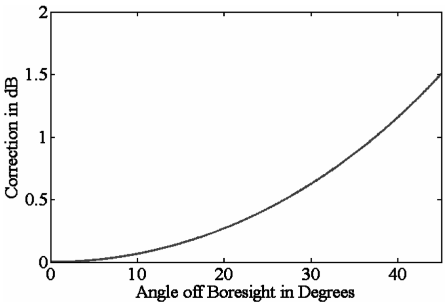

Fig. 4. Reflectivity correction to Probert-Jones equation due to off boresight electronic beam pointing.

D. Sensitivity

The sensitivity of a frequency agile, phased array weather radar will also vary with frequency and beam pointing angle. Sensitivity is frequently specified as the level of reflectivity that will produce a single pulse signal-to-noise ratio (SNR) of 0 dB versus range. While this seems artificial at first, the result it yields is little different from the sensitivity computed when pulses are integrated and a higher SNR is specified, as required for good detection probability. Returning to (20), if one makes the substitution $\overline{P_{\text{out}}} = \overline{N_{\text{out}}}$ where $\overline{N_{\text{out}}} = G_{rx}(kT_sB)$ is the usual system noise floor, we obtain

$$Z = \left[\frac{1024\ln 2\lambda_0^4}{\pi^3(c\tau)|K|^2\lambda_0^2}\right]\left[\frac{1}{(\theta_{B0}^{br}\varphi_{B0}^{br})}\right] \times \left[\frac{L_s}{P_tG_{rx}G_0^2}\right]\left[\frac{L_aG_{rx}(kT_sB)}{(f/f_0)^4\cos\theta}\right](R^2) \qquad (23a)$$

or

$$Z_{\text{dBZ}} = C_{\text{dB}} + 20\log(R) + L_{a(\text{dB})} - 40\log(f/f_0) - 10\log(\cos\theta). \qquad (23b)$$

The first term in (23b) is simply a constant. For the MWR-05XP, for example, $C_{\text{dB}} = -33.5$ dB when R is in km. The first two terms in (23b) give the usual dependence of reflectivity sensitivity on range while the last two terms provide the correction required as a result of beam pointing or frequency hopping. Equivalently, it can be seen that sensitivity will vary depending upon the beam scan angle and frequency within the operating band of the radar. Scanning the beam 45 deg off boresight, for example, reduces sensitivity by 1.5 dB. For a radar with the reference frequency chosen at the high end of its operating band and a 6% fractional bandwidth, the frequency term in (23b) results in a further decrease in sensitivity of about 1.1 dB at the bottom end of the operating band relative to the top end. Thus, the reflectivity sensitivity of a frequency agile, phased array weather radar can easily change by as much as 2 or 3 dBZ under practical operating conditions.

Inspection of (20) and (23a) shows the correction factors for reflectivity and sensitivity are the same. Thus, Figs. 3 and 4 may also be used to determine the sensitivity corrections for frequency hopping and beam pointing.

V. CALIBRATION

The corrected weather radar equation, (20), clearly indicates the calibration requirements for a frequency agile, phase scanned weather radar.

A. Radar System Calibration Constant

The first of the calibration requirements is the measurement of the radar system calibration constant. The measurement should be made using a reference target of known cross section σ. The power P_{out} returned from the reference target at range R should be measured with the antenna beam on boresight ($\theta = 0$), at the reference frequency f_0. The reference target should be located at the point of maximum beam gain. This provides the measurement data required to evaluate the radar system calibration constant,

$$\left[\frac{P_tG_{rx}G_0^2}{L_s}\right] = \left[\frac{(4\pi)^3R^4P_{\text{out}}}{\lambda_0^2\sigma}\right] \qquad (16)$$

that appears in (20). As noted above, if a broadside measurement cannot be made, (15) can be used to correct a measurement made with the beam pointed at an angle θ with respect to the normal to the array face.

B. Broadside Half-Power Beamwidths

The second of the calibration requirements is the measurement of the radar antenna broadside principal plane beamwidths $\theta_{B0}^{br}, \varphi_{B0}^{br}$ (beam on boresight at reference frequency f_0). This provides the measurement data required to evaluate the term

$$\left[\frac{1}{(\theta_{B0}^{br}\varphi_{B0}^{br})}\right] \qquad \text{from (20)}$$

that appears in (20). Unfortunately, an off boresight measurement of beamwidths is problematic. For an arbitrary pointing angle θ, the orientation of the ellipse that defines the beam principal planes changes direction. Thus, the directions of the axes, u and v, along which the principal plane beamwidths θ_u and φ_v must be measured and are not so easily determined. Careful processing and subsequent correction of data obtained from a two-dimensional scan of the reference target would be required.

The remaining terms in (20) can be evaluated from a knowledge of pulsewidth, reference frequency, transmit frequency, receiver average output power, and beam pointing angle. Thus, the calibration requirements for a frequency agile, phased array weather radar are really quite simple in principle though they may be difficult to implement in practice if boresight measurements cannot be made.

C. Beam Solid Angle Correction

The half-power beamwidths in the Probert-Jones equation are present to account for the beam solid angle contributing to backscattered power. The analytical result derived by Probert-Jones [1] is

$$\iint |f(\theta,\varphi)|^4 d\Omega = \frac{\pi}{4}\frac{\theta_B\varphi_B}{2\ln 2}(1-\delta), \qquad \delta \le 0.034 \tag{24}$$

where $f(\theta,\varphi)$ is the antenna pattern function. The bound on δ implies the error in the result is no greater than 0.15 dB. If the antenna pattern is known, either analytically or from measurement, numerical integration can be used to determine any correction that should be applied to (24).

To illustrate, consider a circular aperture of radius a with parabolic taper,

$$E_a(r) = \left[1-\left(\frac{r}{a}\right)^2\right]^n, \qquad r \le a \tag{25a}$$

where r is the distance from the center of the aperture. The normalized far field pattern function for this distribution is [9]

$$f(\theta,n) = \frac{2^{n+1}(n+1)!J_{n+1}(\beta a\sin\theta)}{(\beta a\sin\theta)^{n+1}} \tag{25b}$$

and therefore the required beam solid angle correction factor is

$$k_n = \frac{\displaystyle\int_0^{2\pi}\int_0^{\pi/2} |f(\theta,n)|^4 \sin\theta\, d\theta\, d\varphi}{\displaystyle\frac{\pi}{4}\frac{\theta_B^2}{2\ln 2}}. \tag{26}$$

Consider a circular aperture with a parabolic taper $n = 1$ and principal plane beamwidths $\theta_B =$ 0.9 deg. This taper gives a theoretical sidelobe level of -24.6 dB. Fig. 5 shows the normalized gain of the aperture compared with the Gaussian beam pattern approximation,

$$G(\theta) = e^{-2.776(\theta/\theta_B)^2} \tag{27}$$

assumed by Probert-Jones. It can be seen that the Gaussian approximation to the gain pattern overestimates the backscattered power in the region near the first null. Numerical integration can be used to evaluate (26) to find the correction factor $k_1 =$ -0.76 dB. Other taper and beamwidth combinations yield different correction factors.

Further evidence of this residual error in the Probert-Jones equation can be readily found. The Colorado State University CHILL weather radar uses a published correction factor of +0.5 dB to the radar constant based on pattern integration [10]. This is equivalent to a correction factor of -0.5 dB as defined in (26).

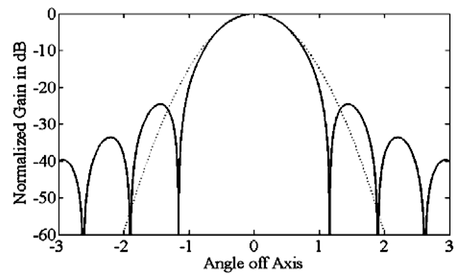

Fig. 5. Gain pattern for circular aperture with $n = 1$ parabolic taper (solid curve) compared with Gaussian pattern approximation (dashed curve). Half-power beamwidth is $\theta_B = 0.9$ deg.

The two examples above show that a reflectivity estimate computed using the Probert-Jones equation will be in error by an amount equal to the beam solid angle correction factor (see (26)) if the correction factor is not determined and included in the equation. While the procedure for correcting for actual beam solid angle is straightforward in principle, it is not so straightforward in practice. It requires knowledge of the antenna pattern function based on either theory, measurement, or numerical modeling.

D. Calibration Alternatives

The sections above describe the direct measurement of the radar calibration constant and broadside beamwidths. That requires a measurement of echo signal power when a reference target is viewed with the main beam in the broadside position. While this is the most straightforward approach, it is certainly possible, as noted, to make measurements with the beam pointed away from the broadside position. In this case, the measurements must be corrected to obtain the desired results using (7) and (10).

The most commonly used reference target is a sphere which has an easily determined radar cross section that is independent of aspect angle. Radar calibration spheres are inexpensive and commercially available. There are other calibration approaches, however, including active calibration and solar calibration. An excellent summary of calibration alternatives and their relative merit is contained in [6]. A detailed discussion of solar calibration is given in [7] and measurement data is presented in [8].

VI. CONCLUSIONS

This paper presents the derivation of a correction to the classical Probert-Jones weather radar equation. It is well known from antenna theory that the gain and beamwidths of a planar array will change with frequency and that these parameters will also vary with beam pointing angle. However, it is not so obvious that the scanned beam of a phased array describes a tilted ellipse on the surface of a sphere. The solid angle of the scanned beam varies with pointing angle and is computed from the beamwidths defined by the major and minor axes of the tilted ellipse. It is shown that the solid angle for an arbitrary beam pointing direction can be related to the solid angle of the more easily visualized and better understood broadside beam. Thus, knowledge of the broadside beam solid angle along with the beam pointing angle, transmit frequency, and reference frequency is sufficient to permit computation of the solid angle when the beam is pointed off boresight and frequency is hopped.

It is also clear that the echo power from precipitation will vary with frequency and that the angular dimensions of the radar resolution cell will change with frequency and beam pointing angle as discussed above. It is not immediately obvious, however, what impact this will have if the classical (uncorrected) Probert-Jones equation is used to compute reflectivity. This paper shows that the application of array fundamentals to the Probert-Jones equation leads to a correction of the weather radar equation that simply and elegantly incorporates the effects of frequency agility and electronic beam pointing. It is important to account for these effects if the reflectivity of precipitation is to be accurately estimated. The accuracy of the reflectivity estimate in turn impacts the estimation of rainfall rate.

The theoretical framework presented here also clearly reveals the calibration requirements for a frequency agile, electronically scanned weather radar. Ideally, two measurements should be made with the beam normal to the array face. First, echo signal power from a reference target of known cross section should be measured to determine the radar constant. Second, the principal plane beamwidths should be measured with the beam pointed in the broadside direction. These beamwidths determine the broadside beam solid angle. Although measurements for an arbitrary pointing angle θ are possible in principle, it is simpler if measurements can be made with the beam normal to the array face.

There remains some residual uncertainty associated with the Probert-Jones factor relating 3 dB beamwidth to the beam solid angle used to determine the effective size of the radar resolution cell. The Probert-Jones equation incorporates a factor $2\ln 2$, which is claimed to be accurate to within 0.15 dB. The exact value of the factor will depend on beam pattern, however, and for a theoretically or experimentally known beam pattern the factor can be determined numerically. The theoretical example included here assumed a circular aperture with a parabolic taper and a 0.9 degree beamwidth, yielding a correction factor of -0.76 dB. The Colorado State University CHILL radar uses a 0.5 dB correction to the Probert-Jones equation based on pattern integration, suggesting a residual error of at least that magnitude. Both the theoretical example and the practical example indicate that the expression for beam solid angle in the Probert-Jones equation results in an uncertainty greater than the 0.15 dB claimed in [1].

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REFERENCES

[1] Probert-Jones, J. R.
The radar equation in meteorology.
Quarterly Journal of the Royal Meteorological Society, **88** (1962), 485–495.

[2] *Weather Radar Technology Beyond NEXRAD*.
National Research Council, ISBN: 0-309-08466-0, 2002.

[3] Elliott, R. S.
Antenna Theory and Design.
Englewood Cliffs, NJ: Prentice-Hall, 1981, 200–206.

[4] Skolnik, M. I.
Introduction to Radar Systems (3rd ed.).
New York: McGraw-Hill, 2001, 6.

[5] PopStefanija, I., Knorr, J. B., and Buczynski, P.
Advanced weather radar algorithms and techniques using a rapid-scanning X-band radar—first results.
Presented at the 32nd Conference on Radar Meteorology, Oct. 24–29, 2005, Albuquerque, NM, Paper P12R.3.
http://ams.confex.com/ams/32Rad11Meso/techprogram/session_18900.htm.

[6] Pratte, F., and Ice, R. L.
External radar calibration options applicable to the WSR-88D network.
Report on special project 20, Engineering Branch, Radar Operations Center, Norman, OK, Apr. 18, 2005.
http://www.roc.noaa.gov/eng/reports.asp.

[7] Sirmans, D., and Urell, B.
On measuring WSR-88D antenna gain using solar flux.
Report, Engineering Branch, Radar Operations Center, Norman, OK, Jan. 03, 2001.
http://www.roc.noaa.gov/eng/reports.asp.

[8] Puhakka, P. V. S.
Solar calibrations with dual polarization weather radar.
Presented at the 32nd Conference on Radar Meteorology, Oct. 24–29, 2005, Albuquerque, NM, Paper P12R.8.

[9] Stutzman, W. L., and Thiele, G. A.
Antenna Theory and Design.
New York: Wiley, 1998, 320, Table 7.1.

[10] Brunkow, D., et al.
Radar equation as applied in CHILL signal processing (PDR 2004).
http://chill.colostate.edu/chill_technical.html, June 14, 2004.

Waveform Design for MIMO Radars

We consider the problem of waveform design for multiple input/multiple output (MIMO) radars, where the transmit waveforms are adjusted based on target and clutter statistics. A model for the radar returns which incorporates the transmit waveforms is developed. The target detection problem is formulated for that model. Optimal and suboptimal algorithms are derived for designing the transmit waveforms under different assumptions regarding the statistical information available to the detector. The performance of these algorithms is illustrated by computer simulation.

I. INTRODUCTION

Recent advances in linear amplifier and arbitrary waveform generation technology, and the ever-increasing processing power, have spawned interest in the development of radar systems which attempt to make full use of the spatial and temporal degrees of freedom available to the radar transmitter. These technological advances make it possible to consider the design of radar systems which allow the transmitter full flexibility in selecting the transmitted waveform (within given bandwidth and power constraints) on a pulse-by-pulse and antenna-by-antenna basis.

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The flexibility of using a multiplicity of transmitted waveforms, and of adaptively adjusting these waveforms, offers significant performance advantages. Fundamentally, the additional degrees of freedom afforded by the ability to vary the transmit waveform can be used to optimize a desired performance criterion. For example, the waveform can be adapted to the target signature to enhance detectability, to increase clutter or interference rejection, or to improve robustness to multipath.

To put these works into perspective we note that the radar design is driven by the assumed models of the target and the interference-plus-noise environment. Targets are often modeled as point scatterers. However, as the resolution of radar systems increases, a better model is that of an extended target which is spread in range, azimuth, and Doppler. The target model can be deterministic or statistical: the former assumes that the target characteristics are fixed and known (possibly up to some unknown parameters which can be estimated), while the latter treats the target as a random variable and attempts to characterize its statistics. Similarly, different models can be used for the interference environment (clutter, jamming, nearby targets).

The work on optimum transmit-receive design in [1], [2], [3], for example, assumes a deterministic target model with a range spread, using a single transmit antenna, or an antenna with multiple polarization modes [4]. In a recent paper [5] we studied optimal waveform design for a single antenna radar. We presented a signal subspace framework which allowed the derivation of the optimal radar waveform for a given scenario and to evaluate the corresponding radar performance.

Recently there has been considerable interest in radar systems employing multiple antennas at both the transmitter and receiver and performing space-time processing on both, commonly referred to as multiple input/multiple output (MIMO) radar. This work has focussed almost entirely on the point target model, and assumes transmission of orthogonal signals on the different antennas. This makes it possible to separate the signals arriving from the different transmit antenna at the receiver, and to perform any transmit array processing functions on the receive side "after the fact." For example, one can scan the transmit beam across the illuminated area within a single dwell time, or perform adaptive beamforming to reduce interference and improve resolution [6–13]. (Note however that the coherent transmitter array gain is lost when doing the transmit beamforming after, rather than during, transmission). Employing adaptive processing it is possible to improve clutter rejection in ways that are not possible in conventional radar [14, 15]. MIMO radar can also provides angular diversity which is useful in some scenarios [16–18].