2009-08

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American Institute of Aeronautics and Astronautics, Inc

AIAA Guidance, Navigation, and Control Conference 10 - 13 August 2009, Chicago, Illinois
http://hdl.handle.net/10945/48624

AIAA Guidance, Navigation, and Control Conference
10 - 13 August 2009, Chicago, Illinois

AIAA 2009-6011

Fuel Optimization of Figure-8 Flight for Unmanned Aerial Vehicles

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This paper investigates characteristics of minimum-fuel figure-8 trajectories for an Unmanned Aerial Vehicle (UAV) at high altitude. Given that loitering over an area of interest (i.e., ground target) falls within the purview of UAV missions, previous research has shown that periodic circling flight, consisting of a boost arc (maximum thrust) and a coast arc (minimum thrust), improves the fuel consumption when compared to steady-state circling. Through numerical simulations, this work investigates the effectiveness of figure-8 flight for optimizing fuel consumption while loitering. The results show that the periodic flight improves the fuel consumption up to 5% when compared to steady-state flight. In addition, the optimal figure-8 trajectory shape is elongated compared to that of the steady-state flight. As demonstrated, this optimal control approach can improve the fuel consumption even while fuel is used during the coast arc.

Nomenclature

C_L	Lift Coefficient
C_D	Drag Coefficient
C_{D0}	Zero-Lift Drag Coefficient
κ	Induced Drag Coefficient Factor
m	Airplane Mass, [kg]
S	Wing Planform Area, [m^2]
T	Thrust, [N]
u	Inertial Velocity, [m/s]
M	Mach Number
$x, y, h(= -z)$	Ground-Fixed Axis, [m]
l	Flight Length, [m]
γ	Flight Path Angle, [deg]
ϕ	Bank Angle, [deg]
ρ	Atmospheric Density, [kg/m^3]
ψ	Flight Direction, [deg]
σ	Thrust Specific Fuel Consumption(TSFC), [kg/N/s]
Subscripts :	
$(\ )_B$	Boost Arc
$(\ )_C$	Coast Arc

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I. Introduction

Until more efficient fuels and alternate propulsion systems are available for aircraft, both manned and unmanned, there will be a need for improved fuel management, or energy management, for both civil and defense applications. Albeit, there are ongoing efforts to improve fuel economy via airframe aerodynamic design changes; however, the recent world economy crisis and the ever-present cyclical oil volatility has lead to more dramatic measures. In particular, long-endurance flights now, perhaps more than ever, demand more aggressive fuel management techniques such as minimizing fuel consumption by way of smart trajectory design. For example, recent U.S. combat operations have identified a significant gap in Intelligence, Surveillance, Reconnaissance, and Target Acquisition (ISR/TA) capability that has confirmed the need for real-time situational awareness throughout the battlespace in order to enhance timely decision making. This gap stems in part from a shortfall in long-endurance Unmanned Aircraft Systems (UAS) needed for persistent surveillance in support of combat operations and planning. To improve this capability, an obvious area of improvement is that of vehicle fuel management. For minimizing fuel use, an optimal steady-state flight is not always sufficient. To improve the fuel consumption, consideration must be given to a periodic flight that switches between maximum and minimum thrust levels.[1,2]

Typically, optimal fuel consumption flights are modeled as long range trajectories, but since UAV missions usually involve some form of circling flight in a prescribed area, such as loitering over a target, then circling or figure-8 trajectories should also be considered. Given the growing need for longer-endurance UAV missions, this is exactly the focus of this research work - figure-8 flight with constant flight length as if loitering over an area of interest. Recent research work has shown that periodic circling flight consisting of a boost arc (maximum thrust) and a coast arc (minimum thrust) improves the fuel consumption more than that of steady-state circling.[3–9] Previous research analyzed the effect of periodic flight for the figure-8 with various flight lengths, but the prescribed minimum thrust value was unrealistic.[10]

It is the purpose of this paper to analyze how optimal periodic flight during figure-8 maneuvers influences fuel consumption compared to steady-state flight. To do so, an optimal control problem is formulated and solved using a pseudospectral-based method. The numerical results include a comparison between minimum-thrust and thrust-specific fuel consumption (TSFC) profiles.

II. Problem Formulation

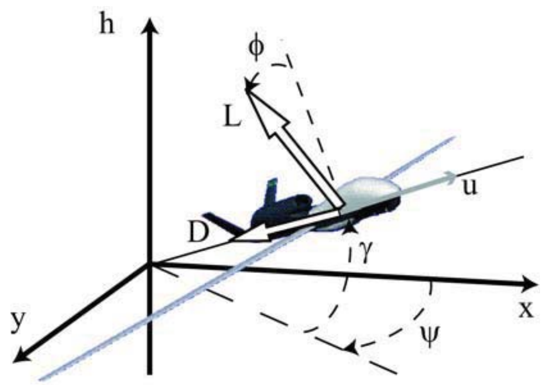

Figure 1. Coordinate System and Reference Frames of the UAV

The cycle of altitude variation for the generated flight trajectories in this paper is greater than the general frequency of a phugoid mode for a rigid body model. Thus, using a point mass model is sufficient for numerical analysis. The point-mass equations of motion for a UAV with respect to Fig.1 are written below as Eq.(1)-(8).

$$\begin{aligned}\dot{u} &= \frac{1}{m}(T - D - mg\sin\gamma) &(1)\\ \dot{\gamma} &= \frac{1}{mu}(L\cos\phi - mg\cos\gamma) &(2)\\ \dot{\psi} &= \frac{L\sin\phi}{mu\cos\gamma} &(3)\\ \dot{x} &= u\cos\gamma\cos\psi &(4)\\ \dot{y} &= u\cos\gamma\sin\psi &(5)\\ \dot{h} &= u\sin\gamma &(6)\\ \dot{l} &= u\cos\gamma &(7)\\ \dot{m} &= -\sigma T &(8)\end{aligned}$$

Here

$$\begin{aligned}L &= \frac{1}{2}\rho u^2 S C_L &(9)\\ D &= \frac{1}{2}\rho u^2 S(C_{D0} + kC_L^2) &(10)\end{aligned}$$

and the control variables are constrained by Eq.(11)-(13). Since variation of altitude with respect to the reference altitude is only ± 600 m, the atmospheric density is defined as a constant value. For this reason, maximum thrust T_{max} and TSFC σ are also assumed as constant values.

$$\begin{aligned}T_{min} &\leq T \leq T_{max} &(11)\\ C_{Lmin} &\leq C_L \leq C_{Lmax} &(12)\\ \phi_{min} &\leq \phi \leq \phi_{max} &(13)\end{aligned}$$

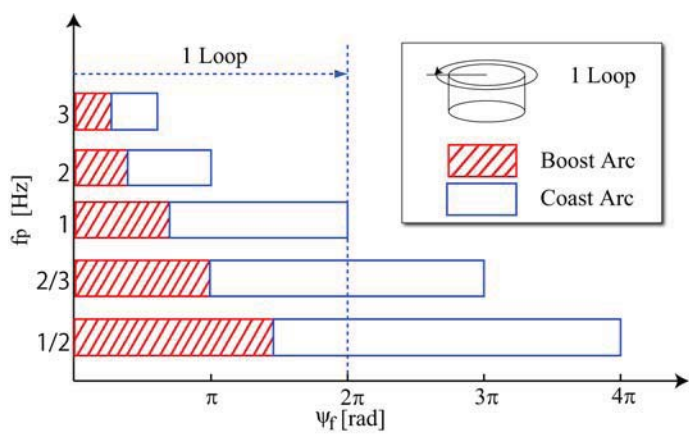

Figure 2. Periodic Flight Frequency f_p [Hz] for Circling Flight

The periodic flight consists of a boost arc (maximum thrust) and a coast arc (minimum thrust). The periodic frequency for a given periodic flight scenario is defined in Fig.2 for the circling flight.[5] Similarly, the periodic frequency of the figure-8 flight can be defined as one periodic flight scenario for each figure-8 shape. In this work, the periodic frequency for flight around one figure-8 is chosen to be $f_p = 2$[Hz] as indicated in Fig.3. The boost arc for the first period is A∼B and the coast arc is B∼C. The second period is C∼D and D∼A, respectively. Since both periods are symmetrical about the x-axis (an artifact from assuming zero wind), numerical computation is only required for the first period, A∼C. As such, the symmetrical

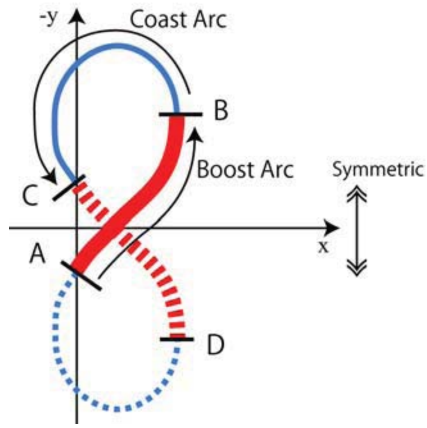

Figure 3. Outline of Boost Arc and Coast Arc Configuration for Figure-8 Flight

connection between the initial point A and the final point C has to satisfy Eq.(14)∼(22).

$$u(0) - u(t_f) = 0 \tag{14}$$
$$\gamma(0) - \gamma(t_f) = 0 \tag{15}$$
$$\psi(0) + \psi(t_f) = 2\pi \tag{16}$$
$$x(0) - x(t_f) = 0 \tag{17}$$
$$y(0) + y(t_f) = 0 \tag{18}$$
$$h(0) - h(t_f) = 0 \tag{19}$$
$$h(0) - h_0 = 0 \tag{20}$$
$$l(0) = 0 \tag{21}$$
$$l(t_f) = l_f \tag{22}$$

The switch point, B, between the boost arc and the coast arc is optimized during numerical computation.

From these assumptions, the optimal control problem (B) to minimize the fuel consumption per flight length l_f is stated as follows.

$$\boldsymbol{X} = \begin{bmatrix} u(t) & \gamma(t) & \psi(t) & x(t) & y(t) & h(t) & l(t) & m(t) \end{bmatrix}^T \in \mathbb{X} \subseteq \mathbb{R}^8 \tag{23}$$
$$\boldsymbol{U} = \begin{bmatrix} T(t) & C_L(t) & \phi(t) \end{bmatrix}^T \in \mathbb{U} \subseteq \mathbb{R}^3 \tag{24}$$

$$(B)\begin{cases} \text{Minimize} & J[\boldsymbol{X}(\cdot), \boldsymbol{U}(\cdot)] = \dfrac{1}{l_f}\displaystyle\int_0^{t_f} \sigma T(t)dt \\ \text{Subject to} & Eqs.(1) - (22) \end{cases}$$

Now with the problem posed as a standard optimal control formulation, it is readily solvable employing a nonlinear optimization tool.

III. Numerical Results

The data for the numerical simulations is based on the Global Hawk RQ-4B, where $m_0 = 9,100$[kg], $T_{max} = 37,000$[N] at sea level, $C_{D0} = 0.017$, $\kappa = 0.016$ and $\sigma = 1.8123 \times 10^{-5}$[kg/N/s]. The initial altitude is constrained at $h_0 = 17,500$[m] and the flight length for the first segment A∼C is $l_f = 60$[km]. Thus, the total flight length of the figure-8 flight is 120 km. The speed of sound is 295.07 m/s and the maximum thrust would be approximately $T_{max} = 4,000$[N] at altitude h_0.

The optimal control problem is solved by a modified method based on a Jacobi pseudospectral collocation technique[11] and the time-axis folding method.[2] Optimality of the solution is verified by the dual values of the optimal control problem which are obtained from the solver.[12]

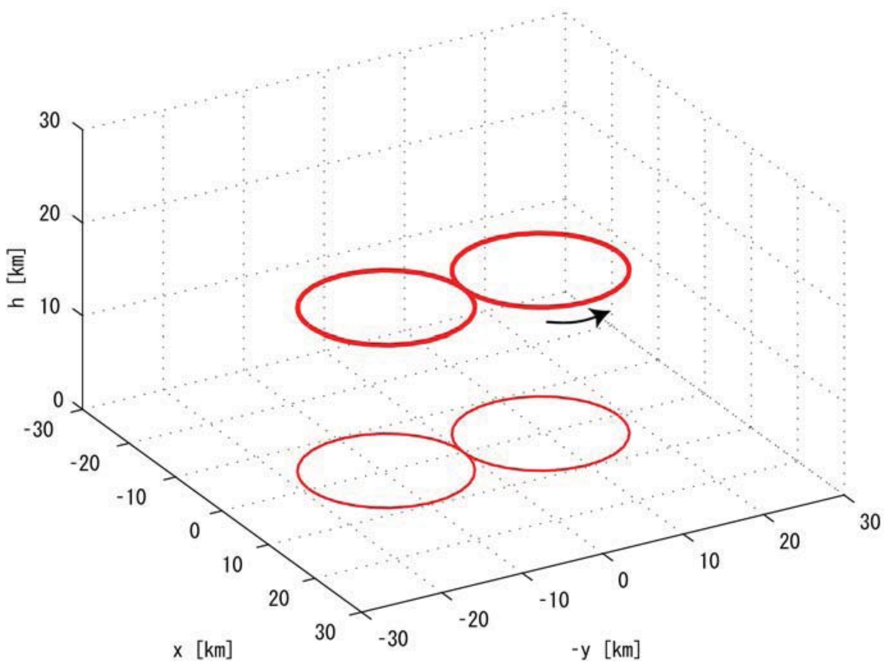

Figure 4. Trajectory of the Steady-State Flight

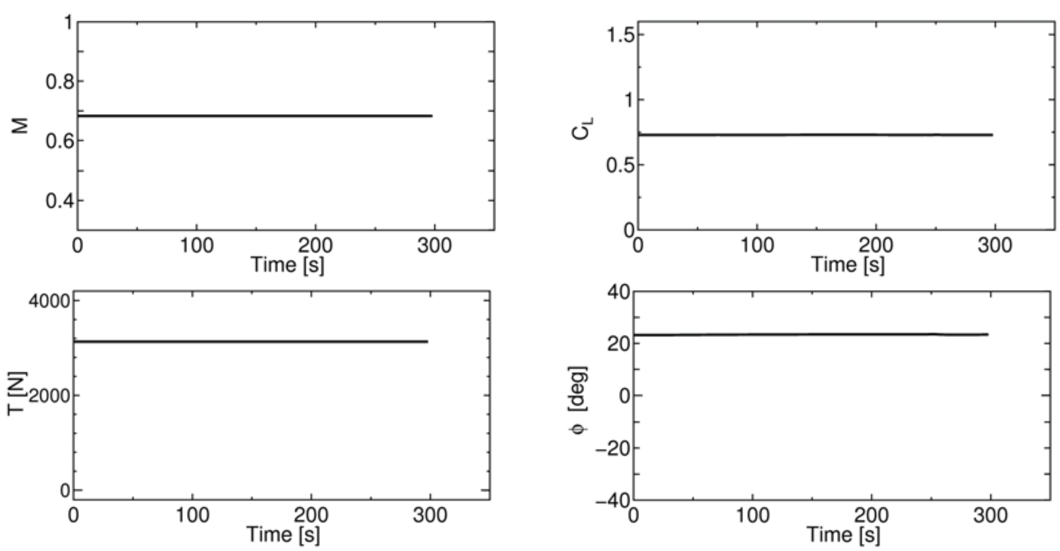

Figure 5. State and Control Variables of the Steady-State Figure-8 Flight for One Circle

III.A. Steady-State Flight

The steady-state figure-8 flight is calculated to compare it with the periodic flight. Fig.4 shows the calculated trajectory with the addition of its symmetrical trajectory. It is clear that the obtained optimal trajectory is a combination of two perfect circles. Fig.5 shows the time response of velocity (in Mach number) and control variables for one circle. Since the bank angle ϕ is held constant during the steady-state turning, it only needs to switch sign at the intersection of the circle trajectories.

For each circle, the thrust is a constant 3133 N which is 78 % of its maximum value, and the corresponding fuel usage is 16.92 kg. The fuel consumption for the periodic flight is expected to be less than this value.

III.B. Periodic Flight

The following three cases are evaluated for periodic flight. Note that for the corresponding figures of each case, the boost arc is indicated by the thick lines and the coast arc by the thin lines.

CASE A : $T_{min} = 0$
CASE B : $T_{min} = 0.25T_{max}$, $\sigma_C = \sigma_B$
CASE C : $T_{min} = 0.25T_{max}$, $\sigma_C = 1.5\sigma_B$

CASE A is a part of the previous work[10] which focused on the various flight lengths for the figure-8 flight.

CASE A

In this case, there is no fuel use during the coast arc as indicative of zero thrust. Fig.6 shows the combined calculated and symmetrical trajectories. The optimal trajectory shows that the boost arcs are arranged around the intersection area and the coast arcs are arranged around the curved ends. Compared to Fig.4, the optimal trajectory is elongated along the y-axis direction. Fig.7 shows the time response of state and control variables for one period.

Since the fuel usage is 16.06 kg for one period, the periodic flight saves 5 % of fuel in comparison with the steady-state flight. Fig.8 shows the fuel rate for periodic flight with respect to various flight lengths.[10] This simple comparsion clearly demonstrates that the periodic flight improves the fuel consumption from that of the steady-state flight.

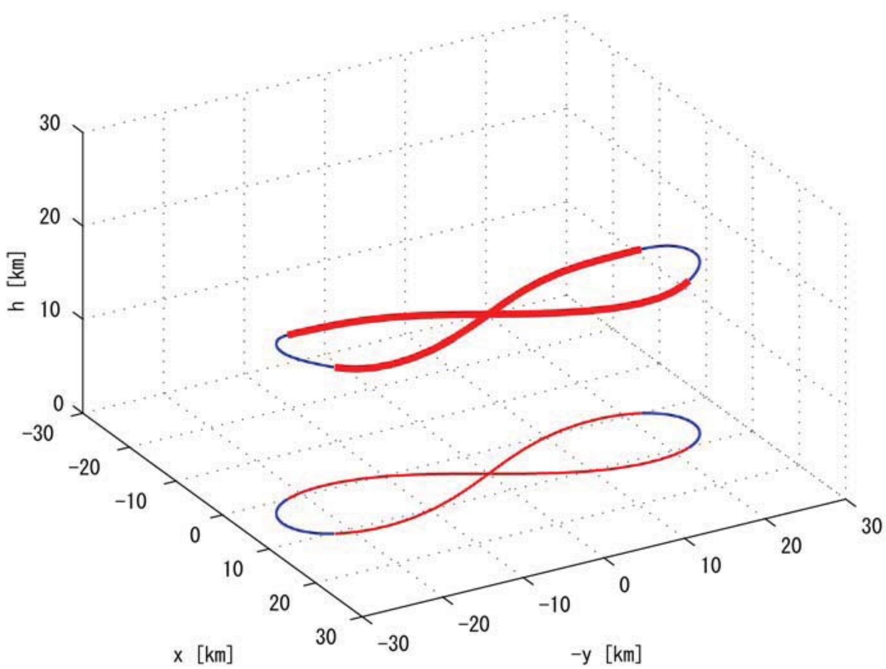

Figure 6. Trajectory of the Periodic Flight; CASE A

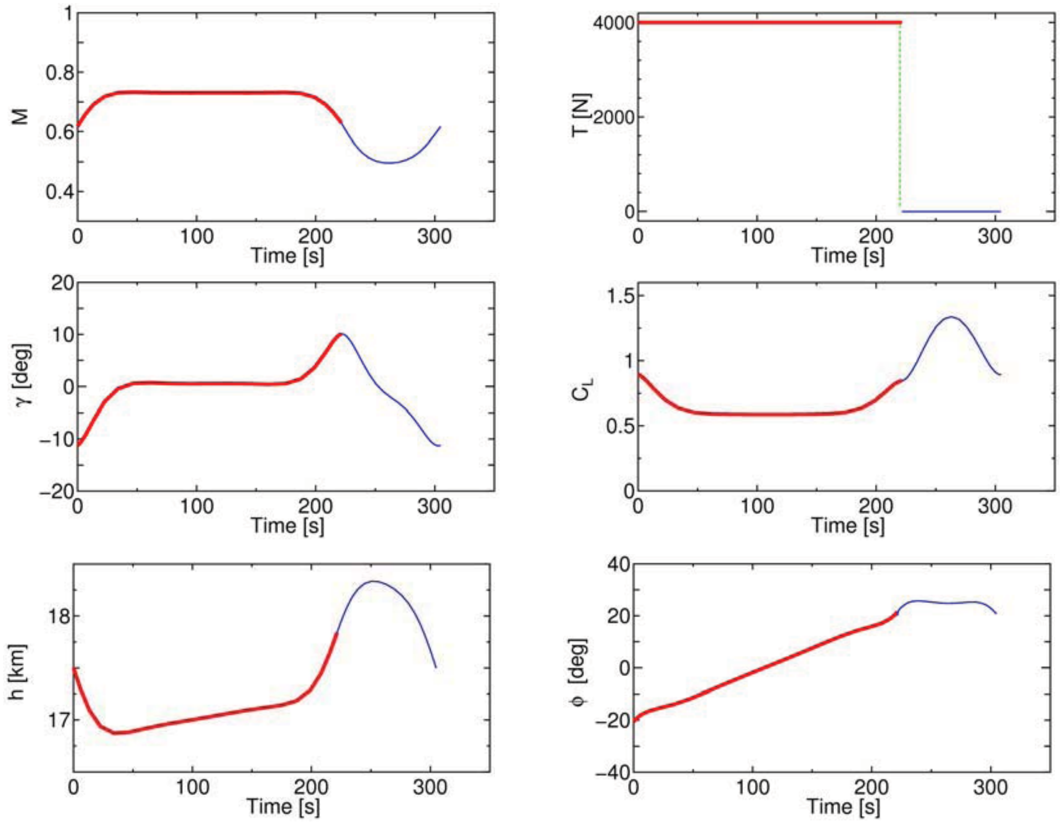

Figure 7. State and Control Variables of the Periodic Flight for First Segment; CASE A

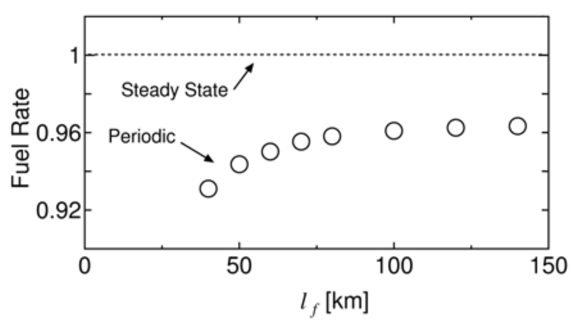

Figure 8. Fuel Rate for Periodic Flight

CASE B

Because fuel is used during the idle setting of the jet engine, CASE A is not realistic for a practical implementation. Therefore, the minimum thrust, which is used during the coast arc, is defined as 25 % of the maximum thrust for CASE B.

Fig.9 shows the calculated trajectory and Fig.10 shows state and control variables. There is no significant difference between CASE A and CASE B. The fuel usage for this case is 16.26 kg. Though using more fuel than CASE A, it still saves 4 % of fuel from the steady-state flight.

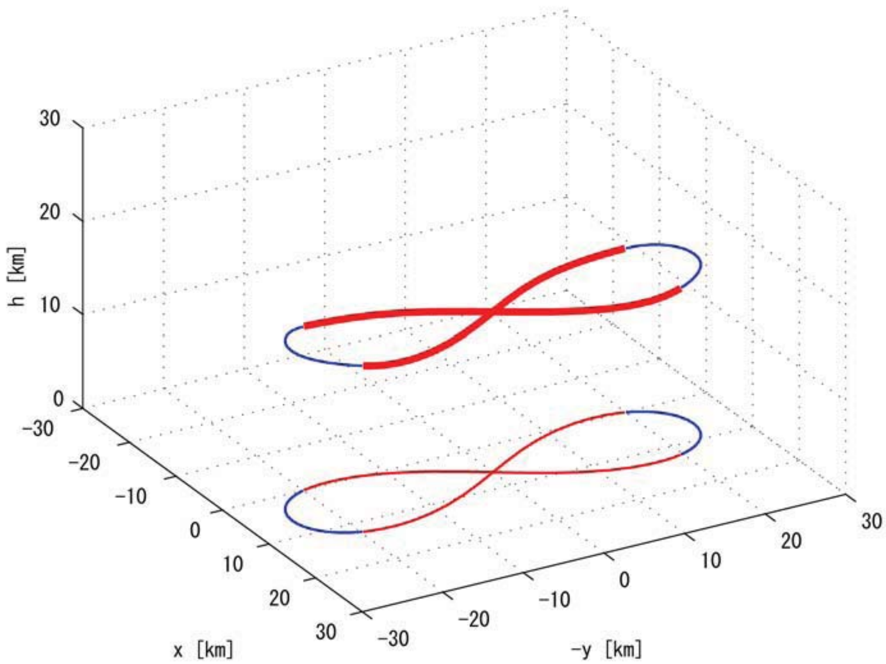

Figure 9. Trajectory of the Periodic Flight; CASE B

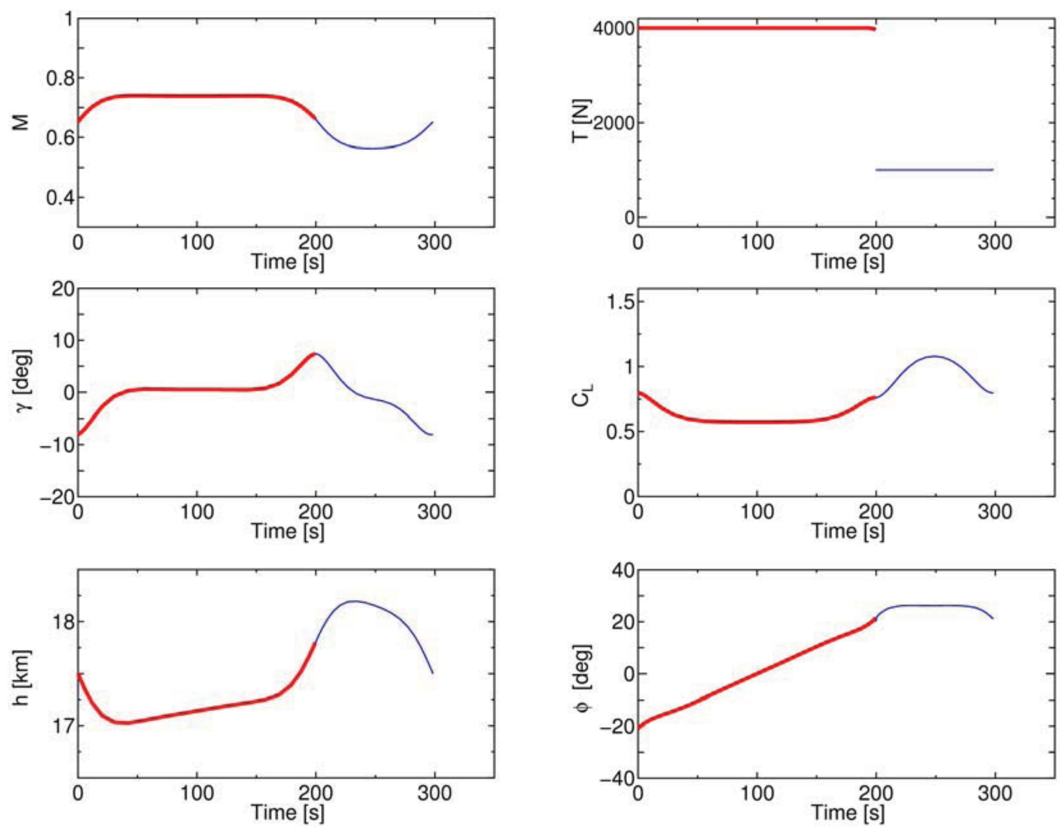

Figure 10. State and Control Variables of the Periodic Flight for First Segment; CASE B

CASE C

At the minimum thrust setting, the TSFC can be more than that at the maximum thrust setting. Therefore, in this case, TSFC of the coast arc σ_C is defined as being larger than the boost arc value ($\sigma_C = 1.5\sigma_B$) and again the minimum thrust is 25 % of the maximum thrust.

Fig.11 shows the calculated trajectory and Fig.12 shows the time response of state and control variables for one period. Compared to CASE A and CASE B, the overall velocity is increased 5 % and consequently this shortens flight time by approximately 20 seconds. In addition, the maximum bank angle during coast arc is increased from 26 deg to 32 deg in comparison with CASE B. Since the fuel usage for this case is 16.43 kg, it still saves 3 % of fuel from the steady-state flight.

IV. Conclusions

The optimal periodic flight for figure-8 maneuvers of a UAV has been analyzed by numerical simulations. As expected preliminary results demonstrate that the periodic flight improves fuel consumption compared to the steady-state flight by at least 3% and up to 5% for a completely unpowered scenario. This advantage still occurs even if the fuel is expended during the coast arc to a certain extent. In addition, these results illustrate the power and relative simplicity of using optimal control techniques, such as the pseudospectral-based method employed in this work, to help investigate how to improve the operation and flight characteristics of aerial vehicles performing periodic figure-8 flight. Overall, the approach used in this work for optimizing fuel utilization has proven to be a viable technique for applications requiring long-endurance flights.

V. Acknowledgement

The authors would like to acknowledge and thank Professor I. Michael Ross, Director of the Control and Optimization Laboratories at the Naval Postgraduate School, Monterey, CA, for providing us the first hand knowledge of pseudospectral-based optimal control and motivating us to explore its versatility in solving numerous problems that demand optimal performance.

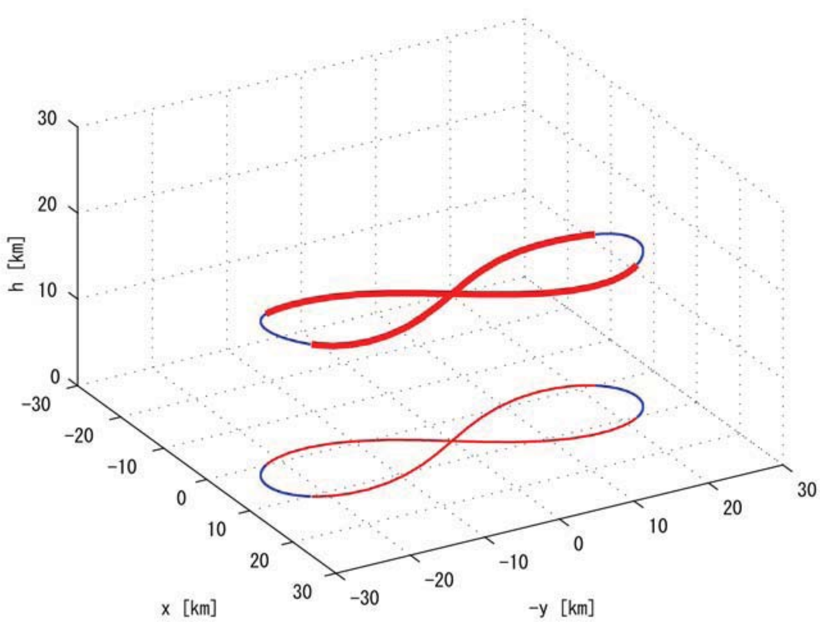

Figure 11. Trajectory of the Periodic Flight; CASE C

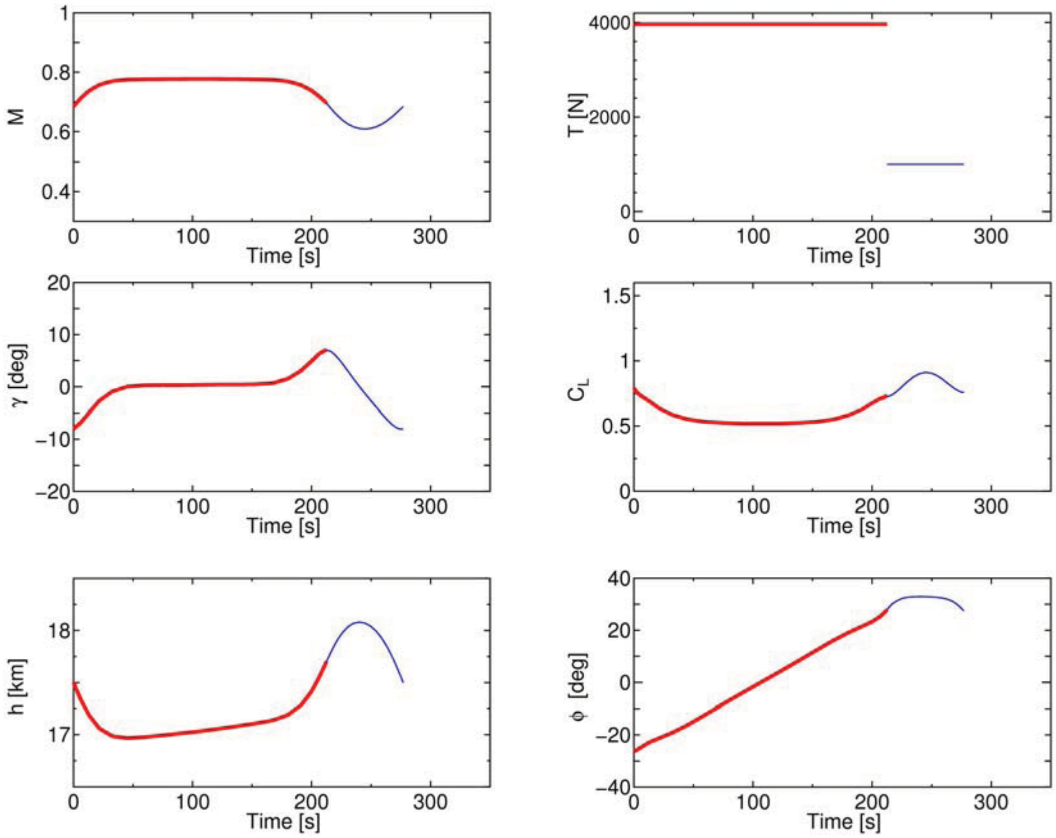

Figure 12. State and Control Variables of the Periodic Flight for First Segment; CASE C

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