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# Photonic analog-to-digital converters preprocessing using the robust symmetrical number system for direct digitization of antenna signals 

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## THESIS



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## 13. ABSTRACT (maximum 200 words)

The need to realize pervasive battlespace awareness is placing an increasing demand on the bandwidth and resolution performance of modern sensors, communication receivers and electronic warfare. Fundamental to realizing this demand is the omnipresent highspeed analog-to-digital converters. The need constantly exists for converters with lower power consumption. To reduce the number of power-consuming components, high-performance ADCs employ parallel configuration of analog folding circuits to symmetrically fold the input signal prior to quantization by high-speed comparators.

In this thesis, a prototype of an optical folding 6 -bit ADC utilizing a 7 -bit preprocessing architecture was implemented using the Robust Symmetrical Number System (RSNS). The RSNS preprocessing architecture is a modular scheme in which the integer values within each modulus (comparator states), when considered together, change one at a time at the next position i.e. Gray-code property. MATLAB simulations are used to help determine the properties of the RSNS. These properties include the dynamic range (largest number of distinct consecutive vectors) and the location of the dynamic range within the number system. Since the waveform repeats every fundamental period, a method that reduces all indexes to the 'lowest common denominator' is developed to find the symmetrical residues of each channel. Using the symmetrical residues determined, the corresponding DC shifts on each waveform can be calculated.

The architecture employs a three-modulus (mod 7, 8, 9) scheme to preprocess the antenna signal. Electro-optic modulation of the input signal to generate the required number of folds within the dynamic range was successfully carried out in the three-modulus realization using modulators with a small half-wave voltage. The detection output are carefully aligned and postprocessed before amplitude analyzing with a high-speed comparator circuit responsible for the sampling and quantization of the signal (designed under a separate thesis). Low frequency analysis of the results using a 1 kHz input signal indicate a 5.42 effective number of bits (ENOB), a signal-to-noise ratio plus distortion (SINAD) of 34.42 dB , and a total harmonic distortion (THD) of 62.84 dB .

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# PHOTONIC ANALOG-TO-DIGITAL CONVERTERS PREPROCESSING USING THE ROBUST SYMMETRICAL NUMBER SYSTEM FOR DIRECT DIGITIZATION OF ANTENNA SIGNALS 

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Submitted in partial fulfillment of the requirements for the degree of

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#### Abstract

The need to realize pervasive battlespace awareness is placing an increasing demand on the bandwidth and resolution performance of modern sensors, communication receivers and electronic warfare. Fundamental to realizing this demand is the omnipresent highspeed analog-to-digital converters. The need constantly exists for converters with lower power consumption. To reduce the number of power-consuming components, highperformance ADCs employ parallel configuration of analog folding circuits to symmetrically fold the input signal prior to quantization by high-speed comparators.

In this thesis, a prototype of an optical folding 6-bit ADC utilizing a 7-bit preprocessing architecture was implemented using the Robust Symmetrical Number System (RSNS). The RSNS preprocessing architecture is a modular scheme in which the integer values within each modulus (comparator states), when considered together, change one at a time at the next position i.e. Gray-code property. MATLAB simulations are used to help determine the properties of the RSNS. These properties include the dynamic range (largest number of distinct consecutive vectors) and the location of the dynamic range within the number system. Since the waveform repeats every fundamental period, a method that reduces all indexes to the 'lowest common denominator' is developed to find the symmetrical residues of each channel. Using the symmetrical residues determined, the corresponding DC shifts on each waveform can be calculated.

The architecture employs a three-modulus $(\bmod 7,8,9)$ scheme to preprocess the antenna signal. Electro-optic modulation of the input signal to generate the required number of folds within the dynamic range was successfully carried out in the threemodulus realization using modulators with a small half-wave voltage. The detection output are carefully aligned and post-processed before amplitude analyzing with a highspeed comparator circuit responsible for the sampling and quantization of the signal (designed under a separate thesis). Low frequency analysis of the results using a 1 kHz input signal indicate a 5.42 effective number of bits (ENOB), a signal-to-noise ratio plus distortion (SINAD) of 34.42 dB , and a total harmonic distortion (THD) of -62.84 dB .


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## EXECUTIVE SUMMARY

The need to realize pervasive battlespace awareness is placing an increasing demand on the bandwidth and resolution performance of modern sensors, communication receivers and electronic warfare. Fundamental to realizing this demand is the omnipresent high speed analog-to-digital converters (ADCs). The technological trend is to move the digital signal sampling up the receiving chain, i.e., sampling at the antenna. Sampling at the antenna eliminates the need for intermediate frequency and baseband processing. High speed ADCs that utilize photonic technology play a critical role in fulfilling this demand.

In this thesis, a prototype of an optical folding 6-bit ADC utilizing a 7-bit preprocessing architecture was implemented using the Robust Symmetrical Number System (RSNS). The architecture employs a three-modulus (mod 7, 8, 9) scheme to preprocess the antenna signal. The preprocessing scheme involves electro-optic modulation and signal alignment using three low half-wave voltage Mach-Zehnder Interferometers (MZIs), a continuous wave distributed feedback laser and three DC sources. To perform electro-optic modulation, an amplified electric field (antenna signal) is applied to each MZI to modulate the optical beam. Each antenna signal is amplified based on the required voltage to produce a modulated signal with a RSNS fundamental period. Signal alignment is completed by adding a DC signal via the DC port into the MZI. This critical process is necessary to achieve the proper alignment of the RSNS signals prior to sampling and conversion into the binary code.

After adjusting the alignment of the signals, each optical signal is converted to an electrical signal using three photodetectors. The detection output is post-processed before amplitude analyzing with a high-speed comparator circuit responsible for the sampling and quantization of the signal (designed under a separate thesis).

Low frequency analysis of the results using a 1 kHz input signal indicate a 5.42 effective number of bits (ENOB), a signal-to-noise ratio plus distortion (SINAD) of 34.42 dB , and a total harmonic distortion (THD) of -62.84 dB .

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## I. INTRODUCTION

## A. ANALOG-TO-DIGITAL CONVERTERS

The need to realize pervasive battlespace awareness is placing an increasing demand on the bandwidth and resolution performance of modern sensors, communication receivers and electronic warfare. Fundamental to realizing this demand is the omnipresent high speed analog-to-digital converters (ADCs). The function of an ADC is to repetitively sample a time-varying waveform, usually at fixed time intervals, and generate a series of digital numbers to approximate the analog sample values [1]. Since the ADC is required to translate the sensor measurements into the digital language used by the computers, ADCs are one of the most widely used electronic component in signal processing systems.

The technological trend is to move the digital signal sampling up the receiving chain, i.e., sampling at the antenna. Sampling at the antenna eliminates the need for intermediate frequency and baseband processing. High speed ADCs that utilize photonic technology play a critical role in fulfilling this demand. These ADCs can digitize the RF signals directly at the antenna using Mach-Zehnder interferometers (MZIs) to amplitude modulate the RF into the optical domain. One of the first known photonic ADC techniques was introduced in 1979 by Taylor [1]. A 4-bit photonic ADC with Gray-code characteristics was implemented using four MZIs. This one bit per interferometer scheme was further improved with a different variation and demonstrated with a sampling rate of 1 gigasamples per second (GS/s) and a 2-bit resolution [2]. The downside to this implementation is the need for $n$ MZIs for an $n$-bit resolution. This presented a significant limitation due to the increased power required to drive the devices in parallel.

## B. PHOTONIC ANALOG-TO-DIGITAL CONVERTERS WITH UNIQUE ENCODING

A unique preprocessing technique is the time-stretch ADC presented by Jalali et al. [3-5]. A sampling rate of $130 \mathrm{GS} / \mathrm{s}$ has been demonstrated with $\sim 7$ effective number of bits (ENOB) using this technique. This is achieved by stretching an analog signal in the time domain to increase the effective sampling rate and the input bandwidth of ADC.

A different approach using an ultra-fast sample and hold circuit was reported by Urata et al. [6-9]. A 160 megasamples/s sampling rate with 3.5 ENOB was achieved using this design. The design uses low-temperature (LT)-grown GaAs metal-semiconductor-metal (MSM) photoconductive switches integrated with a Si-CMOS ADC. Samples are taken using a short-pulse laser to activate the switches and are held with the use of capacitors. Another ADC structure that uses $0.13-\mu \mathrm{m}$ CMOS technology was presented by Wang and Liu. The ADC achieved $5 \mathrm{GS} / \mathrm{s}$ with a 4-bit ENOB for a $200-\mathrm{MHz}$ input signal. This flash ADC design uses an interpolating architecture to reduce the number of amplifiers to enhance the bandwidth [10].

To effectively process the samples in a photonic sampled ADC, a photodiode must turn on and off rapidly [11]. Several proposals ranging from a moderate-speed ADC, time-division and wavelength division techniques have been proposed to ease the problem of ultra-fast sampling. Clark, Kang and Esman [12] introduced a scheme that uses a wavelength-interleaved photonic sampler. The use of wavelength-interleaved pulses achieved a $\sim 7$ bit ENOB with an $18-\mathrm{GHz}$ modulator bandwidth limitation. The system uses a mode-locked laser and multiplexer to perform parallel digitization. Another demonstration using a 1-to-4 optical time-division demultiplexer achieved a high-linearity ADC with a 208 megasamples per sec (MS/s) sampling rate and 12 effective bits [13].

The ADCs that were discussed so far used a combination of optical and electronic devices. An optically sampled and quantized ADC concept has also been studied and simulated [14-15]. It employs a sample and hold technique using a tunable laser to
encode an electrical signal. The concept simulation reported a 4-bit ADC output. An alloptical ADC was proposed by Oda and Maruta [16]. A 2-bit optical quantizer was demonstrated using this all-optical scheme.

An integrated ADC utilizing silicon photonics technology offers higher sampling rates that electronic ADCs cannot accomplish. This may be realized by combining photonic and electronic devices in a single microchip [17].

## C. FOLDING ADC PREPROCESSING ARCHITECTURE: SYMMETRICAL NUMBER SYSTEMS

The need constantly exists for converters with higher resolution, faster conversion speeds and lower power dissipation. To reduce the number of power-consuming components, high-performance ADCs employ a parallel configuration of analog folding circuits to symmetrically fold the input signal prior to quantization by high-speed comparators (analog preprocessing) [18]. Pace et al. reported in 1995 the experimental transfer characteristics of a three-channel 5-bit symmetrical number system (SNS) guided-wave ADC [19] and an 8-bit integrated optical SNS ADC [20]. The SNS preprocessing provides resolution greater than 1 bit per interferometer. The results demonstrated the feasibility of the SNS ADC concept.

Three symmetrical number systems formulations, i.e., SNS, the optimum symmetrical number system (OSNS), and the robust symmetrical number system (RSNS) have been used previously to increase the efficiency of folding ADC converter architectures, efficiently encode digital antenna links, and increase the resolvable bandwidth of two- and three- channel digital intercept receivers [21-28]. Each symmetrical number system is composed of a number of pairwise relatively prime moduli. The integers within each modulus $m_{i}$ are derived from a symmetrically folded waveform. Due to the presence of ambiguities, the set of integers within each symmetrical number system modulus does not form a complete residue system. The ambiguities are resolved by considering the combined values from all channels. The SNS is used to preprocess the analog signal reducing the number of amplitude analyzing comparators that are required. The dynamic range of the SNS is $\hat{M}_{S N S}$. The OSNS is
defined to extend the dynamic range of the SNS [21]. By recombining the $N$ channels, the OSNS is rendered a complete system having a one-to-one correspondence with the residue number system (RNS) with dynamic range $M=\Pi m_{i}>\hat{M}_{\text {SNS }}$ [29]. The SNS and the OSNS, however, have the encoding error characteristics like that of the RNS. Encoding errors can occur when the input signal lies about any code transition point. That is, if one integer within the modulus is incorrect, the resulting value that is decoded has a large error [29].

The RSNS scheme is a modular scheme in which the integer values within each modulus, when considered together, change one at a time at the next position (Gray code properties). This eliminates the possible encoding errors that may result in addition to extending the resolution beyond 1-bit per interferometer. Calculation of the dynamic range and its position within the combined sequences is necessary for any application. Styer and Pace defined the two-channel RSNS and presented a theorem that gives its dynamic range for relatively prime moduli $m_{1}, m_{2}, 5 \leq m_{1} \leq m_{2}$ [29].

Pace et al. presented a folding ADC preprocessing architecture employing a RSNS with Gray-code properties [18]. Closed-form expressions for the dynamic range are also presented for channel moduli of the form $m_{1}=2^{k}-1, m_{2}=2^{k}, m_{3}=2^{k}+1$. The paper highlighted that the selection of the shifts and their permutations among the three moduli have no effect on the dynamic range, $\hat{M}_{\text {RSNS }}$. However, the beginning and ending points are different. For any particular channel, the folding waveform crosses the comparator thresholds in turn every $N$ LSBs. Within the ADC, however, only one comparator threshold is crossed at any LSB code transition point (Gray-code property). The paper noted that, although the dynamic range of the RSNS ADC is not optimum, the absence of encoding errors makes the RSNS encoding scheme an efficient approach for folding ADC designs.
M. Arvizo in her Masters' Thesis demonstrated a full implementation of a 6-bit RSNS ADC concept ( $\bmod 3,4,5$ ) by combining photonic and electronic technologies [30]. The architecture employs a three-modulus scheme to preprocess the antenna signal. An analysis of the results using a $20-\mathrm{kHz}$ input signal indicate a 5.33 effective number of
bits (ENOB), a signal-to-noise plus distortion (SINAD) of 33.8 dB , and a total harmonic distortion (THD) of -43 dB . The importance of accurate electro-optic modulation and signal alignment in order to obtain accurate folding waveforms was noted in the thesis. The input power limitation of the RF port became the basis of using the DC bias port instead. With a summing amplifier, the antenna and DC signal were both injected into the DC port. The DC port, however, has a $40-\mathrm{kHz}$ bandwidth limitation. Although the encoding scheme had minimal errors with low-frequency input signal, the errors increased as the modulated signals became distorted at higher frequencies.

## D. PRINCIPAL CONTRIBUTIONS

In this thesis, a prototype of an optical folding 7-bit preprocessing architecture was designed using the Robust Symmetrical Number System (RSNS). Due to an inconsistent extinction ratio of the modulators, a workaround was proposed and a 6 -bit ADC utilizing the 7-bit design was implemented instead. The architecture employs a three-modulus $(\bmod 7,8,9)$ scheme to preprocess the antenna signal.

A literature search was conducted on the work that has been done. Theoretical calculations were performed to specify the parameters of the hardware components that had to be purchased. MATLAB simulations were carried out on the feasibility and expected results of the proposed design architecture. A formula that reduces the beginning and ending vectors to small index values is used to determine the symmetrical residues of each channel. This provides a fast and easy way to find the integer values of the position index for each channel.

The design efficiently couples an RF signal into the optical domain using a parallel management of three MZIs. The RF signal is preprocessed using an arrangement of three power amplifiers and attenuators. The optical to electrical conversion is performed by three photodetectors.

The three power amplifiers, fixed attenuators and variable attenuators were designed to provide sufficient gain and input voltage to the MZIs to fully exploit the bandwidth range of the MZIs and to accurately preprocess the antenna signal. Due to limitation of the high speed comparators [31] during the integration phase, the design was
changed, and three function generators producing three coherent waveforms had to be used to supply the required voltage. For the amplitude modulation stage, the three low half-wave voltage, wideband MZIs were interfaced with a continuous wave distributed feedback (DFB) laser using a $1 \times 3$ splitter. The preprocessed signal was then applied to the RF ports of the interferometers to amplitude modulate the continuous wave laser signal. A DC signal was applied on the DC bias port of each interferometer to shift and control the alignment of the folded signals. An InGaAs photodetector was connected at the output of each modulator to convert the modulated optical signal from each interferometer into an electrical signal. The detection outputs are carefully aligned and post-processed [30] before amplitude analyzing with a high-speed comparator circuits which sample and quantize the detector output signals (designed under a separate thesis [31]). Low frequency analysis of the results using a $1-\mathrm{kHz}$ input signal indicate a 5.42 effective number of bits (ENOB), a signal-to-noise ratio plus distortion (SINAD) of 34.42 dB , and a total harmonic distortion (THD) of -62.84 dB .

## E. THESIS OUTLINE

The RSNS theory and the steps involved in defining the dynamic range, determining the integer values of the position index for each channel and locating the position index on the RSNS waveform are explained in Chapter II.

A description of how the photonic ADC architecture front-end was designed and implemented is contained in Chapter III.

The characterization, analysis and evaluation of the hardware components and preprocessing architecture results are explained in Chapter IV.

The measurement results, calculations and a discussion of the important parameters that describe the ADC's performance are presented in Chapter V.

In Chapter VI, the conclusions and suggestions for future research are presented.

## II. ROBUST SYMMETRICAL NUMBER SYSTEM

In this chapter, there is an explanation of the basic theory of the RSNS and how the dynamic range for a 3-channel (Mod 7, 8, 9) RSNS is determined. Since the waveform repeats every fundamental period, a formula that reduces the beginning and ending vectors to small index values is used to determine the symmetrical residues of each channel. With the location of the symmetrical residues determined, the corresponding shifts on each waveform can be calculated. This forms the preparatory work needed to support the design of the hardware requirements.

## A. RSNS THEORY

In the RSNS, $N$ different periodic symmetrical waveforms are used to generate a series of integers based on the following sequence [18]:

$$
\begin{equation*}
x_{m(N)}=[0,1,2, \ldots m-1, m, m-1, \ldots, 2,1] \tag{2.1}
\end{equation*}
$$

where $x_{m(N)}$ is a row vector and $m$ is a pairwise relatively prime modulus and is a positive integer greater than 0 . For an $N$-channel RSNS with $N=3$, the sequence is [18]

$$
\begin{equation*}
x_{m(N)}=[0,0,0,1,1,1,2,2,2, \ldots m, m, m \ldots 2,2,2,1,1,1] . \tag{2.2}
\end{equation*}
$$

In each sequence, each value from row vector $x_{m(N)}$ is repeated $N$ times. This results in a periodic sequence with a period of [18]

$$
\begin{equation*}
P_{R S N S}=2 m N . \tag{2.3}
\end{equation*}
$$

The discrete RSNS integrers for a single channel are [18]

$$
g[n]= \begin{cases}\left\lfloor\frac{n-s_{\text {shif }_{i}}}{N}\right\rfloor, & s_{\text {shifi }_{i}} \leq n \leq N m_{i}+s_{\text {shif }_{i}}+1  \tag{2.4}\\ \left\lfloor\frac{2 n m_{i}+N-n+s_{\text {shif }_{i}}-1}{N}\right\rfloor, & N m_{i}+s_{\text {shift }_{i}}+2 \leq n \leq 2 N m_{i}+s_{\text {shif }_{i}}-1\end{cases}
$$

where $g$ is the $n^{\text {th }}$ term of channel $i$ and $m_{i}$ is the channel modulus. The sequence shifts $s_{\text {shift }}$ have values $s_{\text {shifti }} \equiv 0,1,2,3, \ldots, N-1(\bmod N)$, and the number of channels is $N \geq 2$. Therefore, an $N$-channel RSNS contains $N$-row vectors with sequence values based on (2.2). The discrete states of the RSNS are indexed using the $s_{\text {shif }_{i}}=0$ row vector with the index starting from the first zero. The RSNS has a fundamental period of [18]

$$
\begin{equation*}
P F_{R S N S}=\left[2 m_{1} N, 2 m_{2} N, \ldots, 2 m_{N} N\right]=2 N\left[m_{1}, m_{2}, m_{3}\right] \tag{2.5}
\end{equation*}
$$

where $\left[a_{1}, a_{2}, \ldots, a_{N}\right]$ with least common multiple of $a_{1}, a_{2}, \ldots, a_{N}$.
The RSNS is compatible with symmetrical folding waveforms (with folding period $2 m_{i}$ ) where the integers within each sequence are determined by a proper placement of threshold values to amplitude analyze the folding waveform. The threechannel ( $\bmod 7,8,9$ ) RSNS generated (for $h=0$ to 128 ) with the calculated threshold values for the comparators using MATLAB (see Appendix E) are as shown in Figures 1, 2 and 3 . Each folding waveform folds at $2 N m_{i}$. The corresponding symmetrical residue representation for $h=0$ to 36 is as shown in Table 1.


Figure 1. RSNS waveform and thermometer code (left shift), mod 7,

$$
s_{\text {shiffit }_{i}}=\left[\begin{array}{lll}
0 & 1 & 2
\end{array}\right]^{T} .
$$



Figure 2. RSNS waveform and thermometer code (left shift), mod 8,

$$
s_{\text {shift }_{i}}=\left[\begin{array}{lll}
0 & 1 & 2
\end{array}\right]^{T} .
$$

Each of the integer values within the sequence are shown based on (2.2). The integer value in each sequence is generated by comparing the amplitude of the symmetrical folding waveform to the $m_{i}$ comparator threshold values. To ensure that the sequence in (2.2) is followed, the threshold values are set such that the interval between two thresholds will generate integers that are repeated $N$ times. For example, the amplitude of the $m_{1}=7$ signal is less than the seven threshold values T 1 to T 7 at position index $h=0$. Hence, each comparator has an output that corresponds to an integer value of 0 as shown in Table 1. At index position $h=3$, the amplitude of the signal is greater than threshold T 1 , and the output of the corresponding comparator will change to 1 . As expected, the next integer value change occurs at position index $h=6$. This follows the
sequence pattern of having $N$ repeated integers in each row vector. As the signal reaches index position $h=21$, the amplitude of the signal exceeds all seven threshold values. Each of the seven comparators is on and an integer value of 7 is obtained. The sequence continues until the length of the vector reaches the maximum system dynamic range $\hat{M}=133$ for the $\mathrm{N}=3\left(m_{1}=7, m_{2}=8, m_{3}=9\right)$. The maximum system dynamic range is the number of consecutive column vectors that are distinct.

Channel 3 (Modulus 9)


Figure 3. RSNS waveform and thermometer code (left shift), mod 9,

$$
s_{s h i f_{i}}=\left[\begin{array}{lll}
0 & 1 & 2
\end{array}\right]^{T} .
$$

The same process is used to generate the integers contained in $m_{2}=8$ and $m_{3}=9$ row vectors. To maintain the characteristic of the sequence in (2.2), the threshold values must be set correctly. The key to maintaining $N$ repeated integers in each sequence is
having a precise set of threshold values coupled with the three accurately aligned folding waveforms. An illustration of the modulated signals with the threshold values are explained further in Chapter III.

From Table 1, the Gray-code property of RSNS is demonstrated by observing the integer values in each column vector. Observe that for each code transition, only one integer value changes within a column vector. With this property, the RSNS scheme provides better error control during data processing.

Table 1. An illustration of a three-channel RSNS (left shift).

| $\bmod 7$ | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | $\ldots$ |
| $\bmod 9$ | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | $\ldots$ |
| $h$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | $\ldots$ |


| $\bmod 7$ | 6 | 6 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | $\ldots$ |
| $\bmod 9$ | 7 | 7 | 7 | 8 | 8 | 8 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | $\ldots$ |
| $h$ | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | $\ldots$ |

## B. DETERMINATION OF THE DYNAMIC RANGE

The system dynamic range ( $\hat{M}$ ) of the RSNS is the maximum number of distinct vectors without an ambiguity. In any RSNS application it is important to determine $\hat{M}$ in order to determine which combination of moduli form the best RSNS with $\hat{M} \geq 2^{p}$, where $p=$ number of bits needed for the ADC. When three channels are considered and the channel moduli are of the form $m_{1}=2^{k}-1, m_{2}=2^{k}$ and $m_{3}=2^{k}+1$, the closed form expression for $\hat{M}$ is

$$
\begin{equation*}
\hat{M}=\frac{3}{2} m_{1}^{2}+\frac{15}{2} m_{1}+7 \tag{2.6}
\end{equation*}
$$

where $m_{1} \geq 3$ [18].

From (2.6), the system dynamic range calculated for $\bmod (7,8,9)$ is 133 . The key to generating and encoding the RSNS into an EO folding architecture is having a correct input voltage to the MZI, $V_{R F_{i}}$ and proper alignment of signals. The calculations performed in the design of the preprocessing architecture are documented in Chapter III. The starting and ending position vectors for a right and left shift implementation are found using two MATLAB codes as enclosed in Appendix A and Appendix B, respectively. The right shift code employs a naïve approach. This approach is more computationally intensive as compared to the left shift algorithm, which reduces the computational time by several orders of magnitude. In addition, the efficient algorithm uses far less memory than the naïve algorithm and, therefore, can find the $N$-sequence $\hat{M}$ and position for moduli sets with much larger fundamental periods [32].

The outputs from both codes are as shown in Tables 2 and 3. It can be seen that the point indices corresponding to $\hat{M}$ (beginning point and ending point) are different. This represents the useful range of the RSNS that can be used to encode the detected modulation outputs. The three channels need to be properly aligned relative to each other based on the beginning and ending position of the position index prior to sampling.

Table 2. System Dynamic Range for Mod (7, 8, 9) RSNS (right shift).

| Moduli | shift | System dynamic <br> range | Fundamental <br> Frequency | Beginning-Ending of <br> system dynamic range |
| :---: | :---: | :---: | :---: | :---: |
| 7 | 0 |  |  |  |
| 8 | 1 | 133 | 3024 | $649-781$ |
| 9 | 2 |  |  |  |

Table 3. System Dynamic Range for Mod (7, 8, 9) RSNS (left shift).

| Moduli | shift | System dynamic <br> range | Fundamental <br> Frequency | Beginning-Ending of <br> system dynamic range |
| :---: | :---: | :---: | :---: | :---: |
| 7 | 0 |  |  |  |
| 8 | 1 | 133 | 3024 | $733-865$ |
| 9 | 2 |  |  |  |

## C. DETERMINING THE INTEGER VALUES OF THE POSITION INDEX FOR EACH CHANNEL

The beginning and ending points of system dynamic range for the $\bmod (7,8,9)$ RSNS with their corresponding shift are summarized in Tables 2 and 3. Since the waveform for each channel repeats every fundamental period, a method that subtracts out integer multiples of the $2 N m_{i}$ codes within a folding period is developed to find the symmetrical residues of each channel. The formula is

$$
\begin{equation*}
h_{m_{i}}=h-\left\lfloor\frac{h}{2 N m_{i}}\right\rfloor 2 N m_{i} \tag{2.7}
\end{equation*}
$$

where $h_{m_{i}}$ is the index value $h$ of mod $m_{i}$. An example of how the indexes can be derived quickly and accurately without having to map out the entire sequence manually is as shown in Table 4. From (2.7), $h_{7}=20, h_{8}=14, h_{9}=8$. Verification with Table 4 shows that the symmetrical residues determined are all present at $h=62$. This provides a fast and easy way to determine the symmetrical residues at the beginning position index for each channel and is especially useful when the position index is a few hundred positions away. Positive identification of the position index is important as this will enable a quick calculation of the bias voltage value needed for each channel to shift the waveform such that the threshold crossings give the integer values within the sequence at a chosen input voltage.

Table 4. $\quad \operatorname{Mod}(7,8,9)$ RSNS partial structure with $s_{\text {shift }_{i}}=\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]^{T}$.

| $\bmod 7$ | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | $\ldots$ |
| $\bmod 9$ | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | $\ldots$ |
| $h$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | $\ldots$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\bmod 7$ | 6 | 6 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | $\ldots$ |  |
| $\bmod 8$ | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | $\ldots$ |  |
| $\bmod 9$ | 7 | 7 | 7 | 8 | 8 | 8 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | $\ldots$ |  |
| $h$ | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | $\ldots$ |  |


| $\bmod 7$ | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 4 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | $\ldots$ |
| $\bmod 9$ | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | $\ldots$ |
| $h$ | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | $\ldots$ |


| $\bmod 7$ | 4 | 4 | 5 | 5 | 5 | 6 | 6 | $\mathbf{6}$ | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 2 | 3 | 3 | 3 | 4 | 4 | 4 | $\mathbf{5}$ | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | $\ldots$ |
| $\bmod 9$ | 1 | 1 | 1 | 2 | 2 | 2 | 3 | $\mathbf{3}$ | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | $\ldots$ |
| $h$ | 55 | 56 | 57 | 58 | 59 | 60 | 61 | $\mathbf{6 2}$ | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | $\ldots$ |

## D. LOCATION OF POSITION INDEX ON THE RSNS WAVEFORM

The location of the beginning and ending position index and their respective integer values are determined as detailed in section $B$ and $C$. To translate this into voltage shift values, the least significant bit (LSB) needs to be calculated. The LSB represents the minimal DC bias voltage value to be applied to the MZI for every shift in the position index. The size of the LSB within each of the $N$ channels is the same. From (2.5), there are $P_{m_{i}}=2 N m_{i}$ LSBs within a complete fold $2 V_{\pi}$ resulting in

$$
\begin{equation*}
L S B=\frac{V_{\pi_{i}}}{N m_{i}} \mathrm{~V} \tag{2.8}
\end{equation*}
$$

where $V_{\pi_{i}}$ is the effective half wave voltage the MZI.
A complete mapping of the position vectors for left shift $\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]^{\mathrm{T}}$ for the dynamic range from $h=733$ to 860 is shown in Table 5. The starting position is $h=733$ for a 7 -bit
( $2^{7}=128$ ) implementation. From (2.7) for $h=733$, from Table $1, h_{7}=19 h_{8}=13$ and $h_{9}=31$. The DC bias voltage required for channel 1,2 and 3 are 19, 13 and 31 LSBs away, respectively.

Table 5. $\quad \operatorname{Mod}(7,8,9)$ RSNS, $s_{\text {shif }_{i}}=\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]^{T}$, dynamic range $\mathrm{h}=733$ to 861 .

| $\bmod 7$ | 6 | $\mathbf{6}$ | 6 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 4 | $\mathbf{4}$ | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 7 | $\ldots$ |
| $\bmod 9$ | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | $\ldots$ |
| $h$ | 732 | 733 | 734 | 735 | 736 | 737 | 738 | 739 | 740 | 741 | 742 | 743 | 744 | 745 | 746 | $\ldots$ |


| $\bmod 7$ | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | $\ldots$ |
| $\bmod 9$ | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | $\ldots$ |
| $h$ | 747 | 748 | 749 | 750 | 751 | 752 | 753 | 754 | 755 | 756 | 757 | 758 | 759 | 760 | 761 | $\ldots$ |


| $\bmod 7$ | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | $\ldots$ |
| $\bmod 9$ | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | $\ldots$ |
| $h$ | 762 | 763 | 764 | 765 | 766 | 767 | 768 | 769 | 770 | 771 | 772 | 773 | 774 | 775 | 776 | $\ldots$ |


| $\bmod 7$ | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | $\ldots$ |
| $\bmod 9$ | 7 | 8 | 8 | 8 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | $\ldots$ |
| $h$ | 777 | 778 | 779 | 780 | 781 | 782 | 783 | 784 | 785 | 786 | 787 | 788 | 789 | 790 | 791 | $\ldots$ |


| $\bmod 7$ | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | $\ldots$ |
| $\bmod 9$ | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | $\ldots$ |
| $h$ | 792 | 793 | 794 | 795 | 796 | 797 | 798 | 799 | 800 | 801 | 802 | 803 | 804 | 805 | 806 | $\ldots$ |


| $\bmod 7$ | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | $\ldots$ |
| $\bmod 9$ | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | $\ldots$ |
| $h$ | 807 | 808 | 809 | 810 | 811 | 812 | 813 | 814 | 815 | 816 | 817 | 818 | 819 | 820 | 821 | $\ldots$ |


| $\bmod 7$ | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | 2 | 2 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | $\ldots$ |
| $\bmod 9$ | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 9 | 9 | $\ldots$ |
| $h$ | 822 | 823 | 824 | 825 | 826 | 827 | 828 | 829 | 830 | 831 | 832 | 833 | 834 | 835 | 836 | $\ldots$ |
| $\bmod 7$ | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | $\ldots$ |
| $\bmod 8$ | 7 | 7 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | $\ldots$ |


| $\bmod 9$ | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 837 | 838 | 839 | 840 | 841 | 842 | 843 | 844 | 845 | 846 | 847 | 848 | 849 | 850 | 851 | $\ldots$ |


| $\bmod 7$ | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 6 | 6 | 6 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 4 | 4 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | $\ldots$ |
| $\bmod 9$ | 4 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | $\ldots$ |
| $h$ | 852 | 853 | 854 | 855 | 856 | 857 | 858 | 859 | 860 | 861 | 862 | 863 | 864 | 865 | 866 | $\ldots$ |

## E. SUMMARY

In this chapter, the RSNS theory was explained. A detailed discussion of the preparatory work to be done prior to designing the preprocessing architecture was presented. In the next chapter, the design and implementation of a photonic RSNS ADC preprocessing architecture leveraging on the preparatory work done in Chapter II is discussed. Design specifications, calculations and hardware selection will be highlighted and discussed in detail.

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## III. RSNS ADC ARCHITECTURE DESIGN

A background on RSNS theory was presented in Chapter II, and a detailed discussion of the preparatory work to be done prior to designing the preprocessing architecture was presented. This included determining the dynamic range, identifying and locating the symmetrical residues and determining the corresponding shifts required for each channel so that the three channels can be aligned in such a way that the Gray-code properties can be extracted.

The implementation of a RF photonic front-end ADC preprocessing architecture is explained in this chapter. The components selection criteria and effects of each device's operating characteristics and limitations on the system operation are pointed out.

## A. RF PHOTONIC ADC SYSTEM DESIGN

In [30], a full implementation of a 6-bit RSNS ADC concept was constructed by combining photonic and electronic technologies. The architecture employs a threemodulus $(3,4,5)$ scheme to preprocess the antenna signal. In this thesis, an improved architecture employing a three-modulus $(7,8,9)$ scheme to preprocess the antenna signal is implemented and tested. Analysis of results demonstrates an improvement in the dynamic range (from 43 to 133) and reinforces the feasibility of using electro-optic devices to fully implement an optical folding ADC using the RSNS concept.

The photonic ADC front-end prototype was built by combining photonic and electronic devices. It was fully implemented by combining several signal processing stages to produce three aligned electrical signals. These signals are then passed through the RSNS-to-binary conversion stage [31]. Component selection for each signal processing phase are addressed in subsequent sections.

The block diagram of the RF photonic ADC architecture is shown in Figure 4. An RF signal received by the antenna (in this case a signal generator) is split into three channels and amplified individually to the maximum voltage required to generate the folding periods. A $1.55-\mu \mathrm{m}$ continuous wave optical signal from a distributed feedback (DFB) laser is used after going through a $1 \times 3$ splitter. The splitter divides the laser signal
into three beams, providing a signal for each Mach-Zehnder modulator. As the amplified RF signal is applied to the electrodes of the modulator, amplitude modulation of the laser signal occurs. To obtain the necessary phase shift for alignment of the RSNS channels a DC voltage is also applied on the bias port of each modulator. After amplitude modulation of the laser, each modulated optical signal is sent to a photodetector to convert it into an electrical signal. The electrical signal is amplified and further processed using the RSNS-to-binary scheme utilizing an FPGA [31]. The encoding scheme converts the three-modulus RSNS signals to a bipolar 7-bit binary code. Each code within the 7 -bit binary representation corresponds to a quantized value (dynamic range $\hat{M}=128$ ).


Figure 4. Block diagram of RSNS based photonic ADC architecture (From [30]).

## B. RF SIGNAL AMPLIFICATION

Each amplifier has a variable gain to obtain the required voltage for electro-optic modulation. For a three-modulus RSNS structure, each channel requires a different voltage to obtain the following fundamental periods [18]

$$
\begin{equation*}
P F_{R S N S}=2 N\left[m_{1}, m_{2}, m_{3}\right] \tag{3.1}
\end{equation*}
$$

for $\left[m_{1}, m_{2}, m_{3}\right]=[7,8,9]$ where [.] is the LCM operator $=\pi m_{i}$ and $N=3$ $P F_{\text {RSNS }}=3024$. The fundamental frequency for each RSNS channel can be represented as

$$
\begin{equation*}
f_{\text {RSNS }_{i}}=\frac{1}{P_{\text {RSNS }}}=\frac{1}{2 N m_{i}} \tag{3.2}
\end{equation*}
$$

where $i=[1,2,3]$ is the channel index.

From the relationship between the half-wave voltage of the MZ modulator and the period in each channel, the required applied voltage for a bipolar ADC using the RF port is [18]

$$
\begin{equation*}
V_{R F_{i}}=\frac{\hat{M}}{2 N m_{i}} V_{\pi_{i}} \tag{3.3}
\end{equation*}
$$

where $V_{\pi_{i}}$ is the effective half-wave voltage. From here we can see that the most important parameter is the $V_{\pi}$ of each modulator. With a smaller $V_{\pi}$ the required signal amplification $V_{R F_{i}}$ will be lower.

The voltage and power ratings for the three-modulus $(7,8,9)$ channels using (3.3) is as shown in Table 6. It can be seen that for mod 7, the highest power rating is 29 dBm . Although this is higher than the modulator's specification of 27 dBm , it is a significant improvement from the 33.5 dBm requirement in the three-modulus $(3,4,5)$ implementation [30] where the high input power, the DC port had to be used instead of the RF port. This lowered the bandwidth processing capability of the setup. It also meant that the alignment of the three channels via the application of the bias shift voltages had to be coupled to the $V_{R F_{i}}$ via a summing amplifier [30]. This posed a challenge in
adjusting the bias voltage to be applied to each channel when performing the channel alignment in the laboratory. With the improved $V_{\pi}$, the RF port was used and a signal of $1-\mathrm{kHz}$ was successfully preprocessed and encoded. The availability of the DC port allowed the required bias voltage to be applied on the bias port of each modulator to obtain the necessary phase shift in the transmisivity for alignment of the RSNS channels.

Table 6. Calculated applied RF port voltage and power requirements.

| $i^{\text {th }}$ Channel | RF Port Half-wave Voltage <br> $V_{\pi R F_{i}}$ | Calculated RF Port <br> Applied Voltage <br> $V_{R F_{i}}$ | Power Rating |
| :---: | :---: | :---: | :---: |
| $1(\bmod 7)$ | 3 V | 9.14 V | 29 dBm |
| $2(\bmod 8)$ | 3 V | 8.0 V | 28 dBm |
| $3(\bmod 9)$ | 3 V | 7.1 V | 27 dBm |

With the required voltage and power rating, it helped to narrow down the required amplifier and attenuators needed for the signal amplification design. Due to the unavailability of Low Noise Amplifiers (LNA) with power output of at least 0.5 W , a power amplifier with a low noise figure is proposed. To cater for the differing $V_{R F_{i}}$ requirements for the three channels, a flexible and adjustable amplification circuit had to be employed. The amplification design for three channels is as shown in Figure 5.


Figure 5. RF preprocessing circuit design.

The mixture of fixed and variable attenuators allow the tuning of the voltage and power values prior to the input of each modulator. A summary of the components selected is as shown in Table 7.

Table 7. Signal amplification components.

| Component | Model | Remarks |
| :--- | :--- | :--- |
| Power Amplifier | Nextec-RF (NB00415) | Amplification of signal source |
| Fixed Attenuator | Bird Technologies <br> $(100-A-M F A-03 / 06)$ | Provide attenuation to suitable levels <br> for input to the variable attenuator (3 <br> dB and 6 dB$)$ |
| Variable <br> Attenuator | Agilent 8494A | Fine tune $V_{R F_{i}}$ required for each <br> channel |

## C. MODULATION AND RSNS SIGNAL ALIGNMENT

Electro-optic modulation is achieved using an MZI. An electric field is used to amplitude modulate the light from an optical source such as a laser. The basic structure of an electro-optic modulator is as shown in Figure 6. A basic structured modulator [33] is comprised of 1) two waveguides, 2) two Y-junctions and 3) RF/DC electrodes. Optical signals coming from the laser are launched into the modulator through the polarization maintaining (PM) fiber. It is then equally split into two optical waveguides by the first Yjunction on the substrate. When the voltage is not applied to the RF electrode, the two signals are re-combined coherently at the second Y-junction. In this case, the output signal from the modulator is recognized as "ONE." When the voltage $V_{\pi}$ is applied to the RF electrode, due to the electro-optic effects of the modulator substrate, the refractive index is changed, and the propagation constant of the optical signal in one arm is delayed with respect to the other arm. As the two signals are re-combined at the second Yjunction, they are transformed into higher order mode and lost as a radiation mode. When two signals are recombined out of phase, the output signal from the modulator is recognized as "ZERO." The voltage difference which induces this "ZERO" and "ONE" is called the half-wave voltage $V_{\pi}$ of the modulator.


Figure 6. Basic structure of a MZI (From [34]).

For this system, a DFB laser is the optical source. It provides a $1.55 \mu \mathrm{~m}$ continuous wave signal. A laser diode controller module is used to control the intensity. Injecting 150 mA of current into the laser diode, we produce enough intensity for the photodetector to detect.

As the optical beam travels through the crystal lattice of the modulator, an applied electric field causes amplitude modulation on the light beam. The applied electric field must have adequate amplitude to produce the essential RSNS period or frequency for each channel. The intensity of the MZ modulator output is defined as [35]

$$
\begin{equation*}
I=I_{0} \cos ^{2}\left(\frac{\pi V}{2 V_{\pi}}\right) \tag{3.4}
\end{equation*}
$$

where $I_{0}$ is the input optical intensity, $V$ is the applied voltage, and $V_{\pi}$ is the half-wave voltage of the MZ modulator.

The transfer characteristic or transmissivity of an optical modulator, each with a different half-wave voltage is demonstrated in Figure 7 using (3.4) [30]. The plots depict how the period of the modulated optical beam varies as a function of applied voltage. As the applied voltage increases, the period of the modulated signal decreases. Also, each modulated signal is symmetric to the $y$-axis, and the peaks occur when the applied voltage is equal to an integer multiple of $2 V_{\pi}$. Adding a DC voltage into the modulator causes a phase shift in the modulated signal. In such case, the peaks will also shift left and right depending on the applied DC voltage. In optical communications, it is convenient to set the bias point at $50 \%$ transmission (quadrature) point, as shown in Figure 8. The electrical signals are transformed into optical digital signal by electrically switching the voltage from $V_{B}-\frac{V_{\pi}}{2}$ to $V_{B}+\frac{V_{\pi}}{2}$.


Figure 7. Transfer function of a Mach-Zehnder modulator (From [30]).


Figure 8. Electrical-optical plot (From [36]).

The key performance parameters of the modulator chosen for this thesis are as shown in Table 8. The key to encoding the RSNS into an EO folding architecture is having a correct $V_{R F_{i}}$ and proper alignment of the modulation signals. Recall Table 2 summarizes the beginning-ending of system dynamic range for the three modulus RSNS with their corresponding shift.

Table 8. Modulator key performance parameters.

| Model | Operating <br> Wavelength | Insertion Loss | Bandwidth | Driving <br> Voltage $V_{\pi}$ |
| :---: | :---: | :---: | :---: | :---: |
| EOSPACE Inc |  |  |  |  |
| AX-CK5-10- |  |  |  |  |
| PFU-SFU-UV | 1550 nm | $<5 \mathrm{~dB}$ | $>10 \mathrm{GHz}$ | $<3 \mathrm{~V} @ 1 \mathrm{GHz}$ |

From Table 2, the start index of the RSNS for shift $\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]^{\mathrm{T}}$ is 733 and ends at 860. As shown in Table 4, the column vectors for position index 733 and 860 are [64 7] ${ }^{\text {T }}$ and $\left[\begin{array}{lll}6 & 1 & 1\end{array}\right]^{\mathrm{T}}$, respectively. To obtain the integer values contained in the starting column vector, each interferometer transmissivity must be phase shifted by adding either a positive or negative DC voltage. The required DC bias voltages are positive 19, 13 and $31 \mathrm{LSBs}(\mathrm{LSB}=102 \mathrm{mV})$ away, respectively.

The modulated optical signals are aligned to generate column vectors that start with $\left[\begin{array}{lll}6 & 4 & 7\end{array}\right]^{\mathrm{T}}$ using the same process outlined in Chapter II. The MATLAB generated RSNS and thermometer code for left shift $\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]^{\mathrm{T}}$ at starting position index $h=733$ with symmetrical residue [647] ${ }^{\mathrm{T}}$ are as shown in Figures 9, 10 and 11.

Channel 1 (Modulus 7)


Figure 9. RSNS waveform and thermometer code, $\left(\bmod 7, s_{\text {shif }_{1}}=0\right)$.


Figure 10. RSNS waveform and thermometer code, $\left(\bmod 8, s_{\text {shit }_{2}}=1\right)$.

The threshold values are also shown for each channel. With the waveforms properly aligned, the combination of integers in the first fifteen column vectors are shown in Table 9. For example at $h=733$, the amplitude of modulus 7 signal is greater than the threshold value T6. Thus, the integer value at that position is 6 (number of comparators on). With modulus 8 , the amplitude is greater than T1 ... T4. Hence, an integer value of 4 is also obtained. Lastly, the value of modulus 9 signal is greater than its first seven threshold values, which results in integer 7. The integer values represent the symmetrical values within each channel.


Figure 11. RSNS waveform and thermometer code, $\left(\bmod 9, s_{\text {shift }_{3}}=2\right)$.

Observe that the interval between the threshold voltages becomes smaller as it approaches the maximum and minimum point of the folding waveform. The modulus- 9 signal clearly demonstrates this characteristic, which is caused by the non-linearity of the folding waveform and higher fundamental period. Therefore, a careful examination of the threshold values must be made when using higher moduli values to prevent errors caused by added noise. When the threshold interval is less than the amplitude of the added noise, encoding errors occur.

Table 9. First fifteen column vectors.

| $\bmod 7$ | 6 | 6 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bmod 8$ | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 7 | 7 | $\ldots$ |
| $\bmod 9$ | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 3 | 3 | 3 | $\ldots$ |
| h | 733 | 734 | 735 | 736 | 737 | 738 | 739 | 740 | 741 | 742 | 743 | 744 | 745 | 746 | 747 | $\ldots$ |

## D. OPTICAL SIGNAL CONVERSION

After electro-optic modulation, the optical beam must be converted to an electrical signal prior to sampling. This can be done using a photodetector. The photodetector selected needs to have a large response at the wavelength to be detected, a small value for the noise introduced by the detector and sufficient speed of response to follow variations in the optical signal being detected.


Figure 12. Relative responsivity of InGaAs based photodetector (From [37]).

To fulfill the above requirements, the key performance parameters to look out for in selecting a suitable photodetector are the operating wavelength, bandwidth, rise time, responsivity and noise equivalent power (NEP). The relative responsivity of photodetectors made from InGaAs is as shown in Figure 12. It can be seen that the responsivity is almost $100 \%$ for 1550 nm . The response time of the photodetector is characterized by the rise time. A short rise time will enable the detection of fast pulse. From the rise time, we can calculate the high frequency cutoff (equivalent to bandwidth), which is about 39 GHz , quite close to the manufacturer's specification of 45 GHz . The
amount of optical power incident on the surface of a photodetector that produces a signal at the output of the detector just equal to the noise generated internally by the detector is the NEP. A low NEP will allow a good signal-to-noise ratio (SNR), thereby minimizing the quantization errors. The performance parameter of the chosen photodetector is as shown in Table 10. The noise equivalent power (NEP) of the InGaAs photodetector is $<45 \mathrm{pW} / \sqrt{\mathrm{Hz}}$, which is very small. Thus, the noise coming from the photodetector is not the dominant noise source.

Table 10. Key performance parameters of the choice photodetector.

| Model | Wavelength | Rise time | Bandwidth | Responsivity | Noise <br> Equivalent <br> Power |
| :---: | :---: | :---: | :---: | :---: | :---: |
| New Focus <br> 1014 | $950-1650$ <br> nm | 9 ps | 45 GHz | $0.4 \mathrm{~A} / \mathrm{W}$ | $<45 \mathrm{pW} / \sqrt{\mathrm{Hz}}$ |

## E. NOISE REDUCTION

Noise reduction is also important in the preprocessing. Using filters is one of the simplest methods to reduce the noise. A RC low-pass filter was used in the prototype [30]. Depending on the frequency of the signal of interest, the cutoff frequency can be adjusted using

$$
\begin{equation*}
f_{c}=\frac{1}{2 \pi R C} \tag{3.5}
\end{equation*}
$$

where $R$ is the resistor value and $C$ is the capacitance. The capacitance was developed by using several capacitors in parallel with the DC power supply to reduce the noise level prior to amplification.

## F. SUMMARY

The signal processing steps to implement the RSNS preprocessing front-end architecture were described in this chapter, and the characteristics and considerations in the choice of hardware for the RF preprocessing circuit, optical modulation circuit and the optical to electrical conversion or detector circuit were explained. Finally, the chosen hardware was highlighted together with the key performance parameters. A summary of the components selected is shown in Table 11. In the next chapter, a detailed discussion of the results will be presented.

Table 11. Summary of selected components.

| Component Type | Selected Components |
| :--- | :--- |
| Power Amplifier | Nextec-RF (NB00415) |
| Fixed Attenuator | Bird Technologies (100-A-MFA-03/06) |
| Variable Attenuator | Agilent (8494A Manual Step Attenuator) |
| Interferometer | EOspace (Model AX-CK5-10-PFU-SFU-UV Low Vpi 10 Gb/s <br> X-cut Lithium Niobate Intensity Modulator at 1550nm <br> Wavelength) |
| Photodetector | New Focus (45-GHz InGaAs Photodetector Model 1014) |
| Optics Splitter | Ozoptics (1×3 splitter FUSED-13-1550-8) |
| Power Supply | Instek (GPS-3030DD) |

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## IV. PREPROCESSING RESULTS AND DATA ANALYSIS

This chapter begins with the characterization of the hardware components. Limitations leading to design and implementation changes are discussed. The respective RSNS waveforms are then generated aligned with the new design. This is performed through application of the RF voltages and DC shift voltages for the individual RSNS channel.

## A. COMPONENT CHARACTERIZATION

To ensure that the components meet the specifications, a series of experiments are conducted to verify the performance parameters of each component.

## 1. Power Amplifier

The specifications for the power amplifier includes being able to generate a gain of $40-50 \mathrm{~dB}$ within a bandwidth of $1-500 \mathrm{MHz}$. The amplifier output corresponding to three different inputs at 1 MHz is shown in Table 12. It can be seen that for a fixed frequency, the gain is constant for the three input values. The measured gain is within specifications. The measurement setup for the power amplifier and attenuators is as shown in Figure 13.

Table 12. Calculated gain of power amplifier at 1 MHz .

| Input | $\mathrm{S} / \mathrm{N} 0238$ |  | $\mathrm{~S} / \mathrm{N} 0239$ |  | $\mathrm{~S} / \mathrm{N} 0240$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Output |  | Gain | Output |  | Gain |  | Output |  | Gain |
| $10 \mathrm{mV} /$ | 17 dBm | $27+27=$ <br> 44 dB | 19.5 dBm | $27+19.5=$ | 22 dBm | $27+22=$ |  |  |  |  |
| -27 dBm |  |  | 46.5 dBm |  | 49 dBm |  |  |  |  |  |
| $25 \mathrm{mV} /$ | 25 dBm | $19+25=$ <br> 44 dB | 27.5 dBm | $19+27.5=$ <br> -19 dBm | 36.5 dBm | $19+30=$ |  |  |  |  |
| $30 \mathrm{mV} /$ | 29 dBm | $17.5+26.5$ <br> $=44 \mathrm{~dB}$ | 29 dBm | $17.5+29=$ | 32 dBm | $17.5+32=$ |  |  |  |  |
| -17.5 dBm |  | 46.5 dBm |  | 49.5 dBm |  |  |  |  |  |  |



Figure 13. Measurement Setup.

## 2. Fixed and Variable Attenuators

A 1 MHz signal with a 10 mV peak-to-peak voltage was sent to the 3 dB and 6 dB attenuators. The output values are -30 dBm and -33 dBm , respectively, in line with the expected results. The measurement setup is similar to Figure 13.

## 3. Photodetectors

The photodetectors were connected with a 1550 nm laser source and the output was monitored on the oscilloscope. All three photodetectors were within specification, i.e., maximum conversion gain of $10 \mathrm{~V} / \mathrm{W}$.

## 4. Modulators

The modulators were tested with a 1550 nm laser source modulated with a 1 kHz RF signal. DC bias was applied via the DC port. All three modulators showed a $\mathrm{V}_{\pi}$ value $<3 \mathrm{~V}$ for the RF bias port and $<5.5 \mathrm{~V}$ for the DC bias port, in line with the specification.

## B. SETUP CHANGES

The output from the three power amplifiers with a 1 MHz sine function is as shown in Figure 14. The output is generated with a function generator via a 1:3 RF splitter. With the power amplifier in place, the required RF input can be reduced to a minimum (in line with what is happening in the real world), thereby allowing more latitude in the power control of each channel. Due to limitation in the sampling circuit [31], the RF input signal is optimized to 1 kHz . For this setup to function, the input RF signal need to have a minimum of 1 MHz (Bandwidth of power amplifier $1-500 \mathrm{MHz}$ ). Though not implemented in the current setup, this proposed design can be applied to future setup once the limitation on the sampling circuit is overcome.


Figure 14. Synchronized RF output using power amplifier.

A new approach utilizing three synchronized function generators was proposed to provide the power required for each channel. This allows great flexibility in the power control for each channel. The key is to synchronize all three function generators to produce three coherent ramp functions such that they cross the $(0,0)$ point at one-half the
maximum voltage. The setup of the three function generators is as shown in Figure 15. The digital function generator was selected as the clock source while the other two analog function generators were selected due to their higher power output level ( $15 \mathrm{~V}_{\mathrm{p}-\mathrm{p}}$ ) as compared to the digital function generator $\left(10 \mathrm{~V}_{\mathrm{p}-\mathrm{p}}\right)$. The synchronized ramp functions are as shown in Figure 16.


Figure 15. Function Generator Setup.


Figure 16. Coherent $25-\mathrm{kHz}$ Ramp Functions.

## C. EXPERIMENTAL SETUP AND RESULTS

The $\mathrm{V}_{R F_{i}}$ input to the three modulators to derive the number of folds required for each channel is as shown in Figures 17, 18 and 19. The $\mathrm{V}_{\pi}$ measurements are also performed for each modulator. The superimposed RSNS waveform for channels 1, 2 and 3 is as shown in Figure 20. The results are as shown in Table 13.


Figure 17. Aligned RSNS waveform for mod 7 (channel 1).


Figure 18. Aligned RSNS waveform for mod 8 (channel 2).


Figure 19. Aligned RSNS waveform for mod 9 (channel 3).


Figure 20. Aligned RSNS waveform for mod 7, 8 and 9.

Table 13. Experimental results and calculations based on 7-bit implementation.

| Channel/ <br> Modulator <br> $\mathrm{S} / \mathrm{N}$ | $\mathrm{V}_{R F_{i}} \mathrm{~V}$ <br> Calc | $\mathrm{V}_{R F_{i}} \mathrm{~V}$ <br> Meas | $\mathrm{V}_{\pi} \mathrm{V}$ <br> Meas | $\mathrm{V}_{D C_{i}}$ <br> Calc | $\mathrm{V}_{D C_{i}} \mathrm{~V}$ <br> Meas. | $K_{i}$ | LSB mV <br> Calc |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 <br> 203919 | 9.14 | 15.9 | 2.60 | 1.9 | 2 | 1.21 | 102 |
| 2 <br> 203922 | 8 | 15.0 | 2.80 | 1.3 | 1.4 | 1.14 | 102 |
| 3 <br> 203925 | 7.1 | 13.1 | 2.76 | 3.2 | 3.3 | 1 | 102 |

## 1. Changes in Dynamic Range.

From Table 12, the measured $V_{R F_{i}}$ values are approximately double that of the calculated values. This is due to the different terminations (50 $\Omega$ termination was assumed for the calculations but the actual input termination was $1 \mathrm{M} \Omega$ ) as a result of the differing RF port input impedances of the modulators. As can be seen from Figures 17, 18 and 19 , the modulated waveforms exhibit periodic uneven amplitude output. When the aligned waveforms are sampled, the original input RF ramp function could not be recovered completely. This is due to the sampling errors introduced as a result of the uneven waveform. It is suspected that there may be an anomaly in the behavior of the material substrate of the modulators. ${ }^{1}$ To overcome the constraint, an alignment was performed for the dynamic range of 41 instead of 128 using the current design, i.e., 6-bit implementation using a 7-bit design. The new dynamic range is obtained through the optimization of the areas of even modulation of all three channels. Alignment for the new dynamic range was achieved via the DC bias port which allowed phase control of each channel. The aligned waveforms are as shown in Figure 21. The results and calculations are as shown in Table 13. The recovered ramp function is as shown in Figure 22.

[^0]
## 2. $\quad K_{i}$ value derivation and LSB calculation.

The $K_{i}$ value for each channel is calculated based on the ratio of the $V_{R F_{i}}$ applied to channels 1, 2 and 3 to the $V_{R F_{i}}$ applied to channel 3 . This is similar to assuming a single function generator with a 1:3 RF splitter applied with the necessary $K_{i}$ value to generate the necessary $\mathrm{V}_{R F_{i}}$ required for each channel. Based on the above, the $K_{i}$ value for channels 1, 2 and 3 are 1.21, 1.14 and 1, respectively. The LSBs are obtained using (2.8). It can be seen that the $K_{i}$ and LSBs calculated are similar for all three channels in Table 13 and Table 14.


Figure 21. Aligned waveforms based on dynamic range of 41.

Table 14. Results and calculations based on 6-bit implementation.

| Channel/ <br> Modulator S/N | $\mathrm{V}_{R F_{i}} \mathrm{~V}$ <br> Meas | $\mathrm{V}_{\pi} \mathrm{V}$ <br> Meas | $K_{i}$ | LSB mV <br> Calc |
| :---: | :---: | :---: | :---: | :---: |
| 1 <br> 203919 | 4.70 | 2.60 | 1.22 | 102 |
| 2 <br> 203922 | 4.45 | 2.80 | 1.15 | 102 |
| 3 <br> 203925 | 3.86 | 2.76 | 1 | 102 |



Figure 22. Recovered ramp function.

## D. KEY OBSERVATIONS

## 1. Signal alignment process

The importance of the input RF voltage needed to generate the correct number of folds and the DC bias control for proper alignment of the signal cannot be overemphasized. For a more efficient and effective alignment process, the following steps are recommended:
a) Use three synchronized digital function generators to supply the required $V_{R F_{i}}$ to each modulator to generate the required number of folds,
b) Align the three outputs from the modulators based on the MATLAB output and calculate $V_{D C_{i}}$. Keep one of the channel values constant, i.e., minimum modulus channel while tuning the other channels,
c) Ensure that all three waveforms are aligned to the same DC offset on the scope to ensure consistent sampling by the comparators,
d) Adjust up the amplitude of the aligned signals using the post-processing circuit. This allows for a more robust sampled signal output since the amplitude of the signal is a function of the threshold values calculated for the comparators.

## E. SUMMARY

The characterization of the hardware components prior to the build-up of the preprocessing architecture were described in this chapter. The limitations leading to design changes and the results were highlighted and discussed at length. Lastly, key observations and lessons learnt are highlighted and shared for a more efficient and effective alignment process. In the next chapter, the dynamic performance of the ADC will be presented.

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## V. DYNAMIC PERFORMANCE ANALYSIS

In this chapter, a number of important parameters that describe a converter's performance are presented. Differential and integral linearity errors are plotted for the ADC to analyze the linearity errors. The dynamic range is determined using a full-scale sinusoid and a subsequent Fourier spectrum analysis of the noise floor. Finally, the signal's spectrum is characterized by the SNR, SNR plus distortion (SINAD) and the effective number of bits (ENOB).

## A. LINEARITY ERRORS

The characteristic transfer function of a 1 kHz triangular waveform is used to determine the linearity of the ADC. The input voltage and the decimal output voltage plot are as shown in Figure 23. A quantization error is present as there is not a one-to-one correspondence between the input and output voltage values. To quantify the linearity error, the step size is first computed [35]. The differential nonlinearity (DNL) is then

$$
\begin{equation*}
D N L_{k}=V_{k}-V_{k-1}-L S B \tag{5.1}
\end{equation*}
$$

where $V_{k}, V_{k-1}$ are two consecutive code transition points and $V_{k}-V_{k-1}$ is the step size. The DNL is the maximum deviation in the output step size from the ideal value of one LSB. The integral nonlinearity (INL) is then [35]

$$
\begin{equation*}
I N L_{j}=\sum_{k=1}^{j} D N L_{k}=V_{j}-j L S B \tag{5.2}
\end{equation*}
$$

where $V_{j}=\sum_{k=1}^{j}\left(V_{k}-V_{k-1}\right)$ is the sum of the step size from zero to the $j^{\text {th }}$ transition point and $j L S B$ is the ideal value at that transition point. The INL is the maximum deviation of the input/output characteristic from a straight line passed through its end points. A good converter typically has linearity error $\leq 0.5$ LSB [35].


Figure 23. (a) Transfer function of photonic ADC using a 1 kHz triangular wave as input signal, (b) Quantization error.

The linearity errors of the quantized signal in Figure 23 are examined in Figure 24. The step size plot depicts the length of input voltage corresponding to each quantization level as the input voltage increases. From the step size plot, we see that the maximum step size is 2.7 LSB. The plot also shows the DNL and INL with a maximum INL value of 1.7 and 7.8 LSB , respectively.


Figure 24. Linearity parameters showing the step size, differential nonlinearity and integral nonlinearity.

## B. NOISE FLOOR ANALYSIS

To determine the noise floor of the ADC, a full-scale sinusoid is used as an input. In this section, the noise floor of the ADC is analyzed using a Blackman-Harris window. Twenty sets of 4,096 digitized samples were gathered. By performing spectral averaging,
the average magnitude spectrum response was obtained. A sinusoidal signal is processed by the ADC and a white noise process (quantization noise) $\gamma_{k}$ is added to the digitized signal. The window samples are represented by $w_{k}$. Using the discrete Fourier transform (DFT), we obtained the point-by-point spectral average [35]. The process for computing the frequency spectrum and analyzing the noise floor is as illustrated in Figure 25.


Figure 25. Process for examining the noise floor of the photonic ADC (From [35]).

To perform spectral analysis and noise floor examination, a sinusoidal signal was sampled at two different frequencies. Twenty sets of digitized signals were acquired, each containing 4,096 samples for each frequency. A Blackman-Harris window was used to window the collected samples because of the low sidelobe levels achieved [35]. To compute the spectrum of the signal, the fast Fourier transform (FFT) function in MATLAB was used. After computing the transform of each signal, the point-by-point spectral average was calculated. The spectral average of a $1-$ and $2-\mathrm{kHz}$ sinusoidal signal with a Blackman-Harris window and $N=4096$ is as shown in Figures 26 and 27. The noise floor levels are -64.5 dB and -59.5 dB , respectively.


Figure 26. Spectral average of a 1 kHz sinusoidal signal with a Blackman-Harris window and $N=4096$.

The noise floor is evaluated by examining the signal-to-noise ratio (SNR) and considering the presence of quantization noise only. By normalizing the magnitude square spectral average with the fundamental signal, the equation to calculate the noise floor using a Blackman-Harris window is [35]

$$
\begin{equation*}
F_{M 2}=10 \log _{10}\left(\frac{3 M}{4 E_{B}}\right)+6.02 n \mathrm{~dB} \tag{5.3}
\end{equation*}
$$

where $n=\log _{2}(48)=5.58$ bits and $E_{B}=2.0$ is the equivalent noise bandwidth of the Blackman-Harris window. From this expression, the theoretical noise floor is -65.49 dB . The measured noise floor from Figure 26 is -65.5 dB . Since the noise floor is similar to the theoretical noise floor, the systems other additive noise sources (such as thermal noise) are less significant as compared to quantization noise.

## 1. Clock Jitter

Two test frequencies were separately used as input signals to test if clock jitter is a dominant noise source [35]. A 2 kHz signal was sampled to compare its noise floor with the 2 kHz sinusoidal signal. The same process in calculating the magnitude square spectrum of the 1 kHz signal was used for the 2 kHz signal.


Figure 27. Spectral average of the 2 kHz sinusoidal signal using a Blackman-Harris window and $\mathrm{N}=4096$.

The normalized spectral average of a 2 kHz signal using Blackman-Harris window is shown in Figure 27. Comparison of the noise floor of both the 1 kHz and 2 kHz signals shows that the difference is within 6 dB . Thus, the clock jitter is not a dominant noise source [35], which is reasonable since this is a low frequency test signal.

## C. FOURIER SPECTRUM ANALYSIS

The signal's spectrum is examined using the SNR, THD, SINAD and ENOB. Two different frequencies were digitized one at a time, starting with 1 kHz , followed by 2 kHz.

The ideal SNR equation is [35]

$$
\begin{equation*}
S N R(d B)=6.02 n+1.76 \tag{5.4}
\end{equation*}
$$

where $n$ is the number of bits. With $n=\log _{2} \hat{M}$, the calculated ideal SNR is 34.01 dB .
The THD expression used in MATLAB calculation is [35]

$$
\begin{equation*}
T H D(d B)=20 \log \sqrt{\left(10^{2^{n d} H A R / 20}\right)^{2}+\left(10^{3^{r d} H A R / 20}\right)^{2}+\ldots} \tag{5.5}
\end{equation*}
$$

where the input signal's first five harmonics are added and are in decibels. Using the SNR and THD values to express the SINAD equation, we get [35]

$$
\begin{equation*}
S I N A D(d B)=-20 \log \sqrt{10^{-(S N R) / 10}+10^{T H D / 10}} \tag{5.6}
\end{equation*}
$$

After calculating the SINAD, we can express ENOB as [35],

$$
\begin{equation*}
E N O B=\frac{S I N A D-1.76+20 \log \left(\frac{\text { full scale amplitude }}{\text { actual input amplitude }}\right)}{6.02} . \tag{5.7}
\end{equation*}
$$

The calculated ADC parameter values in dB for each input frequency are as shown in Table 15. With the 2 kHz signal, the ENOB is 0.0714 bit less than that for a 1 kHz signal. The THD value for the 2 kHz signal is 5.9922 dB higher than that for the 1 kHz signal. The ENOB and SINAD values decrease as the frequency of the sampled signal increases due to signal distortion. A comparison of the ADC 1 kHz input signal and the FPGA output signal is as shown in Figure 28. It shows that the FPGA output signal is able to follow the input signal, with some quantization errors.

Table 15. Dynamic performance parameters of the ADC.

|  |  | Blackman-Harris Window |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | Ideal SNR | SINAD | THD | ENOB |
| $1-\mathrm{kHz}$ | 34.0125 | 34.4199 | -62.8375 | 5.4247 |
| $2-\mathrm{kHz}$ | 34.0125 | 33.9899 | -56.8453 | 5.3533 |



Figure 28. Comparison of ADC input signal and FPGA output signal.

## D. SUMMARY

The performance of the ADC was described in this chapter. Firstly, the linearity of the transfer function of the ADC was determined. Secondly, the computed spectral average was analyzed and the noise floor was determined. Lastly, the calculated dynamic performance parameters of the ADC using the computed spectral average were presented. In the next chapter, the conclusion and recommendations will be presented.

## VI. CONCLUSION AND RECOMMENDATIONS

The concept of an electro-optic ADC using the RSNS was explained and reiterated in this thesis. The RF front-end preprocessing architecture was designed and fully implemented by combining photonic and electronic devices. Due to uneven amplitude output observed from the output of the modulators, a 6-bit (dynamic range 41) ADC utilizing a 7-bit design was fully implemented. Analysis of results again [30] proves the viability of the electro-optic ADC preprocessing architecture. During the integration stage, we analyzed electro-optic modulation and signal alignment. The FPGA encoding scheme was also examined in a separate thesis [31].

## A. CONCLUSIONS

The design and specifications of the individual hardware components required for the 7-bit electro-optic ADC were discussed in Chapter III. In the design phase, the importance of communication between the hardware and software implementation to understand each other's capabilities and limitations can never be over emphasized. The importance of accurate specification, sourcing and procuring the required hardware was learned. In the implementation phase, it is critical to become familiar with the laboratory equipment prior to connecting the hardware components. The alignment process proved less challenging than anticipated with an accurate MATLAB simulation to generate the aligned waveform, the availability of an individual power supply (i.e., three synchronized function generators) to adjust the required RF port input, plus the availability of an individual DC bias port to accurately align the phase of the waveforms. Due to uneven amplitude output observed from the modulators, a 6-bit (dynamic range 41) ADC utilizing a 7 -bit design was fully implemented. Although the encoding scheme had minimal errors with low-frequency input signal, the errors increased as the modulated signals became distorted at higher frequencies. Signal alignment was also critical in obtaining an accurate encoding scheme.

The integration process also examined the FPGA encoding scheme [31]. After obtaining the accurately folded and aligned signals, the correct threshold values obtained
from simulation were programmed into the FPGA. This was essential in achieving the correct sequence of integers in the RSNS row vectors. In order to prevent errors during sampling, the FPGA internal clock synchronized the FPGA and the comparators [31].

Though the full implementation (i.e., 7-bit electro-optic ADC) was not achieved, a 6-bit ADC utilizing a 7-bit design was fully implemented. This proves the feasibility of the 7-bit implementation. With the uneven amplitude output from the modulators resolved, the 7-bit implementation can be realized. This thesis demonstrated the importance of having a separate RF and DC port input during the alignment process. With the flexibility of a separate power control, the waveform generation and alignment process was made much easier and more efficient. With further upgrades, the system can be used to digitize ultra-high bandwidth RF signals without the need for down conversion.

## B. FURTHER IMPROVEMENTS AND STUDIES

The implementation of the ADC was limited to 1 kHz due to the unavailability of the hardware of the sampling circuit [31]. To harness the full potential of the implementation (at least 1 MHz ), 1) procure a function generator that is able to generate a stable ramp function of at least $1 \mathrm{MHz}, 2$ ) replace the modulators with ones that are optimized for multi-half wave voltage applications and, 3) improve the sampling frequency of the postprocessing circuit by replacing the input module [31].

Further studies can also be focused on the effect of noise on smaller threshold intervals near the maximum and minimum of the folding waveforms.

Key to an upgrade in the electro-optic ADC design is the availability of a low $\mathrm{V}_{\pi}$ modulator. It is the single key enabler for a low power implementation in the RF frontend hardware implementation. The current low $\mathrm{V}_{\pi}$ modulators on the market (utilized in this thesis) have a value of $2.6-2.8 \mathrm{~V}$. For an upgrade of the project to be feasible, the half- wave voltage values must be reduced further.

## APPENDIX A. MATLAB CODE TO DETERMINE START AND END POINT FOR A RIGHT SHIFT SYSTEM

```
% THIS PROGRAM FINDS MAXIMAL STRINGS OF NON-REDUNDANT
% VECTORS FOR THE N (N >2 AND ANY INTEGER) CHANNEL ROBUST SNS FOR A
RIGHT-SHIFT SYSTEM.
% WRITTEN BY D. STYER and P. E. PACE
clear all
% DEFINE THE CHANNEL NUMBER,THE VALUES OF CHANNELS AND
% THE NUMBER OF SEARCH WRT FUNDEMENTAL PERIOD
disp('This program finds the maximal strings of non-redundant
vectors');
disp('for the N channel ROBUST SNS');
chanum=input('Enter the Number of Channels for ROBUST SNS >> ');
M=1;
for i=1:chanum
    m(i)=input(['Enter ' int2str(i) '.Channel Value >> ']);
    M=M*m(i);
end
period=2*chanum*M;
nsearch=period+10;
prompt='y';
% DEFINE THE SHIFT AMOUNT OF CHANNELS
while (prompt=='y')|(prompt=='Y')
    for i=1:chanum
        s(i)=input(['Enter ' int2str(i) '. Channel Shift Value >> ']);
    end
    % INITIATE THE VARIABLES TO ZERO
    i=0;
    ii=0;
    j=0;
    jj=0;
    k=0;
    funper=0;
    dynrange=0;
    % DEFINE THE WAVEFORM FOR CHANNEL m(1),m(2)...m(n) IN THE FORM OF
MATRIX g
    for r=1:chanum
        mm(r,[1 2])=[m(r) s(r)];
        for i=1+s(r):chanum*m(r)+s(r)
```

```
        g(r,i)=floor((i-s(r))/chanum);
        end
        for i=chanum*m(r)+1+s(r): 2*chanum*m(r)+s(r)
        g(r,i)=floor((2*chanum*m(r)+chanum-i+s(r)-1)/chanum);
        end
        g(r,2*chanum*m(r)+s(r)+1:4*chanum*m(r)+s(r))=. . .
        g(r,1+s(r):2*chanum*m(r)+s(r));
        g(r,4*chanum*m(r)+s(r)+1:8*chanum*m(r)+s(r))=. . .
        g(r,1+s(r):4*chanum*m(r)+s(r));
g(r,8*chanum*m(r)+s(r)+1:16*chanum*m(r)+s(r))=...
    g(r,1+s(r):8*chanum*m(r)+s(r));
g(r,16*chanum*m(r)+s(r)+1:32*chanum*m(r)+s(r))=...
    g(r,1+s(r):16*chanum*m(r)+s(r));
g(r,32*chanum*m(r)+s(r)+1:64*chanum*m(r)+s(r))=...
    g(r,1+s(r):32*chanum*m(r)+s(r));
g(r,64*chanum*m(r)+s(r)+1:128*chanum*m(r)+s(r ) )=...
    g(r,1+s(r):64*chanum*m(r)+s(r));
g(r,128*chanum*m(r)+s(r)+1:256*chanum*m(r)+s(r))=...
    g(r,1+s(r):128*chanum*m(r)+s(r));
g(r,256*chanum*m(r)+s(r)+1:512*chanum*m(r)+s(r))=...
    g(r,1+s(r):256*chanum*m(r)+s(r));
end
    % DEFINE MATRIX ga AND THIS MATRIX IS THE TRANSPOSE OF MATRIX g
    ga=g';
    % DEFINE MATRIX gb AND THIS MATRIX GIVES THE ROW NUMBER IN COLUMN
1,
    % AND THE VECTOR FOR THAT ROW IN COLUMNS 2 THROUGH THE NUMBER OF
CHANNEL
    % DEFINED IN MATRIX ga
    gb(:,[2:(chanum+1)])=ga(:, [1:chanum]);
    [sgbr,sgbc]=size(gb);
    gb(:,1)=(1:1:sgbr)';
    % DEFINE THE MATRIX gc.THIS MATRIX GIVES THE ROW NUMBER THROUGH
    % THE NUMBER OF SEARCH (eliminate the parts of the matrix beyond
    % the search length)
    gc=gb;
    gc(nsearch+1:sgbr,:)=[];
    [sgcr,sgcc]=size(gc);
    % FIND THE FIRST REDUNDANCIES IN MATRIX gc
    k=1;
    for ii=2:nsearch; % ii is row index into gc
        xrec=gc(ii,[2:(chanum+1)]);
        for jj=ii+1:sgcr;
```

```
        if gc(jj,[2:(chanum+1)])==xrec
        redun=gc(jj,1);
                        % DEFINE THE MATRIX h WHICH IS THE MATRIX OF
        % FIRST REDUNDANCIES
    h(k,1)=ii;
        h(k, 2)=redun;
        k=k+1;
        break
        end
        end
    end
    % DEFINE THE MATRIX hsort AND SORT BY THE REDUNDANCY COLUMN IN
MATRIX h
    hsort=h;
    [yoy,ioi]=sort(hsort);
    % DEFINE THE MATRIX hsorted
    hsorted=[yoy(ioi(:,2),1) yoy(:,2)];
    hsorted;
    % DEFINE THE MATRIX hreduced.
    % ELIMINATE THE ROWS OF THE MATRIX hsorted THAT DO NOT ALLOW
    % THE FIRST COLUMN TO BE MONOTONE INCREASING
    % ssr - rows of hsorted
    % ssc - columns of hsorted
    % a - value in last row of h (hsort)
    % rx - rows in hreduced
    % cx - columns in hreduced
    [ssr,ssc]=size(hsorted);
    hreduced=hsorted;
    a=hsort(ssr,1);
    [rx cx]=size(hreduced);
    for k=1:ssr
        for i=1:ssr
            if i<rx
                if hreduced(i,1)==a
                    hreduced(i+1:rx,:)=[];
                    break
                elseif hreduced(i+1,1)<hreduced(i,1)
                    hreduced(i+1,:)=[];
                        break
                end
            end
            [rx cx]=size(hreduced);
        end
    end
    hreduced;
```

```
    % DEFINE THE MATRIX H THAT SHOWS WHICH SETS OF ROWS
    % ARE MAXIMAL FOR NO REDUNDANCIES AND THEIR LENGTHS.
    [hsr,hsc]=size(hreduced);
H(1,1)=(chanum-1);
H(2:hsr+1,1)=hreduced(1:hsr,1)+1;
H(1:hsr, 2)=hreduced(1:hsr, 2)-1;
H(hsr+1,2)=nsearch;
H(1:hsr+1, 3)=H(1:hsr+1, 2)-H(1:hsr+1, 1)+1;
% FIND THE DYNAMIC RANGE OF N-CHANNEL RSNS
HH=max(H);
dynrange=HH(:,3);
% DISPLAY A MATRIX THAT SHOWS THE BEGIN-END POSITION,
% DYNAMIC RANGE AND ALSO CHANNEL-SHIFT VALUES
disp(' ')
disp(' ')
disp(['THE FUNDEMANTAL PERIOD IS ',num2str(period),' '])
disp(' ')
disp(' ')
disp(['THE DYNAMIC RANGE IS ',num2str(dynrange),' '])
fprintf('\n BEGIN POSITION END POSITION DYNAMIC RANGE\n')
fprintf(' -------------------------------------------------
fprintf('%11.0f %16.0f %12.0f \n',H')
fprintf('\n CHANNEL VALUES SHIFT VALUES\n')
fprintf(' -------------- ------------\n')
fprintf('%11.0f %17.0f \n',mm')
prompt=input('Would you like to try another shift (y/n) ? >>','s');
end
```


# APPENDIX B. MATLAB CODE TO DETERMINE START AND END POINT FOR A LEFT SHIFT SYSTEM 

```
function [dynamic_range] = DynamicRangeSmartSearch(modli)
```

```
%***************************************************************************
```

%***************************************************************************
****
****
%* DynamicRangeSmartSearch Calculates the dynamic ranges for input set
%* DynamicRangeSmartSearch Calculates the dynamic ranges for input set
of
of
%* moduli. dynamic_range = DynamicRangeSmartSearch(moduli) returns
%* moduli. dynamic_range = DynamicRangeSmartSearch(moduli) returns
%* a matrix whose rows contain information about the of all dynamic
%* a matrix whose rows contain information about the of all dynamic
%* ranges for the input set of moduli. The columns of the matrix
%* ranges for the input set of moduli. The columns of the matrix
%* are as follows: Moduli Sum, Moduli (N of them), Fundamental
%* are as follows: Moduli Sum, Moduli (N of them), Fundamental
Period,
Period,
%* DR lower bound, DR lower bound Case, DR upper bound, DR upper
%* DR lower bound, DR lower bound Case, DR upper bound, DR upper
%* bound Case, and Dynamic Range (DR).
%* bound Case, and Dynamic Range (DR).
%* Author: LCDR Brian Luke
%* Author: LCDR Brian Luke
%* Last Modified: 310CT2008 - added ArcGIS shapefile generation
%* Last Modified: 310CT2008 - added ArcGIS shapefile generation
%* Called Functions: crt calculateRedundancies,
%* Called Functions: crt calculateRedundancies,
Generate_RSNSCircle_Shapefile
Generate_RSNSCircle_Shapefile
%* Generate_CircleArc_Shapefile
%* Generate_CircleArc_Shapefile
%* Calling Functions: startRSNSsearch
%* Calling Functions: startRSNSsearch
%*
%*
%**************************************************************************
%**************************************************************************
****

```
****
```

global CMIN CMAX N Pf MODLIST COUNT COMBINATIONS MIN_DYNRANGE
MAX_DYNRANGE RADIUS
global chan
format compact
$N=$ length(modli);
dyn_ranges = [];
comb_dyn_ranges = [];
intervals = [];
\% Fundamental period for PRP moduli
Pf = 2*N*prod(modli);
\% Set the radius of the RSNS circle
RADIUS $=$ round(Pf/(2*pi));
\% generate RSNS circle plot filename and shapefile
fname $=$ [num2str(N),'Channel'];
for i = 1:N
fname = [fname,'_',num2str(modli(i))];
end

```
fname = [fname,'_CirclePlot'];
%Generate_RSNSCircle_Shapefile(modli,fname);
% This loop forms the circularly shifted and linearly increasing NxN
% channel matrix (chan). Channels are in the rows and sub-channels are
in
% the columns
chan = [];
for i = 1:N
    hshift = [0:-1:-N+1]+i-1;
    if i > 1
        hshift = [hshift(i:max(size(hshift))) hshift(1:i-1)];
    end
    chan(i,:) = hshift;
end
% This loop computes binomial coefficients to determine how many
% combinations are in each case. The result put into a 2^N x N
% matrix of 1's and 0's called sortedBin
binChar = dec2bin([0:2^N-1]',N);
pad = repmat(' ',max(size(binChar)),1);
for i = 1:N
    binChar_pad(:,2*i) = pad;
    binChar_pad(:,2*i-1) = binChar(:,i);
end
binNum = str2num(char(binChar_pad));
binNum(:,N+1) = sum(binNum,2);
sortedBin = sortrows(binNum,[N+1 1:N]);
% This loop computes the combination number for each case
% (Case notation 2nd digit)
for i = 0:N
    [I,J,V] = find(sortedBin(:,N+1)==i);
    V = cumsum(V);
    sortedBinCol(I) = V;
end
sortedBin(:,N+2) = sortedBinCol';
% This loop uses the CRT to find all Case N1X COR, which are the
% fundamental COR shifts for all other cases and sub-cases
% The vector holding the COR is called COR, and the vector redundancyID
% contains the case label of the corresponding COR
COR = [];
redundancyID = [];
for subchan = 0:N-1
    ch = chan(:,subchan+1)';
    m = max(ch);
    a = (m-ch)/N;
    center_of_redundancy = crt(a,modli)*N-m;
    if center_of_redundancy < 0
    center_of_redundancy = center_of_redundancy + N*prod(modli);
    end
    COR = [COR center_of_redundancy];
    redundancyID = [redundancyID ...
        str2num([num2str(N),'1',num2str(subchan)])];
```

```
end
COR
redundancyID
% This loop computes the Case N1X redundancies and adds them to the
vector
% called dyn_ranges, which will be used to compute the zones of maximum
% possible dynamic ranges in which to search for the dynamic range
[CORtemp,index] = sort(COR);
CORtemp = [CORtemp Pf/2];
redundancyID = [redundancyID(index) str2num([num2str(N),'10'])];
for i = 1:length(CORtemp)-1
    dyn_ranges = [dyn_ranges; [CORtemp(i)-N+1 redundancyID(i) ...
                        CORtemp(i+1)+N-1 redundancyID(i+1) ...
                        CORtemp(i+1)+2*N-CORtemp(i)-1]];
end
dyn_ranges_loop = flipud(sortrows(dyn_ranges,5));
% The following statements set the maximum and minimum dynamic range
based
% on the current moduli set and the fact that the minimum dyanamic
range
% for an n-channel RSNS is always larger than the minimum dynamic range
for
% an (n-1)-channel RSNS.
% new algorithm steps
m_i = sortedBin(:,1:N).*fliplr(repmat(modli,max(size(binNum)),1));
%m_i = sortedBin(:,1:N).*(repmat(modli,max(size(binNum)),1)); %Wrong
j=find(m_i==0);
m_i(j) = 1;
prod_m_i = prod(m_i,2);
prod_m_j = prod(modli)./prod_m_i;
redundancy_len = 2*N*prod_m_j;
case_max_dynrange = N*prod_m_i+redundancy_len-1;
case_max_dynrange(2^N) = dyn_ranges_loop(1,5);
[C,I] = min(case_max_dynrange);
MAX_DYNRANGE = C(1,1)
max_dynrange_case =
str2num([num2str(sortedBin(I(1,1),N+1)), num2str(sortedBin(I(1, 1),N+2)),
'0']);
[I,J] = find(redundancy_len<MAX_DYNRANGE);
sortedBin(:,N+3) = prod_m_i;
sortedBin(:,N+4) = prod_m_j;
sortedBin(:,N+5) = redundancy_len;
```

```
sortedBin(:,N+6) = case_max_dynrange;
sortedBin(I,N+7) = 1;
%MIN_DYNRANGE_limits = [1 1 42 116 419 1615]; % values are from
previous runs
%if N > length(MIN_DYNRANGE_limits)
% MIN_DYNRANGE = MIN_DYNRANGE_limits(length(MIN_DYNRANGE_limits));
%else
% MIN_DYNRANGE = MIN_DYNRANGE_limits(N);
%end
% This loop calculates all redundancies for all cases by calling the
% function CalculateRedundancies and stores them in a matrix
redundancy_vector = [];
for i = 1:length(I)
    redundancy_vector = [redundancy_vector; ...
        CalculateRedundancies(COR, sortedBin(I(i),:),modli)];
end
% These statements sort the redundancies and removes redundant COR.
% The first sorts the vectors by increasing COR then sorts
% by increasing Start. The next finds the unique COR with the largest
Start,
% which chooses the "smallest" redundancy for each COR (the others do
not
% affect the Dynamic Range. The next keeps only those vectors found in
the
% previous step. The last statement adds a counter column to the
matrix.
temp_vector = sortrows(redundancy_vector,[2 1]);
[B,I,J] = unique(temp_vector(:,2));
redundancy_vector = temp_vector(I,:)
%redundancy_vector = [redundancy_vector
[1:length(redundancy_vector)]'];
% This next section searches through the matrix of redundancies to find
% consecutive redundancies. Once consecutive redundancies are found,
the
% number of vectors between the endpoints of the redundancies is a
% potential dynamic range. The largest string of such vectors is the
% dynamic range. The search routine finds all intervals and then
chooses
% the largest one(s) as the dynamic range.
% find the first redundancy
%interval_start = redundancy_vector(1,:);
%start_pointer = 2;
```

```
% loop through all of the redundancies and compute the distance bewteen
the
% start and end points of consecutive redundancies (intervals)
% which are all potential dynamic ranges
len = size(redundancy_vector,1);
redundancy_vector = sortrows(redundancy_vector,3);
diff_vector = redundancy_vector(1:len-1,1)-redundancy_vector(2:len,1);
f = find(diff_vector>=0);
while isempty(f)~=1
    redundancy_vector(f+1,:)=[];
    len = length(redundancy_vector);
    diff_vector = redundancy_vector(1:len-1,1)-
redundancy_vector(2:len,1);
    f = find(diff_vector>=0);
end
%intervals = redundancy_vector;
% This section computes the sizes of the intervals and keeps only the
% largest intervals which are the dynamic ranges
len = length(redundancy_vector);
interval_sizes = [redundancy_vector(1:len-1,1)+1
redundancy_vector(1:len-1,4:end) ...
    redundancy_vector(2:len,3)-1 redundancy_vector(2:len,4:end)];
interval_sizes(:,end+1) = (redundancy_vector(2:len,3)-1)-
(redundancy_vector(1:len-1,1)+1)+1;
dynamic_range1 = interval_sizes(find(interval_sizes(:,end)== ...
    max(interval_sizes(:,end))),:)
[rows,cols] = size(dynamic_range1);
dynamic_range = [dynamic_range1 repmat([MAX_DYNRANGE
max_dynrange_case],rows,1)];
% generating a shapefile for plotting all dynamic ranges
dynamic_range_vector = [dynamic_range1(:,1) dynamic_range1(:,3)
repmat(000,size(dynamic_range1,1),1)]
%Generate_CircleArc_Shapefile(dynamic_range_vector,modli, '_DynamicRange
');
```

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# APPENDIX C. MATLAB CODE TO ENCODE RSNS SIGNALS USING THE COMPUTED THRESHOLD VALUES 

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program calculates comparator threshold values for a 3-Channel RSNS
% ADC system with moduli 7, 8 & 9.
%
% It then generates an analog waveform for each channel (with appropriate
% signal shift) to align them according to the starting index of the RSNS
% dynamic range.
%
% The analog waveforms are then compared with the threshold values &
% quantized into thermometer codes.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Authors: Kee Leong TONG Date: 29 Jul }201
% Han Wei LIM
% Electrical & Computer Engineering Department
% Naval Postgraduate School
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
close all;
% Variable Declaration
V=1.25; % Full-Scale Voltage
N=3; % Number of Channels
range=128; % Dynamic Range for Moduli 7, 8, 9
t= 0:pi/range:pi; % Time Index (V_pi defined as a half-swing from 0 to pi)
% Moduli Values
m0=7;
m1=8;
m2=9;
% Period Calculation for each channel
P0}=2*N*m0
P1=2*N*m1;
P2=2*N*m2;
% Comparator Threshold Calculations
% Channel }1\mathrm{ (Mod 7)
k0=0:1:m0-1;
T0=V.*(cos(pi/2+pi.*(k0.*N+1)/P0+pi/2/P0)).^2;
% Channel }2\mathrm{ (Mod 8)
k1=0:1:m1-1;
T1=V.*(cos(pi/2+pi.*(k1.*N+1)/P1+pi/2/P1)).^2;
% Channel 3 (Mod 9)
```

```
k2=0:1:m2-1;
T2=V.*(cos(pi/2+pi.*(k2.*N+1)/P2+pi/2/P2)).^2;
% Generate analog waveform for each channel (with signal shift)
%mod7 = V* cos(range/P0*t).^2;
%mod}8=\mp@subsup{V}{}{*}\operatorname{cos}(\mathrm{ range }/\textrm{P}1*\textrm{t}).^2
%mod}9=\mp@subsup{V}{}{*}\operatorname{cos}(\mathrm{ range/P2*t).^2;
mod7 = V*}\operatorname{cos(range/P0*t - 19*pi/2/range).^2;
mod}8=\mp@subsup{V}{}{*}\operatorname{cos}(\mathrm{ range/P1*t - 60*pi/2/range).^2;
mod}9=\mp@subsup{V}{}{*}\operatorname{cos}(\mathrm{ range/P2*t + 22* pi/2/range).^2;
% Generate arbituary thermometer code for each channel
mod7code = ones(1,range);
mod8code = ones(1,range);
mod9code = ones(1,range);
% Quantize Analog Waveform to Thermometer Code (Mod 7)
for i = 1:range
    if mod7(i) > T0(7)
        mod7code(i)=7;
    else if mod7(i)>T0(6)
            mod7code(i)=6;
        else if mod7(i)> T0(5)
                    mod7code(i)=5;
            else if mod7(i)> T0(4)
                    mod7code(i)=4;
                else if mod7(i)>T0(3)
                            mod7code(i) = 3;
                        else if mod7(i)> T0(2)
                    mod7code(i) = 2;
                        else if mod7(i)> T0(1)
                            mod7code(i)=1;
                            else
                            mod7code(i) = 0;
                        end
                        end
                end
                end
            end
        end
    end
end
% Quantize Analog Waveform to Thermometer Code (Mod 8)
for k=1:range
    if mod8(k) > T1(8)
        mod8code(k) = 8;
    else if mod8(k)>T1(7)
            mod}8\operatorname{code}(\textrm{k})=7\mathrm{ ;
        else if mod8(k)>T1(6)
            mod}8\operatorname{code}(\textrm{k})=6
            else if mod8(k)> T1(5)
                mod8code(k)= 5;
                else if mod8(k) > T1(4)
```

```
                        mod}8\operatorname{code}(\textrm{k})=4
                else if mod8(k)>T1(3)
                    mod8code(k)=3;
                        else if mod8(k)> T1(2)
                        mod}8\operatorname{code}(\textrm{k})=2
                        else if mod8(k)> T1(1)
                        mod8code(k)=1;
                                else
                            mod}8\operatorname{code}(\textrm{k})=0
                        end
                                end
                        end
                end
                end
            end
        end
    end
end
% Quantize Analog Waveform to Thermometer Code (Mod 9)
for l= 1:range
    if mod9(1) > T2(9)
        mod9code(1) = 9;
    else if mod9(l) > T2(8)
            mod9\operatorname{code}(1)=8;
        else if mod9(l) > T2(7)
            mod9code(1) = 7;
            else if mod9(l) > T2(6)
                    mod9code(1) = 6;
                else if mod9(1) > T2(5)
                        mod9\operatorname{code}(1)=5;
                        else if mod9(l) > T2(4)
                            mod9code(1) = 4;
                        else if mod9(1) > T2(3)
                            mod9code(l) = 3;
                                else if mod9(l)}>\textrm{T}2(2
                            mod9\operatorname{code}(1)=2;
                            else if mod9(l) > T2(1)
                        mod9\operatorname{code}(1)=1;
                            else
                            mod9\operatorname{code}(1)=0;
                                    end
                                    end
                                    end
                        end
                        end
                end
            end
        end
    end
end
% Generate Threshold Value indices for each channel
t71(1:range)=T0(1);
```

```
t72(1:range)=T0(2);
t73(1:range)=T0(3);
t74(1:range)=T0(4);
t75(1:range)=T0(5);
t76(1:range)=T0(6);
t77(1:range)=T0(7);
figure(1)
subplot(2,1,1)
hold on
plot(mod7,'b-')
plot(t71,'k--')
plot(t72,'k--')
plot(t73,'k--')
plot(t74,'k--')
plot(t75,'k--')
plot(t76,'k--')
plot(t77,'k--')
title('Channel }1\mathrm{ (Modulus 7)')
xlabel('Index')
ylabel('Analog Voltage Levels')
subplot(2,1,2)
plot(mod7code)
title('Thermometer Code')
xlabel('Index')
ylabel('Number of Comparators On')
t81(1:range)=T1(1);
t82(1:range)=T1(2);
t83(1:range)=T1(3);
t84(1:range)=T1(4);
t85(1:range)=T1(5);
t86(1:range)=T1(6);
t87(1:range)=T1(7);
t88(1:range)=T1(8);
figure(2)
subplot(2,1,1)
hold on;
plot(mod8)
plot(t81,'k--')
plot(t82,'k--')
plot(t83,'k--')
plot(t84,'k--')
plot(t85,'k--')
plot(t86,'k--')
plot(t87,'k--')
plot(t88,'k--')
title('Channel 2 (Modulus 8)')
xlabel('Index')
ylabel('Analog Voltage Level')
subplot(2,1,2)
```

```
plot(mod8code)
title('Thermometer Code')
xlabel('Index')
ylabel('Number of Comparators On')
t91(1:range)=T2(1);
t92(1:range)=T2(2);
t93(1:range)=T2(3);
t94(1:range)=T2(4);
t95(1:range)=T2(5);
t96(1:range)=T2(6);
t97(1:range)=T2(7);
t98(1:range)=T2(8);
t99(1:range)=T2(9);
figure(3)
subplot(2,1,1)
hold on
plot(mod9)
plot(t91,'k--')
plot(t92,'k--')
plot(t93,'k--')
plot(t94,'k--')
plot(t95,'k--')
plot(t96,'k--')
plot(t97,'k--')
plot(t98,'k--')
plot(t99,'k--')
title('Channel }3\mathrm{ (Modulus 9)')
xlabel('Index')
ylabel('Analog Voltage Level')
subplot(2,1,2)
plot(mod9code)
title('Thermometer Code')
xlabel('Index')
ylabel('Number of Comparators On')
```

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# APPENDIX D. MATLAB CODE TO PLOT THE QUANTIZED VALUES AND COMPUTE THE STEP SIZE, DNL AND INL 

```
% PROCESS TRANSFER FUNCTION DATA - RSNS
% M^ = 43 - 3 = 40
clear all;
close all;
load onekhztriangle_2.txt
trans=onekhztriangle_2([74:124],1)-min(onekhztriangle_2([74:124],1)); %
Store one cycle of dynamic range & normalise to start from zero.
[rr,cc]=size(trans);
rrs=1:1:rr;
transb=dec2bin(trans);
transbc=str2num(transb);
LSB=0.102; % Effective V_pi/(N*m_i)
% Convert the signal to analog
Quantized_Data = (trans).*(3.22/39);
% Step Size
step_size = hist(Quantized_Data(1:51),40);
step_size= step_size/1.1; % Convert to LSBs
% DNL
actual_dnl = step_size - 1;
% Process INL
inl(1)=actual_dnl(1);
for ij=2:40
    inl(ij)=inl(ij-1)+actual_dnl(ij);
end
QD=Quantized_Data(1:51);
sample_index=0:50;
ideal_Quantized_Data = 0:3.22/40:3.22;
ideal_sample_index = 0:length(ideal_Quantized_Data)-1;
% Quantization error
idea=[LSB/2:LSB:4.5]';
v=sample_index*(3.22/40);
nums=hist(QD,40);
trans=[v' QD];
ct=1;
for ii=1:40
    for jj=1:nums(1,ii)
        qerror(ct,1)=idea(ii,1)-trans(ct,1);
        ct=ct+1;
    end
end
```

```
% Plot Transfer Function
figure(1)
stairs(sample_index*(3.22/40), QD');
title('ADC Transfer Function');
xlabel('Input Voltage (V)');
ylabel('Decimal Output');
grid on
figure(2)
plot(sample_index*(3.22/40), qerror');
xlabel('Input Voltage (V)');
ylabel( 'Quantization Error');
axis([0 4.5 -0.15 0.2]);
grid on;
% Plot Ideal Transfer Function
figure(3)
hold on
stairs(ideal_sample_index*(3.22/40), ideal_Quantized_Data,'r');
title('ADC Transfer Function');
%xlabel('Input Voltage (V)');
ylabel( 'Binary Code');
grid on
ac=0:1:39;
acc=ac*LSB;
% Plot Step Size as a function of input voltage
figure(3)
subplot(3,1,1)
hold on
stem(acc, step_size);
ylabel('LSB');
title('Step Size');
grid on
%Plot DNL
hold on
subplot(3,1,2)
stem(acc,actual_dnl);
axis([0 4 -1 2]);
title('DNL');
ylabel('LSB');
grid on
%Plot INL
hold on
subplot(3,1,3)
stem(acc,inl);
axis([0 4 - -1 8])
title('INL');
xlabel('Input Voltage(V)');
ylabel('LSB');
```


## APPENDIX E. MATLAB CODE TO COMPUTE THE ADC SPECTRAL AVERAGE AND DYNAMIC PARAMETERS

```
% 1-kHz sine wave input signal
% Read data from file
clear all; close all;
nbis = 6;
m = 2; % specify number of columns
num_samples = 4096; % number of samples
fsample = 100e3;
T = 1/fsample;
fo = 1000;
time = 0:T:(num_samples-1)*T;
freq = [0:num_samples/2 - 1].*fsample/num_samples;
% Create cells for data
Binary_Data = zeros(num_samples,20);
Quantized_Data = zeros(num_samples,20);
sig_fft = zeros(num_samples);
sig_blackmanharris = zeros(num_samples,20);
sig_fft_blackmanharris = zeros(num_samples,20);
Data_Array = zeros(num_samples,20);
% read and process 20 sets of sampled data
for i = 1:20
    a = 'Run';
    b = int2str(i);
    c = '.lvm';
    filename = strcat(a,b,c);
    fid = fopen(filename);
    Data = textscan(fid, '%14f%14f', num_samples);
    Data(:,[1])=[]; % Remove column 1
    Data_cell_array{i,:} = Data{1,:};
    Data_Sc = Data{1,:};
    Data_Array = horzcat(Data_Sc, Data_Array);
% Convert the signal to analog
Quantized_Data(1:num_samples,i) = (Data_Array(:,i)-1).*(3.22/41);
% Use BlackmanHarris/Hamming/Hanning Window
sig_blackmanharris(1:num_samples,i) =
Quantized_Data(1:num_samples,i).*blackmanharris(num_samples);
%sig_hanning = Quantized_Data.*hanning(num_samples);
%sig_hamming = Quantized_Data.*hamming(num_samples);
% Perform FFT on the signals
sig_fft = abs(fft(Quantized_Data(:,1), num_samples)/num_samples);
sig_fft_blackmanharris(1:num_samples,i) =
abs(fft(sig_blackmanharris(:,i), num_samples)/num_samples);
end
```

```
%calculate mean fft value
ave_fft_blackmanharris = mean(sig_fft_blackmanharris');
%calculate magnitude squared spectrum
spectP_sig = (sig_fft.*sig_fft);
spectP_sig_blackmanharris =
ave_fft_blackmanharris.*ave_fft_blackmanharris;
% find maximum other than DC
max_sig = max(sig_fft(10:num_samples/2));
max_blackmanharris = max(ave_fft_blackmanharris(10:num_samples/2));
% normalize mean data
%ave_norm_fft_sig = ave_fft_sig./max_sig;
ave_norm_fft_blackmanharris =
ave_fft_blackmanharris./max_blackmanharris;
% Plot Signals from normalized blackmanharris window
figure(1)
plot(freq/500, db(ave_norm_fft_blackmanharris(1:num_samples/2),
'voltage'));
title('Normalized Magnitude Squared Spectrum Using Blackman-Harris
Window');
xlabel('Frequency(kHz)');
ylabel('Magnitude(dB)');
grid on
hold on
% Plot Signals magnitude spectrum with window
figure(2)
plot(freq/500, db(2*ave_fft_blackmanharris(1:num_samples/2),
'voltage'));
title('Frequency Spectrum Plot Using Blackman-Harris Window');
xlabel('Frequency(kHz)');
ylabel('Amplitude(V)');
grid on
hold on
% magnitude spectrum without window
figure(3)
plot(freq/500, db(2*sig_fft(1:num_samples/2),'voltage'), 'r');
title('Frequency Spectrum Plot Without Window');
xlabel('Frequency(kHz)');
ylabel('Amplitude(dB)');
grid on
hold on
% power spectrum with and without spectral averaging
figure(4)
plot(freq/500, db(spectP_sig(1:num_samples/2), 'power'),'r', freq/500,
db(spectP_sig_blackmanharris(1:num_samples/2), 'power'));
title('Power Spectrum');
xlabel('Frequency(kHz)');
ylabel('Power(dB)');
```

```
legend('Signal', 'Spectral Average Using Blackman-Harris Window');
grid on
hold on
% power spectrum with window
figure(5)
plot(freq/500, db(spectP_sig_blackmanharris(1:num_samples/2),
'power'));
title('Power Spectrum Plot Using Blackman-Harris Window');
xlabel('Frequency(kHz)');
ylabel('Power(dB)');
grid on
hold on
ADC=3.5/2+1.65*sin(2*pi*fo.*time-3);
figure(6)
plot(time(1:120),ADC(1:120),'b-',time(1:120),Quantized_Data(1:120),'r-
');
title('ADC 1kHz Input Signal vs FPGA Output Signal');
xlabel('Time(sec)');
ylabel('Amplitude(V)');
legend('ADC Input Signal','FPGA Output Signal');
grid on
%calculate noise floor
noise_floor_M2 = 10*log10(3*num_samples/8) + 6.02*log2(41)
noise_floor_MC = 10*log10(num_samples/2) + 1.76 + 6.02*log2(41)
noise_floor_MJ = 20*log10(1/(2*pi*std(ave_fft_blackmanharris))) +
20*log10(fsample/fo) + 10*log10(num_samples/8)
% Calculate SNR, SINAD, ENOB, THD and SFDR
fin_sig = find(sig_fft(1:num_samples/2) == max_sig);
fin_blackmanharris = find(ave_fft_blackmanharris(1:num_samples/2) ==
max_blackmanharris);
% Span of the input frequency on each side
span=max(floor(num_samples/250),5);
% Approximate search span for harmonics on each side
spanh=40;
% Determine power spectrum
%spectP=(sig_fft).*(sig_fft);
%figure(6)
%plot(freq/500, db(spectP(1:num_samples/2)));
%title('Power Spectrum Plot');
%xlabel('Frequency(kHz)');
%ylabel('Power(dB)');
%grid on
% Find DC offset power
Pdc=(spectP_sig_blackmanharris(1));
% Extract overall signal power
```

```
%Ps=sum(spectP(fin));
% Vector/matrix to store both frequency and power of signal and
harmonics
Fh=[];
% The 1st element in the vector/matrix represents the signal, the next
element represents % the 2nd
% harmonic, etc.
Ph = [];
Ph_sig = [];
sort_harmonics = sort(spectP_sig_blackmanharris, 'descend');
for num_harmonics = 0:4
    % Extract harmonics
    har_bin = fin_blackmanharris +
(num_harmonics*2*(fin_blackmanharris-1));
    Ph=[Ph spectP_sig_blackmanharris(har_bin)];
    har_bin_sig = fin_sig + (num_harmonics*2*(fin_sig-1));
    Ph_sig=[Ph_sig spectP_sig(har_bin)];
end
% Determine Total Distortion Power and Noise Power
P_thd = (Ph(2:5));
P_thd_sig = (Ph_sig(2:5));
%Pn = sum(spectP(1:num_samples/2)) - Pdc - Ps - Pd;
%Calculate dynamic parameters
SNR_db = 6.02* log2(43) + 1.76
THD_db = 20*log10(sqrt(sum(P_thd)))
THD_sig_db = 20*log10(sqrt(sum(P_thd_sig)))
SINAD_db = -20*log10(sqrt((10^-(SNR_db/10) + 10^(THD_db/10))))
SINAD_sig_db = -20*log10(sqrt((10^-(SNR_db/10) + 10^(THD_sig_db/10))))
ENOB = (SINAD_db - 1.76)/6.0206
ENOB_sig = (SINAD_sig_db - 1.76)/6.0206
SFDR_dB = 10*log10(Ph(1)/max(Ph(2:5)))
%SNR_dB = 10*log10(Ps/Pn)
%SINAD_dB = 10*log10(Ps/(Pn+Pd))
%THD_dB = 10*log10(Pd/Ph(1))
%HD = 10*log10(Ph(1:11)/Ph(1));
%plot(Data_Array);
fid = fclose('all');
```

```
% 2- kHz sine wave input signal.
% Read data from file
clear all;
close all;
nbits = 6;
m = 2; % specify number of columns
num_samples = 4096; % number of samples
fsample = 100e3;
T = 1/fsample;
fo = 2000;
time = 0:T:(num_samples-1)*T;
freq = [0:num_samples/2 - 1].*fsample/num_samples;
% Create cells for data
Binary_Data = zeros(num_samples, 20);
Quantized_Data = zeros(num_samples, 20);
sig_fft = zeros(num_samples);
sig_blackmanharris = zeros(num_samples,20);
sig_fft_blackmanharris = zeros(num_samples,20);
Data_Array = zeros(num_samples, 1);
% read and process 20 sets of sampled data
for i = 1:20
    a = 'Run';
    b = int2str(i);
    c = '.lvm';
    filename = strcat(a,b,c);
    fid = fopen(filename);
    Data = textscan(fid, '%14f%14f', num_samples);
    % Data(:,[1])=[]; % Remove column 1
    Data_cell_array{i,:} = Data{1,:};
    Data_Sc = Data{1,:};
    Data_Array = horzcat(Data_Sc, Data_Array);
% Convert the signal to analog
Quantized_Data(1:num_samples,i) = (Data_Array(:,i)-1).*(3.22/41);
% Use BlackmanHarris/Hamming/Hanning Window
sig_blackmanharris(1:num_samples,i) =
Quantized_Data(1:num_samples,i).*blackmanharris(num_samples);
%sig_hanning = Quantized_Data.*hanning(num_samples);
%sig_hamming = Quantized_Data.*hamming(num_samples);
% Perform FFT on the signals
sig_fft = abs(fft(Quantized_Data(:,1), num_samples)/num_samples);
sig_fft_blackmanharris(1:num_samples,i) =
abs(fft(sig_blackmanharris(:,i), num_samples)/num_samples);
end
%calculate mean fft value
ave_fft_blackmanharris = mean(sig_fft_blackmanharris');
%calculate magnitude squared spectrum
```

```
spectP_sig = (sig_fft.*sig_fft);
spectP_sig_blackmanharris =
ave_fft_blackmanharris.*ave_fft_blackmanharris;
% find maximum other than DC
max_sig = max(sig_fft(10:num_samples/2));
max_blackmanharris = max(ave_fft_blackmanharris(10:num_samples/2));
% normalize mean data
%ave_norm_fft_sig = ave_fft_sig./max_sig;
ave_norm_fft_blackmanharris =
ave_fft_blackmanharris./max_blackmanharris;
% Plot Signals from normalized blackmanharris window
figure(1)
plot(freq/500, db(ave_norm_fft_blackmanharris(1:num_samples/2),
'voltage'));
title('Normalized Magnitude Squared Spectrum Using Blackman-Harris
Window');
xlabel('Frequency(kHz)');
ylabel('Magnitude(dB)');
grid on
hold on
% Plot Signals magnitude spectrum with window
figure(2)
plot(freq/500, db(2*ave_fft_blackmanharris(1:num_samples/2),
'voltage'));
title('Frequency Spectrum Plot Using Blackman-Harris Window');
xlabel('Frequency(kHz)');
ylabel('Amplitude(V)');
grid on
hold on
% magnitude spectrum without window
figure(3)
plot(freq/500, db(2*sig_fft(1:num_samples/2),'voltage'), 'r');
title('Frequency Spectrum Plot Without Window');
xlabel('Frequency(kHz)');
ylabel('Amplitude(dB)');
grid on
hold on
% power spectrum with and without spectral averaging
figure(4)
plot(freq/500, db(spectP_sig(1:num_samples/2), 'power'),'r', freq/500,
db(spectP_sig_blackmanharris(1:num_samples/2), 'power'));
title('Power Spectrum');
xlabel('Frequency(kHz)');
ylabel('Power(dB)');
legend('Signal', 'Spectral Average Using Blackman-Harris Window');
grid on
hold on
```

```
% power spectrum with window
figure(5)
plot(freq/500, db(spectP_sig_blackmanharris(1:num_samples/2),
'power'));
title('Power Spectrum Plot Using Blackman-Harris Window');
xlabel('Frequency(kHz)');
ylabel('Power(dB)');
grid on
hold on
figure(6)
plot((0:100)*T, Quantized_Data(1:101));
title('Ramp Function with 2kHz Frequency');
xlabel('Time(sec)');
ylabel('Amplitude(V)');
grid on
%calculate noise floor
noise_floor_M2 = 10*log10(3*num_samples/8) + 6.02*log2(41)
noise_floor_MC = 10*log10(num_samples/2) + 1.76 + 6.02*log2(41)
noise_floor_MJ = 20*log10(1/(2*pi*std(ave_fft_blackmanharris))) +
20*log10(fsample/fo) + 10*log10(num_samples/8)
% Calculate SNR, SINAD, ENOB, THD and SFDR
fin_sig = find(sig_fft(1:num_samples/2) == max_sig);
fin_blackmanharris = find(ave_fft_blackmanharris(1:num_samples/2) ==
max_blackmanharris);
% Span of the input frequency on each side
span=max(floor(num_samples/250),5);
% Approximate search span for harmonics on each side
spanh=40;
% Determine power spectrum
%spectP=(sig_fft).*(sig_fft);
%figure(6)
%plot(freq/500, db(spectP(1:num_samples/2)));
%title('Power Spectrum Plot');
%xlabel('Frequency(kHz)');
%ylabel('Power(dB)');
%grid on
% Find DC offset power
Pdc=(spectP_sig_blackmanharris(1));
% Extract overall signal power
%Ps=sum(spectP(fin));
% Vector/matrix to store both frequency and power of signal and
harmonics
Fh=[];
```

```
% The 1st element in the vector/matrix represents the signal, the next
element represents % the 2nd
% harmonic, etc.
Ph = [];
Ph_sig = [];
sort_harmonics = sort(spectP_sig_blackmanharris, 'descend');
for num_harmonics = 0:4
    % Extract harmonics
    har_bin = fin_blackmanharris +
(num_harmonics*2*(fin_blackmanharris-1));
    Ph=[Ph spectP_sig_blackmanharris(har_bin)];
    har_bin_sig = fin_sig + (num_harmonics*2*(fin_sig-1));
    Ph_sig=[Ph_sig spectP_sig(har_bin)];
end
% Determine Total Distortion Power and Noise Power
P_thd = (Ph(2:5));
P_thd_sig = (Ph_sig(2:5));
%Pn = sum(spectP(1:num_samples/2)) - Pdc - Ps - Pd;
%Calculate dynamic parameters
SNR_db = 6.02*log2(41) + 1.76
THD_db = 20*log10(sqrt(sum(P_thd)))
THD_sig_db = 20*log10(sqrt(sum(P_thd_sig)))
SINAD_db = -20*log10(sqrt((10^-(SNR_db/10) + 10^(THD_db/10))))
SINAD_sig_db = -20*log10(sqrt((10^-(SNR_db/10) + 10^(THD_sig_db/10))))
ENOB = (SINAD_db - 1.76)/6.0206
ENOB_sig = (SINAD_sig_db - 1.76)/6.0206
SFDR_dB = 10* log10(Ph(1)/max(Ph(2:5)))
%SNR_dB = 10*log10(Ps/Pn)
%SINAD_dB = 10*log10(Ps/(Pn+Pd))
%THD_dB = 10*log10(Pd/Ph(1))
%HD = 10*log10(Ph(1:11)/Ph(1));
%plot(Data_Array);
%fid = fclose('all');
```


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[^0]:    ${ }^{1}$ Screen captures of the results were sent to the manufacturer and they have acknowledged that the output is not normal. Due to time constraint, the modulators will be sent back to the manufacturer for evaluation.

