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Navy Operational Planner: anti-submarine warfare with time-dependent performance

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THESIS

NAVY OPERATIONAL PLANNER: ANTI-SUBMARINE WARFARE WITH TIME-DEPENDENT PERFORMANCE

by

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September 2017

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The Navy Operational Planner project develops tools that help the Navy optimize operational planning in terms of time, manpower, and resources. Its purpose is to help decision makers use available platforms efficiently to accomplish as many missions as completely as possible over a finite time period. We adapt the anti-submarine warfare (ASW) component of this model by adding time-dependent performance data that reflect ocean or atmospheric conditions that vary over the planning horizon. We develop three test cases with varying degrees of time-dependence in the performance data, and show that taking this new information into account changes the operational plans generated and can lead to better employment of ASW platforms due to the more realistic representation of platform performance.

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NAVY OPERATIONAL PLANNER: ANTI-SUBMARINE WARFARE WITH TIME-DEPENDENT PERFORMANCE

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ABSTRACT

The Navy Operational Planner project develops tools that help the Navy optimize operational planning in terms of time, manpower, and resources. Its purpose is to help decision makers use available platforms efficiently to accomplish as many missions as completely as possible over a finite time period. We adapt the anti-submarine warfare (ASW) component of this model by adding time-dependent performance data that reflect ocean or atmospheric conditions that vary over the planning horizon. We develop three test cases with varying degrees of time-dependence in the performance data, and show that taking this new information into account changes the operational plans generated and can lead to better employment of ASW platforms due to the more realistic representation of platform performance.

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LIST OF ACRONYMS AND ABBREVIATIONS

ASW	anti-submarine warfare
DDG	guided missile destroyer
KT	knot (measure of speed)
MW	mine warfare
NM	nautical mile (measure of distance)
NMP	Navy Mission Planner
NOP	Navy Operational Planner
P-8	Boeing P-8 Poseidon
SSN	nuclear-powered attack submarine
NOP-ASW	Navy Operational Planner – Anti-submarine Warfare

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EXECUTIVE SUMMARY

Operational planning compares to economics in one key principle: scarcity. In this realm, time and resources are scarce, and therefore require the majority of a planner's attention. This continues to make the task of operational planning a particularly difficult challenge for the United States Navy. One mission area of interest to the Navy is anti-submarine warfare (ASW). Submarines pose a unique threat to military and commercial vessels in times of conflict. Due to their clandestine nature, the techniques used to detect submarines are intensive and time consuming, and can strain resources available and make operational planning even more difficult.

In order to help make better decisions, many tools have been developed over time. The Navy Mission Planner and the Navy Operational Planner are two such tools. Each model has its own aspects that make it useful in operational planning. This thesis focuses on the Navy Operational Planner and expansions that have been made to its ASW model. In particular, our goal is to make the model more effective in accounting for time-varying performance. We add time-varying performance data to see how changing ocean conditions, planned equipment downgrades, and atmospheric conditions can affect resource use.

In its most basic form, the Navy Operational Planner is a mixed-integer linear program. Its objective is to maximize achievement, a measure of how well a mission's requirements have been completed. Specifically, in the ASW portion of Navy Operational Planner, this indicates the level of confidence achieved in ensuring an area is clear of submarines. Our primary addition to this model is a new simulated data table that mimics some effects search capabilities may have during different time periods. This new information has been formatted to function within the model and affects calculations that define important index sets.

We test our model in a contrived scenario that takes place in the South China Sea. The area is simplified in many respects, including travel distances, platforms available, and logistical constraints. Three main trials are run: “NONE,” “LIGHT,” and “HEAVY.” Each has more significant temporal variability of performance.

We find that the new model addition does influence the outcomes in each scenario case. Results from the “NONE” and “LIGHT” cases are very similar, but the new data do completely change how missions are prioritized. The “HEAVY” case significantly changes how achievement is increased, resulting in a plan that relies heavily upon platform combinations that work with the best available time-performance data. These outcomes will vary greatly with changes to other inputs to the model, such as commander’s preference for higher confidence, but the addition of time-varying performance to the Navy Operational Planner increases the accuracy of the scenarios modeled. This enables better choices in the face of scarce time and resources.

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I. INTRODUCTION

A. PROBLEM STATEMENT

Operational planning compares to economics in one key principle: scarcity. In this realm, time and resources are scarce, and therefore require the majority of a planner's attention. This continues to make the task of operational planning a particularly difficult challenge for the United States Navy. One mission area of interest to the Navy is anti-submarine warfare (ASW). Submarines pose a unique threat to military and commercial vessels in time of conflict. Due to their clandestine nature, the techniques used to detect submarines are intensive and time consuming, and can strain resources available and make the operational planning process even more difficult. The Navy Operational Planner (NOP), a set of operational planning models developed at the Naval Postgraduate School (Dugan 2007, Deleon 2015, Molina 2016, Kim 2017), has a module dedicated to the ASW problem; we refer to this module and the related optimization model as NOP-ASW. The focus of this thesis is improving NOP-ASW; in particular, we have added features that model time-dependent performance of the various platforms in each of the regions in a given scenario, allowing greater accuracy of our modeling of the changing ASW conditions they would face over the planning horizon.

B. MOTIVATION

Currently, NOP-ASW is functioning well enough to provide helpful insights to decision makers. Planners can still use it to see which combinations of ships and aircraft are ideal for achieving the ASW mission most effectively in terms of time and achievement; however, there are more factors that can be considered.

Although NOP-ASW now considers different platform types, it does not allow time-varying performance of those platforms, and consequently is limited in its fidelity. With the addition of time-varying performance data, NOP-ASW will be able to model changing ocean conditions, planned equipment downgrades (for maintenance, training, or any other reason), atmospheric conditions, and time-of-day effects such as visibility.

With this increased resolution, NOP-ASW is able to model more realistic scenarios, which in turn will generate more realistic operational ASW plans. This provides better guidance to operational planners who need this information to make the best decisions regarding the platforms they have available and their limited time horizons, both of which are scarce and the keys to successful operational planning.

II. BACKGROUND

A. NAVY OPERATIONAL PLANNER EXPLAINED

1. Origin and Initial Work

Prior to the Navy Operational Planner (NOP), Dugan (2007) presented the Navy Mission Planner (NMP). The goal of his work was “to develop a decision aid that quickly provides a face-valid optimal solution” (Dugan 2007). NMP was able to give solutions, but it was attempting to perform many different missions at once. This resulted in partial completion of missions, but not necessarily satisfactory progress in any one specific area.

The NOP was first developed by Deleon (2015). Deleon laid the groundwork for the concepts used in future iterations of the project. The goal of Deleon’s work was for “NOP [to advise] how to allocate multiple ships to multiple missions in order to accomplish those missions to a prescribed level of completion as quickly as possible, to allow a transition to the next phase of a larger operation” (Deleon 2015). In his thesis, he developed an integer linear program that would attempt to provide an optimal or nearly optimal solution. The overall goal of NOP was to determine how long it would take to achieve one specific mission to an acceptable level in the overall operation. Once one mission had been completed, the time used to complete it could be considered by the planner and used to begin planning the next mission area. Accordingly, his example scenario focused on maritime mine warfare (MW) (Deleon 2015). This would be the first piece of the project that could encompass as many of the Navy’s warfare areas as possible.

The primary difference between NMP and NOP is the end of the time horizon. NMP views the end of the time horizon as a constraint, while NOP uses the end of the time horizon as a decision variable. This is because the goal of NOP is to accomplish the mission to a certain level of accomplishment, and therefore credit is not given until the mission has reached said level. One of the most powerful aspects of NOP is its flexibility. Its code can be easily modified to either perform similar to the NMP or to the newer objective of NOP.

2. ASW Module Creation

Deleon’s work was successful, and the NOP was next studied by Molina. This work was the first attempt at a NOP ASW-specific solution (Molina 2016). Molina’s work was instrumental in setting up the mathematical environment that we still follow. He pointed out the differences that Deleon’s MW module contained that would not be useful for ASW. Of particular note by Molina was that many different platforms would be used in ASW: ships, submarines, and aircraft. Each of these platforms would have to be accounted for in terms of its constraints, as well as their overall effectiveness in completing the ASW mission. This was enough to warrant a reformulation of the problem when compared to Deleon’s work (Molina 2016). The new formulation was successful in providing an optimal or nearly optimal solution, and the ASW module could now be included in the NOP.

3. ASW Module Expansion

The most recent addition to the NOP-ASW module was contributed by Kim in 2017. His work focused on three new areas: mission values, solo aircraft search, and synergistic effort (Kim 2017). Mission values are a new input to the NOP-ASW model that attempt to influence the selection of a certain mission area based on its assigned value, which simulates a commander or planner’s priority. Solo aircraft search is simple; it allows aircraft in the model to operate without the addition of other platforms, such as ships or submarines. Synergistic effort is the most influential of Kim’s contribution to the model: “when platforms are working in the same mission area, we add synergistic effects to attempt to model the positives of communication between platforms to the overall search effort” (Kim 2017). This synergistic effect influences how well probability of detection is influenced, and is discussed in Chapter III. Overall, Kim’s expansion of the NOP-ASW module was successful, leading to the present work.

B. SEARCH AND DETECTION

1. Choice of Search Method

There are many different types of search algorithms available for search and detection models. For the purposes of this thesis and model, continuous random search is used. The equation, simplified into its most basic, individual components, is as follows:

$$P(d) = 1 - e^{-\left(\frac{vw}{A}\right)(t)}$$

where $P(d)$ is the probability of detection, v is the velocity of the searching platform, w is the search width of the platform, A is the area to be searched, and t is the number of time periods spent searching (Chung 2011). The probability of detection therefore grows with each additional time period spent searching, but depending on the size of the area searched, it may take many, many time periods to effectively search an area. This algorithm is simple but effective for modeling a search pattern over a static area (Chung 2011).

2. Platforms Inventory

For the purposes of this research, only three types of platforms are being used in the ASW effort: ships, submarines, and aircraft. The ships considered here are guided missile destroyers (DDG), the submarines are of the fast attack variety (SSN), and the aircraft are Boeing P-8 Poseidons (P-8). Each platform has the following characteristics when used in the model (see Table 1).

Table 1. Platform Search Specifications

Platform	Velocity (KT)	Sweep Width (NM)
DDG	15	2
SSN	10	2.5
P-8	240	0.5

The P-8s effectiveness comes from more than its sweep width alone. The P-8 also employs sono-buoys. One P-8 is capable of monitoring 64 sono-buoys at any given time, but no more than 100 sono-buoys can be employed in one mission area. Because of the sono-buoys, Kim utilizes a different detection equation for the P-8 Platforms. This is because sono-buoys can be laid out in a specific geometric pattern that is static, even while the P-8 itself is in motion. Their probability of detection is based on the following equation:

$$P(d) = \frac{nw_p}{A}$$

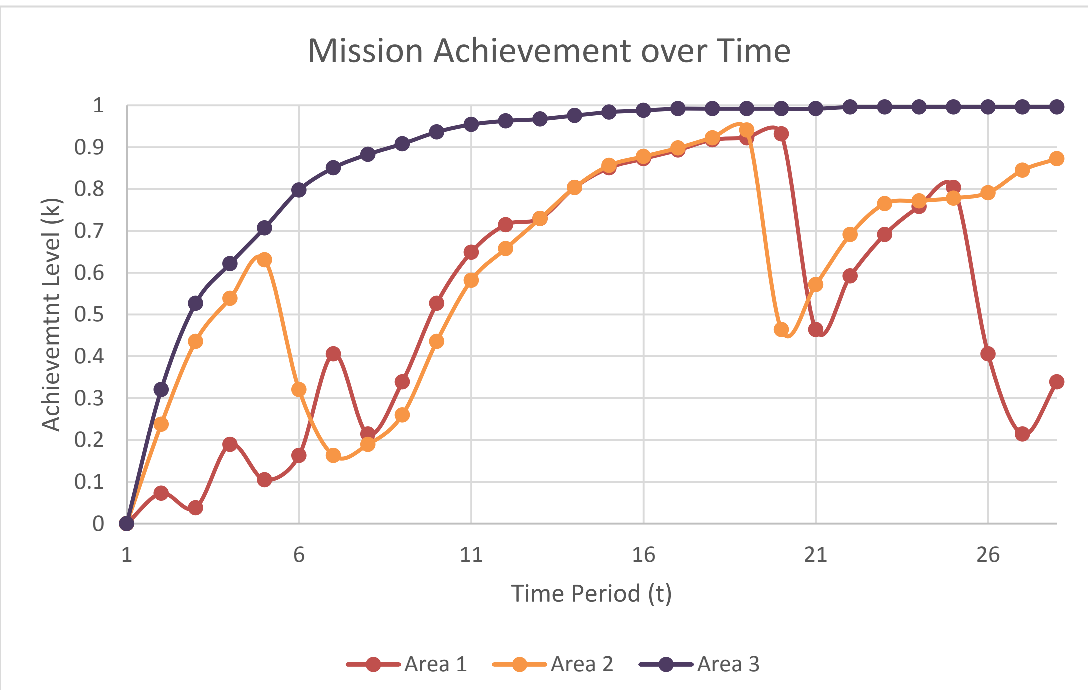
where n is the number of sono-buoys, w_p is the sweep width of each buoy, and A is the overall search area (Kim 2017). This gives the P-8 considerably more power in searching for submarines when utilizing the buoys, as opposed to when there are no buoys.

The velocity and sweep width considerations are assumptions of reasonable values for these platforms. As with most of the data used in this model, it can all be updated by the planner in order to be much more accurate for a specific scenario.

III. MODEL AND ALGORITHMS

A. ACHIEVEMENT

The most important concept for understanding this model's function has been named *achievement*. Achievement is a measure of how well a mission's requirements have been completed, on a scale of zero to one. In the ASW portion of NOP, this number indicates the probability of cleared water that has been achieved. An achievement value close to zero would suggest that the area still has large potential for submarine threats, whereas achievement closer to one indicates that the entire area is almost certainly clear of submarines. Figure 1 is a graph of how this may appear.



In this example, there are three mission areas. Increases in achievement indicate that a particular set of platforms was assigned during that time period, whereas decreases show there were no platforms assigned, resulting in decay of achievement. Note that mission area 3 would have a combination assigned every time period, as achievement never decreases.

Figure 1. Example of Achievement over Time

B. NOP-ASW

The model used to solve this problem is detailed in the following sections. This model has been largely adapted from the prior work discussed in Chapter II (Molina 2016, Kim 2017).

1. Sets and Indices [Cardinality]

$p \in P$	Platforms [3]
$t \in T$	Time Periods [28]
$m \in M$	Mission Areas [3]
$c \in C$	Platform Combinations [10]
$p \in CP_c$	Platform types p in combination c
$p \in TP_t$	Platforms p available in period t
$(k', c, m, t) \in PRE_k$	Tuples that determine achievement level k

2. Derived Sets

$c \in TC_t$	Combinations c available in period t $(t, c) \in TC \iff (t, p) \in TP \ \forall \ p \in c$

3. Data

a_k	Value of Achievement [0.0-1.0]
$val_{t,m}$	Priority value of mission m in period t [1-5]
$thresh_m$	P_d threshold for accomplishing mission m [0.0-1.0]
$avail_{p,t}$	Number of platforms of type p available in period t
$req_{p,c}$	Number of platforms of type p required in combination c

4. Decision Variables [Domain]

$ASGND_{t,m,p}$	Number of platforms of type p assigned to mission m at period t [Integer]
$CACT_{t,m,c}$	Combination of platforms c is chosen for mission m in period t [Binary]
$KACH_{t,m,k}$	Achievement level k is feasible at period t for mission m [Binary]
$KCACT_{k,c,t,m}$	Mission m achievement level is at or above level k in t and combination c is applied to m in t [Binary]
$DONE_{t,m}$	Mission m achievement level meets or exceeds its threshold in period t [Binary]
$MACT_{t,m}$	Mission m has combination assigned at time t [Binary]

5. Formulation

$$\max \quad \sum_{t,m} val_{t,m} \left(DONE_{t,m} + \sum_{k} (.1) a_k KACH_{t,m,k} + (.01) MACT_{t,m} \right) \qquad \text{(T0)}$$

$$s.t. \quad KCACT_{t,m,k,c} \le KACH_{t,m,k} \qquad \forall k,t,m,c \in TC_t \qquad \text{(T1)}$$

$$KACH_{t,m,k} \le \sum_{(k',c):(k',c,m,t)\in PRE_k} KCACT_{t-1,m,k',c} \qquad \forall k,t>1,m \qquad \text{(T2)}$$

$$\sum_{k} KACH_{t,m,k} = 1 \qquad \forall t,m \qquad \text{(T3)}$$

$$KCACT_{t,c,m,k} \le CACT_{t,m,c} \qquad \forall k,t,m,c \in TC_t \qquad \text{(T4)}$$

$$req_{p,c} CACT_{t,m,c} \le ASGND_{t,m,p} \qquad \forall t,m,c \in TC_t,\ p \in TP_t \qquad \text{(T5)}$$

$$\sum_{c\in TC_t} CACT_{t,m,c} \le 1 \qquad \forall t,m \qquad \text{(T6)}$$

$$MACT_{t,m} \le \sum_{c\in TC_t} CACT_{t,m,c} \qquad \forall t,m \qquad \text{(T7)}$$

$$\sum_{m} ASGND_{t,m,p} \le avail_{p,t} \qquad \forall t,\ p \in TP_t \qquad \text{(T8)}$$

$$ASGND_{t,m,p} \le req_{p,c} \sum_{c\in TC_t} CACT_{t,m,c} \qquad \forall t,m,\ p \in TP_t \qquad \text{(T9)}$$

$$\sum_{(k',c):(k',c,m)\in PRE_k} KCACT_{t-1,m,k',c} \le 1 \qquad \forall k,t,m \qquad \text{(T10)}$$

$$DONE_{t,m} \le \sum_{k:a_k \ge thresh_m} KACH_{t,m,k} \qquad \forall t,m \qquad \text{(T11)}$$

$$DONE_{t,m} \in \{0,1\} \qquad \forall t,m$$

$$MACT_{t,m} \in \{0,1\} \qquad \forall t,m$$

$$KCAC_{t,k,m,c} \in \{0,1\} \qquad \forall t,m,k,c \in TC_t$$

$$KACH_{t,m,k} \in \{0,1\} \qquad \forall t,m,k$$

$$CACT_{t,m,c} \in \{0,1\} \qquad \forall t,m,c \in TC_t$$

$$ASGND_{t,m,p} \ge 0, Integer \qquad \forall t,m,\ p \in TP_t$$

6. Discussion

The formulation in Section 5 focuses on *achievement*, a value that represents the ASW effort that has been applied to a particular ASW mission area, *m*, up to a particular time period, *t*. The objective function (T0) calculates the total achievement over all mission areas, over the time horizon of the model, and includes for each mission and each time period a reward for having reached a high enough level of achievement for the mission to be considered complete in that period, a smaller reward for the actual achievement level in that period, and, finally, a very small reward for having an active combination of platforms performing ASW in the area in that period.

As described by Kim (2017), each constraint (T1)-(T2) defines the values of the achievement for a mission in each period based on the achievement in the prior time period and the combinations that were active in the previous period, and (T3) ensures that exactly one achievement level is selected from those that are attainable. Each constraint (T4) and (T5) requires that exactly one combination is working in an area in any period, and that all appropriate platforms are assigned that make up that combination. Each constraint (T6) and (T7) control whether a mission is active based on whether there is (exactly) one combination active in the mission area. Constraint (T9) ensures that platforms are not over-assigned to combinations. Constraint (T10) limits the model to choosing exactly one precursor achievement level to justify the current achievement level. Finally, each constraint (T11) prevents the model from claiming a mission has been completed unless it has reached a level of achievement that has been marked as being at or above the achievement threshold for that mission.

The update this thesis makes is contained primarily within (T2), with particular emphasis on how the PRE_k set is calculated. We have endowed the transitions between achievement levels with a time-dependent structure, which means that the same combination of platforms working in the same mission area can attain a different achievement level based on *the time period in which they operate*. If no combination of platforms is assigned to an area, then the achievement of that area will decay. This decay represents the uncertainty that comes with no longer searching an area, as enemy submarines may have reentered the area undetected.

C. PROGRAM AND CODE CONSIDERATIONS

This model formulation is completed using Pyomo, "a Python-based open-source software package that supports a diverse set of optimization capabilities for formulating, solving, and analyzing optimization models" (Hart et al. 2011). As it is based in Python, the code is written in Python version 3.5, and the solver used is IBM's CPLEX. The previous code has been worked on by Carlyle, Molina, and Kim.

D. CALCULATED DATA

This model uses two different calculated bonuses. The first of these bonuses is the combination bonus that was developed for the most recent iteration of NOP-ASW. This is the synergistic effect that was briefly discussed in Chapter II. The synergistic effect is another probability of detection equation.

$$P_c(d) = 1 - e^{-(1+\beta_c)t\sum_p P_p(d)}$$

In this equation, $P_c(d)$ is the combination's probability of detection. β_c is the bonus coefficient applied to the sweep widths of the platforms in the selected combination. The summation is the $P_p(d)$ from the individual platforms in the combination itself. Kim notes that "this added bonus to sweep width encourages the model to add multiple synergistic platforms to mission areas to achieve and maintain the threshold level quicker, and can discourage the use of two platforms that might impeded each other's progress" (Kim 2017). The combinations can be found in Table 2. The bonus values are located in the far right of the table.

Table 2. Combinations

Combination	DDG	SSN	P-8	Bonus
c0	0	0	0	0
c1	1	0	0	0
c2	0	1	0	0
c3	0	0	1	0
c4	1	1	0	0
c5	1	0	1	10
c6	0	1	1	0
c7	1	1	1	10
c8	1	1	2	15
c9	2	0	0	10
c10	2	0	1	15
c11	1	0	2	15
c12	2	0	2	20

The newest addition to the model is a boost to performance for each combination based on time period. This addition attempts to bring some time-based influence into the model, as mentioned in the motivation section from Chapter I. Because the time frame of the model is incredibly short, at no more than 7 days with each run of the model, water temperature change affecting SONAR performance is limited. Still, the idea is to mimic changing conditions throughout each 24-hour period. With each period accounting for six hours, this means that a repetitive pattern would develop every four time periods. The environmental effects would most likely effect each platform differently—SONAR remaining unaffected by lighting, while visual aids might be influenced.

Table 3 provides a set of values, $c\_bonus_{c,t}$, for each combination in each time period. From these values we calculate the $ct\_boost_{c,t}$ values using the formula:

$$ct\_boost_{c,t} = \sum_{p \in c} v_p w_p (1 + c\_bonus_{c,t})$$

where v_p is the platform velocity, and w_p is the platform sweep width.

Despite each platform having its own properties, the combinations are still useful here, and their mutual effects are blended together by using a simple averaging of the platform values in Table 3. The current values are speculative and not based on empirical experience. The idea is to induce a rotating effect based on modular patterns to simulate how a day might affect detection abilities of different platforms without disrupting the model's function. In turn, these values affect the updates of probability of detection and therefore achievement in the model

Once the *ct_boost* values have been calculated, they are used to help determine the next level of achievement, by finding the discrete level that most closely corresponds to the following calculation:

$$ach' = \begin{cases} ach/2 & \text{if } c = c0 \\ 1-\left(1-ach\right)e^{-6ct\_boost_{c,t}/3600} & \text{if } c \neq c0 \end{cases}$$

where *ach* is the current value of the achievement level of a given mission in period *t*, and *ach'* is the resulting level after using combination *c* in period *t*. The first portion explains how the decay works. If the combination that contains no platforms ("*c*0," in Table 2) is assigned, then the achievement value for that period will be half of what it was previously. If a nonempty combination is used, its *ct_boost* value is used to reduce the difference between perfect achievement (1.0) and the current achievement value, *v*, at an exponential rate, corrected for the six-hour duration of one time period. This calculation allows the model to find the next level of achievement for one specific area, for any possible combination assigned to that area, in each time period. In this way we can create the transitions in the set PRE_{k}.

Table 3. Combination-Time Boost Values

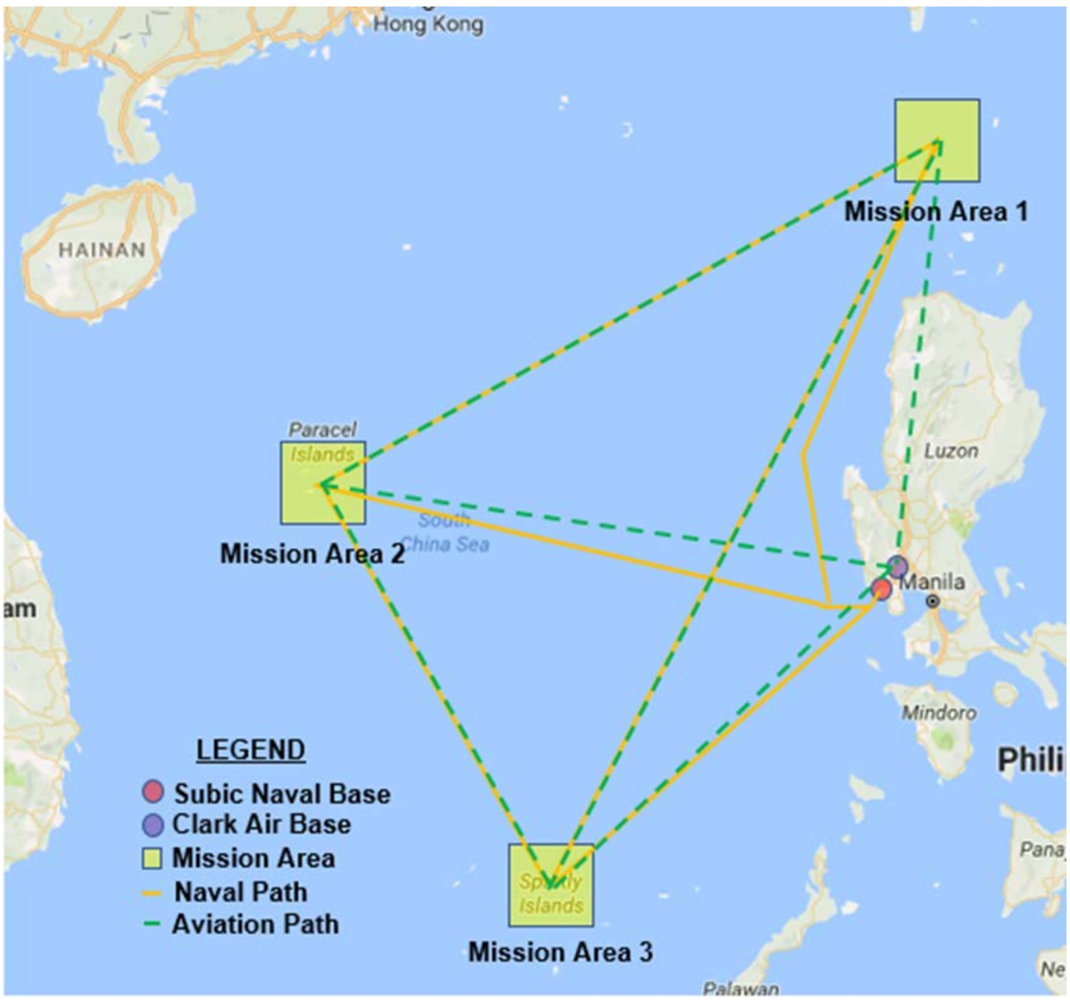
Time Period	Combination												
	c0	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12
1	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
2	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
3	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
4	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9
5	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
6	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
7	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
8	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9
9	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
10	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
11	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
12	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9
13	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
14	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
15	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
16	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9
17	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
18	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
19	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
20	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9
21	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
22	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
23	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
24	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9
25	1	0.9	0.9	0.8	0.9	0.85	0.85	0.85	0.85	0.9	0.85	0.85	0.85
26	1	1	1	1.2	1	1.1	1.1	1.1	1.1	1	1.1	1.1	1.1
27	1	1.1	1.1	1.2	1.1	1.15	1.15	1.15	1.15	1.1	1.15	1.15	1.15
28	1	1	1	0.8	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9

E. TESTING SCENARIO

In order to test our additions to the model, the following scenario was developed. It is strictly hypothetical. All data in the following sections are rough estimates. The purpose of this section is to test the model, as opposed to providing an incredibly accurate depiction of a real-world scenario.

1. Location

The scenario will involve three mission areas around the South China Sea that are of interest: the Luzon Strait, the Paracel Islands, and the Spratly islands. The base of operations for naval assets (DDGs and SSNs) will be Subic Bay, to the west of Manila. P-8s will be flying from Clark Airbase. The two are separated only by a small distance, but this has been noted for distance considerations for the purpose of estimating transit times, as seen in Figure 2.



Original map obtained from Google Maps. All marks created by the author.

Figure 2. Scenario Area of Operations

Mission Area 1 is roughly based in the Luzon Strait. Mission Area 2 is centered on the Paracel Islands. Finally, Mission Area 3 is based on the Spratly Islands. The solid lines represent the routes that DDGs and SSNs will utilize for transit from Subic Naval Base to Mission Areas, or between Mission Areas. The dotted line represents the routes that P-8s will use between Clark Air Base to Mission Areas, or between Mission Areas. We provide travel distances for both modes of travel (air and sea) in Tables 4 and 5, for future work, but we do not use these values directly in the current version of the model.

Table 4. Distances from Base to Mission Areas (in NM)

Platform	**DDG**	**SSN**	**P-8**
Mission Area	**Base**	**Base**	**Base**
1	391	391	308
2	443	443	456
3	348	348	400

Table 5. Distances between Mission Areas (in NM)

Platform	**DDG**	**DDG**	**DDG**	**SSN**	**SSN**	**SSN**	**P-8**	**P-8**	**P-8**
Mission Area	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
1	-	543	626	-	543	626	-	543	626
2	543	-	352	543	-	352	543	-	352
3	626	352	-	626	352	-	626	352	-

2. Platforms

Table 6 shows platform availability. In this scenario, more DDGs enter after the first two days, and more still after the first five days. The same goes for SSNs, although only one more submarine will be added throughout the remainder of the operation. The number of P-8s available does not change in this scenario, as the aircraft available at the beginning of the operation are the only ones that will be there in time for the completion of the first seven days.

Table 6. Platform Availability

Time Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14
DDG	2	2	2	2	2	2	2	2	3	3	3	3	3	3
SSN	1	1	1	1	1	1	1	1	2	2	2	2	2	2
P-8	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Time Period	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
DDG	3	3	3	3	3	3	4	4	4	4	4	4	4	4
SSN	2	2	2	2	2	2	2	2	2	2	2	2	2	2
P-8	2	2	2	2	2	2	2	2	2	2	2	2	2	2

3. Mission Area Priority

The next item of interest is how to prioritize mission areas. This ranges on a simple scale of one to four, with one as being least important and four being most important. These values change at different time periods during the mission. For this scenario, the Straits of Luzon are prioritized first (Mission Area 1). After that, Mission Area 3 gains importance. It is not until the second half of the time horizon that Mission Area 2 is stressed over the other two. This relationship is depicted in Table 7.

Table 7. Mission Importance by Time Period

Time Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Area 1	4	4	4	4	4	4	4	4	4	3	3	3	3	3
Area 2	1	1	1	1	2	2	2	2	2	2	2	2	3	3
Area 3	2	2	2	3	3	3	3	4	4	4	4	4	4	4
Time Period	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Area 1	3	3	3	3	3	2	2	2	2	1	1	1	1	1
Area 2	3	4	4	4	4	4	4	4	4	4	4	4	4	4
Area 3	3	3	3	2	2	2	2	2	2	2	2	2	2	2

IV. RESULTS

A. SCENARIO OUTCOMES

The goal of the testing scenario is to examine the effects of the combination-time boost. Three different scenarios are performed on a Dell Inspiron 3558 with an Intel Core i3-5015 2.1GHz processor. Each run takes approximately two-to-three hours, solving with CPLEX (IBM 2017). The first run (NONE) employs no effects, to serve as a base case. The second run (LIGHT) employs light time-combination boost effects. Finally, a third run (HEAVY) employs heavy effects.

Figure 3 displays the achievement of the base case. Mission areas 1 and 3 are improved at nearly identical rates. Area 2 begins improving in time period eight, after stuttering below 0.1 for the first seven periods.

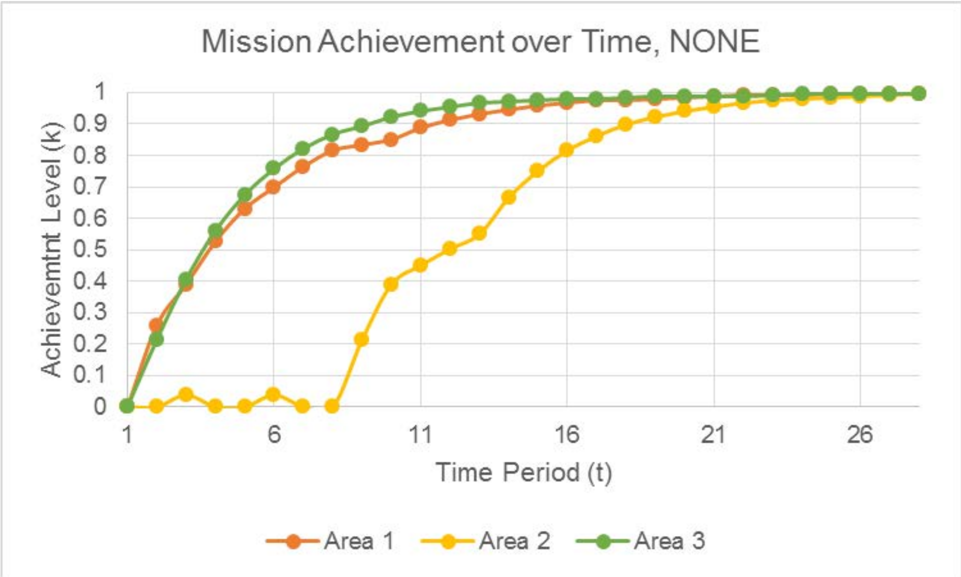


Figure 3. NONE: Scenario with No Time-Based Variation in Performance

Table 8 gives the raw breakdown of the numbers for Figure 3, as well as the combinations that were assigned in each time period to achieve them. The small wave effect seen in mission area 2 is explained here by the lack of platforms assigned early on (denoted by c0).

Table 8. NONE Tabular Results

Time Period	Mission Areas					
	Area 1		Area 2		Area 3	
	Combo	Achieve	Combo	Achieve	Combo	Achieve
1	c7	0	c0	0	c5	0
2	c3	0.260	c1	0	c7	0.214
3	c5	0.390	c0	0.038	c7	0.406
4	c5	0.527	c0	0	c7	0.561
5	c3	0.631	c1	0	c7	0.675
6	c5	0.699	c0	0.038	c7	0.758
7	c5	0.765	c0	0	c7	0.822
8	c4	0.816	c5	0	c3	0.867
9	c4	0.834	c5	0.214	c7	0.893
10	c7	0.851	c4	0.390	c5	0.922
11	c10	0.888	c4	0.450	c6	0.941
12	c5	0.913	c4	0.503	c7	0.954
13	c3	0.932	c7	0.550	c4	0.967
14	c5	0.945	c7	0.666	c4	0.971
15	c3	0.959	c7	0.751	c4	0.976
16	c5	0.967	c7	0.816	c2	0.980
17	c1	0.976	c7	0.862	c5	0.980
18	c4	0.976	c5	0.898	c3	0.984
19	c3	0.980	c10	0.922	c1	0.988
20	c7	0.984	c6	0.941	c9	0.988
21	c5	0.988	c7	0.954	c9	0.988
22	c1	0.992	c10	0.967	c7	0.988
23	c9	0.992	c6	0.976	c7	0.992
24	c7	0.992	c7	0.980	c9	0.996
25	c7	0.996	c7	0.984	c9	0.996
26	c6	0.996	c7	0.988	c1	0.996
27	c7	0.996	c7	0.992	c9	0.996
28	c0	0.996	c0	0.996	c0	0.996

Figure 4 displays the achievement results of the LIGHT effects graphically. There is small influence with the combination-time boost values from Table 3, which differ from NONE, because NONE uses a value of one for every single combination-time boost. We can see that there is a small degree of separation between the achievement levels of mission areas 1 and 2, while mission area 3's achievement does not significantly increase until after time period eight. Overall, this graph is very similar to Figure 3; however, there are two key differences. First, at no point does achievement ever dip back to zero, even if it is very low for mission area 3. Secondly, LIGHT effects cause a switch in prioritization between areas 2 and 3.

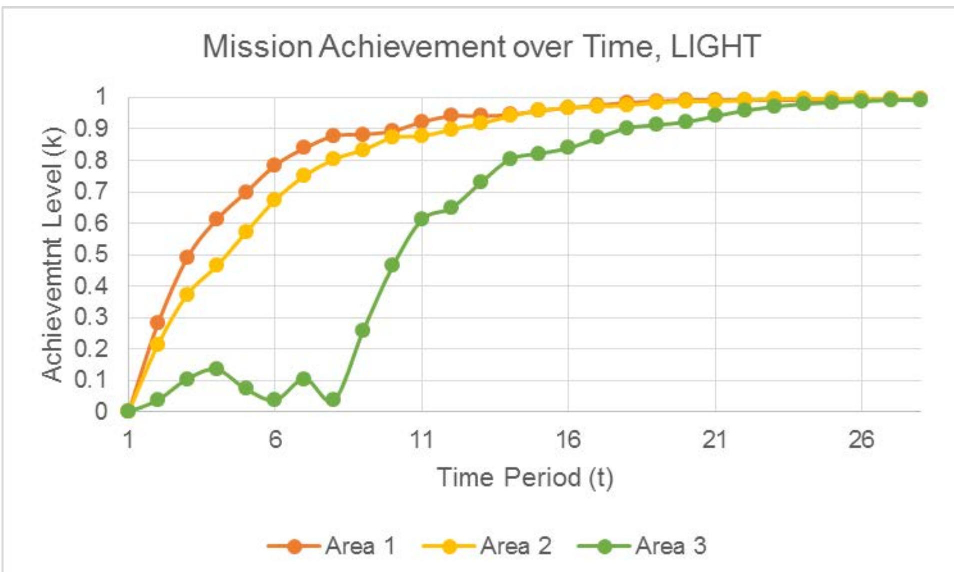


Figure 4. LIGHT: Scenario with Significant Time-Based Variation in Performance

Table 9 outlines the details of how Figure 4 is created. It provides more insight than the graph alone because the combinations assigned in each period are clearly displayed for each period. The explanation for never reaching zero is seen here in area 3. Even though there are no platforms assigned three times, there is enough achievement to avoid a decay back to zero in this area.

Table 9. LIGHT Tabular Results

Time Period	Mission Areas					
	Area 1		Area 2		Area 3	
	Combo	Achieve	Combo	Achieve	Combo	Achieve
1	c7	0	c3	0	c1	0
2	c7	0.281	c3	0.214	c1	0.038
3	c7	0.490	c3	0.374	c1	0.105
4	c7	0.612	c5	0.464	c0	0.135
5	c7	0.699	c5	0.571	c0	0.073
6	c5	0.785	c6	0.675	c1	0.038
7	c7	0.839	c5	0.751	c0	0.105
8	c1	0.878	c3	0.804	c7	0.038
9	c4	0.883	c5	0.834	c7	0.260
10	c7	0.893	c1	0.872	c7	0.464
11	c7	0.922	c3	0.878	c4	0.612
12	c2	0.941	c5	0.898	c7	0.649
13	c1	0.941	c7	0.918	c7	0.729
14	c5	0.945	c7	0.941	c4	0.804
15	c7	0.959	c5	0.959	c4	0.822
16	c7	0.967	c4	0.967	c5	0.839
17	c7	0.976	c4	0.971	c5	0.872
18	c3	0.984	c7	0.976	c4	0.903
19	c5	0.988	c6	0.984	c4	0.913
20	c1	0.992	c1	0.988	c8	0.922
21	c9	0.992	c6	0.988	c7	0.941
22	c1	0.992	c7	0.992	c7	0.959
23	c1	0.992	c1	0.996	c8	0.971
24	c1	0.992	c1	0.996	c10	0.980
25	c7	0.992	c1	0.996	c6	0.984
26	c1	0.996	c6	0.996	c7	0.988
27	c11	0.996	c9	0.996	c4	0.992
28	c0	0.996	c0	0.996	c0	0.992

Following this, HEAVY effects are examined. HEAVY effects are amplified temporal effects, similar to those found in Table 3. Table 10 shows the specific changes made to these initial LIGHT effect values. There are now periods with large bonuses and others with smaller ones that will make searching much less beneficial.

Table 10. HEAVY Combination-Time Boost Values

Time Period	Combination												
	c0	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12
1	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
2	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
3	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
4	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65
5	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
6	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
7	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
8	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65
9	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
10	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
11	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
12	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65
13	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
14	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
15	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
16	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65
17	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
18	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
19	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
20	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65
21	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
22	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
23	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
24	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65
25	1	0.4	0.4	0.3	0.4	0.35	0.35	0.35	0.35	0.4	0.35	0.35	0.35
26	1	1	1	1.7	1	1.35	1.35	1.35	1.35	1	1.35	1.35	1.35
27	1	1.6	1.6	1.7	1.6	1.65	1.65	1.65	1.65	1.6	1.65	1.65	1.65
28	1	1	1	0.3	1	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65

Figure 5 shows that the HEAVY time-combination effects result in a completely different graph than in Figure 3 and 4. Area 1 now lags behind the other two, but quickly catches up immediately after time period five. The increases in achievement also appear to be much more step-wise oriented, jumping up quickly in places but then only making small increases in each time period thereafter.

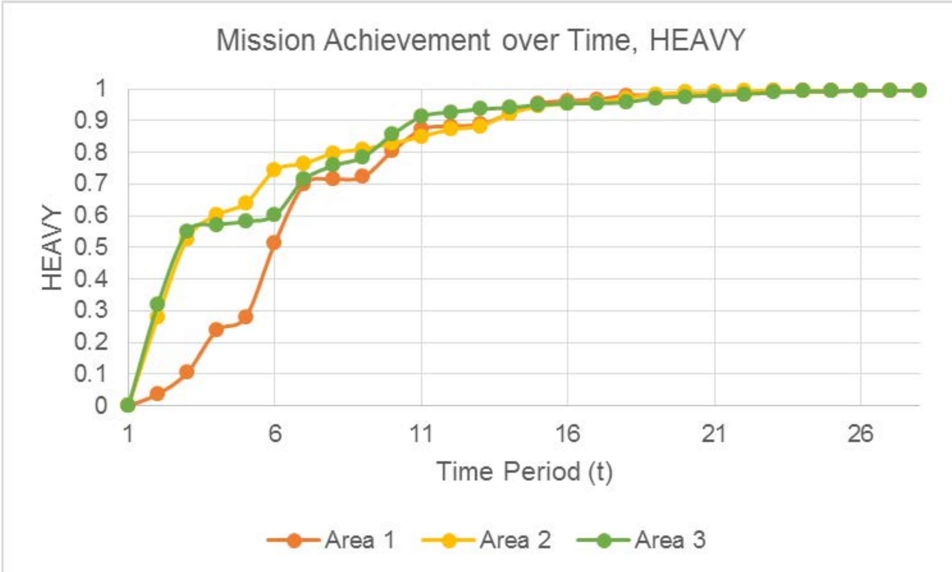


Figure 5. HEAVY: Scenario with Significant Time-Based Variation in Performance

Table 11 gives the details for Figure 5. The combinations chosen for this run are different than the light combinations in nearly every time period, particularly in the beginning of each run. The different time effect values force a completely different selection of platform resources, but also result in more gradual increases in achievement overall.

Table 11. HEAVY Tabular Results

Time Period	Mission Areas					
	Area 1		Area 2		Area 3	
	Combo	Achieve	Combo	Achieve	Combo	Achieve
1	c1	0	c3	0	c7	0
2	c2	0.038	c5	0.281	c5	0.320
3	c5	0.105	c5	0.527	c2	0.550
4	c3	0.237	c7	0.602	c1	0.571
5	c7	0.281	c3	0.640	c1	0.582
6	c7	0.515	c1	0.744	c3	0.602
7	c1	0.699	c6	0.765	c5	0.714
8	c1	0.714	c3	0.797	c7	0.758
9	c3	0.722	c4	0.810	c7	0.785
10	c5	0.804	c4	0.828	c7	0.856
11	c4	0.872	c6	0.851	c10	0.913
12	c4	0.883	c5	0.872	c7	0.927
13	c3	0.888	c7	0.883	c1	0.936
14	c7	0.922	c3	0.922	c4	0.941
15	c7	0.954	c6	0.945	c9	0.950
16	c7	0.963	c6	0.954	c9	0.954
17	c7	0.967	c3	0.959	c1	0.954
18	c9	0.980	c7	0.971	c6	0.959
19	c5	0.980	c6	0.984	c4	0.971
20	c7	0.984	c1	0.988	c7	0.976
21	c9	0.984	c10	0.988	c3	0.980
22	c7	0.984	c10	0.992	c4	0.984
23	c4	0.992	c1	0.996	c7	0.988
24	c2	0.992	c10	0.996	c1	0.992
25	c10	0.992	c1	0.996	c5	0.992
26	c1	0.996	c4	0.996	c11	0.996
27	c6	0.996	c1	0.996	c9	0.996
28	c0	0.996	c0	0.996	c0	0.996

B. SCENARIO ANALYSIS

The three different applications of the time-combination boost yield differences in their outputs. At the very least, it is clear that the addition of time-dependent performance data to the model does influence how platforms will be allocated to complete the mission. These differences provide some insights into how the module takes its inputs and returns results.

Each of the three runs optimizes mission areas differently than the other two. The “NONE” run follows the mission priorities established in Table 7 exactly. Mission areas 1 and 3 are prioritized early on, and then mission area 2 follows. The “LIGHT” run appears almost identical to the “NONE” run, but there is a clear switch between mission area 2 and 3. Even more interesting is that the “HEAVY” results prioritize mission area 1 – supposedly the most important mission area at the beginning – *less* than the other two areas. By time period 11 this difference is negligible, but that is nearly half of the time horizon. This shows that the availability of a boost may take priority over mission area importance, if it helps greater achievement overall.

Platforms chosen are also of great importance. We observe that combination bonuses are influential. Whenever a mission area needs to gain more achievement quickly, the more potent combinations, such as c7 (containing one DDG, one SSN, and one P-8) or even c8 (similar to c7 but with two P-8s) are chosen. These provide bonuses that are more helpful in boosting achievement. They are particularly useful when paired with a more beneficial combination-time boost period, allowing for a double boost in the time period effects.

Clearly the higher the boost value is, the more useful that combination will be. This explains why c7 and c8 appear frequently. The P-8 is a constraining resource, as it is in high demand for both of these combinations, and only two P-8s are available for the entirety of this scenario. The DDG is the most readily available platform throughout the scenario, and it is often used as a placeholder to keep the achievement in an area from decaying. The c1 combination is therefore one of the most frequently used in the solutions, as there are as many as four DDGs ready starting in time period 21, and continuing to the model's end. With at least one DDG for each area, decay is no longer an issue in any of the scenario runs, and this can be seen from the results in all three figures.

Towards the end of the scenario, when there are more platforms available, it is much easier to push the achievement up in all three mission areas, in all three runs. By time period 16, in all three figures, each mission area has reached an achievement level of at least 80 percent. This would allow a decision maker who wanted at least 80 percent confidence in mission completion to consider the ASW mission accomplished. The next phase of the operation could then begin.

Of course, commanders may prefer much higher levels of achievement for each area. This solution is not necessarily optimal, but it is a good starting point for the planning process. Clearly, more platforms early on in a mission will make it easier to boost achievement to the required levels. This is not the only option, however. Taking advantage of the larger improvements in performance can be equivalent to having more capable platforms or a larger quantity of platforms available to accomplish the mission. This result has better face value for warfighters, and is especially useful in the beginning of the scenario, when platforms are scarce.

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V. FUTURE WORK AND CONCLUSIONS

A. MODEL IMPROVEMENTS

1. Data

The importance of data pedigree, or model input, cannot be overstated; as such, gathering the most accurate information possible and applying it to this model ensures the veracity of the results and insights. For this thesis, our main goal is to ensure that such data can be incorporated, as explained in Chapter IV. One of the most influential aspects of meteorological and oceanic data is that it varies greatly based on different locations, and will have to be adapted accordingly for each new location that is tested. Existing systems potentially could be used, especially those with automation. Importing data from weather, geographic, and bathymetric information systems could populate future iterations of the model parameters.

2. Logistics

While this thesis does consider how many platforms are available for ASW, it does not effectively track each individual platform. To increase the model's effectiveness, this problem needs to be specifically addressed. One way to possibly do this is to make every platform unique, even if it has the same performance specifications. For instance, two P-8s can be treated as separate entities and tracked as such, even though their speed, on-station time, and sweep widths are identical. This would increase the size of the resulting model, and most likely increase run time considerably with new constraints.

In spite of the complications, the benefits would be considerable. The most useful application of individually tracking platforms would be for cost considerations, most notably in terms of fuel. Additional constraints could then be added to the model to try to control how much money is spent on an operation—another important consideration in the face of scarcity and something that every operational planner is interested in.

3. Time Considerations

The current NOP-ASW model handles 28 time periods efficiently. For this thesis, each accounted for six hours, giving seven-days' worth of planning resolution. For meteorological effects, it may be desirable to reduce the amount of time covered by each period. For instance, the time periods may need to be in three-hour or four-hour, or possibly even one-hour increments for a very specific model. 28 one-hour periods is not very useful for ASW, however. As such, expanding the number of time periods the model can handle efficiently could increase the model's resolution on a short-time scale, or even a long-time scale if four weeks were to be examined, as opposed to only one week.

B. CONCLUSION

NOP-ASW is now better ready to provide more accurate information to operational planners. The addition of a time and platform combination boost improves the resolution of the model by allowing for time-dependent conditions to affect the usefulness of combinations. Some time periods are seen as better or worse based upon them, and the decision maker can choose which assets to use accordingly. More efficient use of the limited available platforms increases the amount of time saved. Overall, this helps deal with the problem of scarcity and improves operational effectiveness in the long run.

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