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Detection Performance of Matched Transmit Waveform for Moving Extended Targets

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***Abstract*—Depending on the radar-target dynamics, the time extent and amplitude of a moving extended target from a radar's perspective may actually change as a function of relative motion. It follows that waveform design should accommodate for the increase or decrease of a target's time extent and changes in amplitude as the target moves towards or away from a radar or vice versa. This paper shows the performance gain and/or degradation of both matched transmit waveform (called eigen-waveform) and the classical wideband pulsed transmit waveform when the effect of motion on target's time extent and amplitude changes are considered.**

***Index Terms*—Extended Target, Moving Target, Eigenwaveform, Cognitive Radar, Waveform Design, Matched Waveform**

I. Introduction

It is widely known that a moving target's return may change with respect to a stationary radar. Of course, the same is true if the radar is moving and the target is static. This is true for both point target and extended target. There are two related phenomena of importance to consider. First, the range loss is increased or decreased due to range change as dictated by the radar range equation. The second phenomenon due to relative motion is that the target's effective impulse response changes. For a point target, the amplitude changes, i.e. the radar cross section (RCS) changes. For an extended target, its impulse response changes both in amplitude and time. For an extended target moving away from the radar, the amplitude and time extent become smaller and they become larger when the target or radar is approaching. Matched waveform design for extended targets was first tackled in [1] using both the SNR and mutual information (MI) metrics considering a static target. For signal-dependent interference, the work in [2] obtained optimal spectra for transmit waveforms matched to deterministic and stochastic targets using the two metrics mentioned above. Recently, waveform design has found its application in cognitive radar (CRr). We use the acronym CRr to differentiate it to cognitive radio (CR). The earlier notions of a cognitive radar (CRr) is introduced in [3] and a more formalized treatment is described in [4]. In [5], a closed-loop CRr platform is introduced for target classification with the use of adaptive matched waveforms which are created real-time to interactively respond to each received measurement.

If the target moves, it follows that waveform design should accommodate for the changes of a target's amplitude and time extent as the target (or radar) moves. If the increase in extent is accommodated, this paper shows the increase in detection probability of a matched waveform. We compare that performance gain to the performance gain of traditional wideband pulse (which also accommodates for the increasing extent). If the target is moving away, then the matched waveform should also be adjusted to minimize the loss in detection probability. We compare its performance to the performance of the wideband waveform. In this paper, we assume that both systems account for proper sampling time (due to timing changes of relative motion between target and radar).

II. Signal Model and the Eigenwaveform

Let $\mathbf{h}$ be the complex-valued target response and let $\mathbf{x}$ be an arbitrary complex-valued transmit waveform. Let complex-valued vector $\mathbf{w}$ be the additive white Gaussian noise from the receiver hardware. Thus, the received signal plus noise is $\mathbf{y} = \mathbf{s} + \mathbf{w}$ where $\mathbf{s} = \mathbf{h} * \mathbf{x}$ and $(*)$ designates the convolution operation. For convenience, we can specifically describe $\mathbf{h} = \sqrt{E_h}\overline{\mathbf{h}}$ such that E_h is the target response energy and $\overline{\mathbf{h}}$ is a unit energy vector. It follows that we can let $\mathbf{x} = \sqrt{E_x}\overline{\mathbf{x}}$ such that E_x is the transmit waveform energy and $\overline{\mathbf{x}}$ is a unit energy vector. Then $\mathbf{y} = \sqrt{E_h}\overline{\mathbf{h}} * \sqrt{E_x}\overline{\mathbf{x}} + \mathbf{w}$. If we let $\overline{\mathbf{H}}$ be the target response convolution matrix, then $\mathbf{y} = \sqrt{E_h}\sqrt{E_x}\overline{\mathbf{H}}\overline{\mathbf{x}} + \mathbf{w}$. Using the proper matched filter to the received signal plus noise i.e. $(\mathbf{H}\mathbf{x})^{\dagger}\mathbf{y}$, then the received energy due to the echo $\mathbf{s} = \mathbf{h} * \mathbf{x}$ is given by

$$E_s = E_x E_h \left(\overline{\mathbf{H}}\overline{\mathbf{x}}\right)^{\dagger} \overline{\mathbf{H}}\overline{\mathbf{x}} = E_x E_h \overline{\mathbf{x}}^{\dagger}\overline{\mathbf{H}}^{\dagger}\overline{\mathbf{H}}\overline{\mathbf{x}} \quad (1)$$

where $\dagger$ represents the conjugate-transpose or Hermitian operation. If we let $\overline{\mathbf{R}} = \overline{\mathbf{H}}^{\dagger}\overline{\mathbf{H}}$ be the autocorrelation of the target convolution matrix then the received energy due to the target echo for any transmit waveform is given by

$$E_s = E_h E_x \overline{\mathbf{x}}^{\dagger}\overline{\mathbf{R}}\overline{\mathbf{x}}. \quad (2)$$

Using eigenvalue decomposition, we realize

$$E_{s,\lambda} = E_h E_x \overline{\mathbf{q}}^{\dagger}\lambda\overline{\mathbf{q}} = E_h E_x \lambda \quad (3)$$

where $E_{s,\lambda}$ corresponds to a particular eigenvalue λ and its corresponding unit-energy eigenvector $\overline{\mathbf{q}}$. Thus, we can maximize the received energy E_s by choosing the eigenvector corresponding to the maximum eigenvalue to be our transmit waveform. Thus, the maximum received echo energy is given by

$$E_{s,\max} = E_h E_x \lambda_{\max} \overline{\mathbf{q}}_{\max}^{\dagger} \overline{\mathbf{q}}_{\max} = E_h E_x \lambda_{\max} \quad (4)$$

where the matched transmit waveform that maximizes the received echo energy is clearly $\mathbf{x} = \sqrt{E_x}\overline{\mathbf{q}}_{\max}$, i.e. $\overline{\mathbf{x}} = \overline{\mathbf{q}}_{\max}$ which we now call the eigenwaveform.

III. Probability of Detection

A. Eigenwaveform

Here we are interested in the detection performance of the eigenwaveform design in comparison to an arbitrary waveform. The detection hypotheses are

$$\mathcal{H}_0 : \mathbf{y} = \mathbf{w}$$
$$\mathcal{H}_1 : \mathbf{y} = \mathbf{s} + \mathbf{w} = \sqrt{E_h}\overline{\mathbf{H}}\sqrt{E_x}\overline{\mathbf{q}}_{\max} + \mathbf{w}.$$

With $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$, the corresponding *pdf*s are given by

$$p(\mathbf{z}|\mathcal{H}_0) = \frac{1}{\pi^N \sigma^{2N}} \exp(-\frac{1}{\sigma^2}\mathbf{y}^{\dagger}\mathbf{y}) \quad (5)$$
$$p(\mathbf{z}|\mathcal{H}_1) = \frac{1}{\pi^N \sigma^{2N}} \exp\left(-\frac{1}{\sigma^2}(\mathbf{y}-\mathbf{s})^{\dagger}(\mathbf{y}-\mathbf{s})\right).$$

Utilizing the "matched eigenfilter" to the received signal i.e. $(\mathbf{H}\mathbf{q}_{\max})^{\dagger}\mathbf{y}$, the decision statistic for fixed threshold γ' is

$$T(\mathbf{y}) = \mathrm{Re}\left((\mathbf{H}\mathbf{q}_{\max})^{\dagger}\mathbf{y}\right) \geq \gamma' \quad (6)$$

where $\mathbf{H} = \sqrt{E_h} \cdot \overline{\mathbf{H}}$ and $\mathbf{q}_{\max} = \sqrt{E_x}\overline{\mathbf{q}}_{\max}$. It can be shown that the expected values under two hypotheses are

$$E(T;\mathcal{H}_0) = E[(\mathbf{H}\mathbf{q}_{\max})^H\mathbf{w}] = 0 \quad (7)$$
$$E(T;\mathcal{H}_1) = E[(\mathbf{H}\mathbf{q}_{\max})^H\mathbf{H}\mathbf{q}_{\max}) + \mathbf{H}\mathbf{q}_{\max}{}^H\mathbf{w}] = E_h E_x \lambda_{\max}.$$

Despite lengthy derivation, it can be shown that the variance under the null hypothesis is the same as the variance under the target present hypotheses which is given by

$$\mathrm{var}(T;\mathcal{H}_0) = \mathrm{var}(T;\mathcal{H}_1) = \frac{\sigma^2}{2} E_h E_x \lambda_{\max}. \quad (8)$$

Given the arbitrary threshold, the probability of detection is

$$P_D = \Pr(T \geq \gamma'; \mathcal{H}_1) \ = Q\left(\frac{\gamma' - E_h E_x \lambda_{\max}}{\sqrt{\frac{\sigma^2}{2} E_h E_x \lambda_{\max}}}\right). \quad (9)$$

Since the probability of false alarm (P_{FA}) is given by

$$P_{\mathrm{FA}} = Q\left(\frac{\gamma'}{\sqrt{\frac{\sigma^2}{2} E_h E_x \lambda_{\max}}}\right), \quad (10)$$

the threshold is simply

$$\gamma' = Q^{-1}(P_{\mathrm{FA}})\sqrt{\frac{\sigma^2}{2} E_h E_x \lambda_{\max}}. \quad (11)$$

Thus,

$$P_{\mathrm{D}} = Q\left(Q^{-1}(P_{\mathrm{FA}}) - \sqrt{\frac{2}{\sigma^2} E_h E_x \lambda_{\max}}\right). \quad (12)$$

B. Classical Pulsed Waveform

Notice there are plenty of waveforms where the received echo energy E_s may actually be less than that of the multiplication of transmit waveform energy and target response energy i.e., $E_s < E_x E_h$. This is clearly an undesirable condition. To achieve $E_s = E_x E_h$, it can easily be shown that a transmit waveform that meets this requirement is the classical pulsed waveform whose idealization is an impulse wideband waveform. For an arbitrary waveform $\sqrt{E_x}\overline{\mathbf{x}}$, the detection hypotheses are

$$\mathcal{H}_0 : \mathbf{y} = \mathbf{w} \quad (13)$$
$$\mathcal{H}_1 : \mathbf{y} = \mathbf{s} + \mathbf{w} = \sqrt{E_h}\overline{\mathbf{H}}\sqrt{E_x}\overline{\mathbf{x}} + \mathbf{w}.$$

The corresponding matched filter is given by $(\mathbf{H}\mathbf{x})^{\dagger}$ and thus received peak energy E_s out the matched filter can be calculated by $E_s = E(T;\mathcal{H}_1) = (\mathbf{H}\mathbf{x})^{\dagger}\mathbf{H}\mathbf{x} = (\mathbf{x})^{\dagger}\mathbf{R}_h\mathbf{x}$. It can be shown that the diagonal elements of the autocorrelation of the target convolution matrix is E_h. Thus, if we let $\mathbf{x}_p = \sqrt{E_x}\overline{\mathbf{x}}$ be the impulse waveform, then the energy received is given by

$$E_s = \mathbf{x}_p^{\dagger}\mathbf{R}_h\mathbf{x}_p = E_x E_h. \quad (14)$$

For the pulsed waveform, P_{D} is then given by

$$P_{\mathrm{D}} = \Pr(T \geq \gamma'; \mathcal{H}_1) \quad (15)$$
$$= Q\left(\frac{\gamma' - E_s}{\sqrt{\sigma^2 E_s/2}}\right) = Q\left(Q^{-1}(P_{FA}) - \sqrt{\frac{2E_h E_x}{\sigma^2}}\right).$$

In the case of traditional waveform design for a point target or wideband pulse for extended target, the probability is purely a function of $E_s = E_x E_h$ rather than $\lambda_{\max} E_x E_h$. In eigenwaveform design, the received energy is amplified by the maximum eigenvalue and as such has better performance than the wideband pulsed waveform.

IV. Extent as Function of Simple Motion Model

Regardless whichever one is moving, when the motion between target and radar results in increased proximity (decreased range), then the amplitude scale and temporal extent of the target are increased in the radar's perspective. Thus, the effective energy of the target response increases and therefore by (12) the probability of detection increases if the changes are properly accounted for by the radar. That is, the transmit waveform has to be changed on-the-fly; the matched filter has to be modified; and the sampling instant has to adjusted for the increased time extent and increased amplitude scale to ensure peak detection. These adjustments ensure the probability increase promised by (12) can be realized. If the modifications are not made during motion then serious detection performance is compromised. When the motion between target and radar results in decreased proximity (increased range), then the target's amplitude scale and temporal extent are decreased. Thus, the effective target energy decreases. Therefore by (12) for eigenwaveform and (16) for wideband pulsed waveform the probability of detection also decreases. Despite this decrease

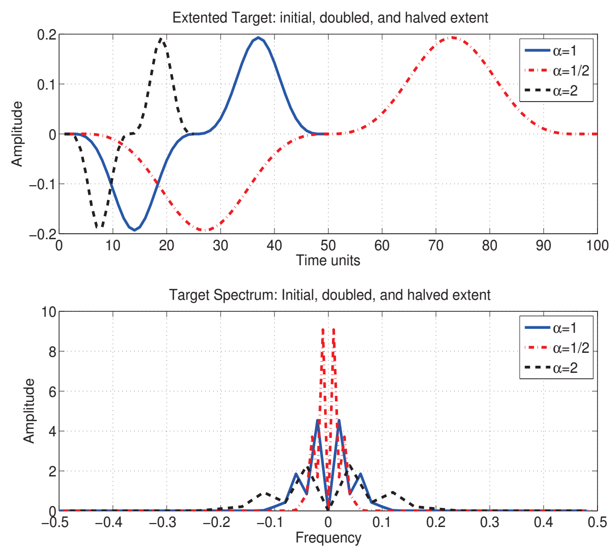

Fig. 1. Example of an extended target response. Top panel: real part of a complex-valued response for $\alpha = 1, 1/2, 2$. Bottom Panel: Corresponding magnitude of amplitude spectra for $\alpha = 1, 1/2, 2$.

however it is clear that P_{D} for eigenwaveform is always larger than the performance (15) of wideband pulsed waveform (as long as the $\lambda_{\max}$ is larger than 1).

The proper change of amplitude scale and temporal extent of a target response depends on how the target and/or radar is/are moving with respect to each other. In other words, it depends on the motion model. For example, a straightforward but simplistic model would be

$$\mathbf{h}_{\alpha}[n] = \beta \mathbf{h}[\alpha n] \tag{16}$$

where n is the time index and $\mathbf{h}_{\alpha}[n]$ is the stretched target response. The variable α accounts for the temporal "stretching" of target response and β accounts for the "amplitude scaling". Clearly, $\alpha = 1$ and $\beta = 1$ indicate no relative motion between target and radar. An $\alpha < 1$ would mean increased extent where there would theoretically be an increase in β. An $\alpha > 1$ would mean decreased extent where there would theoretically be a decrease in β. For convenience, we will set amplitude scale $\beta = 1$ (since the effect of β on target energy is straightforward and can easily be incorporated if needed). In practice if $\beta = 1$ does not change during relative motion, then there is no discernable change in target amplitude but discernable change in temporal extent if α changes. In this case, the increase or decrease in effective target energy is a function of α.

For illustration, we consider an arbitrary complex-valued target response. In the top panel of Figure 1, the real part of the target response is shown. The response corresponding to $\alpha = 1$ represents a static or initial target response; the response corresponding to $\alpha = 1/2$ represents a doubling of the extent; and the response corresponding to $\alpha = 2$ represents a halving of the extent. We let the sampling frequency be normalized. Thus $\alpha = 1/2$ corresponds to a doubling of the effective target energy and $\alpha = 2$ corresponds to halving of the effective target energy. In the bottom panel of Fig. 1, the corresponding magnitudes of the amplitude spectra are shown. As expected, the stretching of the target response in the time domain corresponds to the opposite stretching in the frequency domain. Thus, the effective target energy as a function of α is given by

$$E_{\alpha} = \frac{1}{\alpha} E_h \tag{17}$$

where the original target energy is given by $E_h = \mathbf{h}[n]^{\dagger}\mathbf{h}[n]$.

As a function of α, the probability of detection is modified to

$$P_D(\alpha) = \Pr(T \geq \gamma^{'} \; ; \mathcal{H}_1) \; = Q\left(\frac{\gamma^{'} - \frac{E_h}{\alpha} E_x \lambda_{\max}}{\sqrt{\frac{\sigma^2}{2}\frac{E_h}{\alpha} E_x \lambda_{\max}}}\right). \tag{18}$$

The probability of false alarm (P_{FA}) is also a function of α and is given by

$$P_{\text{FA}}(\alpha) = Q\left(\frac{\gamma^{'}}{\sqrt{\frac{\sigma^2}{2}\frac{E_h}{\alpha} E_x \lambda_{\max}}}\right), \tag{19}$$

where the corresponding threshold is now

$$\gamma^{'} = Q^{-1}(P_{\text{FA}}(\alpha))\sqrt{\frac{\sigma^2}{2}\frac{E_h}{\alpha} E_x \lambda_{\max}}. \tag{20}$$

Finally, the detection probability is given by

$$P_{\text{D}}(\alpha) = Q\left(Q^{-1}(P_{\text{FA}}) - \sqrt{\frac{2}{\sigma^2}\frac{E_h}{\alpha} E_x \lambda_{\max}}\right). \tag{21}$$

For the unit-energy target shown in Fig. 1, the maximum eigenvalue happens to be 15.6334. In Figure 2, for a given probability of false alarm (0.1), the probability of detection curves for $\alpha = 1, 1/2, 2$ corresponding to the eigenwaveform and wideband pulsed waveform are shown. The dashed lines correspond to the wideband waveform and solid lines correspond to the eigenwaveform. For $\alpha = 1$ (which corresponds to the initial target response) and $P_{\text{D}} = 0.9$, the required target-energy-to-noise (TxNR) ratio (E_x/σ^2) for the wideband pulsed waveform is 5.2 dB. For the same detection probability, the required TxNR is approximately -6.7 dB for the eigenwaveform. This is an amazing performance advantage of 11.9 dB! This happens to be the dB value of $\lambda_{\max} = 15.6336$. In other words, in the linear regions of the detection probabilities, the performance gain of using the eigenwaveform over a wideband waveform is $\lambda_{\max,\text{dB}} = 10\log_{10}\lambda_{\max}$. This of course assumes receiver detection timing is also adjusted for the corrected time extent. Notice the increase in P_{D} for $\alpha = 1/2$ and decrease in P_{D} for $\alpha = 1/2$ for both waveforms. The performance increases for both waveforms are both 3 dB which is of course the effective target energy increases for both cases. It follows that the 3 dB performance decreases for both waveforms for $\alpha = 2$ are due to the 3 dB effective

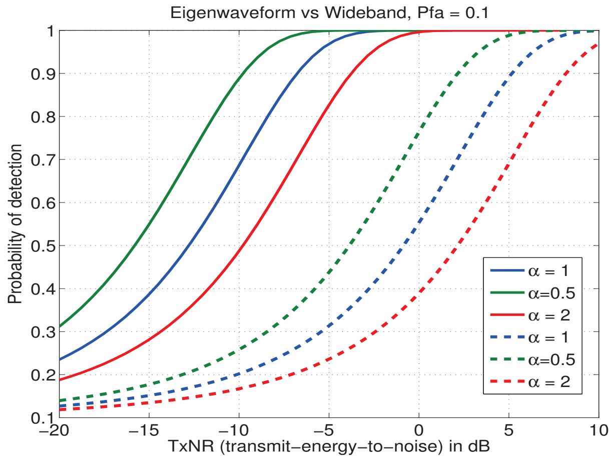

Fig. 2. Target response. Top panel: Increasing extent. Bottom Panel: Decreasing Extent.

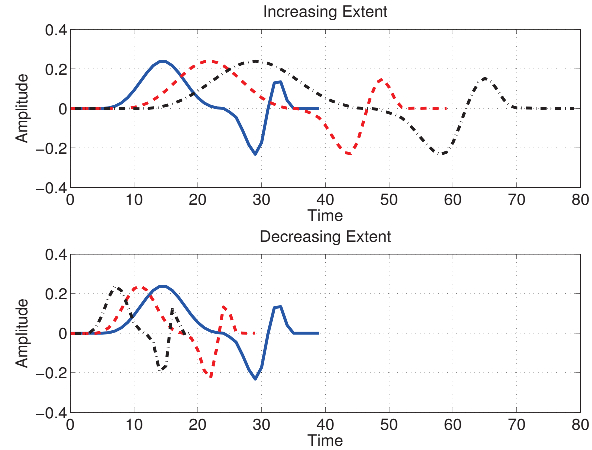

Fig. 3. Eigenwaveform vs Pulsed Waveform detection performances as function of α. Solid lines for eigenwaveform and dashed lines for wideband waveform.

target energy decreases. In summary (with a simplistic motion model for target response), given a constant false alarm rate (CFAR), the effective target energy is higher for a target of increased temporal extent ($\alpha < 1$), and thus the probability of detection increase is $-10\log_{10}\alpha$ dB. Conversely, for a target of decreased temporal extent ($\alpha > 1$), the effective target energy is lower and thus the probability of detection is lower as dictated by $-10\log_{10}\alpha$.

V. Target Response as Function of Complicated Motion Models

Generally, the motion model between radar and target can be very complex and depends on complexity of the actual relative motions between the two. For example, a target may be approaching the radar with constant acceleration (therefore non-constant velocity) and thus the model in (16) is too simplistic. Indeed, even the acceleration towards the target may not be constant. For example, a target may perform various evasive maneuvers against a radar in pursuit which renders the rate of change of the time extent to be highly changing i.e., there are instants in time where the increase/decrease in extent is greater than other times. In practice, complicated motions also result in slight changes to the actual amplitude response of the target response itself. This means that the amplitude spectrum is simply not stretched as shown in bottom panel of Fig. 1 but rather may have a modified version of the static or initial target spectrum. For example, a sharp turn by a target may lead to a different look angle from radar to target. This almost guarantees a correlated but different response let alone the change in extent. Motion modeling for various situations is very dependent on the radar-target motion dynamics and is therefore beyond the scope of this paper. It suffices to say that a radar tracking a target should be able to update estimates on target track parameters (such as distance and velocity in relation to the radar). In other words, a radar (a CRr in particular) should be able to change its transmit waveform, matched filter, and adjust sampling time to accommodate a moving target. For this section, we have chosen a target depicted in Figure 3 (real part of response shown). In Fig. 3, a target whose extended responses are slightly more complicated than the simplistic model in (16) is shown. Clearly the responses are highly correlated but a close look reveals that amplitude responses are slightly different for various α values specifically at the tail end of the responses. As mentioned, such changes in the shape of target amplitude response may easily occur in practice.

A. Increasing Extent - Decreasing Range

The top panel in Figure 3 shows the real part of a 40 unit-time length complex-valued target. Here we use α to simply indicate the stretch in time extent. The real part of target responses are shown for $\alpha = 1, 0.75, 0.5$ (which corresponds to 40, 60, and 80 time lengths). Note the obvious increase in energy of the target responses. In Figure 4 detection performance curves corresponding to $P_{\mathrm{FA}} = 0.1$ as function of TxNR (E_x/σ^2) for wideband pulsed waveform and eigenwaveform are shown. Again, consider the P_{D} of both waveforms for $\alpha = 1.0$ (initial target response) where we note the superior performance of the eigenwaveform compared to the wideband waveform. Now let's consider detection performance of the wideband waveform at $P_{\mathrm{D}} = 0.9$. There is a 3 dB performance increase from $\alpha = 1$ to $\alpha = 0.5$ as expected. The increase is simply due to the doubling of effective target energy. Now, consider detection performance due to eigenwaveform at $P_{\mathrm{D}} = 0.9$. There is a 9.2 dB increase from $\alpha = 1$ to $\alpha = 0.5$. The increase in performance would only have been 3 dB if the target followed the simplistic model in (16). Although the increase in effective target energy is only 3 dB, in a sense the net detection performance increase is 6.2 dB! Unlike the wideband waveform, the increase in performance is due to the increase in maximum eigenvalue as the target's amplitude shape and temporal extent is changed.

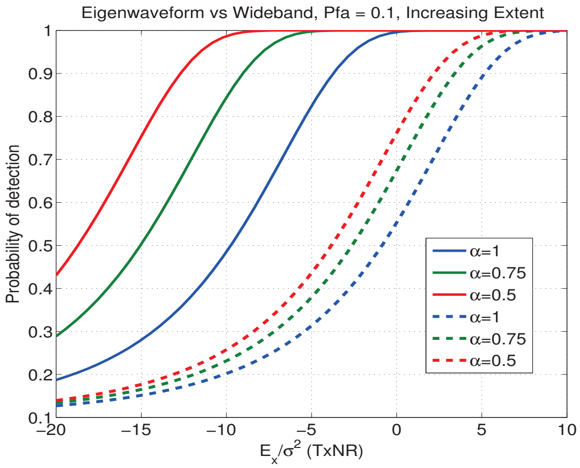

Fig. 4. Eigenwaveform vs Pulsed Waveform detection performances as decreasing function of α. Solid lines for eigenwaveform and dashed lines for wideband waveform.

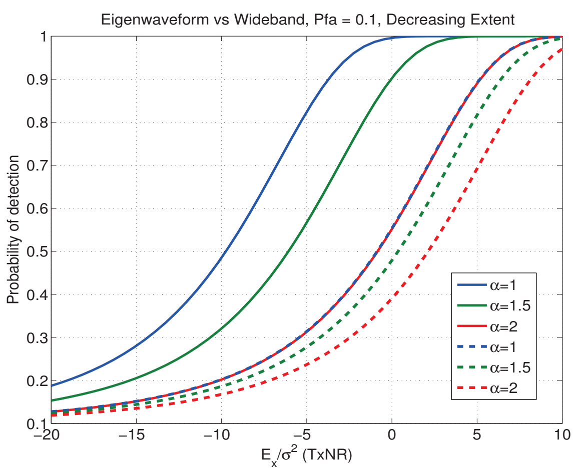

Fig. 5. Eigenwaveform vs Pulsed Waveform detection performances as function of increasing α. Solid lines for eigenwaveform and dashed lines for wideband waveform.

In other words, the increase in detection probability not only depends on the increased target effective energy but also on the change in maximum eigenvalues. The larger the increase in eigenvalue translates to larger detection performance gain. Of course a low increase in eigenvalue translates to lower detection performance gain.

B. Decreasing Extent - Increasing Range

In the bottom panel of Figure 3, the real part of target responses are shown for $\alpha = 1, 1.5, 2.0$ (which corresponds to 40, 30, and 20 time lengths). We use the complex-valued target (with length 40 time units) shown in top panel of Fig. 3 as the initial response. Again, notice the highly correlated nature of the responses albeit slightly different at the tail ends. Again, we use α to simply indicate the stretch in time extent. Note the obvious decrease in effective target energy. In Figure 5, detection performances corresponding to $P_{\text{FA}} = 0.1$ as function of TxNR (E_x/σ^2) for wideband pulsed waveforms and eigenwaveforms are shown. Notice the decrease in detection probability for both waveforms as a function of decreasing temporal extent. The performance decrease ($P_{\text{D}} = 0.9$) for the wideband pulsed waveform from $\alpha = 1$ to $\alpha = 2.0$ is 3 dB as expected. The performance decrease is larger for the eigenwaveform. Clearly, this is because there is a larger maximum eigenvalue decrease due to the slight change in target's amplitude response. Nevertheless, it's performance compared to the wideband waveform is always better. In the limit, when the extent becomes very small, the target becomes a point target. Assuming unit impulse, the received waveform is merely a scaled version of the transmit waveform. For convenience, assume a scaling of 1. Then the receive energy is equal to the transmit energy. It is widely known that any receive waveform regardless of its shape produces the same detection probability as any other so long as a matched filter is used and the receive energy is the same. In other words, when the target is a point target with a fixed RCS, detection is a function of transmit (or receive) energy rather than waveform shape (again so long as a matched filter is used).

VI. Summary

In this paper, we evaluated the detection performance of matched waveform which we termed as the eigenwaveform and compared it to the wideband pulsed waveform. We considered the performance gain and degradation for moving extended targets. Motion leads to stretching of the target's time extent as well as amplitude changes. We considered a simple signal model for motion and note that the performance gain and degradation is a simple function of the stretching constant α for both waveforms. We considered an example extended target whose amplitude slightly changes with α and note that its performance gain and degradation becomes a function of the modified maximum eigenvalues $\lambda_{\max}$. In other words, the performance gain can be larger than $-10\log_{10}\alpha$ for targets moving towards the radar. Of course, the performance degradation can also be worse for targets moving away from the radar. Nevertheless, the performance gain for the eigenwaveform is always better than that of the wideband pulsed waveform provided $\lambda_{\max}$ is larger than 1.

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