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Auction and Game Theory Based Recommendations for DOD Acquisitions

Blott, Justin; Boardman, Nicholas; Cady, Andrew; Elliott,
Jake; Griffin, William; Mastronardi, Nick; Quinn, Parker

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Auction and Game Theory Based Recommendations for DOD Acquisitions

24 March 2015

**Justin Blott, 2d Lt; Nicholas Boardman, 2d Lt; Andrew Cady, 2d Lt;
Jake Elliott, 2d Lt; William Griffin, Capt; Nick Mastronardi, Maj;
and Parker Quinn, 2d Lt**

United States Air Force Academy

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Abstract

This paper synthesizes auction and game theory literature into specific military acquisition improvement recommendations. We characterize acquisition environments into distinct categories, present the results of seminal literature that pertains to each category, and translate the literatures' recommendations for military contracting practitioners. The relevant categories are procurement with unknown cost and no risk, item(s) with known costs and existent but understood stochastic risk, and item(s) with unknown costs and/or unknown stochastic risk. We break out these three categories into sub-categories depending on whether there are one or multiple potential competing vendors, and, if multiple, by whether we must buy one lot or potentially a schedule of lots from a host of vendors.

Keywords: game theory, auctions, contracting, principle-agent models, Major Defense Acquisition Programs, off-the-shelf, mechanism design.



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About the Authors

Lt Blott graduated from the US Air Force Academy in May 2013 and joined the Department of Economics and Geosciences faculty in July 2013 as a Casual Lieutenant researcher. He is currently a flight training student at Laughlin AFB.

Justin Blott, 2d Lt
APT Student, Air Education Training Command 47th Student Squadron/DOT; formerly, United States Air Force Academy/DFEG
E-mail: justin.blott.1@us.af.mil

Lt Boardman graduated from the US Air Force Academy in May 2014. He is currently a graduate student at the Air Force Institute of Technology (AFIT).

Nicholas Boardman, 2d Lt
Graduate Student, AFIT IN/ENS; formerly, United States Air Force Academy
E-mail: nicholas.boardman.2@us.af.mil

Lt Cady graduated from the US Air Force Academy in May 2014. He is currently a doctoral candidate at the Pardee RAND Graduate School and an assistant policy analyst at RAND.

Andrew Cady, 2d Lt
Student, Student, AFIT-CI; formerly, United States Air Force Academy
E-mail: Andrew_Cady@rand.org

Lt Elliott graduated from the US Air Force Academy in May 2014. He is currently researching game theory applications at Rome Labs in Rome, NY.

Jake Elliott, 2d Lt
Operations Research Analyst, AFRL/RITC; formerly, United States Air Force Academy
E-mail: jacob.elliott.5@us.af.mil

Capt Griffin was a member of the US Air Force Academy faculty in the Department of Management until January 2015. He is currently Deputy Program Manager for Aviation Resource Management System (ARMS), an ACAT III software system, at Air Force Lifecycle Management Center, Maxwell-Gunter AFB, AL.

William Griffin, Capt
Deputy Program Manager, ARMS, AFLCMC/HIBD; formerly, United States Air Force Academy/DFM
E-mail: william.griffin.14@us.af.mil

Maj Mastronardi was a member of the US Air Force Academy faculty in the Department of Economics and Geosciences through June 2014 and is still a member of the Air Force Reserves for US Cyber Command. He is currently a Senior Economist for Amazon (Quidsi, a NY/NJ subsidiary) where he leads teams for



conducting big data predictive analytics for pricing, marketing, and business intelligence in general.

Nick Mastronardi, Maj (USAFR)

Senior Economist, Amazon and Major, US Air Force Reserve Command; formerly, United States Air Force Academy/DFEG

nickmastronardi@gmail.com

Lt Quinn graduated from the US Air Force Academy in May 2014. He is currently an Operations Research Analyst for Air Mobility Command at Scott AFB.

Parker Quinn, 2d Lt

Operations Research Analyst, AMC/A9L; formerly, United States Air Force Academy

E-mail: parker.quinn.1@us.af.mil





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Disclaimer: The views represented in this report are those of the author and do not reflect the official policy position of the Navy, the Department of Defense, or the federal government.



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Introduction

Secretary of Defense McNamara once famously said, “The government is not in the business of putting business out of business,” implying that when the government conducts private-sector contracting arrangements, the government should not abuse its monopsonist position to the detriment of American private-sector defense contracting corporations. Back then – when federal budgets were better padded, our defense industrial base could stand to grow, and most competing defense contractors were 100% U.S. corporations – McNamara’s perspective was a luxury we pursued. Today, however, the landscape is quite different. Federal budgets are lean, our warfighting capabilities hinge greatly on the marginal dollar, our defense industrial base has grown to a more mature warfighting-sustaining size, and most defense contractors work closely with a network of international partners and subcontractors resulting in U.S. payment largess disseminated throughout that network. Also, since the time of McNamara there has been an explosion in auction and game theory literature. The private sector has worked diligently to implement these findings in its contracting practices. The time has now come for the Department of Defense (DOD) to also cull the auction and game theory literature holistically and systematically for practices that can help reduce costs, reduce risk, increase timeliness, and increase quality.

In this paper, we categorize the DOD acquisitions landscape into three main categories: procurement of item(s) with unknown cost and no risk, item(s) with known costs and existent but known stochastic risk, and item(s) with unknown costs and/or unknown stochastic risk. We break out each of these three categories into a couple sub-categories depending on whether there are one or multiple potential competing vendors, and by whether we must buy one lot or potentially a schedule of lots from a host of vendors.

One can argue that competitive auction environments may incite irrational bidding behavior. Perhaps bidders may get caught up in the game and deviate from theoretical predictions, or they may not have the underlying knowledge necessary to devise optimal bidding behavior in the first place. We believe that it is indeed reasonable to expect rational bidding. As stakes get larger, bidding behavior trends more closely to theoretical predictions. In addition, even if bidders may have never been formally educated in bidding behavior, they can arrive at the same results through experience and corporate lessons learned. Just as shoppers at a grocery store don’t compute constrained optimization Lagrangians before checking out, yet economists still find that shoppers’ purchasing behavior matches well with the predictions of constrained optimization models, we argue that DOD contracting practitioners seeking to implement novel auction and game theoretic models should expect highly rational bidder responses, and especially as stakes increase.



Our hope is that DOD acquisition leadership is sensitive to the arguments made in this paper: that the distinct environments we outline warrant distinct acquisition approaches. The acquisition approaches we recommend, specific as they may be, are all available through the current contracting quiver of contract types and conditioning. Our subsequent hope is that contracting practitioners seek to execute the approach recommendations. We are seeking funding for this next fiscal year to facilitate the implementation of this research by translating our recommendations specifically for the T-38 replacement acquisition environment. The files we would publish from the follow on study, a contracting practitioner handbook that succinctly characterizes the contracting categories and recommendation highlights devoid of the justifications, should make these contracting approach recommendations readily implementable by the practitioner.

Federal Acquisition Types

Before developing the discussion of each distinct environment, a brief discussion of core underlying principles of the Federal Acquisition Regulation (FAR) and associated contracting principles is warranted. This discussion will focus on three areas: the general contract categories applicable to federal contracts, federal contract types, and bid/negotiation regulations.

There are multiple federal contract types, but two general contract categories are applicable when referring to Major Defense Acquisition Programs: Cost Reimbursement and Fixed Price. In general terms, a Cost Reimbursement contract means the government will pay all of a contractor's allowable costs plus a pre-negotiated fee, while a Fixed Price contract means the government will pay an exact pre-negotiated amount for delivery of a completed project or service (Rumbaugh, 2010, p. 72). Generally, Fixed Price contracts are favored by the government, since they involve little or no price risk for the government (Rumbaugh, 2010, p. 78)]. Conversely, Cost Reimbursement contracts tend to be favored by contractors, since they involve little or no profit risk for the contractor.

Within each of these general contract groups there are multiple contract types. The Fixed Price contract group can be broken down into several contract types, including Firm Fixed Price (FFP), Firm Fixed Price with Economic Price Adjustment (FP/EPA), and Fixed-Price Incentive (FPI). Generally, the least risky contract type for the government is Firm Fixed Price. Under the Firm Fixed Price contract type, contractors theoretically bear 100% of the risks of performance. Figure 1 depicts this inverse risk relationship, while excluding contract types not significant to this paper.



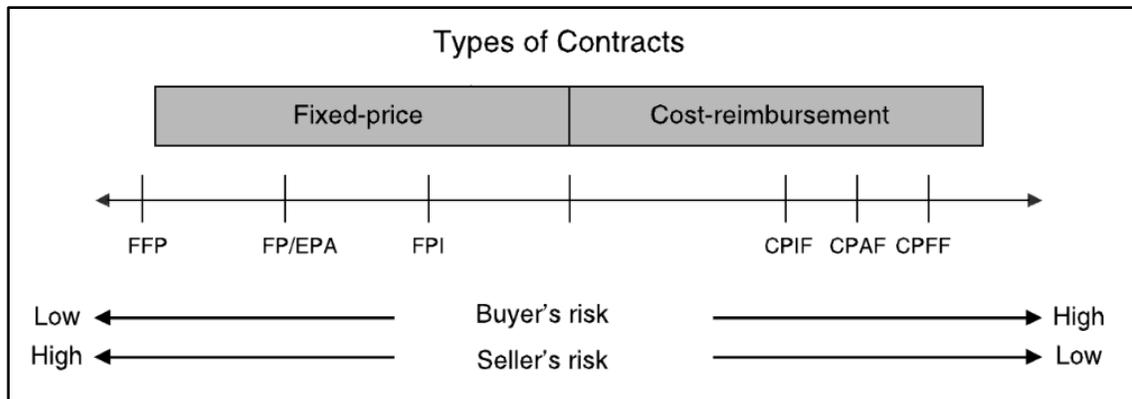


Figure 1. Types of Contracts by Risk (adapted from Griffin & Schilling, 2011, p. 24)

In the reverse of Firm Fixed Price, under the Cost Plus Fixed Fee (CPFF) contract type, the government bears 100% of the risks of performance, in practical terms. Generally, the government bears less risk under the Cost Plus Award Fee (CPAF) contract type, and still less under the Cost Plus Incentive Fee (CPIF) contract type.

There are three methods of acquisition prescribed in the FAR: simplified acquisition procedures (FAR Part 13), sealed bidding (FAR Part 14), and contracting by negotiation (FAR Part 15). Only sealed bidding and contracting by negotiation are used in a significant manner for MDAPs, since simplified acquisition procedures are limited to contracts too small (FAR 2.101) to factor in seriously to the total cost of an MDAP. Additionally, acquisitions under FAR Part 12, commercial acquisition, are sometimes viewed as a fourth acquisition method; in fact, “the rules for each of the three methods also govern commercial item procurements, but they are simplified... to encourage broader participation by commercial contractors” (Rumbaugh, 2010, p. 67).

Under sealed bidding, only price and price-related factors stated in the Invitation for Bid can be used as decision factors for any bid conforming to the minimum requirements of the acquisition (Rumbaugh, 2010, p. 70; FAR 14.408-1). This necessarily restricts the use of sealed bidding to FFP and FP/EPA contracts (FAR 14.104). Per FAR 6.401, sealed bidding is required when

- there is enough time for soliciting, submitting, and evaluating sealed bids;
- the award will be made on the basis of price and other price-related factors;
- it is not necessary to conduct discussions with the responding bidders about their bids; and



- there is a reasonable expectation of receiving more than one sealed bid.

This necessitates that the specifications are precise enough to permit price and price-related factors to be the determining criteria for award (Rumbaugh, 2010, p. 70) and means that a conforming bid will win only on the basis of price; no price and performance trade-offs are permitted between conforming bids.

Under contracting by negotiation, only factors stated in Request for Proposal (RFP) can be used as decision factors for a proposal, but, in contrast to sealed bidding, non-price factors may be used. Using factors stated in the RFP, the source selection authority can select between those offerors who are not dominated in the trade-space between all decision factors. For example, in the case of an RFP where only the factors of cost and past performance were considered and the submitted RFPs were rated as shown in Table 1, only offers 1, 2, and 3 could be considered by the source selection authority (SSA), assuming the SSA agreed with the ratings of each offer.

Table 1. Notional Offeror Ratings With Only Two Factors Considered

	Offeror 1	Offeror 2	Offeror 3	Offeror 4	Offeror 5
Cost	\$100M	\$120M	\$150M	\$175M	\$120M
Past performance rating	Good	Very Good	Excellent	Excellent	Good

Offer 4 would be dominated by Offer 3, while Offer 5 would be dominated by offer 2. According to FAR 15.101-1, a tradeoff process is appropriate when the government will consider contract award to other than the lowest priced offeror or other than the highest technically rated offeror.

Similar to the risk continuum for contract types, the factors rated as important in the RFP can be considered along a “best value continuum,” as stated in FAR 15.101,

In different types of acquisitions, the relative importance of cost or price may vary. For example, in acquisitions where the requirement is clearly definable and the risk of unsuccessful contract performance is minimal, cost or price may play a dominant role in source selection [contractor selection]. The less definitive the requirement, the more development work required, or the greater the performance risk, the more technical or past performance considerations may play a dominant role in source selection.

One can see, then, how the risk of poor technical and schedule performance relative to a contractor’s offer is traded off against offer price similarly to the implicit tradeoff made in selecting a contract type.



This brief discussion targeted the core principles of fixed price and cost plus contract types and reviewed bid/negotiation regulations found primarily in FAR Parts 14 and 15.

Imperfectly Known Vendor Cost, No Risk (OTS): Auctions

If the government knows the product or service it seeks, that solution exists without any development or delivery risk (the item is available off the shelf [OTS]), and the contractor cost is well gauged, then the cost and cost risk minimizing contracting approach is for the government buyer to price discriminate: charge a contractor exactly his cost or willingness to sell. As this is sometimes viewed as anticompetitive in the private sector, it might not always be good practice for the government to pursue this tactic. If so, then the next best approach from a cost and cost risk minimizing framework would be the same as the approach should the government not know the contractor seller cost: auctions. There are many kinds of auctions and other similar truth-revealing incentive mechanisms. Solving for optimal bidding behavior, and thus, auctioneer surplus, is complicated and dependent on many factors. Fortunately Myerson, Vickrey, Clark, Groves, and others have made incredible academic progress characterizing optimality conditions for auctions and these other incentive mechanisms. It is worthwhile to note that even though the auction formats we recommend consist of a single buyer facing many sellers as compared to the more traditional examples with a single seller facing many buyers (e.g., eBay), the analysis is analogous and the results generalize.

No Product Differentiation: Identical Competing Products

In this section we start with the simplest case where the competing products or services have no differentiation in either quality or timeliness. Once we describe the optimal auction and incentive mechanisms for this identical product(s) and service(s) world, we will discuss how to generalize these mechanisms should there be differences in the product/service quality and timeliness.

Procurement of a Single Item

This section pertains to the situation in which the government seeks to buy one single item, such as one generator. If the government seeks to buy a batch of identical OTS items, it may want to procure this as a single batch, but may also want to allow for divided contractor supply of the desired capacity depending on a couple factors yet. We characterize those factors later in Procurement of Multiple Items.

The first case we will look at is the simplest case for product procurement. In this case, the product that is needed has already been developed and is readily available for purchase from numerous companies. It is a case where only a single



product is needed and the military would simply like to acquire it for the lowest possible price.

In this situation of a no risk off-the-shelf solution with multiple competing vendors offering identical products/services with identical time lines and the government only seeking one, the government cost minimizing procurement method is a second-price auction. Second price auctions can either be implemented via sealed bid when not all bidding vendors are available at the same moment, or as a descending price event (single buyer version of the ascending English oral auction). We describe both in more detail immediately below.

A sealed bid second price auction traditionally consists of a single seller soliciting sealed bids from competing bidding potential buyers. The seller allocates the item to the highest bidder, but does not charge that bidder his bid, but instead charges him the second highest bid. This may at first cause the reader or contracting practitioner consternation: Why not charge the highest bidder more—his bid, for instance? The answer is that if the auction had been specified to charge the highest bidder his bid, this would have resulted in different bidding behavior, and as proven by Myerson, the seller revenue is greater in the second price auction (1981). The intuition is that in order to generate higher bidding closer to the buyers' true valuations, we need to break a bidders' payment from his bid. Excepting some technical implementation discrepancies, this is the same auction format eBay provides, why sellers flock to eBay, and why eBay has had the success they've had. It is worth noting that there is some foregone seller revenue, the difference between the high bidder's bid and the second high bid, and economists generally refer to this difference as the value of the information paid by the seller to learn the bidders' true valuations or willingness to pay (to learn the demand curve).

In addition to proving that the sealed bid second price auction maximizes seller revenue, Myerson also proved that it is equivalent to the ascending English oral auction where a seller starts with a low price and then incrementally raises the price until only one buyer remains. The intuition behind the equivalence is that the last remaining bidder receives the item at the price one increment above the second highest valuation, where the other last bidder dropped out.

In the government buyer setting, the government should solicit vendor sealed bids, buy from the low bidder, and pay the low bidder the next highest bid. This government buyer sealed bid auction would also be equivalent to a descending oral auction where the government would start by announcing a really high price where presumably all contractors would be eager to take the contract, and then incrementally lowering the price until one contractor remains who will get the contract for one increment below the next lowest contractor's willingness to sell.



After having taught Game Theory and Auctions for several years, I (Mastronardi) can attest to the fact that even though second price and English oral auctions are common online (eBay), at county fairs, and other settings, the Myerson's result is rarely intuitively obvious, and those new to auctions often innately assume that a first price auction can generate the auctioneer greater surplus. The first price auction in a standard selling auctioneer setting is where the item is allocated to the highest bidder for his bid. Fortunately for naïve auctioneers, as the number of bidders increases in an auction, the difference between bidders' bids and their true valuations, how much they will shave their bid away from their valuation to increase their surplus, decreases and asymptotically approaches the second price auction bidding behavior and auction surplus allocations. Many DOD procurements are effectively first price auctions (item allocated to lowest bidding vendor at his bid) with a few bidding vendors, and so there are in general cost savings left on the table compared to DOD procurements where the contract would go to the lowest bidder for the second lowest bid because the second price auction will induce lower vendor bids.

Finally, the government can also utilize auction reserve prices for even greater potential cost savings. By announcing that the government will award a contract to the lowest bidder at the second lowest bid only if the lowest bidder is below a specified reserve, the government can decrease its expected cost. Having a reserve price introduces a trade-off between auctioneer surplus and the possibility that no bidder will meet the reserve and the good goes unallocated. Depending on the acquisition environment, this may be a trade-off the government is sometimes willing to take to a degree. For instance if the government has flexibility in the need or the timing of the need, it may choose to invoke a reserve, and if bidders satisfy the reserve with low bids, great, and if not, not a big deal.

To assess the optimal auction mechanism, we may look to an analogous situation where the government is the sole seller of an object (the contract) with possibly many buyers (as opposed to the monopsonist environment wherein the government is the only purchaser of an object). This problem is thoroughly addressed by Myerson, and begins with a few definitions and assumptions (1981).

Again, there is only one seller with only a single object to sell, who is also facing n bidders. Let N represent the set of bidders:

$$N = \{1, \dots, n\}. \tag{1}$$

We distinguish between two bidders in N with i and j ($i, j \in N$). For each bidder i , the value estimate is represented by t_i , and is the maximum amount for which i would be willing to pay for the object. Furthermore, the seller does not know the exact value of t_i . We assume that the seller's uncertainty about i 's value estimate is described by



the continuous probability distribution $f_i: [a_i, b_i] \mathbb{R}_+$, where a_i is the lowest possible value of t_i and b_i represents the maximum value of t_i (Myerson, 1981).

To be clear, an auction mechanism is any such auction game that includes a description of the strategies which the bidders are expected to use. We therefore describe the auction mechanism as a pair of outcome functions (p, x) where $p_i(t)$ is the probability that i gets the object and $x_i(t)$ is the price that i expects to pay, given t_i . We then define

$$Q_i(p_i, t_i) = \int_{T_i} p_i(t) f_{-i}(t_{-i}) dt_{-i} \quad (2)$$

to be the conditional probability that i will get the object from the auction mechanism (p, x) , given the value estimate of t_i .

Using this strategy, Myerson (1981) moved to maximize the seller's expected utility U_0 , and showed that it is determined entirely by the probability function p (this determines which bidder gets the object in every situation) and by the numbers $U_i(p, x, a_i)$, which represent the expected utility of each bidder if his value estimate were at its lowest. Essentially, the seller gets the same expected utility from any two auction mechanisms which have the following two properties: (a) the object always goes to the highest bidder, as long as his bid is higher than the seller's own value estimate; and (b) every bidder can expect a zero utility if value estimates were at the lowest possible level. Furthermore, Myerson (1981) commented that the auction studied and proposed by Vickrey (1961), wherein the winning bidder pays the second highest price, is the special case of this general solution. Vickrey's auction is optimal if and only if the bidders are symmetric (i.e., there exist universal high and low valuations and each bidder has the same probability distribution), and $a_i = 0$ and $e_i = 0$ (i.e., bidder i 's valuation would not change if he were to become aware of bidder j 's valuation).

It is important to note that these results do not hold in a common value auction (with imperfect information). In this case the item being auctioned has a common value (i.e., it is of roughly equal value to all bidders) and the bidders do not know the item's market value. Each player's bid then comes primarily from his or her own private estimation of the market value. If we assume that the average bid is accurate, the auction winner—the individual with the highest bid—will tend to overpay. This phenomenon is known as the *winner's curse*. The severity of the winner's curse is exacerbated by higher numbers of bidders; as the number of bidders increases it becomes more likely that some have overestimated the item's value. However, overpayment will occur only if the winner has failed to account for winner's curse when placing the bid. It is reasonable to expect that a rational bidder



will avoid the winner's curse by bid shading, or placing a bid that is lower than his ex ante estimation of the item's value.

Procurement of Multiple Items

A natural extension to the case of Procurement of a Single Item discussed in the prior section is when multiple identical items must be procured—such as Meals Ready to Eat (MREs). We assume that the military has a very large demand for MREs such that one company may not be able to supply all the demand or that it would be most cost-effective for multiple companies to supply the government. Here we reach this subset of the environment addressed in the Procurement of a Single Item section where we do not assume a single bidder, but rather multiple bidders. A spectrum auction is a classic example of this environment, where a regulatory agency (such as the government) sells the rights to use sections of bandwidth to multiple agents.

Though it may seem logical to use the same auction described in the previous section (Procurement of a Single Item), it would not be the appropriate mechanism. Applying a sealed bid second price auction to this situation would be known as a generalized or uniform second-price auction. Unlike in the previous section, however, this auction structure does not induce multiple bidders to bid for multiple items truthfully. As a result, this auction structure is problematic for an auction environment with multiple items.

In order to allow multiple items to be procured from a multitude of bidders while still maintaining a truth-telling mechanism, theory recommends the use of an auction structure that implements what is known as a Vickrey-Clarke-Groves (VCG) mechanism. In essence, the VCG is designed to minimize the price of the items to the auctioneer by having each bidder pay the cost that his participation introduces to the other players. This structure would allow the auctioneer to keep the necessary truth-telling mechanism that he desires, while allowing the auctioneer to procure multiple similar items from multiple vendors at the lowest price.

Google and Facebook both use VCG auctions when allocating space on their websites for advertisements. Companies looking to post ads report values for real estate on the site, then the VCG mechanism is used to determine the appropriate allocation. This is appropriate because there are multiple areas for advertisement, all of which are the same—multiple identical items—and require a VCG auction to induce truth-telling by bidders.

William Vickrey made his original contribution through his design of what later became known as the “Vickrey auction.” In the case described in the previous section, where a single item is auctioned, the Vickrey auction is equivalent to the second-price sealed-bid auction. These terms are used differently in this case,



where multiple identical items are sold. Vickrey won the Nobel Prize for his design, which is the foundation of auction theory. Vickrey’s original auction design has been melded with the Clarke-Groves design for public goods (Clarke, 1971). The resulting auction works for heterogeneous goods and has looser requirements for bidders’ value functions (Ausubel & Milgrom, 2006).

The essence of the VCG auction is that the winning bidder pays the “opportunity cost” for the units won, and his payment depends only on his opponent’s reported values. By entering into the auction, each bidder alters the benefit received by all other participants, usually to a detriment: described by the allocation and payment rules.

The allocation rule says that the mechanism will allocate resources in a way that maximizes total reported utility. The allocation rule can be shown mathematically:

$$\chi(\hat{v}) \in \arg \max_x \sum_i \hat{v}_i(x) \tag{3}$$

The payment rule stipulates that the payment made by each bidder in the auction reflects his social impact on the other bidders. In VCG auctions, a bidder’s payment depends on whether he is a pivotal player—one whose participation affects the utility received by the other players. Specifically, each bidder pays the value of the other participants had he not taken part in the auction minus their values when he does participate. Shown informally,¹ $\text{payment}_i = [\sum U_i \mid i \text{ does not participate}] - [\sum U_i \mid i \text{ participates}]$, or formally,

$$p_i(\hat{v}) = \max_x \sum_{j \neq i} \hat{v}_j(x) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v})) \tag{4}$$

A simple example of the VCG mechanism may further demonstrate the theory. Consider the situation described above where an agency is auctioning bandwidth to two agents (a spectrum auction). There are two spectrum units to auction and there is no difference between them. The agents’ true values are shown in Table 2.

Table 2. Spectrum Auction

	Agent 1	Agent 2
1 Unit	8	11
2 Units	18	20

¹ An expanded description of the VCG mechanism can be found in Appendix, item 1.



Using Equation 4 or the informal equation immediately above it, we can calculate the payments for both players:

$$\text{payment (1)} = 20 - 20 = 0$$

$$\text{payment (2)} = 18 - 0 = 18$$

In this case, Agent 2 wins 2 units at a price of 18, and Agent 1 gets nothing and pays 0. Looking at each payment equation, we can see that Agent 1 pays the value of Agent 2 had he not participated in the auction (20) minus the value of Agent 2 when he does participate (20). Alternatively, Agent 2 pays the value Agent 1 receives when Agent 2 does not participate (18) minus the value of Agent 1 when Agent 2 does participate (0).²

When applying this mechanism to a military acquisition setting, there are some important considerations. The most important point is that for procurement auctions, the military is looking for the lowest price bid by participants. The VCG concepts and process still apply, but instead act to find the most reduced and truthful price for the military. We can go back to the example in which the military is looking to acquire MREs from multiple companies. To do this, the military would ask for a demand schedule from each company. This would include different prices for different amounts of MREs that they are willing to supply (most likely, they would offer a lower price per MRE as quantity increases). The military would then find the social impact of each company in order to determine how many MREs to buy from each company to minimize cost.

Ausubel and Milgrom (2006) said that the VCG mechanism has several important strengths. The first is the dominant-strategy property, which reduces the cost of the auction by making it easier for bidders to determine their optimal bidding strategies and by eliminating their incentives to try to learn about competitor's values or strategies. This also adds to the efficiency of the auction. Another virtue of VCG is its scope of application. The basic rules of the auction can be altered slightly if the auctioneer wishes to impose some extra constraints. For example, the seller may want to limit the concentration of ownership of an item. Or, the buyer in a procurement auction might want to limit its total purchases from first-time bidders or require that total capacity of suppliers is some percentage of the amount ordered. These constraints can be added without affecting the theory in any essential way. A final advantage to VCG is that average revenues are not less than that from any other efficient mechanism. The VCG mechanism is the only direct reporting mechanism (the buyers themselves report values) with dominant strategies, efficient outcomes, and zero payments by losing bidders.

² A more complex and concrete illustration of the VCG mechanism can be found in the Appendix, item 2.



However, Ausubel and Milgrom (2006) also explained that the VCG mechanism has several possible weaknesses, including low (or zero) seller revenues, non-monotonicity of the seller's revenues in the set of bidders and the amount bid, vulnerability to collusion by a coalition of losing bidders, and vulnerability to the use of multiple bidding identities by a single bidder. When every bidder has "perfect substitutes" preferences, the listed weaknesses will never occur. However, if there is even one bidder whose preferences violate the substitutes condition, and the remaining bidders choose values appropriately, all of the listed weaknesses will be present. There are other drawbacks to VCG. The Vickrey theory assumes that bidders' payoffs are linear under certain arguments and requires that there is no effective budget limit to constrain bidders and that the buyer, in a procurement auction, has no overall limit on its cost of procurement. The dominant strategy property breaks down when bidders have limited budgets. The VCG auction may also present a privacy preservation problem. Bidders may rationally be reluctant to report their true values, fearing that the information they reveal will later be used against them. Without private values, VCG auctions immediately lose their dominant-strategy property. These weaknesses limit the practicality of using VCG auctions in Federal procurement.

Ausubel (2004), on the other hand, explained that there may be an alternative efficient ascending-bid auction for multiple similar objects that is simpler for bidders to understand and has the advantage of assuring the privacy of the upper portions of bidders' demand curves. This "dynamic" auction yields the same outcome as the sealed-price Vickrey auction when bidders have pure private values. With interdependent values, the proposed auction may still yield efficiency, whereas the Vickrey auction fails. The auctioneer calls a price, bidders respond with quantities, and the process iterates with increasing prices until demand is no greater than supply. However, a bidder's payment does not equal his final quantity times the final price. Rather, at each price p , the auctioneer determines whether, for any bidder i , the aggregate demand $x - i(p)$ of bidder i 's rivals is less than the supply M . If so, the difference is deemed "clinched," and any goods newly clinched are awarded to bidder i at price p .

For example, suppose that five objects are available and that four bidders A, B, C, and D initially bid for quantities of 3, 2, 2, and 2, respectively. The collective demand of the "losing" bidders is six, which exceeds the supply of five, so the auctioneer must raise the price. Suppose that bidders continue to bid these quantities until price p , when Bidder D reduces from 2 units to 0, dropping out of the auction. While there continues to be excess demand in total, Bidder A's opponents now collectively demand only four units, while five are available. Bidder A therefore clinches one unit at price p , and the auction (for the remaining objects) continues.



This process continues until all the items are clinched. This is easily applied to a military setting (see the MRE example above).

Ausubel showed that there are some key advantages to this type of auction over the former Vickrey auction. The new (dynamic) auction format outperforms the (static) Vickrey auction on efficiency. Moreover, there are at least two sets of examples within this model for which the dynamic auction raises higher expected revenues. Simplicity and privacy-preservation are also key strengths, as well as robustness to interdependent valuations.

While the VCG mechanism is an excellent auction to use for multiple similar items because it is an efficient truth-telling mechanism, it is also very complicated. For the military's purposes, the Dynamic Vickrey auction proposed by Ausubel may be a better option because it is simpler for bidders to understand and provides privacy-preservation.

Differentiated Products: Heterogeneous Competing Products

In this next section we examine a further subset of the imperfectly known cost, no risk area of the defense acquisition marketplace consisting of differentiated, or heterogeneous product acquisition. In this subset we assume that the government is buying existing off-the-shelf products as in the previous two subsections, but in this case these products will be differentiated from each other on one or more key attributes. An excellent example of this is the purchase of computers for a government organization. While these products will be off-the-shelf and thus carry no developmental risk, the government does not know the cost of these computers and could choose any one of a number of alternatives, which have different costs and quality aspects. Continuing with the example of computers, these attributes would consist of things such as hard drive space, screen size, weight, the presence and number of USB drives, or the speed at which they could be delivered, while an alternative would be a model of computer, such as the Fujitsu Lifebook, the Lenovo ThinkPad, or the Apple MacBook. In order to decide which computers to buy, the government will need to run some type of competitive decision-making mechanism, such as an auction, to decide on the product with the ideal combination of attributes and cost. This last point is the crux of this section, and is expanded considerably.

Currently the government's preferred mechanism for procuring differentiated goods is a method generally referred to as the Request For Proposals, or RFP, mechanism (FAR 15.203). With an RFP, the government predetermines several attributes (cost not among them) which it considers important and which it will "grade" the various alternatives on. The government then makes these important attributes public and requests a proposal from any contractor willing to submit one. The government then waits until a predetermined deadline for submission passes



and evaluates the proposals which have been submitted. If any one aspect of a proposed alternative in a proposal does not meet the requirements set forth on the key attributes, the alternative is often thrown out entirely. The government then decides from the remaining alternatives on the “ideal” alternative satisfying the requirements at a low cost.

Several problems present themselves in this mechanism. First, there is a danger of misspecification by the government on its requirements, leading to alternatives being thrown out when they deserve consideration. This danger is compounded by the fact that many purchases are made by surrogate actors, in this case acquisitions officers, rather than by the users of the products themselves. Furthermore, by letting producers know the exact specifications of a needed product, each producer is able to judge not only how capable they are of meeting that specification, but also how capable each of their competitors are, thus creating a situation where only a single producer will submit a bid that just undercuts the next best bid from a competitor, thus reducing competition and increasing the expected price paid by the government (Athey, Levin, & Seira, 2004). Finally, RFP, the mechanism currently used by the DOD, will generally procure items which exceed the quality, and as a result, cost, of that which was required by the initial request (Che, 1993).

In order to correct these deficiencies with the RFP mechanism, we turn to the available academic literature for alternative mechanisms which the government could use in future acquisitions. Though these mechanisms are largely theoretical and each present problems of their own, they present a promising direction in which to take future acquisitions strategy.

Che (1993) argued that the optimal scoring rule in a multidimensional auction will discriminate against quality (such that a high quality item will be penalized), but that this scoring rule requires strong commitment power, which is nearly impossible in defense acquisitions. Also important to note is the tendency for a surrogate actor to procure quality in excess of what is desired. In this case the second preferred offer auction may be optimal (where the winning bidder must match the second bidder’s cost and quality). Finally, Che stated that either first or second price auctions will achieve this outcome.

Branco (1997) further analyzed multidimensional score auctions in a manner similar to Che, with the critical exception that he assumed costs between bidders to be correlated as opposed to the independent costs assumed in Che. This changes the optimal structure to be that of a two stage first or second price multidimensional auction, in which the first round consists of a score auction in the style of Che to determine the most efficient alternative, then a second round of bargaining between



the bidder and the buyer to determine the specific quality to be supplied. This conclusion is ratified by Wise and Morrison (2000).

Melese, Richter, and Simon (2011) further expanded the model, exploring a mechanism by which cost is excluded from the decision variables entirely. Instead, bidders are asked to submit a quality bid on different factors based on a range of different budgets. This will result in the buyer having information on the bidders' quality capabilities over a range of different budgets, which will then allow the buyer to select based on a probability distribution of expected future budgets. This is particularly valuable in the realm of large-scale, long-term military acquisitions where budgets are constantly changing as it would allow the buyer to select a less risky bid proposal with respect to future price increases.

Another possible auction design theory, a bid credit auction, is described by Shachat and Swarthout (2003). First they described an RFP in which buyers provide evaluation criteria on quality and receive prices and qualities from the various bidders. They likened this situation to a sealed bid first price auction with private values. The differential between cost and quality will be used to determine a winner. Second they described an English auction with bidding credits (EBC auction) where the buyer provides evaluation criteria to bidders, who then provide descriptions of their products. The buyer then distributes bidding credits depending on the observed relative quality. This credit is then used by the bidders in a second round of auctioning, in which an English, descending price auction is held until the lowest price bid wins. However, upon empirical experimentation, the authors concluded that real-world actors don't follow the Nash equilibrium in the RFP and that buyers assign overly generous bidding credits in the EBC. This behavior results in the EBC actually outperforming the RFP in terms of both bidder and buyer surplus.

From these selected articles, the next step is to determine the most effective mechanism for procuring items in a differentiated market for the DOD. Currently, as the vast majority of contracts in this market are what are called "fixed price" contracts, in which the item, quantity, schedule, and price are all determined at the outset of the contract, the most effective mechanism is a combination of the auction types described above, depending on the exact product to be procured.

The first and most logical step to formulating an auction is to set a variety of attributes which the government will be evaluating different proposals from the sellers. The government should inform the potential sellers of these key attributes, but should not tell them exactly what the weights for each key attribute will be. For a smaller item, such as the batch of pens described above, this will be a simple one or two variable request. However, for a more complex differentiated item, such as computers, there will be many attributes by which each seller's alternative will be



evaluated. In each case, however, cost will be considered as an independent variable with a weight more or less than the other attributes of the product.

In addition to this scoring rule, if negotiations are expected to be repeated every year for additional procurement, an additional level of complexity should be added to the request. This would likely be the case in a large organization, for example, where different portions of the organization will need to purchase computers each year. If that is the case, the government should require each seller to submit a bid for several possible budget levels. The government should specify which of those possible budget levels will be most likely in the current budget cycle, but given the constant budget uncertainty, the government should reserve the right to change the budget level in future years' negotiations. Thus, the government will require each potential seller to submit a bid filling the budget level for this year, as well as a schedule of potential future budget levels. It is important to note here that, as opposed to the more simplistic model above, cost would be excluded from the decision entirely, as it would instead be captured by the budget schedule. The information provided to the government by sellers with this variety of budget levels will enable this budget flexibility without the constraints under a fixed price contract.

This mechanism is similar to that described by Melese, Richter, and Simon, who first proposed its use in their 2011 paper. Under this mechanism, the optimal bidding rule will be as described by the authors and of the following form:

$$\text{Max } V(A_i) = \sum_{j=1}^n w_j a_{ij} \quad (5)$$

where V is the bidder's valuation of the contract, A is the bid amount, n is the number of bidders, w is the weight the buyer places on attribute j , and a is the level of attribute j offered by vendor i . From here, each vendor's supply problem is modeled with the microeconomic supply function. Specifically,

$$\text{Max } Q(A_i) = \sum_{j=1}^n w_{ij} a_{ij}, j = 1, \dots, m \quad (6)$$

$$\text{s. t. } TC_i = \sum_{j=1}^m c_{ij}(a_{ij}) \leq B \quad (7)$$

where $Q(A)$ is the value function to the bidder, $c(a)$ is the cost as a function of the amount of attribute a placed by bidder j , and B is the hypothetical budget set forth by the government. In this situation the problem becomes a Lagrangian function, and results in each bidder bidding the amount of attribute a which it believes the government values. Though this result is intuitive, it suggests that the



bidding market will function as an efficient competitive marketplace despite the exclusion of cost from the decision variables, especially in an environment with a large amount of bidders such as in off-the-shelf goods.

Throughout these solutions we are making several key assumptions which should undergo further scrutiny. The first is that there are a relatively large number of well-endowed firms willing to bid. In practice the government often deals with only a handful of firms, or in some cases only one firm. Oftentimes, these firms also have vastly different resources—one needs only to compare the Big Five defense firms to the rest of the industry to appreciate this reality. The second key assumption is that of zero costs to entry or to bid preparation. In our models, introducing barriers to entry would restrict competition and influence bidding behavior in a complex manner, and would vary some of the conclusions of this paper. However, in the defense industry, there are often significant costs to bid preparation—sometimes ranging into the millions of dollars for a single project. While the government does make some effort to compensate for these bid preparation efforts, oftentimes it fails to fully recoup firms for their costs. Also important are the high barriers to entry in the defense market. Firms must not only have considerable infrastructure in place to bid on larger, more capital intensive projects for the DOD, but oftentimes must go through a lengthy approval process to do business with the government in the first place. When working with classified materials, this process is even longer and more costly, and represents another considerable barrier to entry which would affect the conclusions of this paper.

In summary, there are several different options available to the government in the acquisition of differentiated multi attribute goods. The ideal mechanism is that which captures cost as a constraint outside of the model, rather than included in the model as an independent variable. However, there are limitations to this approach and the government would do well to consider alternatives described in this section if any of the assumptions prove grossly false. However, one common theme of each mechanism described is clear: the current RFP mechanism leaves much to be desired and should be modified to achieve ideal efficiency in government acquisition of differentiated goods.



Product Procurement in Principal-Agent Models

In this section of the paper, the government is no longer acquiring an item already in existence and must work with a contractor to procure the item or service. Accordingly, the government is acquiring item(s) with known or unknown costs and/or known or unknown stochastic risk. Currently, the military is the entity that assesses the risk of each proposal through a committee process that takes into account many factors, and not all of them are certain. In the current environment, each bidder simply submits a bid and it is up to the military to determine if the company can really deliver the needed item up to the bid's specifications. We feel that this is an unnecessary risk on the part of the military and that each bidder can better assess their own project's risk and, if given the right incentive, would be willing to pass on this more precise information to the military's item scoring committee.

Procurement with a Single Agent

In the first environment we assumed that the military knew both the cost and risk of the items they wished to procure; however, in this next section we look at how the situation changes when the military does not know the risk involved of the project. An example of this environment would be if the military wished to work with a company in the development of a new technology. These types of situations are often referred to as principal-agent problems, where the military acts as the principal and the contractor is the agent. A simple diagram illustrating the basic frame work of the problem is shown in Figure 2 where the principal wishes to extract the benefits from the agent doing the work; however, the principal is not able to perfectly observe the effort of the agent. v_h and v_l represents the value of a good/bad outcome to the principal, w_h and w_l are the wages offered to the agent for a good/bad outcome, and e_h and e_l are the cost to the agent of putting forth high/low effort. p and q represent the probability of a good outcome depending on the effort level of the agent. The principal is the first party to act, deciding whether or not to offer a contract, and if they do offer, how much to offer for a good outcome (G) or a bad outcome (B). The agent then has to decide if she wants to accept or decline the contract, and should she accept, she then has to decide if she wants to devote high or low levels of effort. However there is a stochastic element present in that high levels of effort by the agent will not guarantee a good outcome, and low levels of effort will not guarantee a bad outcome, so the agent could devote low levels of effort and still achieve a good outcome.



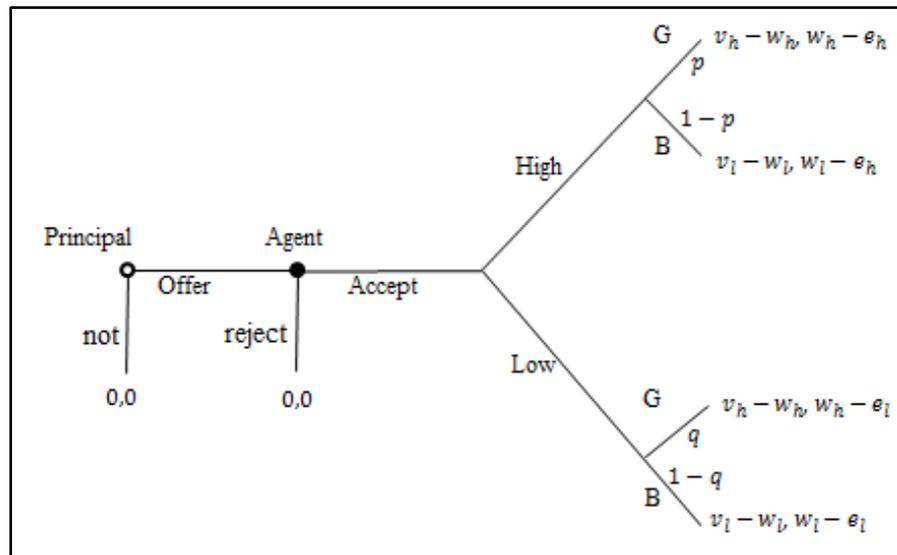


Figure 2. Principal-Agent Problem

There is also imperfect monitoring present as the principal cannot measure the effort of the agent and only the final outcome, introducing an element of risk to the interaction. As a result, the principal does not know if there was a good outcome, because the agent put forth high effort or put in low effort and still achieved a favorable outcome. The principal then must design the contract to entice the agent to put in the required effort in order to reach the desired outcome (called the incentive constraint). The principal may not always want to induce high effort from the agent, though, as the principal must look at the expected productivity gained from the agent putting forth high effort. If there is a large difference between the cost of high versus low effort to the agent, the agent may not be willing to put forth the extra effort if the expected wage gain is relatively small. The principal must also consider how risk averse the agent is when deciding how to offer the contract. As the agent becomes more risk averse the principal may want to make w_h closer to w_l so there is more of a “guaranteed” payment to the agent.

There are several different studies that have looked at how this incentive constraint impacts the behavior of the principal and the agent, as well as ways to help reduce or eliminate these effects. When there is a collaborative effort between the principal and the agent, Iyer, Schwarz, and Zenios argued that the principal should provide some of her own resources (equipment, capital, etc.) to reduce the cost of the project (2005). While this may seem strange, it is advantageous to the principal in that it helps her gain hidden information about the supplier’s capability by the exchanging of information helping remove some of the uncertainty in observing the agent’s effort (Iyer et al., 2005). This resource allocation depends on whether the principal’s involvement is a substitute or a complement to the supplier’s capability (a substitute in that the principal’s resources reduce the number of steps while the

supplier's resources influence the cost of each step). Iyer et al. found that when the principal's resources act as substitutes, that increasing their commitment lowers the supplier's marginal capability, thus the principal should commit more resources (2005). They also stated that the buyer should commit all their resources and offer a fixed price to achieve the optimal contract. However, when the resource commitment acts as a complement, it is advantageous to scale back their contribution. The principal offers contracts that either involve little resource commitment, or a larger resource commitment by the principal but as a price discount. The more capable agent will then find it beneficial to choose the contract with a higher resource commitment from the principal because it will mean a larger cost reduction and a larger profit (Iyer et al., 2005).

Sappington (1991) looked at a similar situation where the agent has more knowledge about the environment, and how the principal can use this to help shape the interaction. One way of doing this is by asking the agent to give a prediction of the environment she expects to be in and then link the contract back to the forecast. This can also be extended to a competitive environment by asking each agent to predict not only her own environment, but also that of her competitor. The effort of the agent also depends on the length of the interaction, since if it is only a one time deal then agent does not have as much incentive to provide the desired level of work. However, if the agent knows it will be a repeated interaction this can help remove some of the risk (Sappington, 1991).

Of significant interest is the role project scope, the effect that newly designed parts have in terms of time and cost, has on the process. Clark (1989) focused on this area and how this affected various projects across different relationships. Clark found that a large reason for the difference in time and cost of projects could be attributed to project scope, specifically, that it could be advantageous to design and develop new products rather than adapting existing ones. A small change in the scope of a project was shown to reduce the development time of the project by several months. Clark also argued that the better the relationship is between the two parties, the more profound this effect is.

Bromiley (1991) looked at the effect that risk had on performance and how past performance as well as industry conditions affected future risk-taking by firms. The main finding was that performance and past industry performance both had strong negative effects on risk-taking (low industry performance increases risk-taking by firms), while expectations and aspirations had positive effects, increasing risk-taking (Bromiley, 1991). While initially, only the previous year was considered when looking at risk, the long-term effects from two to four years ago were also analyzed. Bromiley found that the influence of risk on performance from four years ago was



greater than the influence from the previous year, which illustrates the long-term lingering effects of risk on firms.

An example of this environment would be if the military wishes to work with a company in the development of a new technology, such as new computer software. The military, in this case the principal, would hire the contractor, the agent, to develop this desired software. However, there is always the possibility that complications will arise or problems develop that are unforeseen by the contractor and, as a result, the contractor may be hesitant to provide their full effort in development. The military could help this situation, though, by providing some of its own engineers to work on the project in conjunction with the contractor. The military could also ask the contractor to provide estimates for the probabilities of different problems that may arise and base part of the contract on these predictions to prevent them from over- or underestimating to the contractor's advantage. For example, the military could provide a higher contract award amount to a software development contractor if the contractor believes that problems are very unlikely to occur but decrease payment if problems not predicted do occur during development. The contractor now has an incentive to accurately predict what they expect to happen. It would also benefit the military for the contractor to believe that the military will need to procure related items in the future, which leads to the belief of a possible repeated interaction. Knowing that current success could affect future contracts, the contractor now has a larger incentive to perform well and devote more effort in hopes of gaining a good reputation with the military.

Based on the research of Iyer et al.; Sappington; Clark; and Bromiley, the interaction between the principal and the agent relies on several different conditions. First, it is beneficial for the principal to share the risk with the agent in terms of how the contract is structured. It is also advantageous to the principal to allocate some of their own resources to help the agent in order to reduce cost and gain information. The principal can also attempt to reduce the knowledge gap by asking the agent what she expects to happen and link the contract to this prediction. Past performance, industry conditions, and company goals can also be looked at to help determine risk attitudes of different firms. The principal can use all these tools to help gain a better understanding of the environment, but the optimal contract will depend on the degree of risk sharing and knowledge gap between the parties.

A final note worth mentioning is that it may be worthwhile for the principal to ask the agent to provide estimates for the cost and risk of the project and use these in contract development. An example of such estimates is shown in Figure 3, where Agent 1 predicts a lower cost but a larger variance, while Agent 2 predicts a higher average cost with smaller variance.



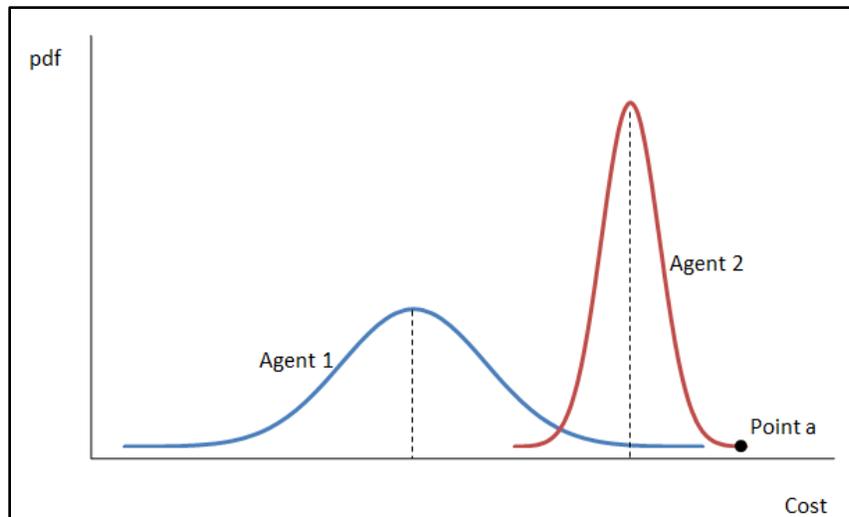


Figure 3. Variance in Cost Predictions

While Agent 2 predicts a higher cost, she would only be penalized if she ended up at an event that she predicted was not likely, but still ended up there (i.e., Point a). Because she is more confident of her price range, she should be penalized if this unlikely event occurred, whereas Agent 1 predicts a larger variance resulting in a larger spread in cost, so Agent 1 should be penalized only at an equally unlikely point.

Procurement with Multiple Agents

In Procurement with a Single Agent, we assumed the military was working with only one agent. In this next section, we relax this assumption and allow the military to work with a variety of contractors (i.e., agents).

With the introduction of multiple agents (or bidders), the principal's problem now becomes twofold: First, the principal must structure the optimal contract while considering the effects of moral hazard, risk sharing, and competition amongst the bidders; and second, the principal must then structure the competition environment so as to reveal the best bidder (note: the definition of *best* is purposely vague here, and is elaborated on later).

In constructing the optimal contract, we start with modeling the ex post cost of the project or task. We assume n agents ($n > 1$), and that the ex post cost to agent i has three components (McAfee & McMillan, 1986):

$$c_i = c_i^* + w - \xi \quad (8)$$

where c_i is the i^{th} agent's expected cost, w represent the unpredictable costs which may be incurred during the course of the project, and ξ is the effort put forth by the agent to reducing costs (this cannot be observed by the principal). Furthermore, we assume the principal to be risk-neutral and to design a contract to minimize his

expected payment to the agent. We also assume that the contract's payment, P , is contingent on both the ex post cost of the project, c , and the winning agent's bid, b , and is also linear (McAfee & McMillan, 1986):

$$P = \alpha c + \beta b + \gamma \quad (9)$$

for some constants α , β , and γ .

This yields three distinct types of contracts. If $\alpha = 1$, and $\beta = 0$, (9) defines a cost-plus contract. That is, the principal agrees to cover all of the costs associated with the project. If $\alpha = 0$ and $\beta = 1$, (9) defines a fixed-price contract, in which the principal agrees to pay a fixed ex ante price. Now, if $0 < \alpha < 1$, and $\beta = 1 - \alpha$, (9) defines an incentive contract (i.e., the agent is responsible for a fraction, $1 - \alpha$, of any cost overrun; McAfee & McMillan, 1986). Note that in practice, the government uses two types of incentive contracts: cost-plus-incentive and fixed-price-incentive, but that McAfee and McMillan (1986) did not differentiate between the two in their analysis (this prompts further research).

After solving for the expected-utility-maximizing choice of b and ξ , McAfee and McMillan (1986) then moved on to find the principal's expected-payment-minimizing choice of contract (given the agent's optimizing behavior). In the first portion, they found that the larger the share of costs paid for by the government, the less effort any individual firm is willing to exert towards lowering costs (as α increases, ξ decreases). Furthermore, bids decline as α rises, ceteris paribus. The explanation is that an increase in the winning bidder's costs covered by the principal is similar to a reduction in the variance of expected costs for all bidders, and so competition will drive down the bids. They also found that $\alpha = 1$ (cost-plus contract) can never be an optimal solution for the principal. When $\alpha = 1$, bids bear no relationship to expected costs, and so the lowest-cost firm is likely not selected by the principal. Moreover, they concluded that incentive-contracts are usually the optimal choice.

The second half of a principal-(multiple) agent problem is shaping the bidding environment itself; Asker and Cantillon (2008) offered valuable insight. They focused their analysis on auctions which offer flexibility in terms of contract specification. They attributed three types to this category: (1) "beauty contest," wherein the principal gives the agents a range of attributes that she cares about, but request only a single offer; (2) "menu auction," in which the agents submit menus of price and nonmonetary attributes, and the principal picks the one which best suits her needs; and (3) scoring auction—the principal announces the way she ranks different attributes ex ante, the agents then submit offers on all dimensions of the product, and then the bid which scored the highest according to the principal's ranking schedule is chosen (Asker & Cantillon, 2008).



Asker and Cantillon found that, from the buyer’s perspective, scoring auctions strictly dominate price-only auctions, and they weakly dominate menu auctions and beauty contests when an open ascending format is used. With a sealed-bid “second price” format, scoring auctions weakly dominate a menu auction and strictly dominate a beauty contest. Moreover, first price menu auctions were found to always be inefficient. They concluded that scoring auctions generally provide higher utility to the principal.

McAfee and McMillan provided very useful results, but their research is limited strictly to linear contracts. In the case where the agent can choose to exert more than two levels of effort, non-linear contracts may be more efficient. Instead of just high or low effort the agent may choose from a schedule of effort levels $e_n, n = \{1, 2, \dots, n\}$, or the agent may choose from a continuous distribution of effort rather than just discrete choices. An example contract follows:

$$p = \begin{cases} 10, & \text{where } q(e) < 10 \\ 25, & \text{where } 10 \leq q(e) \leq 20 \\ 30, & \text{where } 20 \leq q(e) \end{cases} \quad (10)$$

where p is the payment to the agent and $q(e)$ is the output produced by the agent, dependent on e effort. The payment schedule can also be a continuous function of (e) (e.g. $p = \sqrt{q(e)}$). The payment schedule should be optimized to minimize cost of incentivizing effort to the principal while maximizing the agent’s effort.

Conclusion

We have described multiple markets in which the government engages in acquisitions activities and possible auction-based or principle agent approaches to each. In each market, the solution identified is mathematically superior to the existing acquisition mechanism, suggesting that modifying acquisitions to match the recommendations of this paper will result in greater efficiency and a cost savings to the government.

Several environments favoring auction-based acquisition were examined. The first market analyzed, those of simple goods purchased individually used a classic auction type known as a Vickery (or second-price) auction, which reveals bidders’ true values and results in an efficient allocation of bids. The second market, simple goods purchased from a multitude of firms, opts for a dynamic Vickery auction as it achieves a similarly efficient result as other auction types while being easy to understand and implement. The third market analyzed, that of differentiated goods, opts for a multi attribute scoring auction with budget considered an independent variable in single negotiations and as a variable constraint in situations where renegotiation is likely. This allows for greater flexibility and less risk in future years, despite several possible limitations.



Next we examined principle-agent relationships, where items are procured amongst unknown cost and risk or known cost and existent but understood stochastic risk, which generally is the realm of developmental goods. Among these, the fourth market analyzed is of interactions of the principle with agents in single agent procurements, where the government is executing a contract with only one offeror. In this environment the optimal contract will depend on the degree of risk sharing and knowledge gap between the parties. The fifth market consists of multiple agent procurements, where the government is executing a contract with multiple independent offerors. In this case, scoring auctions generally provide higher utility to the principle. These suggestions look to narrow the knowledge and risk gap between the government and the contractor to reduce cost and risk in future years and throughout the life of the contract.

Based on the above analysis, it is clear that the contracting environments need to be approached differently in order to maintain competition and encourage truthful reports from participants. The current bidding structure often does not provide adequate incentives for bidders to accurately report their costs; however, simple modifications could be made for this to happen. This would help increase competition and allow the government to get better contracts while still acquiring all necessary items and services.

Authors' Note

At multiple points in this paper the use of singular pronouns (e.g. him, her, and she) was best for clarity. Desiring to show no bias, we favored male pronouns in the auctions portion of the paper and female pronouns in the principle-agent portion of the paper for simplicity in editing.



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Appendix

Appendix items 1 and 2 are taken directly from “The Lovely but Lonely Vickrey Auction” by Ausubel and Milgrom (2006). They include form and illustrative examples of the VCG mechanism.

(1) Formal VCG mechanism:

Formally, the VCG mechanism is described as follows. Let x be a vector of goods that a seller has on offer and let $v_n(x_n)$ denote bidder n 's value for any nonnegative vector x_n . Each bidder $n = 1, \dots, N$ reports a value function \hat{v}^n to the auctioneer. The auctioneer then computes a value-maximizing allocation: $x^* \in \arg \max_{x_1, \dots, x_N} \sum_n v_n(x_n)$ subject to $\sum_n x_n \leq x$. The price paid by a bidder n is then $p_n = \alpha_n - \sum_{m \neq n} \hat{v}^m(x_{m^*})$, where $\alpha_n = \max\{\sum_{m \neq n} \hat{v}^m(x_m) \mid \sum_{m \neq n} x_m \leq x\}$. Notice that α_n depends only on the value reports of the other bidders and not on what bidder n reports (Ausubel & Milgrom, 2006).

(2) Illustrative VCG mechanism:

To illustrate the VCG mechanism, suppose that there are two items for sale (A and B) and two bidders. Each bidder $n = 1, 2$ submits bids: $\hat{v}^n(A)$ for item A; $\hat{v}^n(B)$ for item B; and $\hat{v}^n(AB)$ for the two items together. Assume without loss of generality that $\hat{v}^1(AB) \geq \hat{v}^2(AB)$ and $\hat{v}^1(A) + \hat{v}^2(B) \geq \hat{v}^1(B) + \hat{v}^2(A)$. If $\hat{v}^1(AB) > \hat{v}^1(A) + \hat{v}^2(B)$, then the outcome is that bidder 1 wins both items. Applying the formula, his payment is $\hat{v}^2(AB)$. However, if $\hat{v}^1(AB) < \hat{v}^1(A) + \hat{v}^2(B)$, then the outcome is that bidder 1 wins item A (with an associated payment of $\hat{v}^2(AB) - \hat{v}^2(B)$) and bidder 2 wins item B (with an associated payment of $\hat{v}^1(AB) - \hat{v}^1(A)$). In each case, the winner pays the opportunity cost of the items won, and his payment depends only on his opponent's reports (Ausubel & Milgrom, 2006).

The final appendix item is taken from Ausubel's “An Efficient Ascending-Bid Auction for Multiple Items” (2004).

(3) An illustrative example of the Dynamic Vickrey auction:

I will illustrate my proposal for an ascending-bid, multi-unit auction with an example loosely patterned after the first U.S. spectrum auction, the Nationwide Narrowband Auction. There are five identical licenses for auction. Bidders have taste for more than one license, but each is limited to winning at most three licenses. There are five bidders with values in the relevant range, and their marginal values are given as in Table 1 [on the following page] (where numbers are expressed in millions of dollars):



	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
Marginal Value (1 unit)	123	75	125	85	45
Marginal Value (2 units)	113	5	125	65	25
Marginal Value (3 units)	103	3	49	7	5

Table 1: Bidder Valuations in Illustrative Example

For example, if Bidder A were to purchase two licenses at prices of 75 each, her total utility from the transaction would be computed by: $u_A(1) + u_A(2) - 75 - 75 = 123 + 113 - 150 = 86$. In this example, bidders are presumed to possess complete information about their rivals' valuations.

The proposed auction is operated as an ascending-clock auction. The auctioneer announces a price, p , and each bidder i responds with a quantity, $q_i(p)$. The auctioneer then calculates the aggregate demand and increases the price until the market clears. Payments are calculated according to a "clinching" rule. Suppose that the auction begins with the auctioneer announcing a price of \$10 million ($+\epsilon$). Bidders A–E, if bidding sincerely according to the valuations of Table 1, would respond with demands of 3, 1, 3, 2 and 2, respectively. The aggregate demand of 11 exceeds the available supply of 5, so the auction must proceed further. Assume that the auctioneer increases the price continuously. Bidder E reduces his quantity demanded from 2 to 1 at \$25 million, Bidder E drops out of the auction completely at \$45 million and Bidder C reduces his quantity demanded from 3 to 2 at \$49 million, yielding:

Price	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
49	3	1	2	2	0

The aggregate demand, now 8, continues to exceed the available supply of 5, so the price must rise further. When the price reaches \$65 million, Bidder D reduces her demand from 2 to 1, but the aggregate demand of 7 continues to exceed the available supply of 5:

Price	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
65	3	1	2	1	0



However, let us examine this situation carefully from Bidder A's perspective. The demands of all bidders other than Bidder A (i.e., $1 + 2 + 1 + 0$) total only 4, while 5 licenses are available for sale. If Bidders B–E bid monotonically, Bidder A is now mathematically guaranteed to win at least one license. In the language of this paper (and in the standard language of American sports writing), Bidder A has clinched winning one unit. The rules of the auction take this calculation quite literally, by awarding each bidder any units that she clinches, at the clinching price. Bidder A thus wins a license at \$65 million.

Since there is still excess demand, price continues upward. With continued sincere bidding relative to the valuations in Table 1, the next change in demands occurs at a price of \$75 million. Bidder B drops out of the auction, but the aggregate demand of 6 continues to exceed the available supply of 5:

Price	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
75	3	0	2	1	0

However, again examine the situation from Bidder A's perspective. Her opponents collectively demand only $0 + 2 + 1 + 0 = 3$ units, whereas 5 units are available. It may now be said that she has clinched winning 2 units: whatever happens now (provided that her rivals bid monotonically), she is certain to win at least 2 units. Hence, the auction awards a second unit to Bidder A at the new clinching price of \$75 million. By the same token, let us examine this situation from Bidder C's perspective. Bidder C's opponents collectively demand only $3 + 0 + 1 + 0 = 4$ units, whereas 5 units are available. He has now clinched winning 1 unit: whatever happens now (provided that his rivals bid monotonically), he is certain to win at least 1 unit. Hence, the auction awards one unit to Bidder C at a price of \$75 million.



There continues to be excess demand until the price reaches \$85 million. Bidder D then drops out of the auction, yielding:

Price	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
85	3	0	2	0	0

At \$85 million, the market clears. Bidder A, who had already clinched a first unit at \$65 million and a second at \$75 million, wins a third unit at \$85 million. Bidder C, who had already clinched a first unit at \$75 million, wins a second unit at \$85 million. In summary, we have the following auction outcome:

	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
Units Won	3	0	2	0	0
Payments	65 + 75 + 85	0	75 + 85	0	0





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