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Using data envelopment analysis and the analytical hierarchy process to find node influences in a social network

William P. Fox and Sean F. Everton

Abstract

In a social network analysis the output provided includes many measures and metrics. For each of these measures and metrics, the output provides the ability to obtain a rank ordering of the nodes in terms of these measures. We might use this information in decision making concerning disrupting or deceiving a given network. All is fine when all the measures indicate the same node as the key or influential node. What happens when the measures indicate different key nodes? Our goal in this paper is to explore two methodologies to identify the key players or nodes in a given network. We apply two procedures to analyze these outputs to find the most influential nodes as a function of the decision makers' inputs. We use data envelopment analysis as a method to optimize efficiency of the nodes over all criteria and use the analytical hierarchy process (AHP) as a process to consider both subjective and objectives inputs through pairwise comparison matrices. We illustrate our results using two common networks from the literature: the kite network and the information flow network. We discuss some basic sensitivity analysis that can be applied to the methods. We find the AHP method as the most flexible method to weight the criterion based upon the decision makers' inputs or the topology of the network.

Keywords

Social network analysis, data envelopment analysis, analytical hierarchy process

1. Introduction to social network analysis

Social network analysis (SNA) is the methodical analysis of social networks in general and dark networks in particular.[1,2] SNA is a collection of theories and methods that assumes that the behavior of actors (individuals, groups, organizations, etc.) is profoundly affected by their ties to others and the networks in which they are embedded. Rather than viewing actors as automatons unaffected by those around them, SNA assumes that interaction patterns affect what actors say, do, and believe. Networks contain *nodes* (representing individual actors or entities within the network) and *edges and arcs* (representing relationships between the individuals, such as friendship, kinship, organizational position, sexual relationships, communications, tweets, Facebook friendships, etc.). These networks are often depicted in two formats: graphically or as a matrix. We might call the graph a social network diagram, where nodes are represented as *points* or *circles* and *arcs* are represented as lines that interconnect the nodes.

We will provide only a little background on SNA here. More precisely, we introduce some of the more common measures and their definitions that are used for exploratory SNA of networks. In this paper we assume we are only looking for the *powerful* and *influential* players in a network.

There are a multitude of measures (metrics) that are found in most SNA software. We begin by defining a few metric terms or measures in SNA that we use in our analysis.[1,3,4]

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- Betweenness is a measure of the extent to which a node lies on the shortest path between other nodes in the network. This measure takes into account the connectivity of the node's neighbors, giving a higher value for nodes which *bridge clusters*. The measure reflects the number of people who a person is connecting indirectly through their direct links.
- An edge is said to be a bridge if deleting it would cause its endpoints to lie in different components of a graph.
- Centrality is the measure which gives a rough indication of the social power of a node based on how well they 'connect' the network. 'Betweenness', 'closeness', 'degree', and 'eigenvector' are all measures of centrality.
- Centralization is the difference between the numbers of links for each node divided by maximum possible sum of differences. A centralized network will have many of its links dispersed around one or a few nodes, while a decentralized network is one in which there is little variation between the numbers of links each node possesses.
- Closeness is the degree an individual is near all other individuals in a network (directly or indirectly). It reflects the ability to access information through the 'grapevine' of network members. Thus, closeness is the inverse of the sum of the shortest distances between each individual and every other person in the network. The shortest path may also be known as the 'geodesic distance'.
- Degree is the count of the number of ties to other players in the network.
- Density is a measure of network cohesion that is equal to the actual number of ties in a network divided by the total possible number of ties, which means that density scores range from 0.0 to 1.0.
- Eigenvector centrality is a variation on degree centrality that assumes ties to central actors are more important than ties to peripheral actors and thus weights an actor's summed connections to others by their centrality scores. Google's Page rank score is a variation on eigenvector centrality.

2. Examples of metrics for influential players in networks

2.1. Example 1. The kite network

We begin by looking at a classic network from SNA literature. We look at the 'kite network' (see Figure 1), which was developed by David Krackhardt,[5] a leading social network analyst. The nodes are connected by some sort of relational tie between the actors. For example, two nodes

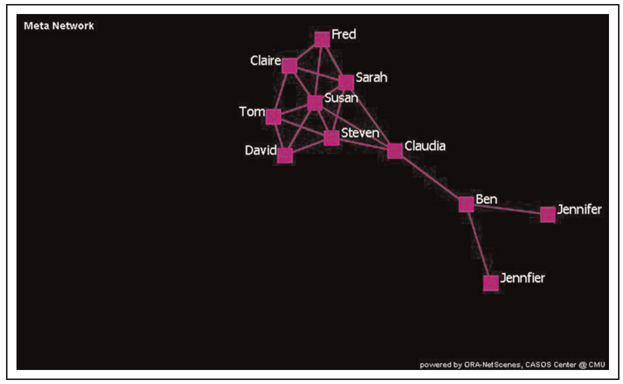

Figure 1. 'Kite network' from Organizational Risk Analyzer (ORA).

are connected if they regularly talk to each other or interact in some way. So, if Tom regularly interacts with Susan but not with Fred, Tom and Susan are connected, but there is no link drawn between Tom and Fred. This network is useful because it effectively demonstrates the distinction between the three most popular individual centrality measures that might indicate an influential node: *degree centrality, betweenness centrality*, and *closeness centrality*.

2.1.1. Degree centrality. Social network researchers measure network activity for a node by using the concept of degrees – the number of direct connections a node has. For each member of the network, we find the number of connections to other members:

> *Fred 3, Claire 4, Tom 4, David 3, Steven 5, Susan 6, Sarah 5, Claudia 3, Ben 1, and Jennifer 1.*

In the Kite network, Susan has the most direct connections (6) in the network, making hers the most active node in the network. She is a 'connector' or 'hub' in this network. It is often assumed that in personal networks 'the more connections, the better', but this is not always so. What really matters is to where those connections lead – and how they connect the otherwise unconnected! Here Susan has connections only to others in her immediate cluster – her clique. She connects only those who are already connected to each other.

2.1.2. Betweenness centrality. While Susan has many direct ties, Claudia has few – less than the average in the network – 3 as compared to the average of 3.5. Yet, in many ways, she has one of the best locations in the network – she is *between* two important constituencies. She is in a position to play a 'brokerage' role in the network. The good news is that she plays a powerful role in the network; the bad news

Table 1. Information exchange matrix for input into ORA.

	Count.	Comm.	Educ.	Indu.	Mayr.	Wro.	News	Uway.	Welf.	West.
Count.	*0*	*1*	*0*	*0*	*1*	*0*	*1*	*0*	*1*	*0*
Comm.	*1*	*0*	*1*	*1*	*1*	*0*	*1*	*1*	*1*	*0*
Educ.	*0*	*1*	*0*	*1*	*1*	*1*	*1*	*0*	*0*	*1*
Indu.	*1*	*1*	*0*	*0*	*1*	*0*	*1*	*0*	*0*	*0*
Mayr.	*1*	*1*	*1*	*1*	*0*	*0*	*1*	*1*	*1*	*1*
Wro.	*0*	*0*	*1*	*0*	*0*	*0*	*1*	*0*	*1*	*0*
News	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*
Uway.	*1*	*1*	*0*	*1*	*1*	*0*	*1*	*0*	*1*	*0*
Welf.	*0*	*1*	*0*	*0*	*1*	*0*	*1*	*0*	*0*	*0*
WEST.	*1*	*1*	*1*	*0*	*1*	*0*	*1*	*0*	*0*	*0*

is that she is a single point of failure. Without her, Ben and Jennifer would be cut off from information and knowledge in Susan's cluster. A node with high *betweenness* has great influence over what flows – and does not flow – through a network. Claudia may *control* the outcomes in a network.

2.1.3. Closeness centrality. Sarah and Steven have fewer connections than Susan, yet the pattern of their direct and indirect ties allow them to access all the nodes in the network more quickly than anyone else. They have the *shortest paths* to all others – in terms of path length, they are, on average, closer to everyone else. They are in an excellent position to monitor the information flow in the network – they have the best visibility into what is happening in the network.

In summary, from these three found measures, we found Susan was most important from degree centrality. Claudia was most important when we consider between centrality. Sarah and Steven were equally most important in closeness centrality. So who is the most powerful and influential person in this network? We will provide a model to examine this issue.

2.2. Example 2. Information flow network

In 1978, Knoke and Wood collected data from workers at 95 organizations in Indianapolis. Respondents indicated with which other organizations their own organization had any of 13 different types of relationships. Knoke and Kuklinski selected a subset of 10 organizations and two relationships, money and information.[6] We will examine only the information exchange in this example. The value "1" implies there is a direct relationship/connection and "0" there is not a direct relationship/connection. If a node is directly connected to another node, we give it a value of "1" otherwise we give it a value is "0". Nodes are not considered connected to themselves. The resulting network matrix (Table 1) and diagram (Figure 2) are in the format required by Organizational Risk Analyzer (ORA).[7]

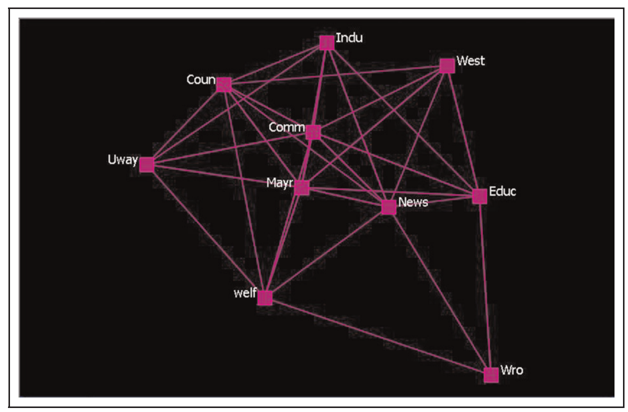

Figure 2. Information exchange diagram from ORA for the information flow network.

We use the matrix and Figure 2 to conduct SNA. We examine this network and make some useful observations about the network and the players in the network. We use ORA to analyze the network and obtain some important measures.

We begin by calculating the network density, which, as defined earlier, equals the number of actual connections divided by the total number of possible connections. These connections are lines in our network. The number of possible connections is found by the formula: $PC = n(n-1)/2$. For our network of 10 nodes, we have $(10 \times 9)/2 = 45$. We count the number of actual connections insuring not to count the same line in both directions. There are 49 connections and each goes both ways, so we use 24.5 one way connections.

$$\text{Density} = \#\text{lines}/(\#\text{possiblelines}) = 24.5/(45) = 0.544$$

Observation: The literature states the maximum density is 1. Our density is greater than 50%.

In SNA, multiple measures are calculated and analysis made. We briefly summarize these results.

Degree: *Mayr* and *Comm* have the greatest out-degrees and might be regarded as the most influential. These two players are joined by *News* when we examine in-degree. That other organizations share information with these three would seem to indicate a desire on the part of others to exert influence.

Path distances: Since the information network is considered a directed graph, separate closeness and farness can be computed for sending and receiving. We find that *Wro* has the largest sum of geodesic distances from other players and to other players.

Closeness: An index of the 'reach distance' from each player to (or from) all others is calculated. Here, the maximum score (equal to the number of nodes) is achieved when every other node is one-step from ego. The reach closeness sum becomes less as players are two steps, three steps, and so on (weights of 1/2, 1/3, etc.). These scores are then expressed in 'normed' form by dividing by the largest observed reach value. The two tables are quite easy to interpret. The first of these shows what proportion of other nodes can be reached from each player at one, two, and three steps (in our example, all others are reachable in three steps or less). The last table shows what proportions of others can reach ego at one, two, and three steps. Note that almost all nodes can contact *News* in one step.

The next few measures are performed with specialized social network software.

Eigenvector: We turn our attention to the scores of each of the cases on the 1st eigenvector. Higher scores indicate that players are 'more central' to the main pattern of distances among all of the players, lower values indicate that players are more peripheral. The results are very similar to those for our earlier analysis of closeness centrality, with *News, Mayr*, and *Comm* being most central, and players *Wro* being most peripheral. Usually the eigenvalue approach will do what it is supposed to do: give us a 'cleaned-up' version of the closeness centrality measures, as it does here.

Betweenness: *Comm, Educ*, and *Mayr* appear to be relatively a good bit more powerful than others by this measure. Clearly, there is a structural basis for these players to perceive that they are 'different' from others in the population. Indeed, it would not be surprising if these three players saw themselves as the movers-and-shakers, and the deal-makers that made things happen. In this sense, even though there is not very much betweenness power in the system, it could be important for group formation and stratification.

2.3. Information network summary

SNA methods provide some useful tools for addressing one of the most important (but also one of the most complex and difficult) aspects of social structure: the sources and distribution of power. The network perspective suggests that the power of individual players is not an individual attribute but arises from their relations with others. Whole social structures may also be seen as displaying high levels or low levels of power as a result of variations in the patterns of ties among players. And, the degree of inequality or concentration of power in a population may be indexed.

2.4. Power in a network

Power arises from occupying advantageous positions in networks of relations. Three basic sources of advantage are high degree, high closeness, and high betweenness. In simple structures (such as the star, circle, or line), these advantages tend to co-vary. In more complex and larger networks, there can be considerable disjuncture between these characteristics of a position– so that a player may be located in a position that is advantageous in some ways, and disadvantageous in others.

We have reviewed three basic approaches to the 'centrality' of individuals' positions, and some elaborations on each of the three main ideas of degree, closeness, and betweenness. This review is not exhaustive. The question of how structural position confers power remains a topic of active research and considerable debate. As you can see, different definitions and measures can capture different ideas about where power comes from, and can result in some rather different insights about social structures.

In the information exchange network, we find different key players depending on which metric we examine. We will present some methodologies and models to help access the 'key' player modeling across all metrics.

3. Methodologies to find key players across many metrics: application of DEA and AHP

3.1. Data envelopment analysis (DEA)

Data envelopment analysis (DEA) is a relatively new 'data input-output driven' approach for evaluating the performance of entities called decision making units (DMUs) which convert multiple inputs into multiple outputs.[8] The definition of a DMU is generic and very flexible. It has been used to evaluate the performance or efficiencies of hospitals, schools, departments, US Air Force wings, US armed forces recruiting agencies, universities, cities, courts, businesses, banks, countries, regions, etc. DEA has been used to gain insights into activities that were previously analyzed by other methods.[8,9]

In 1978, Charnes et al. described DEA as mathematical programming model applied to observational data – providing a new way of obtaining empirical estimates of relations.[8] It is formally defined as a methodology directed to frontiers rather than central tendencies.

The model in simplest terms is a linear programming problem.[10–12] Although several formulations for DEA exist, we seek the most straightforward formulation in order to maximize an efficiency or DMU as constrained, as shown in equation (1). We suggest normalizing the metric outputs for the alternatives within each criterion measure. We will call this normalized matrix, **X**, with entries x_{ij}. We define an efficiency unit as E_i *for i=1,2,…,nodes.* We let w_i be the weights or coefficients for the linear combinations. Further, we strict any efficiency from being larger than one. This gives the following formulation:

$$\mathit{Max}\ E_i$$

$$\mathit{subject\ to}$$

$$\sum_{i=1}^{n} w_i x_{ij} - E_i = 0, j = 1, 2, \ \ldots \tag{1}$$

$$E_i \leq 1 \ \mathit{for\ all}\ i$$

3.2. Analytical hierarchy process (AHP)

The analytical hierarchy process (AHP) is a multi-objective decision analysis tool first proposed by Satty.[13] It is designed when either subjective and objective measures or just subjective measures are being evaluated in terms of a set of alternatives based upon multiple criteria, organized in a hierarchical structure. At the top level, the criteria are evaluated or weighted, and at the bottom level the alternatives are measured against each criterion. The decision maker assesses their evaluation by making pairwise comparisons in which every pair is subjectively or objectively compared. The subjective method involves a nine-point scale, as we will explain.

3.2.1. AHP Background. We only desire to briefly discuss the elements in the framework of AHP. AHP can be described as a method to decompose a problem into subproblems. In most decisions, the decision maker has a choice among several to many alternatives. Each alternative has a set of attributes or characteristics that can be measured, either subjectively or objectively. The attribute elements of the hierarchal process can relate to any aspect of the decision problem – tangible or intangible, carefully measured or roughly estimated, well or poorly understood – anything at all that applies to the decision at hand.

In its simplest sense we can state that in order to perform AHP we need an objective, a set of alternatives, each with attributes to compare. Once the hierarchy is built, the decision maker(s) systematically evaluate its various elements pairwise (by comparing them to one another two at a time), with respect to their impact on an element above them in the hierarchy. In making the comparisons, the decision makers can use concrete data about the elements, but they typically use their judgments about the elements' relative meaning and importance. It is the essence of the AHP that human judgments, and not just the underlying information, both can be used in performing the evaluations.

The AHP converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This capability distinguishes the AHP from other decision making techniques.

In the final step of the process, numerical priorities are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action.

3.2.2. Uses and applications. While it can be used by individuals working on straightforward decisions, the AHP is most useful where teams of people are working on complex problems, especially those with high stakes, involving human perceptions and judgments, whose resolutions have long-term repercussions. It has unique advantages when important elements of the decision are difficult to quantify or compare, or where communication among team members is impeded by their different specializations, terminologies, or perspectives.

Decision situations to which the AHP can be applied include the following where we desire *ranking*:

- Choice – The selection of one alternative from a given set of alternatives, usually where there are multiple decision criteria involved.
- Ranking – Putting a set of alternatives in order from most to least desirable
- Prioritization – Determining the relative merit of members of a set of alternatives, as opposed to selecting a single one or merely ranking them
- Resource allocation – Apportioning resources among a set of alternatives
- Benchmarking – Comparing the processes in one's own organization with those of other best-of-breed organizations
- Quality management – Dealing with the multidimensional aspects of quality and quality improvement
- Conflict resolution – Settling disputes between parties with apparently incompatible goals or positions

Table 2. Saaty's nine-point scale.

Intensity of importance in pairwise comparisons	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements *i* and *j*, if *i* is 3 compared to *j*, then *j* is 1/3 compared to *i*.
Rational	Force consistency; measure values available

3.2.3. Using the AHP. The procedure for using the AHP can be summarized as:

Step 1 Build the hierarchy for the decision.

Goal	*Select the most influential node*
Criteria	*Metrics from ORA*
Alternatives:	*Nodes:* $a_1, a_2, a_3, \ldots, an$

Step 2 Judgments and comparison. Build a numerical representation using a 1–9 point scale in a pairwise comparison for the attributes criterion and the alternatives. The goal, in AHP, is to obtain a set of eigenvectors of the system that measures the importance with respect to the criterion. We can put these values into a matrix or table based on Table 2.

Insure that this matrix is consistent according to Saaty's scheme to compute *CR*, which must be less than or equal to 0.1 to be considered consistent. Saaty's computed *RI*'s for random matrices for up to 10 criteria are as shown below.[13]

N	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
RI	*0*	*0*	*0.52*	*0.89*	*1.1*	*1.24*	*1.35*	*1.4*	*1.45*	*1.49*

Next, we approximate the largest eigenvalue, using the power method.[15] We compute the consistency index, *CI*, with the formula:

$$CI = (\lambda - n)/(n - 1)$$

We compute *CR* using:

$$CR = CI/RI$$

If $CR < 0.1$, then our pairwise comparison matrix is consistent and we may continue. If not, we must go back to our pairwise comparison and make fix the inconsistencies.

Step 3 Using all the eigenvectors combined in order to obtain a comparative ranking. Since our students at the Naval Postgraduate School have had only college algebra, covering matrices, matrix operations, eigenvalues, and eigenvectors is quite a stretch. However, we have previously covered discrete dynamical systems (DDS) in our first course. Students build DDS models and find solutions numerically through iteration and graphically. They have a good grasp of the concept of stability. What they were not taught was the relationship between stable solutions and the use of eigenvalues and eigenvectors in a closed-form solution. We take advantage of the concept of stable solutions to obtain our stable set of values. Some additional background of DDS is provided elsewhere.[14,15]

Step 4. After the $m \times 1$ criterion weights are found and the $n \times m$ matrix for n alternatives by m criterion, we use matrix multiplication to obtain the $n \times 1$ final rankings.

Now we apply both the DEA and AHP techniques to our two networks.

3.3. Applications of DEA to find influences on a network

Assume all we have are the outputs from ORA which we do not show here due to the volume of output produced. We take the metrics from ORA and normalize each column. The columns for each criterion are placed in a matrix X with entries, x_{ij}. We define wj as the weights for each criterion. We set up the linear program using equation (1) with the output from the kite network.

We formulate the linear program and present the output. We interpret the linear program's output, shown in Table 3 as follows: Player 1, Susan, is rated most influential followed closely by Sarah and Steven. Additionally, we see the most important criterion in solving the optimal problem was the eigenvectors of the network.

We apply the same DEA methodology to the information exchange network.

3.3.1. Information exchange network. Here we find applying the normalized SNA metric outputs as the input data to our LP formulation in equation (1). The rank ordering for the key nodes (players) in the network is as follows:

> *Mayr 1, Comm 2, Educ 3, News 4, Welf 5, Indu 6, West 7,Wro 8, Coun 9, Uway 10,*

We find that *betweeness, W6*, is the most important criterion used for the linear program to rank the efficiencies of the alternatives, as shown in Table 4.

Table 3. Linear program output for kite network.

Variable	Value
Objective Function	1
E1 (Susan)	1
E2 (Sarah)	0.785511
E3 (Steven)	0.785511
E4 (Claire)	0.653409
E5 (Fred)	0.653409
E6 (David)	0.535511
E7 (Tom)	0.5535511
E8 (Claudia)	0.59517
E9 (Ben)	0.137784
E10 (Jennifer)	0.02983
W1	0
W2	5.711648
W3	0
W4	0
W5	0
W6	0

Table 4. Linear programming output for information network.

Variable	Value
Objective function	1
E1 (Mayr)	1
E2 (Comm)	0.68952
E3 (News)	0.15323
E4 (Educ)	0.65323
E5 (West)	0.00403
E6 (Indu)	0.004435
E7 (Uway)	0.0
E8 (Welf)	0.06855
E9 (Wro)	0.02016
E10 (Count)	0.02016
W1	0
W2	0
W3	0
W4	0
W5	0
W6	2.65232

In DEA, the process compares all the nodes to the node which exhibits the highest efficiency value. That node (and all ties) are given a value, an efficiency value, of 1. The values of the other nodes are percentages compared to 1.

We next apply AHP to each network and determine the rankings.

3.4. Applications of AHP to find key nodes

Next, we assume we can obtain pairwise comparison matrix from the decision maker concerning the criterion. We use the output from ORA and normalize the results for AHP to rate the alternatives within each criterion. We provide a sample pairwise comparison matrix for weighting the criterion from the kite example using Saaty's nine-point scale; see Table 5.

The *CR* is 0.0828, which is less than 0.1, so our pairwise matrix is consistent.

We obtain the steady state values that will be our weights, where the sum of the weights equals 1.0. There exist many methods to obtain these weights. The methods used here are the power method from numerical analysis or discrete dynamical systems.[14,15]

0.1532	**0.1532**	**0.1532**	**0.1532**	**0.1532**	**0.1532**
0.1450	**0.1450**	**0.1450**	**0.1450**	**0.1450**	**0.1450**
0.1194	**0.1195**	**0.1194**	**0.1194**	**0.1194**	**0.1194**
0.0672	**0.0672**	**0.0672**	**0.0672**	**0.0672**	**0.0672**
0.1577	**0.1577**	**0.1577**	**0.1577**	**0.1577**	**0.1577**
0.3575	**0.3575**	**0.3575**	**0.3575**	**0.3575**	**0.3575**

These values provide the weights for each criterion: *centrality* = 0.1532, *eigenvectors* = 0.1450, *in-centrality* = 0.1194, *out-centrality* = 0.0672, *information centrality* = 0.1577, and *betweenness* = 0.3575. We multiply the matrix of the weights and the normalized matrix of metrics from ORA to obtain our output and ranking are:

Table 5. Pairwise comparison matrix.

	Centrality	Eigenvector	In-degree	Out-degree	Information centrality	Betweenness
Centrality	1	3	2	2	½	1/3
Eigenvector	1/3	1	1/3	1	2	½
In-degree	½	3	1	½	½	¼
Out-degree	½	½	1	1	¼	¼
Information centrality	2	2	4	4	1	1/3
Betweenness	3	2	4	4	3	1

Susan	**0.159421**	**2**
Steven	**0.132728**	**3**
Sarah	**0.113323**	**4**
Tom	**0.075717**	**6**
Claire	**0.075717**	**6**
Fred	**0.061358**	**8**
David	**0.061358**	**8**
Claudia	**0.175856**	**1**
Ben	**0.109015**	**5**
Jennifer	**0.035507**	**10**

For this example of AHP, Claudia, *cl*, is the key node. However, the bias of the decision maker is important in the analysis of the criterion weights. Betweenness is 2 to 3 times more important than the other criterion.

In the information flow network, we apply a similar approach. We provide both the criterion weights that sum to 1.0 and final rankings of nodes.

Criterion weights:

0.078256
0.101696
0.084758
0.425158
0.172091
0.138041

Ranking:		
Mayr	**0.17055**	**1**
Comm	**0.14976**	**2**
News	**0.11061**	**4**
Educ	**0.11796**	**3**
Coun	**0.08742**	**5**
Indu	**0.08463**	**6**
Uway	**0.08131**	**7**
Welf	**0.07968**	**8**
West	**0.0734**	**9**
Wro	**0.04468**	**10**

3.5. Sensitivity analysis

From DEA we can examine the reduced costs to obtain information concerning sensitivity analysis in a similar fashion as a normal LP. In AHP we need to use a controlled 'trial and error' method to find the impact of the decision maker on the criterion weights, and thus the effect of changes in the criterion weights to the altering of the ranks. We revisit the kite network. Our decision maker has changed their pairwise comparison, and now we have criterion weights of six metrics as:

0.153238
0.211213
0.104478
0.076881
0.345447
0.108743

These new criterion weights provide a new ranking ordering with Susan as the most influential node:

Susan	**0.142161**	1
Steven	**0.124408**	3
Sarah	**0.117025**	4
Tom	**0.096293**	5
Claire	**0.096293**	5
Fred	**0.084394**	7
David	**0.084394**	7
Claudia	**0.125052**	2
Ben	**0.082076**	9
Jennifer	**0.047903**	10

Continued trial and error can eventually find the breakpoint that causes the change in rank orderings.

4. Discussion

We have shown two distinct multi-attribute decision making applications to identifying key nodes in social network analysis. Each has advantages and disadvantages. For data envelopment analysis, it is easy to use and it utilizes the data in its original format as well as the linear programming formulation is easy to follow. The limitations include the requirement that all other nodes are compared in efficiency to the most efficient node. The value of one given to the most efficient node is misleading. The AHP method is also easy to set up and use. The main drawback or limitation is the subjectivity of the pairwise comparison to obtain the decision makers weights. Our pairwise comparison one day may not be the same as the next day. However, the ability to do trial and error sensitivity analysis, although tedious, may be done using technology.

5. Conclusion

We have provided two separate methodologies, DEA and AHP, to ranking influential nodes (players) in a given social network. We have illustrated these methods through two separate examples, the kite and information exchange networks. We believe that the incorporation of decision maker weights with the metrics of a social network is invaluable to analysis in finding the *key* and *influential* players.

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Declaration of conflicting interest

The author declares that there is no conflict of interest.

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