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Collection probability with discrete random signal-to-interference power ratios in multiple interferences for electronic intelligence receivers

Ric A. Romero, Tri T. Ha, Stylianos Lioulis

ECE Department, Naval Postgraduate School, 833 Dyer Road, Monterey, CA 93943, USA
E-mail: rnromero@nps.edu

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Abstract: In certain applications, signal transmit power is adjusted in discrete levels. Moreover, interference powers themselves can be discretely random. For electronic intelligence applications, the authors refer to the signal to be collected as signal of interest (SoI). Thus, for a passive receiver, the resulting instantaneous SoI-to-interference power ratio (SIR) in multiple interferences is a discrete random variable. This means signal detection or collection is not guaranteed. They proposed collection probability (CP) as a metric when the SIR is random. In this study, the authors evaluate CP probabilistically and show CP against SIR threshold degradation and/or improvement as a function of increasing number of interferences, increasing signal power range and increasing distribution gain.

1 Introduction

1.1 Background

It has long been known that the ability to detect and finally decipher a received, collected or intercepted signal is directly related to signal-to-noise power ratio (SNR) [1]. It follows that in the case where the receiver is interference-limited rather than noise-limited, detection is a function of signal-to-interference power ratio (SIR) instead. Thus, a certain required detection probability corresponds to a required or needed SIR. Although this study applies to both passive and intended receivers alike, we use the perspective of a passive receiver. Passive receivers are usually used in electronic intelligence (ELINT) which is concerned with the collection or interception of various signals such as radar, radio and some data link (or communication-type) signals [2]. Of course for a signal to be deemed detected, the passive receiver systematically varies various signal collection parameters such as bandwidth and sweep time, until it correlates to the signal of interest (SoI) [3]. When this happens, the passive receiver is said to be matched to the SoI signal. Unfortunately, a few signal parameters may still be unknown. Exact signal power is an example although at times it can be estimated. However, estimation of SoI power becomes difficult when the signal power itself is random. Also, SoI power estimation becomes more difficult in the case where the passive receiver is in interference environment. In this study, we address the issue of SoI power being random. Specifically, we assume that the SoI power is discretely random. Making matters more difficult, we also assume the presence of multiple interferences and that their powers themselves are also discretely random. Varying transmit power signals is not uncommon and of course cellular base station power control is a good example. Indeed varying power levels (discrete or continuous) are used in transmitter gain control and automatic gain control. There are many references to list but [4, 5] are good introductions to the novice reader, while an earlier text by Hughes [6] deals with detailed design. More modern texts such as [7, 8] are also excellent references. Power control in communications and networking has been an active research area for a long time and thus there are too many of references to list but the works in [9, 10] are good starters for the interested reader. Variable power may take continuous or discrete values. The discrete nature of transmit power levels is also utilised for other purposes such as clustering in wireless sensor networks [11, 12]. Indeed, a specific set of variable gain amplifiers (VGA) known as digitally controlled amplifiers and variable attenuators are readily available and specifically built to facilitate this particular design. VGAs and variable attenuators are common radio frequency (RF) components and so is the literature that pertains to them, but we list a few interesting devices here [13–16]. It should be noted that the same devices may also be used for receive functionalities.

1.2 Motivation

Consider first the case when the SoI power and interference power are deterministic. Then we can easily calculate the SIR power value. If it is high compared to some SIR threshold, then the SoI collection will be successful. Otherwise it will not be. Now consider the case when the SoI is transmitted in different possible power discrete levels. For a passive receiver with no a priori knowledge of how the transmit power is varied, the collected signal power is then discretely random. Here, we assume that the signal and interference (or multiple interferences) are collected by a single passive receiver. Since the received powers are random, the power of an interference (or more interferences) may exceed the SoI power, that is, a SIR realisation could easily be low, and therefore could prevent the reliable collection and eventual detection of the SoI waveform. Thus, meeting a certain required SIR is not guaranteed, that is, meeting the SIR requirement is probabilistic. For ELINT applications, there is very little in the literature that addresses a proper metric in terms of signal collection when the received signal power is random and the interferers' powers are discretely random. Thus, we introduce collection probability (CP) as a metric when SIR is random. As in any ELINT receiver, our main interest is in the interception of various signals such as radar, radio or communications-type signals, that is, RF modulated signals. Indeed, our formulation also applies to random process-like signals such as noise radar waveforms. For the intended receiver, it is usual to specify the metric of bit error rate (BER) for communications or probability of detection (P_{d}) for radar and a corresponding required SIR is needed to meet either of these required probabilities. For an intended receiver (such as in communications applications), the power schedule may be known. For a passive receiver, the power schedule may not be known and thus SIR is random. We perform probabilistic analysis to characterise SIR. Since the SIR is discretely random, its distribution is a probability mass function (pmf). The pmf or its corresponding cumulative mass function (cmf) quantifies the probability distribution of SIR. We recall that in a random variable (RV), the cmf quantifies the probability of that RV taking on a range of values.

For example, consider a SIR random variable S with cmf $F(s)$, then $F(s) = P\{S \le s\}$. In other words, $F(s)$ quantifies the probability corresponding to the SIR less than or equal to a given value s. If we are interested in meeting a required SIR, then we are interested in SIR being greater than some s (i.e. $P\{S > s\}$). Thus, we need the complement 'one minus the cmf'. In other words, if we require a certain SIR threshold to be exceeded, then we desire to quantify the probability that the SIR is greater than that threshold. We refer to this probability as collection probability (CP) and thus the 'one minus the cmf' is the CP against SIR threshold distribution. The term 'collection probability' is a good choice since it distinguishes it from a different metric in electronic warfare (EW) called probability of intercept (POI) [17, 18]. In EW, the definition of POI is very much dependent on what is known, unknown and/or measured about the SoI, which includes the emitter–receiver range, antenna gains and pattern alignment, waveform duty cycle, interception time and so on. Thus, formally we define collection probability as that probability that a desired performance (BER, P_d, or POI) may be supported when the SIR is random. Alternatively, the original SIR cmf mentioned above quantifies a form of outage probability [19]. The original SIR cmf specifically describes the quantity as collection outage probability (COP) against SIR threshold. Thus COP may be chosen instead of CP as the metric of performance. Since CP and COP are complement, it is easy for a designer to switch from one to the other in forming a system specification. In our results section, we will present CP against SIR threshold remembering that COP against SIR threshold is just the complement. CP may also be a function of what type of detector is used. For example, an energy detector requires a higher SIR than a correlator receiver. Thus, a passive receiver that uses an energy detector may require a higher SIR than an intended receiver which uses a correlator assuming all other parameters are equal. Detector design in various forms of interferences and its effect on detection probability has a rich literature and would not be covered here. It suffices to say that once the needed performance to be supported and its corresponding SIR requirement are specified by a system designer for the passive receiver, then the designer can perform probabilistic analysis to find the corresponding CP.

A more concrete example of a signal collection problem occurs in the use of a high altitude platform to collect signals from the ground emitters. The emitter of interest may be surrounded by co-channel emitters (as in cellular communications) that transmit in the same frequency band causing severe interference. The formulation in this paper deals with both non-identical and identical distributions and non-uniformly spaced and uniformly spaced discrete power RVs, which are reflections of what is encountered in practical scenarios. However, we also consider the special case where the interferers have identical uniformly distributed and uniformly spaced discrete powers, that is, the powers are distributed from 1, 2, …, L and thus the sum power RV (of the interferences) is the sum of the individual interference powers, which are in themselves RVs. One may think of this sum as a discrete version of the Irwin–Hall distribution, but we point out the obvious differences. If the RVs were continuous and a RV is only constrained in [0, 1], then the sum interference power is given by the Irwin–Hall distribution [20, 21]. In our case, our RVs are discrete in nature (because of the nature of the discrete power transmit levels). The power values may be uniformly spaced, that is, 1, 2, …, L; although this may not necessarily be the case specially for particular applications. For a continuous RV, the exact probability at a certain power value is undefined while the probability of a discrete power level for the discrete RV is clearly $1/L$. Thus, we derive the particular discrete equivalent of the Irwin–Hall distribution which to our knowledge has not been reported before. Using the general expression for discrete ratio RV, we can generate a pmf of the SIR. In practical designs (for intended receivers), it is usually ensured that maximum signal power received is greater than that of the interference power to ensure minimum specified performance. Thus, we allow for a signal distribution where discrete powers are in the set 1, 2, …, K, where $K > L$. Then, we investigate the performance improvement as a function of increasing the maximum signal power (or increasing power range) allowed in the distribution. In practice, mitigation techniques for co-channel interference are applied. One example is beamforming which means increasing the distribution gain. The effect of increased distribution gain in performance is also investigated.

The contributions of this paper are as follows: (a) the introduction of the metric collection probability for ELINT passive receivers when SIR is discretely random due to discrete but random nature of SoI and interference powers; (b) the discrete version of the Irwin-Hall distribution which to our knowledge has not been published before; (c) the simple realisation that a window of SoI collection is created despite the SIR being random when SoI and interferer powers are equally (identically) distributed compared to when the SoI and interferer powers are equal and deterministic; and (d) the numerical evaluation of CP against SIR threshold via probabilistic analysis as a function of increasing number of interferers, increasing SoI power range (i.e. increasing maximum gain) and increasing distribution gain. This paper is outlined as follows. Section 2 discusses preliminary modelling of discrete power RV. Section 3 discusses sum of discrete power RVs, which we need to sum interference powers. Here, the discrete equivalent of Irwin–Hall distribution is derived and reported. For discrete RVs that are not uniformly distributed and non-uniformly spaced, we present an iterative equation to derive the pmf of the sum of ratio discrete RVs which is used for finding sum power distributions. Section 4 discusses the ratio of discrete RVs which we need to model SIR distribution and which allows us to calculate collection probability given a SIR threshold. In Section 5, we consider various signal and interference distribution possibilities and number of interferences to quantify CP performance against SIR threshold. Section 6 contains our conclusions.

2 Discrete power random variable

Let X be the power RV which represents the set of possible received discrete power levels of the SoI. We can assume the SoI to be a communications signal (modulated waveform) or noise-like (as in noise radar waveforms). Let Y be the RV which represents the set of possible received discrete power levels of an interference signal. We assume that the two are independent RVs. We point out that variation in power is a large-scale effect; that is, which although random, the RV takes on discrete power value and may stay on that value for time much greater than a symbol time. This is to clearly differentiate it from small-scale effects such as Rayleigh fading, whose time duration effects are in the order of symbol duration [22]. The latter is not our interest but there are plenty of references addressing how to improve detection via diversity in this case. Again, our interest is the former, that is, large time-scale changes on the random discrete power. Let the signal be $v(t)$, T is the collection time and the energy in that duration is designated as E_v. Then the set of discrete possible modulated signals is given by $a_1 v(t)$, $a_2 v(t)$, …, $a_L v(t)$. Since

$$\frac{1}{T}\int_0^T \left|a_1 v(t)\right|^2 \mathrm{d}t = \left|a_1\right|^2 \frac{E_v}{T} = \left|a_1\right|^2 P$$

where P is clearly the average power in duration time T, then ensemble of power signals is simply the set

$$X \in \left\{x_1 = \left|a_1\right|^2 P, \quad x_2 = \left|a_2\right|^2 P, \ \ldots, x_L = \left|a_L\right|^2 P\right\} \qquad (1)$$

each of which clearly has a probability$1/L$. If P is normalised

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($P = 1$), then

$$X \in \left\{x_1 = |a_1|^2, \quad x_2 = |a_2|^2, \ldots, x_L = |a_L|^2\right\} \tag{2}$$

If the SoI is noise-like much like a noise radar, then the power or variance of a zero-mean $v(t)$ with variance σ_v^2 is given by

$$E\left[|av(t)|^2\right] = |a|^2 E\left[|v(t)|^2\right] = |a|^2\sigma_v^2$$

and thus

$$X \in \left\{x_1 = |a_1|^2\sigma_v^2, \quad x_2 = |a_2|^2\sigma_v^2, \ldots, x_L = |a_L|^2\sigma_v^2\right\} \tag{3}$$

If the variance is normalised where $\sigma_v^2 = 1$, then X is simply described by (2). We can assume the interference to be a waveform with structure (e.g. co-channel interference in cellular communications) or noise-like (e.g. broadband jamming). Consider the case of one interference and that interference is a co-channel interference. Let the interference signal set be $b_1 r(t)$, $b_2 r(t)$, …, $b_K r(t)$, then

$$Y \in \left\{y_1 = |b_1|^2 P_Y, \quad y_2 = |b_2|^2 P_Y, \ldots, y_K = |b_K|^2 P_Y\right\} \tag{4}$$

each of which clearly has a probability $1/K$. If the power is normalised, then

$$Y \in \left\{y_1 = |b_1|^2, \quad y_2 = |b_2|^2, \ldots, y_K = |b_K|^2\right\} \tag{5}$$

If we let Y be a zero-mean noise interference (as in broadband jamming) with $E\left[|br(t)|^2\right] = |b|^2 E\left[|r(t)|^2\right] = |b|^2\sigma_r^2$, then

$$Y \in \left\{y_1 = |b_1^2|\sigma_r^2, \quad y_2 = |b_2|^2\sigma_r^2, \ldots, y_L = |b_K|^2\sigma_r^2\right\} \tag{6}$$

If $\sigma_r^2 = 1$, then the set of Y possible values reduces to (5). So, a particular SIR in an ensemble set of many possible ratios would be $w = x/y$ and it is clear that there are many discrete ratios (LK for just one interference) and each of which has a probability associated with it, depending on the power distributions of the SoI and interferer. More interferers result in more SIR elements in the set of the resulting SIR random variable. If the signal powers or variances are normalised, then it is simply $w = |a^2|/|b^2|$ regardless whether the signal or interference is noise-like or not.

3 Sum of interference powers

3.1 Distribution of sum of RVs

Before we discuss ratio distribution, we need to discuss sum distribution in the case of multiple interferences. The sum of two distributions is widely known [23, 24] and need not be covered here. In general, the power distribution of an individual interference may not be uniformly spaced. Moreover, each distribution may be different from others but is independent from others. Thus, a closed-form expression for the sum power of all the interferences depends on the individual power interference distributions. However, we can form an iterative equation that allows us to form the sum distribution if the individual distributions are given. Let $S = \sum_{i=1}^{L} S_i$ be the sum distribution. Then, the sum is given by the following recursive equation

$$p_S(s_n) = \sum_{s_{m,L}} \sum_{\hat{s}_{k,L-1}} p_{S_L}(s_{m,L}) p_{\hat{S}_{L-1}}(\hat{s}_{k,L-1}) \times \delta\left[s_n - \left(\hat{s}_{k,L-1} + s_{m,L}\right)\right] \tag{7}$$

where $\hat{S}_{L-1} = \sum_{i=1}^{L-1} S_i$ is the recursive RV sum of $L-1$ signal terms, k is its resulting index, S_L is the last signal power RV, and m is its index.

3.2 Sum of identically uniform and uniformly spaced distributions

Consider the case where the power distributions of the interferers are uniformly and identically distributed. Moreover, consider the distributions to be uniformly spaced. Of course various distributions are obviously possible. Here we consider one special case for illustration and recall that (7) can be used for the general case. Let the sum of independent and identically distributed (iid) uniform RVs be $W = \sum_{i=1}^{M} V_i$, where $V_i \in \{1, 2, \ldots, N\}$, and clearly $p_V(v_n) = 1/N$. Our goal is to derive a closed-form expression for the distribution. We recall the z-transform [25] of a particular discrete RV V_i is given by

$$P_V(z) = \sum_{n=1}^{N} p_V(v_n) z^{-n} = \frac{1}{N}\sum_{n=1}^{N} z^{-n} \tag{8}$$

Using the identity $\sum_{n=1}^{N} a^n = a\left(1 - a^N\right)/(1 - a)$ and setting $a = z^{-1}$, (8) reduces to

$$P_V(z) = \frac{1}{N} z^{-1}\frac{1 - z^{-N}}{1 - z^{-1}} = \frac{1}{N} z^{-N}\frac{z^N - 1}{z - 1} \tag{9}$$

The z-transform of the sum random variable W can be calculated and is given by

$$\begin{aligned} P_W(z) &= N^{-M} z^{-MN}\frac{\left(1 - z^N\right)^M}{(1 - z)^M} \\ &= (-1)^{-M} N^{-M} z^{-MN}\frac{\sum_{k=0}^{M}\binom{M}{k}(-1)^k z^{kN}}{(z - 1)^M} \\ &= \left[(-1)^{-M} N^{-M}\sum_{k=0}^{M}\binom{M}{k}(-1)^k z^{N(k-M)-1}\right]\frac{z}{(z - 1)^M} \end{aligned} \tag{10}$$

Note that the term in the bracket of the last line of (18) represents a delay in the discrete w domain.

It is difficult to obtain the inverse z-transform of (10). However, the distribution $P_W(z)$ can be derived by cleverly using two z-transform pairs [26], which are given by the following

$$\begin{aligned} \frac{\prod_{j=1}^{M} n - j + 1}{a^M M!} a^n u(n) &\leftrightarrow \frac{z}{(z - a)^{M+1}} \\ u(n) &\leftrightarrow \frac{z}{z - 1} \end{aligned} \tag{11}$$

where $u(n)$ is the discrete unit step function. With some effort, we obtain the following closed-form expression for the discrete distribution of the sum random variable W as given by

$$\begin{aligned} p_W(w_n) &= (-1)^{-M} N^{-M} \\ &\times \sum_{k=0}^{M}\binom{M}{k}(-1)^k\frac{\prod_{j=1}^{M-1}\left(w_n - MN + kN - j\right)}{(M - 1)!} \\ &\times u\left(w_n - MN + kN - j\right) \end{aligned} \tag{12}$$

One may think of this as the Irwin–Hall distribution equivalent for discrete RVs (uniformly spaced from 1, 2, …, N) which to our knowledge has not been reported before.

4 Signal to multiple interference power ratio

Now, we formalise the SIR ratio RV to be given by

$$W = X/Y \tag{13}$$

Again, the continuous ratio counterpart is well-documented in the literature [23, 24]. The distribution $p_W(w_n)$ is given by

$$\begin{aligned} p_W(w_n) &= E\left[p_W(w_n|x_k, y_m)\right] \\ &= \sum_{y_m}\sum_{x_k} p_Y(y_m)p_X(x_k)\delta\left[w_n - x_k/y_m\right] \end{aligned} \tag{14}$$

where $E[\bullet]$ stands for the expectation operator and n, k and m serve as indexes for W, X and Y random variables, respectively.

Using the identity $v(n) = \Sigma_k v(k)\delta[n-k]$, it can be shown that (14) simplifies to

$$p_W(w_n) = \sum_{y_m} p_Y(y_m)p_X(w_n y_m) \tag{15}$$

Again, it should be noted that (14) is the general expression for the distribution of a ratio power RV where X and Y need not be identical and need not be uniformly spaced.

4.1 Multiple interferences

If the power RVs are independent, the sum interference power RV is the sum of the individual interference RVs. Thus if we let $W = \sum_{i=1}^{M} V_i$ be the interference power sum, then the SIR ratio is $S = Q/W$, where Q is the SoI power RV and W is the sum power of the interferences whose distribution is given by (12) if the individual RVs are uniformly spaced and identically uniformly distributed. In general, the closed-form distribution $p_S(s_n)$ of S depends on the resulting distribution of W. Even if we consider the special case where Q and W are uniformly spaced, it is easy to verify that S in general is not uniformly spaced. As such the theory of z-transform cannot easily be used to the resulting SIR ratio. However, we have the general expression (14) for the distribution of a ratio RV. Thus, we can easily use (15) in obtaining numerical results; that is, we can easily calculate ratio distributions given specific examples. The pmf is then given by

$$p_S(s_n) = \sum_{w_m} p_W(w_m)p_Q(s_n w_m) \tag{16}$$

Thus, we can investigate the effect of increasing the number of interferences in W on SIR. In other words, we quantify the performance degradation of CP against SIR threshold due to increased number of interferences. Since (16) is general, we can accommodate not only the special case of discrete sum (Irwin-Hall equivalent) in (12) but also cases where the interference power distributions are non-identical and non-uniformly spaced. Indeed, we can also consider where the power range of SoI is increased. This means the maximum power of the SoI exceeds either the maximum interference power or the maximum interference power sum, which is more in line with practical scenarios. An interference mitigation technique is to allow for gain in the signal distribution. Of course, this is indeed the case in practice. The passive receiver may steer the beam towards the direction of the SoI to ensure good SoI power reception in the antenna main beam. We also investigate this case in the results section.

5 Numerical results

In this section, we present several examples from various signal and interference power distribution possibilities. However, first consider the simplistic case when SoI power has probability 1 and interference power has probability 1, that is, both powers are deterministic. If they are also equal, then the distributions are identical (trivial but nonetheless identical). Regardless of actual power values, there is only one SIR value and it is clearly 0 dB. As a passive receiver, if we require a SIR threshold to be of specified value, for example, $\text{SIR}_{\text{req}} = 7$ dB, then we deem that the collection is unsuccessful or that collection probability is 0 since $\text{SIR} < \text{SIR}_{\text{req}}$. However, our interest is when SIR is random because of random SoI power and interference power. Consider the case where SoI and interference powers are random but with identical distribution (similar to the deterministic case above only this time the signal and interference discrete powers are truly random). Then we ask the question: will collection probability be equal to 0 as in the deterministic and equal power case? Amazingly, the collection probability is better than 0! Thus something is gained when the SIR is random! We will illustrate this in the first example below. More practical possibilities and scenarios are considered in the subsequent examples.

5.1 Example 1

For this example, we set both the SoI power and the interference power to be identically distributed. For the purposes of generating results, assume both are uniformly spaced: 1, 2, …, 10 power units, that is, each signal power has a probability of 0.1. Of course the power units could be watts or milliwatts. Since our measure of interest is SIR, all that matters is the power ratio. Consider Fig. 1. In Fig. 1*a*, we calculate and show the resulting SIR pmf distribution (in dB) with one interference. Notice the 'jagged' nature of the resulting distribution. Had the SoI power distribution and interference power distribution been continuous, the resulting SIR distribution would have been smooth and the closed-form expression could have been easily derived. Instead, since the distributions are discrete, the resulting SIR is calculated via the algorithm dictated by (14) where a closed-form expression is not easily found. In Fig. 1*b*, we show the corresponding 'one minus cmf of SIR' distribution which represents the collection probability (CP) against SIR threshold. Notice that at SIR = 7 dB threshold, CP is 0.05, which means that signal collection becomes possible when the signal and interference powers are identically distributed compared to when the signal and interference are equal and deterministic. Recall we previously mentioned that in the case of deterministic and equal SoI and interference powers, collection cannot be supported since SIR is always 0 dB which is lower than the required threshold of 7 dB!

The fact that CP decreases as SIR threshold is increased is intuitively satisfying. The more we demand a higher SIR to be received, the lower the probability becomes. The lower we require the SIR threshold to be, the higher CP becomes. However, lowering SIR threshold has consequences. Recall the SIR threshold actually corresponds to a detection performance that is required. Consider the case if the system designer explores to change the SIR threshold to be 3 dB. From Fig. 2*b*, we see that CP increases. However, realistically SIR of 3 dB is low in most practical situations. In Examples 3 and 4, we consider more practical cases where SoI has more power (albeit still random) compared to interference and thus CP against SIR threshold curves are better. But first in Example 2, we investigate the performance impact due to increasing number of interferences whose received powers are random.

5.2 Example 2

For this example, we again set both the SoI power and the interference power to be identically distributed and that both are uniformly

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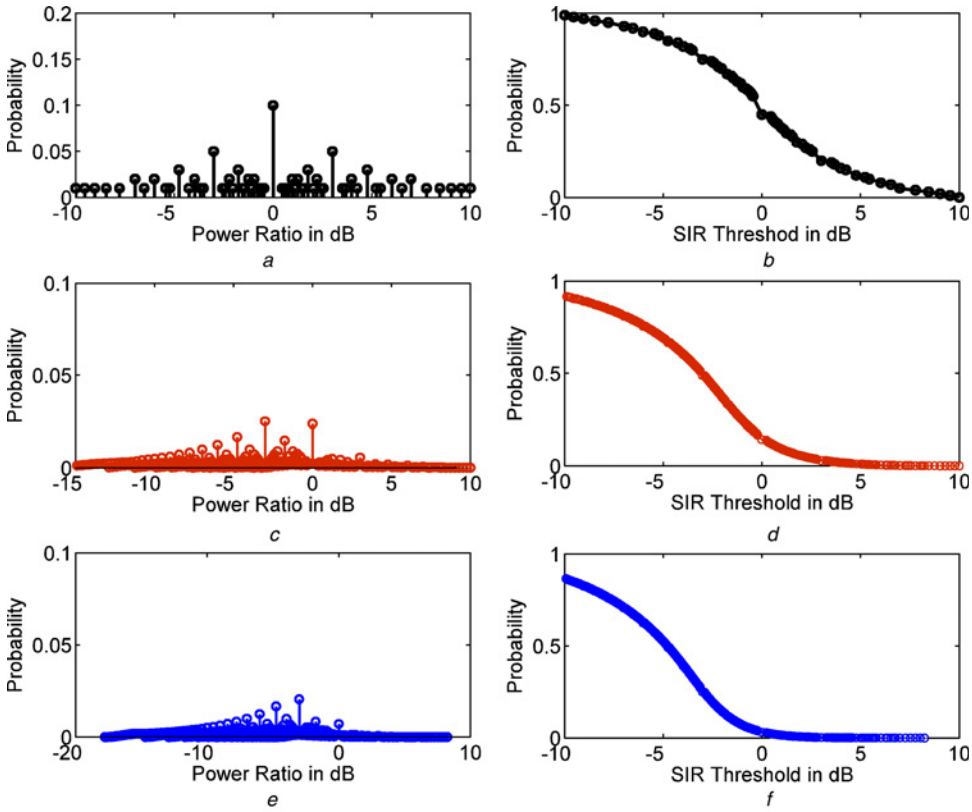

Fig. 1 *CP versus SIR threshold with increasing number of interferences*
a pmf of a SIR ratio with interference
b CP against SIR threshold with one interference
c pmf of a SIR ratio with two interferences
d CP against SIR threshold with two interferences
e pmf of a SIR ratio with three interferences
f CP against SIR threshold with three interferences

spaced: 1, 2, …, 10 power units, that is, each signal power has a probability of 0.1. Again, consider Fig. 1. This time we consider increasing the number of interferences. Recall that Figs. 1*a* and *b* correspond to one interference.

In Figs. 1*c* and *d*, we show the SIR distribution with two interferences and the CP against SIR threshold curve, respectively. In Figs. 1*e* and *f*, we show the SIR distribution with three interferences and the CP against SIR threshold, respectively. Here, it is clear that CP suffers as a function of increasing number of interferences. For example, if we choose a SIR threshold of 5.3 dB, then the CP with one interference is 0.11. For two interferences, CP is 0.0089. For three interferences, CP is 0.000034. In other words, increasing the number of interferences reduces collection probability as expected. Here, the reduction is drastic but should not be surprising since the CP for one interference is 0.05. Indeed, the maximum-SoI-to-maximum-interference ratio ($SIR_{mx/mx}$) is only 0 dB ($SIR_{mx/mx} = 10/10$). In the next case, we investigate the case when the SoI power range is increased which is more in line with practice.

5.3 *Example 3*

For this example, we consider the case where SoI power range is increased, that is, the maximum signal power is greater than that of interference power. Again interference power is uniformly distributed and uniformly spaced: (1, 2, …, 10). The maximum signal power is increased. The distribution is uniform with ten discrete power steps, that is, each signal power again has a probability of 0.1. Thus, the signal power distribution used for each sub-figure in Fig. 2 is as follows: (1, 2, 3, …, 10), (1, 3.11, 5.22, …, 20), (1, 12, 23, …, 100) and (1, 23.11, 45.22, …, 200). Since the maximum sum interference power is (10, 20, 30, 40) for one, two, three and four interferences, respectively, the $SIR_{mx/mx}$ are (0, 3, 10, 13) dB, (−3, 0, 7, 10) dB, (−4.8, −1.8, 5.2, 8.2) dB and (−6, −3, 4, 7) dB, respectively. Thus, for one interference, in Fig. 2*a*, we show the CP against SIR threshold as a function of increasing range of the SoI power RV. Choosing a threshold to be approximately 5 dB (recall that power levels are discrete), the CP corresponding to $SIR_{mx/mx}$ of 0, 3, 10 and 13 dB are approximately 0.11, 0.27, 0.79 and 0.86, respectively. In other words, the improvement is fairly pronounced for 10 and 13 dB $SIR_{mx/mx}$, which in practice are modest SIRs. The next three figures correspond to increasing number of interferences. Notice the graceful degradation of CP performance of the SoI with the largest increase in power range. For example, in the case of four interferences, where the maximum interference sum power is 40 and the SoI maximum power is 200, the CP is 0.77! The CP performance being referred to is the curve $SIR_{mx/mx} = 7$ dB in Fig. 2*d*.

5.4 *Example 4*

Here we consider the case where the random SoI power is scaled by a scalar or constant. This is a practical representation of a gain of the system. For example, the passive receiver may be pointed to the emitter of interest and as such has a receiver antenna gain. Therefore, the signal power RV is multiplied by a gain constant. The SoI power and interference powers are again uniformly distributed and uniformly spaced: (1, 2, …, 10). The signal power gains are 3, 6, 10 and 13 dB. In Fig. 3, CP is plotted against SIR threshold for one interference. As might be expected, CP improves with

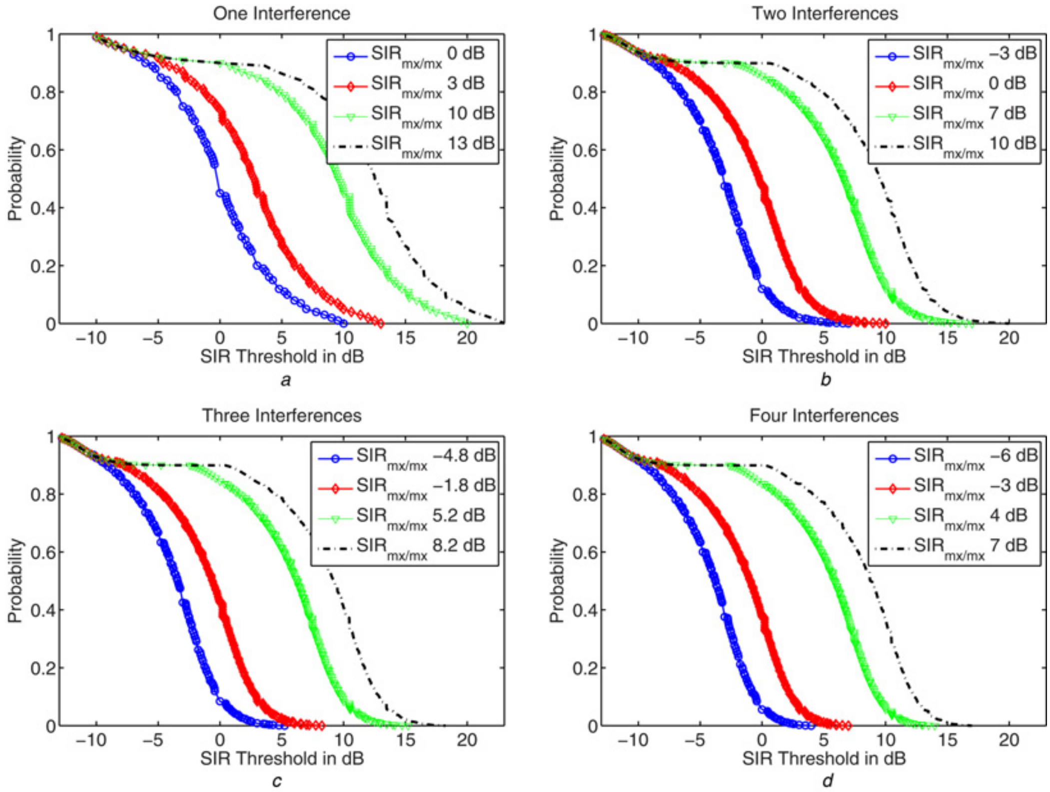

Fig. 2 *CP against SIR threshold with increasing SoI power range*
a CP against SIR threshold with increasing SoI power range for one interference
b CP against SIR threshold with increasing SoI power range for two interferences
c CP against SIR threshold with increasing SoI power range for three interferences
d CP against SIR threshold with increasing SoI power range for four interferences

increasing gain and we note the unmistakable performance trend. The performance trend is the same for multiple interferences and as such CP against SIR for those are not plotted here. A more interesting and intuitive comparison however is to compare the performance where the SoI power range is increased (as in Example 3) and where the SoI gain is such that the maximum signal power is the same as the former. For example, with one interference, we compare the performance when $SIR_{mx/mx} = 13$ dB (this is shown in Fig. 2*a*) and the gain is 13 dB (in Fig. 3 labelled gain 13 dB). For a threshold of approximately 5 dB, CP for the former is about 0.86 and 0.95 for the latter. This is intuitively satisfying, that is, having gain in the SoI distribution instead of merely increasing the power range would perform better. This is because the gain also increases the minimum signal power (and others) in the uniform distribution while in the case of increased signal power range the minimum power stays the same.

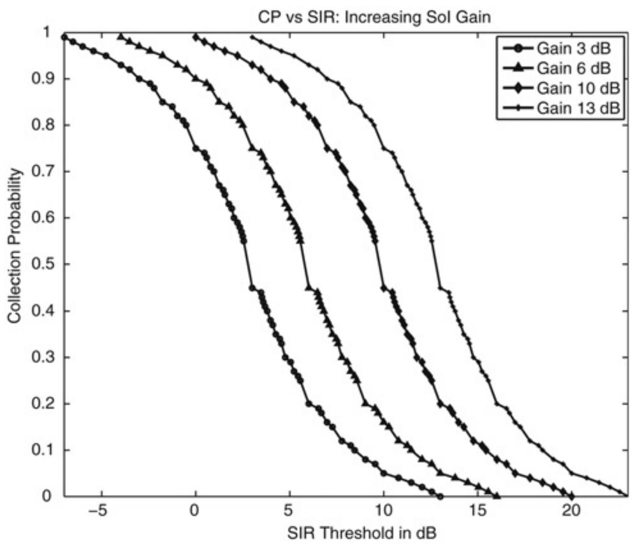

Fig. 3 *CP against SIR threshold with increasing gain to the signal power distribution*

5.5 Example 5

To show the flexibility of the probabilistic analysis for SoI and interferences with discrete power levels, we finally consider the case where their distributions are arbitrary (to reflect any practical transmitter or emitter power control strategies). In other words, the received distributions are not necessarily identical nor uniformly distributed. We plot CP against SIR as a function of number of interferences. For the purposes of simulating results, arbitrary power pmfs of SoI and interferences are generated and shown in Fig. 4*a*. We assume the powers to be Watts. In Fig. 4*b*, we plot the resulting sum interference distributions. Notice the increased number of possible sums in the set of sum possibilities. In Fig. 4*c*, the pmfs of the three possible SIR RVs are shown. The algorithm dictated by (16) allowed us to find these distributions numerically with great convenience as oppose to difficulties of finding closed-form solutions for continuous distributions. However, notice in the case of three interferences, the number of possible discrete ratios in the ratio set is significant. This is important in terms of numerical analysis. This means that if the number of elements in the SoI power and sum interference power are substantial, the elements in the ratio set could be very large. As such, care must be exercised in implementing the algorithm dictated by (16). Indeed, if two

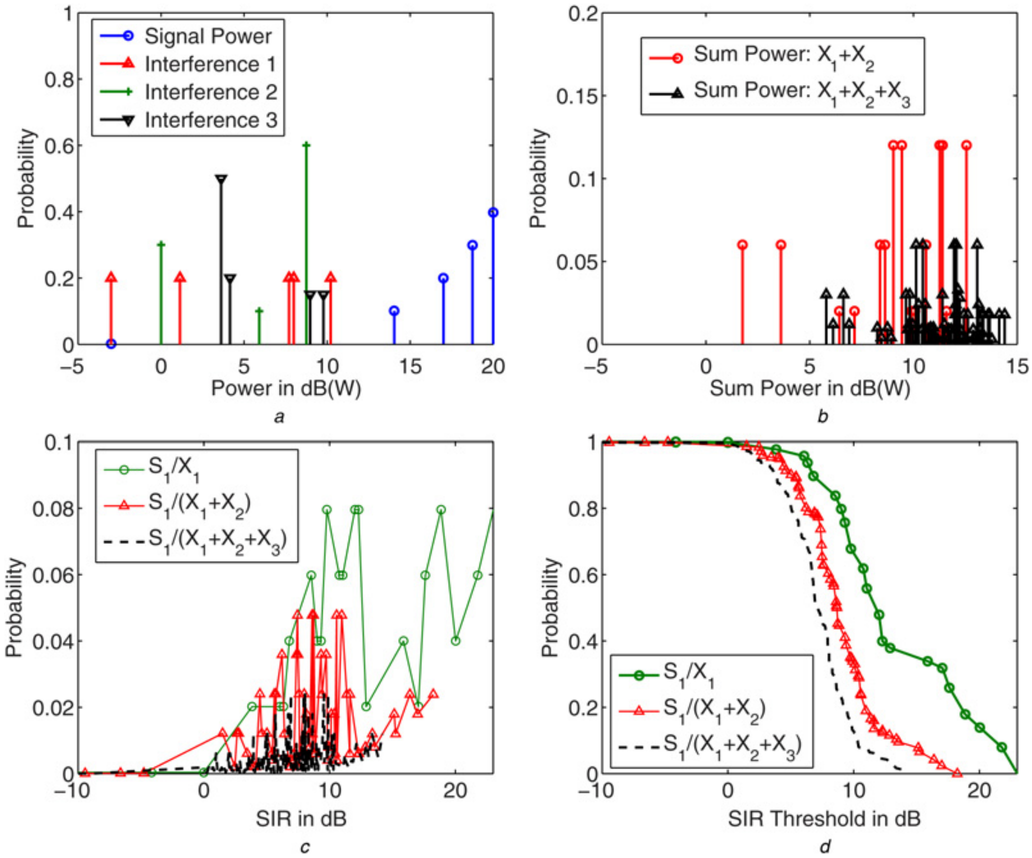

Fig. 4 *CP analysis for arbitrary power pmfs of SoI and interferences*
a Power distributions of signal and interferences
b Sum interference distribution for two and three interferences
c SIR pmf's for one, two and three interferences
d CP against SIR threshold for one, two and three interferences

possible ratios are very close, numerical issues may arise. In Fig. 4*d*, CP against SIR threshold curves are plotted as function of number of interferences. Notice the interesting form of CP against SIR threshold curve for one interference. This non-smooth nature of the distribution attests to the discrete nature of the power distributions. However, as we increase the number of interferences, the CP curves get smoother despite the fact the corresponding pmfs are still pretty jagged as shown in Fig. 4*c*. This is to be expected since CP is a cmf, that is, an integrated distribution. As expected, the CP against SIR performance curve for one interference is better than for two and three interferences as shown in Fig. 4.

6 Conclusion

In this paper, we presented a probabilistic approach to analyse SIR of a received signal in multiple interferences when the SoI received power is random with discrete power levels for ELINT passive receiver with no a priori knowledge of power variation. We also assumed that the received powers of the interferences are discretely random and independent from each other. We derived a closed-form expression for the distribution of the sum of the interference powers when the interference powers are independent, identically uniformly distributed RVs, resulting to the discrete version of the Irwin-Hall distribution which is previously unreported. The SIR ratio analysis yielded the metric of CP against SIR threshold as a measure of performance. We presented numerous examples for various signal and interference distribution possibilities. We showed how a collection opportunity exists for the case where SoI and interference powers are random with identical distribution compared to when SoI and interference powers are equal and deterministic. We showed how CP performance degrades as a function of the number of interferences. We also investigated the SIR improvement as a function of increasing the signal power range and increased SoI gain. Lastly, we considered arbitrary distributions to show utility and flexibility of the probabilistic analysis in quantifying CP as a function of the SIR.

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