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Logistic Model for Distributed Lethality

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Abstract

In this study we evaluate a Gas Station (GS) resupply method for Distributed Lethality (DL) scenarios, and obtain some insights about the interplay among spatial, temporal and capacity parameters related to this mode of resupply. Our motivating questions are: Where should the GS be located in the communication zone? How does the capacity of the GS affect the combat readiness of the Adaptive Force Packages (AFPs)? What is the impact of the number of AFPs served by the GS on their time off station? Shall the Gas Station be a shuttle (i.e., a ship that goes back and forth to port to replenish) or a delivery ship that is being resupplied by a separate shuttle? Our results do not necessarily establish a specific blueprint for logistic planning, but rather point out at key factors and considerations that affect logistic responsiveness in a DL setting.

Keywords: distributed lethality, Gas Station resupply, logistics, stochastic theory

1 Introduction

Changes in the global political and strategic environments have resulted in some modifications in US defense strategies. In particular, these changes have led to the emergence of a new naval operational concept: *Distributed Lethality* (DL) [1]. From the force-employment point of view, the DL concept calls for fragmenting the traditional naval battle/strike groups into smaller, more agile and lethal Surface Action Groups (SAGs) or Adaptive Force Packages (AFP) comprising a small number of surface vessels. The SAGs and AFPs operate in a distributed manner over a relatively large area of operations. Tactical implications of the DL concept have been studied by the Commander, Naval Surface Forces, Surface and Mine War-fighting Development Center Command, and Naval War College (NWC). The studies are based on a Distributed Lethality Task Force and war games conducted at the NWC. The DL concept brings about a serious logistic challenge: how to effectively and efficiently satisfy logistic demands at different times and many locations dispersed over a large area. A fundamental dilemma in this context is choosing between two logistic principles: flexibility, derived from concentrating resources, and attainability obtained from distributing them [2]. Concentrating resources at the operational level – either on land or afloat – enhances logistic flexibility by directing resources only to areas of need. This principle has two important benefits. First, similarly to the inventory-pooling principle in commercial supply chain management [3], operational flexibility saves resources and enhances efficiency. Second, holding resources in a central location at the back of the AOI minimizes the logistic tail of the forward deployed AFPs, and thus reduces the AFPs' signature as targets, and make them more tactically agile. Third, keeping the inventories afloat in the relatively safe communication zone – beyond the threat area – enhances the survivability of the supplies. However, these positive features of concentrating logistics in the communication zone come at the cost of timeliness; the lead time required for shipping supplies from a central theater source at the back of the theater to the DL tactical units may be long.

SAGs or AFPs are small and agile; they may find it difficult to "drag" a logistic tail and protect it. Thus, the DL concept implies the transition from the traditional *shuttle ship – delivery ship* setup, typical to CVN battle groups, where a delivery ship is attached to the battle group and a shuttle ship resupplies it with resources pulled from the theater logistic base, to a new approach that we denote the *Gas Station* setup. In the *Gas Station* approach, the AFPs need to travel back from the combat zone to meet a resupply source – e.g., an AOE or T-AO ship – in the communication zone.

The objective of this study is to evaluate the Gas Station resupply method for DL scenarios and obtain some insights regarding the interplay among spatial, temporal and capacity parameters related to this mode of resupply. In the rest of this report we use the terms *AFP* and *AOE* for the combat and resupply ships, respectively. We develop an analytic model for studying the relations among the aforementioned parameters and answering questions such as: What is the effect of the ratio between supply ship capacity and combat ship on-board capacity on logistic responsiveness? How does the location of the Gas Station affect that responsiveness? What is the impact of the number of AFPs served by the Gas Station? Should the Gas Station be a shuttle (i.e., a ship that goes back and forth to port to replenish) or a delivery ship that is being resupplied by a separate shuttle?

The model and the answers we provide to these questions are general; they may apply to any type of logistic supplies. The observations and insights obtained from the model are universal. They do not establish a specific blueprint for logistic planning, but rather highlights key factors and considerations that affect logistic responsiveness in a DL setting.

2 Scenarios

The general logistic chain in the DL setting is depicted in Figure 1. It comprises three types of entities: (1) a *port*, which is the theater logistic base and the source of the logistic chain, (2) an AOE (or any other resupply ship) that operates as a Gas Station, and (3) AFPs, which are the consumers to be resupplied by the AOE. The AOE is located in the communication zone at the rear of the combat zone. We call the location of the AOE – the *resupply point*. AFPs arrive at the resupply point to be replenished by the AOE. The location of the resupply point is affected by the layout of the DL posture, distance to the port and the vulnerability of the AOE to adversary's hostile actions. The AFPs are scattered over a large area in the theater of operations, each covering a " tactical station", or in short – *station*, which is a local area of interest (AOI). The consumption rate by the various AFPs may fluctuate based on the activity of the AFP in the AOI and the type of supply. We consider two basic resupply scenarios: (1) a single mobile AOE, and (2) two AOEs – a *delivery* ship and a *shuttle* that replenishes the delivery ship. In the first scenario, once its inventory is depleted, the AOE leaves the resupply point and heads to the port to reload supplies. This is the case where the AOE is a mobile Gas Station. During the trip of the AOE to port, the resupply point remains vacant and AFPs needing supply have to wait for the AOE's return.[1]In the second scenario, the gas station is continuously operational; one AOE – the delivery ship – is essentially static and is replenished by the shuttle ship. Obviously, scenario 2 provides better logistic responsiveness than scenario 1, but the questions is: by how much?

We assume that the durations of resupply events at the port and resupply point, as well as vessels' velocities, are deterministic and known. In the next section we start off with a simple situation where there is only one AFP in the theater. In Section 4 we consider the more realistic case where multiple AFPs operate in the theater.

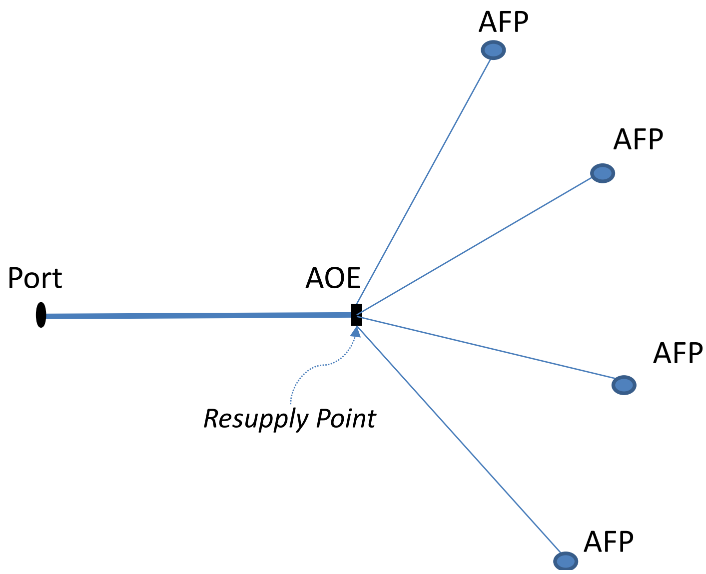


Figure 1: Gas Station Logistic Chain

3 Single AFP

We start off with the simple case where only one AFP operates in the theater at a station a distance R from port. After operating at the station for a certain, randomly distributed, time period T, the AFP needs to be replenished. The AOE is located at a resupply point a distance a from port. When needed, the AFP travels to the resupply point where it rendezvous with the AOE. The duration of the resupply process at the resupply point is S_a and after it is completed, the AFP heads back to its station. The AOE carries enough supply for k replenishments of the AFP. The velocities of the AOE and the AFP are u and v, respectively. The duration of the resupply process of the AOE at port is S_p.

[1]Although an AFP may travel, in that case, towards the port and either meet the AOE on its return trip from port or get resupplied directly at the port, we assert that such a practice is marginally beneficial at best, and therefore discard it

3.1 Scenario 1

In this scenario the single AOE is mobile. After the k'th replenishment of the AFP the AOE heads to port to load fresh supplies. Once reloaded, the AOE returns to the resupply point ready to service the AFP. While the AOE is away, off its resupply point (a), the AFP cannot be replenished. The measure of effectiveness (MOE) used here is the *utilization rate* which is the ratio η between the expected time the AFP is on station and the expected duration of a replenishment cycle (the time between two consecutive replenishments). The goal is to maximize this MOE. During a replenishment cycle of the AFP, it is off-station a minimum of $2(R-a)/v + S_a$ as it travels back and forth to the resupply point and gets replenished there. However, after k replenishment cycles, there may be an additional off-station time for the AFP if the AOE has not returned to the resupply point from port before the AFP is ready for replenishment.

First note that if $2a/u + S_p \leq 2(R-a)/v$, which means that the logistic cycle time of the AOE is shorter than the operational cycle time of the AFP, then the AFP never waits. In that case the off-station time is deterministic and only comprises the travel and replenishment times: $2(R-a)/v + S_a$. Otherwise, the expected time of replenishment cycle $k+1$ is affected by the on-station time T, which is a random variable with a probability distribution $F(t)$ (e.g., uniform, exponential, etc.). The time T is measured from the moment the AFP returns to its station. If $T + 2(R-a)/v > 2a/u + S_p$, then the AOE arrives back to the resupply point before the AFP, and thus the AFP does not wait to be resupplied. Let W denote the additional time the AFP is off station while waiting for the AOE during replenishment $k+1$, when the AOE needs to go back to port to reload. We have that

$$\begin{aligned} P[W=0] &= P[\text{no waiting}] \\ &= P[T + 2(R-a)/v > 2a/u + S_p] \\ &= 1 - F(2a/u + S_p - 2(R-a)/v). \end{aligned}$$

Thus, W has a point probability mass at $W = 0$ equal to $1 - F(2a/u + S_p - 2(R-a)/v)$. Otherwise, the AFP is off station an additional time period of length $2a/u + S_p - (T + 2(R-a)/v)$. The worst case scenario for the AFP in terms of off-station time is when $T = 0$. In that case the AFP stays off-station the longest: $2a/u + S_a + S_p$.

Define g_W as the density of the random variable W. If $2a/u + S_p > 2(R-a)/v$,

$$g_W(t) = f(2a/u + S_p - 2(R-a)/v - t), \quad \text{for } t \in (0, 2a/u + S_p - 2(R-a)/v],$$

and the CDF of W:

$$G_W(t) = 1 - F(2a/u + S_p - 2(R-a)/v - t), \quad \text{for } t \in (0, 2a/u + S_p - 2(R-a)/v].$$

If $2a/u + S_p \leq 2(R-a)/v$, the AOE always returns before the AFP so $W = 0$ with probability 1. For example, when T is exponentially distributed with rate parameter $\lambda > 0$, we have (if $2a/u + S_p > 2(R-a)/v$)

$$G_W(t) = \exp(-\lambda(2a/u + S_p - 2(R-a)/v - t)), \quad \text{for } t \in (0, 2a/u + S_p - 2(R-a)/v]$$

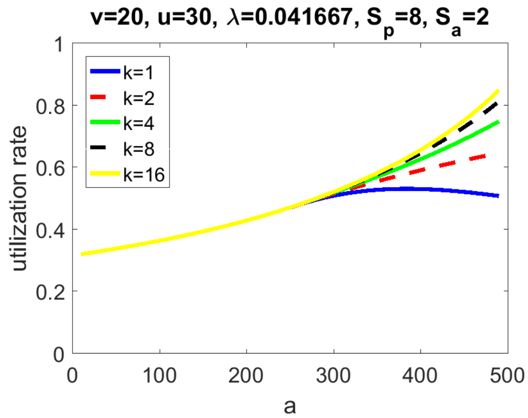
and the expected wait time is

$$\mathbf{E}[W] = \frac{2a}{u} + S_p - 2(R-a)/v - \frac{1}{\lambda}\left(1 - \exp\left(-\lambda(2a/u + S_p - 2(R-a)/v)\right)\right).$$

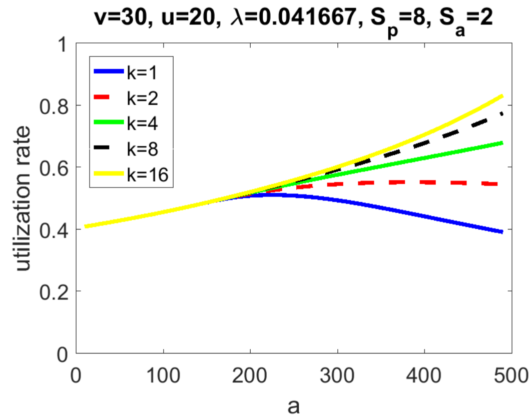
The long-run expected time off-station per replenishment is $E[OS] = \frac{2(R-a)}{v} + S_a + \frac{1}{k}\mathbf{E}[W]$. This follows because in one AOE cycle, replenishments 2 through k will never require waiting, but the first replenishment may wait if the AOE does not return from port before the AFP. Putting the pieces together, our MOE, the utilization rate, is $\eta = \frac{E[T]}{E[OS]+E[T]}$. For the exponential distribution this simplifies to $\eta = \frac{1}{\lambda E[OS]+1}$.

Figure 2 presents the utilization rate for the single-AOE scenario, as a function of the distance a of the resupply point, where $R = 500$, $S_a = 2$, $S_p = 8$. The first column of Figure 2 corresponds to a case where the AOE is faster than the AFP, that is, $v = 20$, $u = 30$. The second column flips those velocities and the AFP is faster: $v = 30$, $u = 20$. Throughout, distances are measured in NM, velocities in knots, and time in hours. The three rows in Figure 2 correspond to three values of the expected duration on station: $E[T] = 24$, 48, 72. The x-axis of Figure 2 is the distance a of the resupply point from port. For relatively small values of a, when the resupply point is closer to port, all lines coincide. That is, the utilization rate is not affected by the capacity of the AOE. In this case, the travel time of the AFP dominates that of the AOE and therefore the AFP never has to wait; the AOE always returns from port before the AFP returns for replenishment. We see however that in general, the closer the resupply point is to the AFP station the higher the utilization rate. Only when the AOE has very limited capacity (i.e., $k = 1$, the Blue curve), and the expected duration on station $E[T]$ is relatively short (see Figures 2a and 2b), the optimal distance a will be somewhere close to the midpoint. Also we note that beyond a certain capacity of the AOE (e.g., $k = 4$) additional capacity has marginal effect, in particular when a is constrained due to tactical or operational considerations, and the expected duration on station $E[T]$ is relatively long. Finally when the AFP never waits (smaller value of a), the utilization rate is higher for faster AFPs (column 2) as the AFP wants to travel back and forth from the resupply point as quickly as possible. However, for larger a, when AFP may have to wait for the AOE, the utilization rate is higher for faster AOEs (column 1). Even though the travel time for the AFP is larger in this case, the total time off station is shorter because the AOE returns from port faster.

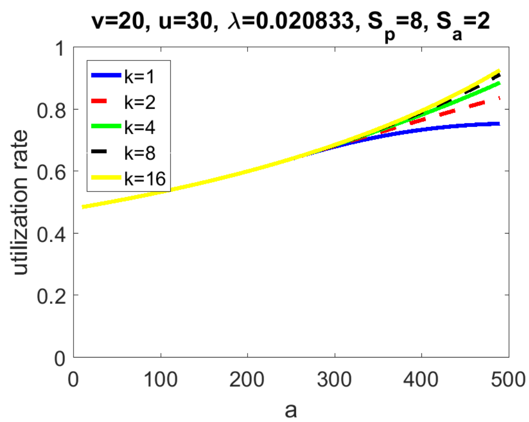
Figure 2 suggests that in most scenarios the optimal placement of the resupply point a is as close to the AFP station R as is operationally allowable. For even moderate AOE capacity k, waiting happens infrequently and its impact is minimal. However, the utilization rates in Figure 2 are based on expected values, and in some scenarios the commander may want to consider the potential risk when long wait times do occur and therefore a probability-based MOE would be in order. Figure 3 displays several cumulative distribution functions (CDFs) of the total time the AFP is off station for the $k+1-th$ replenishment, the situation when the AOE may not be available. We use similar combinations of velocities v and u and $\mathbf{E}[T]$ as in Figure 2 except we skip the middle row where the expected on-station time between replenishments is $\mathbf{E}[T] = 48$, and only consider the extremes: $\mathbf{E}[T] = 24, 72$. Each plot in a frame contains several curves corresponding to different values of the distance a between the port and the resupply point. The distance a impacts both the travel time of the AFP to resupply and the likelihood of waiting and length of wait W. The thin black dashed line is the (trivial) CDF when there is no AOE, so the AFP has to go back and forth from its station to port, with total (deterministic) off-station time $2R/v + S_a$. The other vertical lines represent the trivial CDFs (deterministic times) that correspond to situations where the AFP does not need to wait because the AFP travel time is longer than the AOE resupply



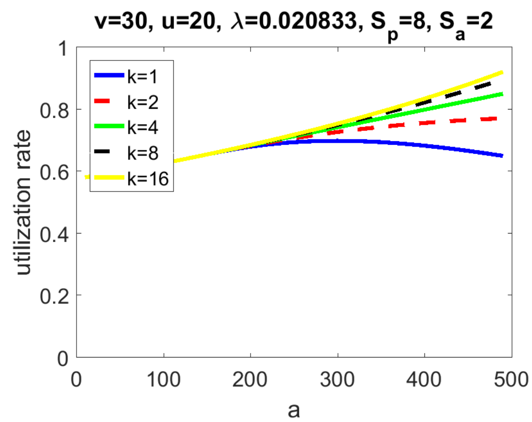
(a) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 24$



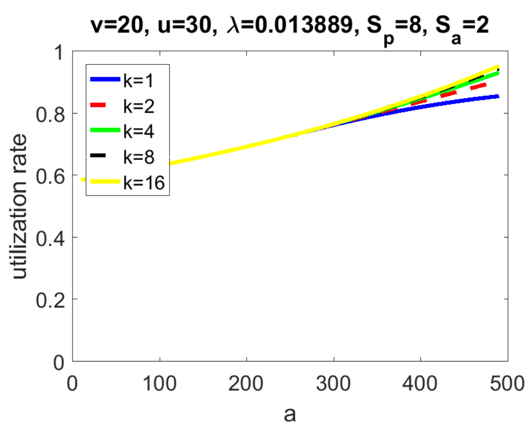
(b) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



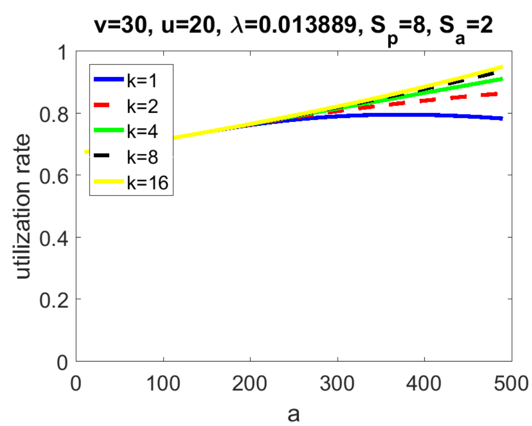
(c) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 48$



(d) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 48$



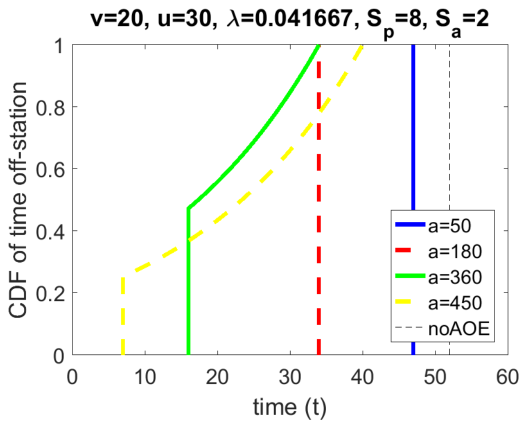
(e) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 72$



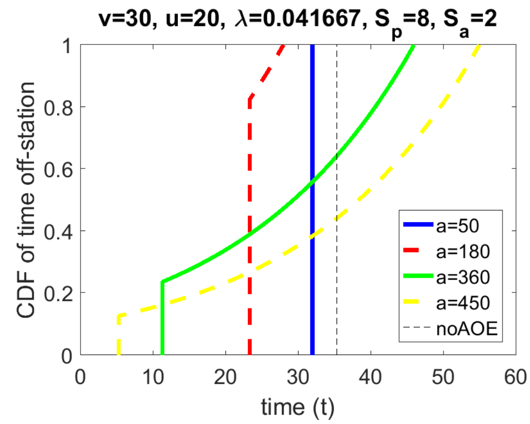
(f) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 72$

Figure 2: Utilization Rate

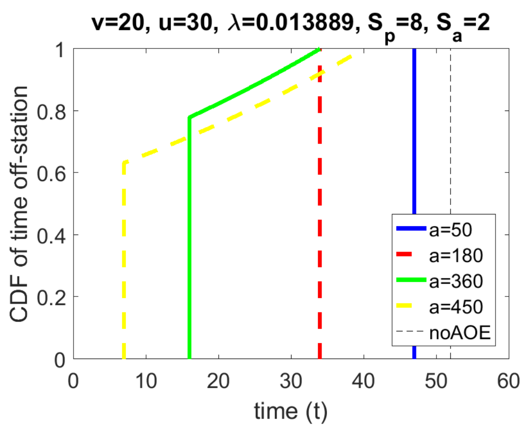
cycle, that is $2a/u + S_p \leq 2(R-a)/v$. As we increase a, eventually there is some possibility that the AFP has to wait for the AOE. For the slower AOE scenarios (column 2 of Figure 3), the time off-station can be substantial when a is large (green and yellow lines). Figures 3b and 3d both show that the wait time to the AOE would be so long such that the total time off-station may even exceed the off-station time when the AFP has to sail all the way back to port (see the portions of the yellow and green lines to the right of the black dashed line).



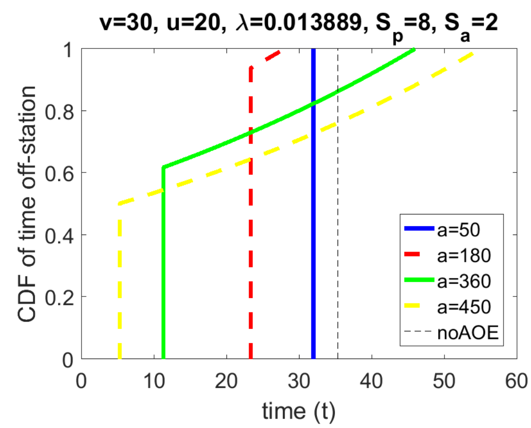
(a) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 24$



(b) Relatively fast AFP: $v = 20$, $u = 30$, $E[T] = 24$



(c) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 72$



(d) Relatively fast AFP: $v = 20$, $u = 30$, $E[T] = 72$

Figure 3: CDF for time off-station

We conclude this section by briefly discussing the choice of resupply point a. We return to this more formally in Section 3.3. In some situations, a may be dictated as the threshold between the communication (relatively safe) zone and the combat zone. In other situations the commander may have discretion to decide where he should place the resupply point. If the commander chooses, he should set a at least to the maximum value where the AFP never has to wait (i.e., the value a such that $2a/u + S_p = 2(R-a)/v$). In Figure 2 this point occurs where the curves diverge. Increasing a beyond this point has potential benefits with smaller off-station times because the AFP has to travel short distances to the resupply point. However, there is risk (illustrated in Figure 3), especially if the AOE is slow (small

u), that the AFP may have to wait a while for the AOE to transit back.

3.2 Scenario 2

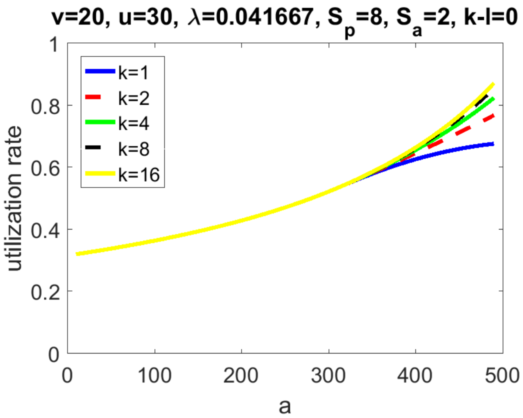
In this scenario we assume that there are two AOEs: A delivery ship stationary at the resupply point a and a shuttle ship that travels between the resupply point and the port. The shuttle is dispatched from the port to resupply the delivery ship when the supply level in the delivery ship reaches a certain level. Suppose that the shuttle is sent out at the time when the delivery ship executes its l-th resupply operation to the AFP, $l = 1, ..., k$. Unlike scenario 1 where we only had to deal with one random on-station time T, here we need to deal with several random variables T_i, $i = 1, 2...$, which are independent and identically distributed with CDF $F(t)$. The off-station time of the AFP for each one of the first k replenishment events is $2(R - a)/v$. The off-station time of the $k + 1$ replenishment instance may be longer due to the fact that the delivery ship is depleted, waiting to be refilled by the shuttle ship. Let W denote the wait time of the AFP for the $(k + 1)$th replenishment. The AFP will have to wait if the total time of $k - l + 1$ resupply cycles of the AFP is shorter than the travel time of the shuttle, that is, $T_1 + ... + T_{k-l+1} + \frac{2(k-l+1)(R-a)}{v} + (k - l)S_a \leq \frac{a}{u} + S_D$ where here u is the velocity of the shuttle and S_D is the time it takes the shuttle to refill the delivery ship. Thus, we have that

$$G(t) = Pr[W \leq t] = 1 - F^{*(k-l+1)} \left(\left[\frac{a}{u} + S_D - \frac{2(k - l + 1)(R - a)}{v} - (k - l)S_a \right] - t \right)$$

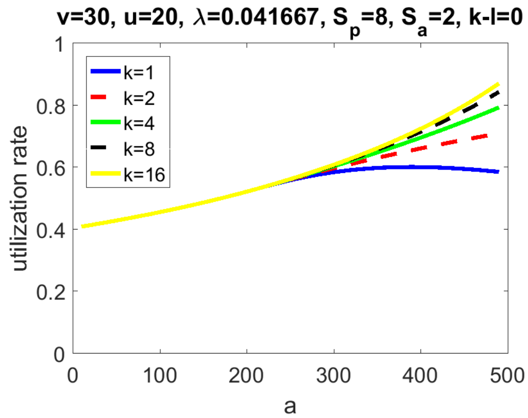
where F^{*m} is the $m - th$ convolution of F. It follows that $\mathbf{E}[OS] = \frac{2(R-a)}{v} + S_a + \frac{1}{k}\mathbf{E}[W]$.

Figure 4 is the analogous figure to Figure 2. We set $S_D = S_p = 8$ for comparison to Figure 2. This scenario requires specification of l – the number of replenishments the delivery ship executes for the AFP before the shuttle is dispatched from the port to resupply the delivery ship. The worst case situation from a utilization standpoint is when $l = k$, that is, when the shuttle is dispatched only when the delivery ship (AOE) has exhausted its supplies. The utilization curves in Figure 4 will be even higher if we use smaller values of l, which means sending out the shuttle ship to the resupply point more often. Of course, using smaller values of l imposes its own operational burden as the shuttle is being utilized more intensively. Comparing Figure 4 to Figure 2, we see that the shuttle provides no benefit for small values of a, because the AFP will never wait if the supply point is close enough to port. For larger a the benefits are small for large k (large capacity of the delivery ship) and large $\mathbf{E}[T]$ (expected time on station). However, for $k = 1$ or 2, and $\mathbf{E}[T] = 24$ the benefit of using a shuttle is significant. For example comparing Figure 4b to 2b, the $k = 1$ scenario provides an increase in utilization rate from 0.4 to 0.6. Arguably, the case of $k = 1, 2$ is quite unrealistic; a delivery ship should be able to supply many more than 2 replenishments of the AFP.

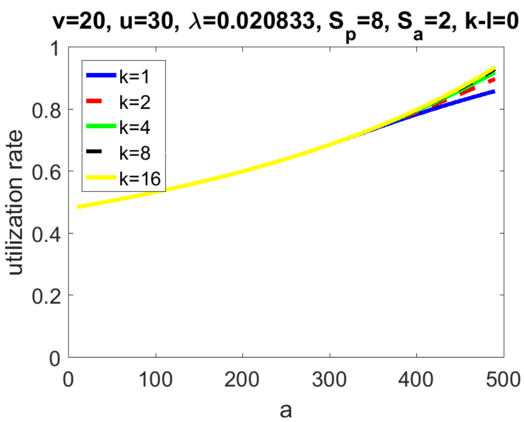
We next analyze the distribution for the off-station time by comparing Figure 5 to Figure 3. As discussed previously there is potential risk that the total off-station time will be very large for a near R. This occurs in Scenario 1 when the AFP has to wait for the round-trip of the slow AOE (see Figures 3b and 3d). With the shuttle in Scenario 2, there is much less risk of large off-station times because the shuttle only needs to make one leg of the transit



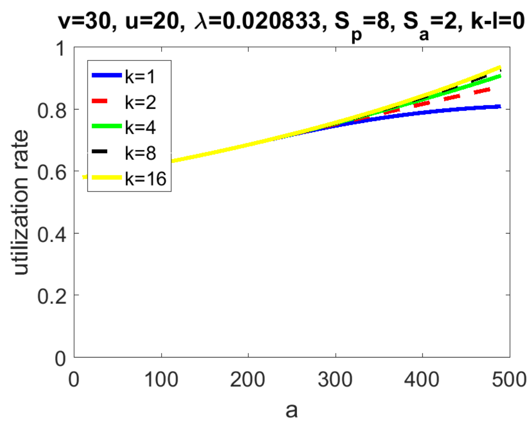
(a) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 24$



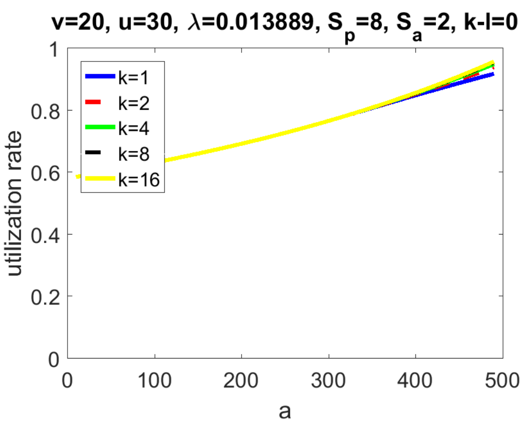
(b) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



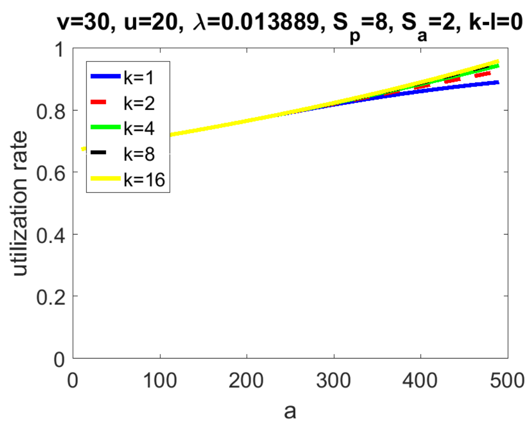
(c) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 48$



(d) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 48$



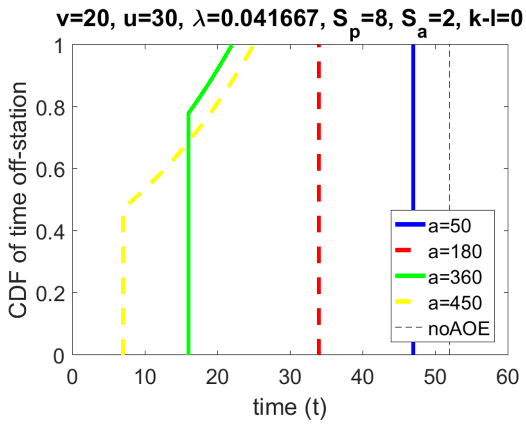
(e) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 72$



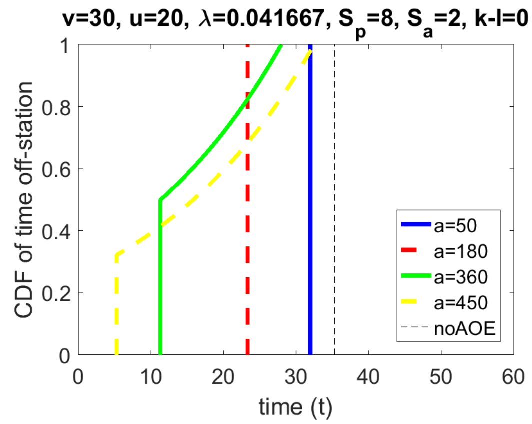
(f) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 72$

Figure 4: Utilization Rate, $l = k$

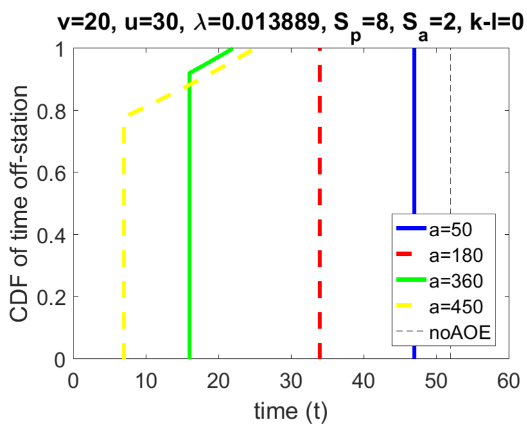
to resupply the AOE. Comparing Figures 3b and 3d with Figures 5b and 5d, we see that Scenario 2 produces much lighter tails of the off-station CDFs. In particular, notice that in all four cases of Figure 5 the dotted black vertical line – corresponding to the case where there is no Gas Station – is entirely to the right of all other curves. That is, any resupply point is better than no Gas Station.



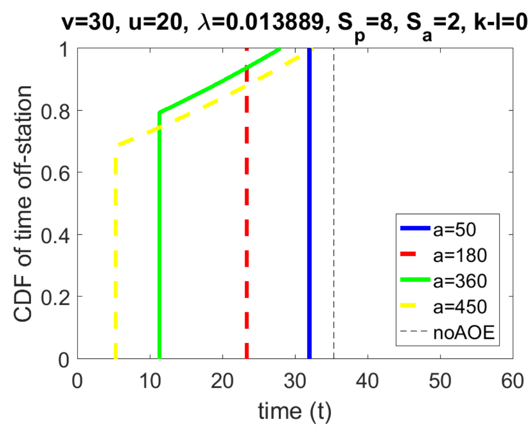
(a) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 24$



(b) Relatively fast AFP: $v = 20$, $u = 30$, $E[T] = 24$



(c) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 72$



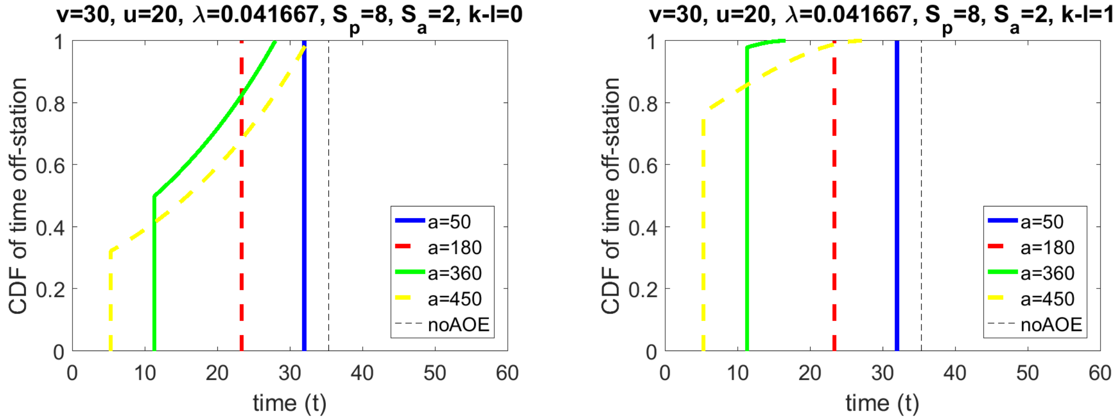
(d) Relatively fast AFP: $v = 20$, $u = 30$, $E[T] = 72$

Figure 5: CDF for time off-station: $l = k$

The shuttle scenario can perform even better if we send the shuttle for $l < k$. Figure 6 compares the time off-station CDF when $l = k$ vs. $l = k - 1$. When the shuttle leaves earlier, we can position the AOE at a near R with little risk.

3.3 Location of the resupply point

In this section we model the optimal location of the resupply point a for Scenario 1 (see Section 3.1). As shown in Section 3.1, if the AOE has enough capacity ($k > 2$) a resupply point that is closer to the station results in higher utilization rate. However, this desired property may come at the cost of increased risk to the AOE of being attacked at the resupply point. It is this trade-off that we analyze in this section.



(a) $l = k$,Relatively fast AFP: $v = 20$, $u = 30$, $E[T] = 24$

(b) $l = k - 1$, Relatively fast AFP: $v = 20$, $u = 30$, $E[T] = 24$

Figure 6: CDF for time off-station: Comparison for $l = k$ and $l = k - 1$

As before, the port is located at point 0, the resupply point is at a, and a the AFP's station is located at R. The resupply ship moves in a back-and-forth manner between point the port at 0 and the resupply point a. The risk of having the resupply point at location a is represented by a function $f(\cdot)$. The function $f(\cdot)$ captures the cost of defensive measures and expected losses if there is an attack on the AOE at the resupply point.

Without loss of generality, we define the point $a^{(-)}$ as the rightmost point of no risk; that is $f(a^{(-)}) = 0$, and $f(x) > 0$ for $x > a^{(-)}$. Realistically, the risk increases as the resupply point gets closer to the station location R, meaning that $f(\cdot)$ is non-decreasing. The decision variable in this model is the resupply point, a. As we move the resupply point a toward R, the downside is that the cost increases; the benefit is that the utilization rate increases. The question is: How far forward of $a^{(-)}$ should the resupply point a be set? Call that location a^*.

It is reasonable to assume that f is convex; the marginal cost is increasing as the AOE gets closer to the combat zone. Recall from Section 3.1 that the long-run expected time off-station per replenishment is

$$E[OS] = \frac{2(R-a)}{v} + S_a + \frac{1}{k}\mathbf{E}[W],$$

where for T having an exponential distribution yields

$$\mathbf{E}[W] = \frac{2a}{u} + S_p - 2(R-a)/v - \frac{1}{\lambda}\left(1 - \exp\left(-\lambda(2a/u + S_p - 2(R-a)/v)\right)\right).$$

Omitting constant terms, the optimization problem, in the variable a, is

$$\min_{a^{(-)} \le a \le R} 2a\left(\frac{-1}{v} + \frac{1}{ku} + \frac{1}{kv}\right) + \frac{\exp(-\lambda(S_p - 2R/v))}{k\lambda}\exp\left(-2a\lambda(1/u + 1/v)\right) + \gamma f(a),$$

The objective function is convex because it is the sum of a linear function plus two convex functions in a. Hence, a sufficient condition for optimality is to set the first derivate with respect to a equal to zero. If that is not possible, then the solution is an extreme point, either $a^{(-)}$ or R, depending on whether the derivative is positive at $a^{(-)}$ or negative at R, respectively. The first derivative with respect to a set to zero satisfies

$$2\left(\frac{-1}{v}+\frac{1}{ku}+\frac{1}{kv}\right)-2\lambda(1/u+1/v)\frac{\exp(-\lambda(S_p-2R/v))}{k\lambda}\exp\left(-2a\lambda(1/u+1/v)\right)+\gamma f'(a)=0.$$

This equation can be solved numerically for a given set of parameters using standard methods (e.g., Newton's method).

4 Multiple AFPs

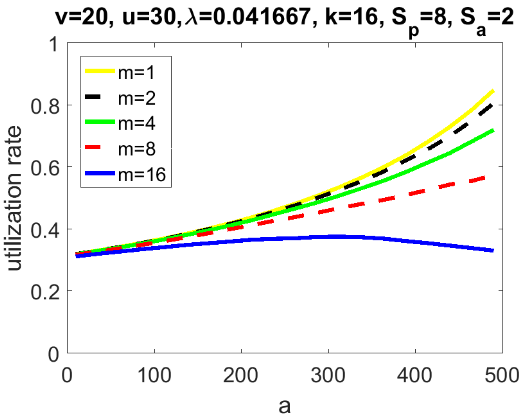
We move on now from a single AFP to the case of multiple AFPs and consider Scenario 1 where a single AOE resupplies the AFPs. Suppose there are m identical AFPs, which are scattered in the theater of operations. The resupply point is at distance a from port and all the AFPs are at approximately equal distance $R-a$ from it. The capacity of the AOE facilitates the resupply of m AFPs. Thus, each AFP has, on average, k/m resupply opportunities from the AOE before the latter needs to go back to port and get resupplied.

With multiple AFPs, the problem becomes much more difficult to analyze with purely analytic methods. In general setting with $S_a > 0$, we have a spatial queue: an AFP may have to "wait in line" for other AFPs to refuel at the AOE (in addition to possible waiting for the AOE to return from port). Therefore we use simulation to create similar figures to Figures 2 and 3. In the special case where $S_a = 0$, there is no queuing and the only additional time off-station occurs when waiting for the AOE to return from port. We can make analytic headway for $S_a = 0$, but we defer that analysis for the Appendix and just provide the main general results here.

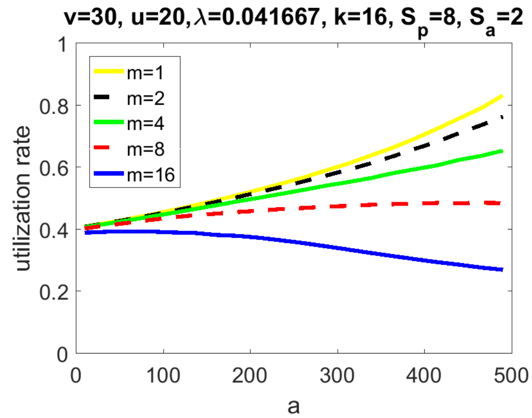
Figure 7 is the utilization figure analogous to Figure 2. We fix $k = 16$ in all figures and plot one curve for several different values of m. The other parameter combinations coincide exactly to the parameters in Figure 2. As discussed, each AFP receives on average k/m resupply opportunities from the AOE. Therefore, we might expect that the m curve in Figure 7 will be similar to the k/m curve in Figure 2. The curves are quite close except the $m = 16$ curve in Figure 7 is a reasonable amount less than the the $k = 1$ curve in Figure 2. This occurs because with $m = 16$ AFPs and only $k = 16$ resupplies there will be significant queueing, which increases off-station time and lowers utilization.

We next compare the risk associated with long off-station times in the multiple AFP scenario versus the single AFP. Figure 8 displays the probability an AFP will need to wait for replenishment. Waiting occurs if time off-station exceeds $\frac{2(R-a)}{v} + S_a$ and includes both queueing and waiting for the AOE to return from port. The parameter combinations are the same as in Figure 7, except we only include $E[T] = 24$ and $E[T] = 72$. We see that for $m \geq 4$, the wait probability can be significant as a approaches R.

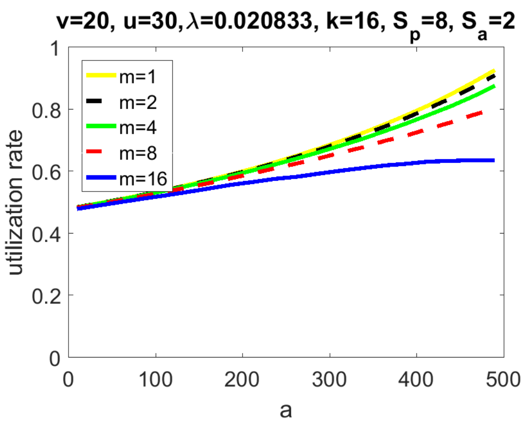
We conclude by showing a CDF figure similar to Figure 3. Figure 9 considers parameters directly comparable to Figure 3b: short time on-station ($E[T] = 24$) and a relatively slow AOE ($v = 30$, $u = 20$). Each figure fixes $k = 16$ and m to a specific value. Figure 9 presents the off-station time distribution for all AFP resupplies, whereas Figure 3 displays the distribution just for the $k+1th$ resupply. The important comparison to make is the tail of the distribution. In Figure 9d for $m = 16$, the tail is much heavier for larger a than any other curves. This is due to the significant waiting caused by queueing. Also, we observe



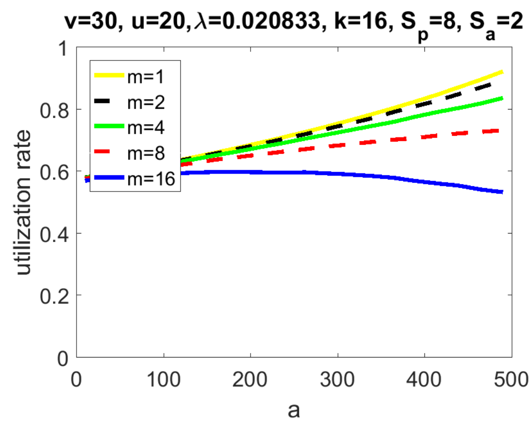
(a) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 24$



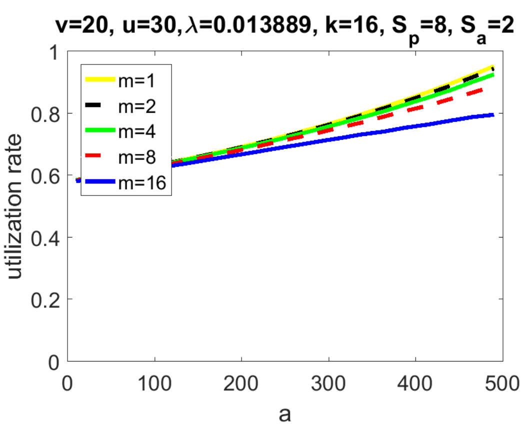
(b) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



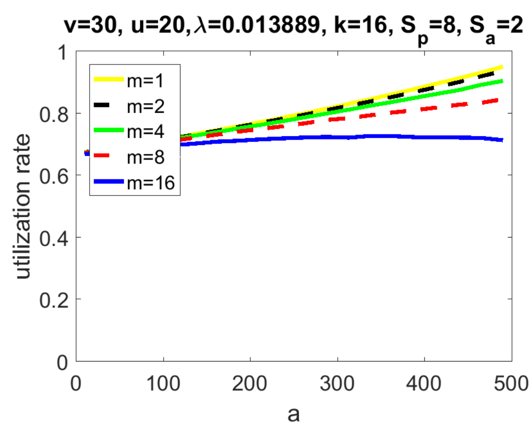
(c) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 48$



(d) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 48$

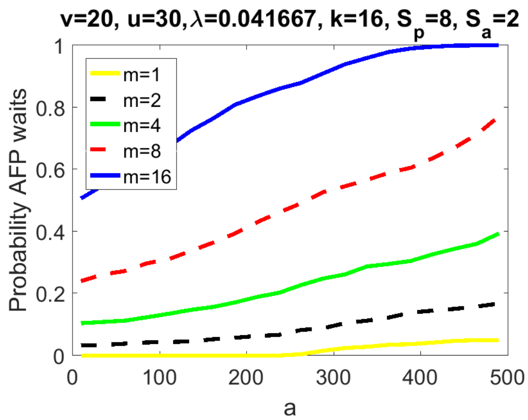


(e) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 72$

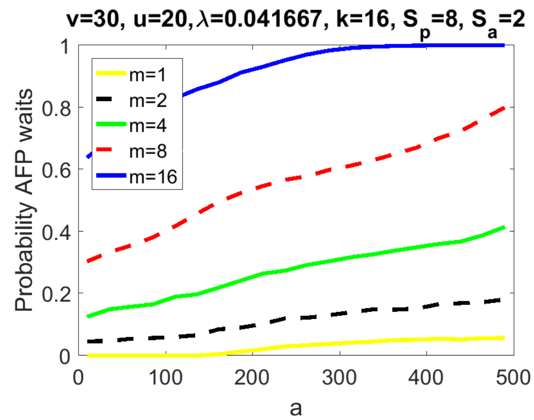


(f) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 72$

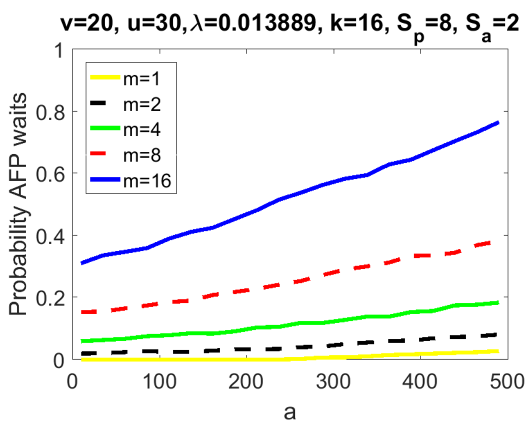
Figure 7: Utilization Rate: Multiple AFPs



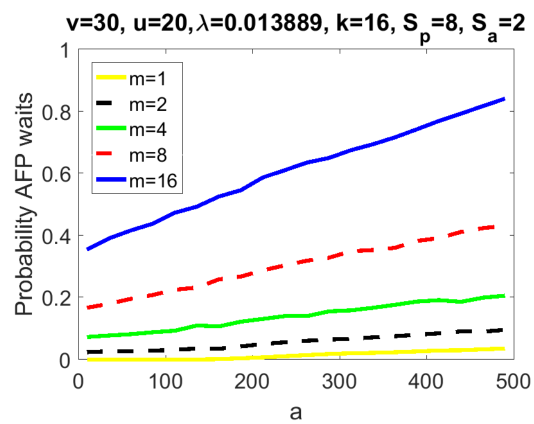
(a) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 24$



(b) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



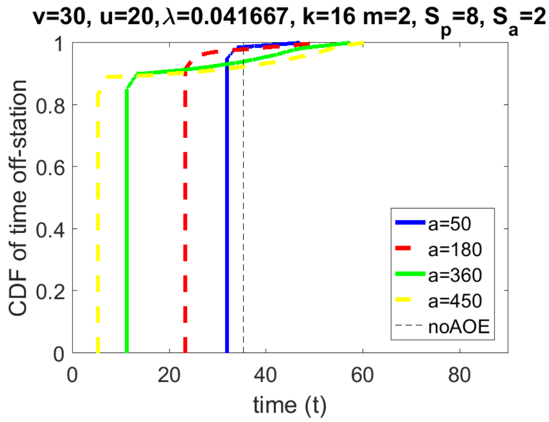
(c) Relatively slow AFP: $v = 20$, $u = 30$, $E[T] = 72$



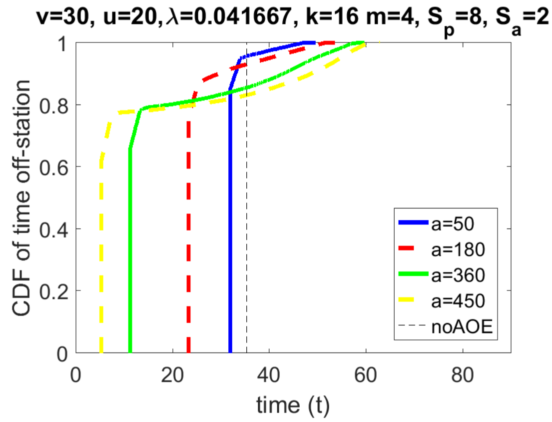
(d) Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 72$

Figure 8: Probability AFP Waits: Multiple AFPs

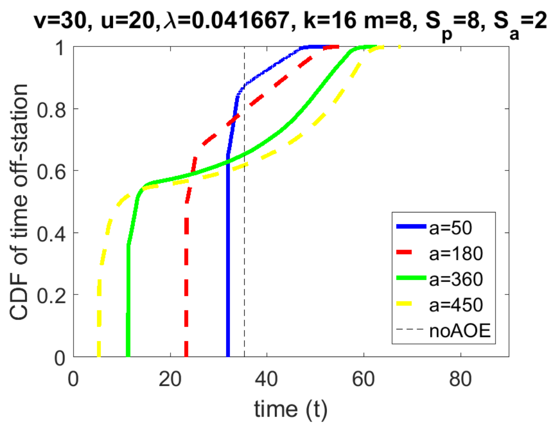
that the farther the resupply point from port (larger a) The heavier the tail of the CDF. We conclude that the Gas Station should be located closer rather than farther from port.



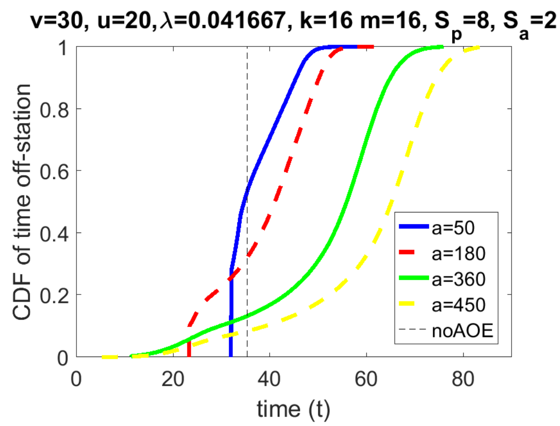
(a) $m = 2$, Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



(b) $m = 4$, Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



(c) $m = 8$, Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$



(d) $m = 16$, Relatively fast AFP: $v = 30$, $u = 20$, $E[T] = 24$

Figure 9: CDF for time off-station for different values of m: Multiple AFPs

5 Conclusion

Distributed lethality is an operational concept that embodies significant logistic implications. The existing logistic system, supporting carrier battle group, where the logistic tail is an integral part of the tactical force – the shuttle-delivery ship setup – will clearly be inappropriate when the force structure is fragmented into small AFPs. Attaching a logistic tail to each such AFP is neither economically viable nor operationally feasible. A new logistic structure is required that adequately responds to the new naval force layout.

In this report we model and analyze the Gas Station setup where an AOE is deployed at a certain resupply point in the communication zone and AFPs travel from their stations to that point to be resupplied. We define the utilization rate – the fraction of time the AFP is on station - as a measure of effectiveness. From a relatively simple model, with only one AFP, we see that for resupply points relatively far away from the AFP stations, the utilization rate is in sensitive to the capacity of the AOE; the dominant factor is the travel time of the AFP to the resupply point. If the AOE is deployed closer to the combat area we see some differences in utilization rates for different capacities; for high capacities the utility is monotone increasing as the AOE gets closer to the combat zone but for low capacity there is an optimal resupply point closer to port. In the more realistic, and relevant, case when there are multiple AFPs we see that the utilization rate, as well as the probability an AFP has to wait for the AOE, are sensitive to the size of the operating force in the theater, but only when the force is relatively large. For any distance of the resupply point a, the utilization rate is very similar when the force size is between 1 and 4 AFPs.

The work presented in this report can be extended in several directions. The linear situation described in this report (optimizing the value of a on a line) can be extended to a two-dimensional situation, taking into account spatial considerations, as well as variable demand scenarios. Also, the underlying queuing problem may be addressed with more rigorous mathematical tools. A challenge would be modelling of specific commodity types (e.g., fuel, ammunition, repair parts) as independent drivers of needing to leave station for resupply. Interrelations among temporal parameters (e.g., longer time on station T implies longer replenishment time S_a) is another modeling issue that may be explored. Finally, while we addressed attritional aspects in Section 3.3, embedding the logistic model in a combat one may produce additional insights.

Acknowledgement:

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APPENDIX

A Analytic Approximation for $S_a = 0$

For $S_a = 0$ we do not need to consider queueing aspects. The only possible time an AFP waits is when an AFP arrives to a before the AOE returns from port. In this section we derive analytic expressions for the time off-station and utilization rate. We assume for simplicity that the resupply time at the port for the AOE negligible ($S_p = 0$) and $k \geq m$.

A.1 Probability of Waiting

When the AOE returns to the gas station, there will be $Z \leq m$ AFPs waiting at the gas station for immediate supplies, where Z is a random variable. The remaining $k - Z$ supply units will be dispensed later without the AFPs having to wait. If the process continues in this fashion, with Z AFPs waiting at the gas station for the return of the AOE, then we can compute the probability of waiting

$$\mathbf{P}[\text{AFP waits}] = \frac{\mathbf{E}[Z]}{k}$$

We next estimate $\mathbf{E}[Z]$. To do this we assign a Bernoulli random variable to each AFP and estimate the probability the AFP reaches the AOE location a before the AOE returns. When the AOE leaves a to resupply, one AFP just left the AOE. This AFP will be back to point a before the AOE if $2(R-a)/v + T < 2a/u$, and hence the probability associated with the Bernoulli for this AFP is $F(2a/u - 2(R-a)/v)$. However, to determine the Bernoulli probability for the other $m-1$ AFPs, we need to know where those AFPs are when the AOE returns to port for resupplies.

There are three possible stages an AFP can be in: returning to AO from AOE, heading to AOE from AO, or waiting at AO for next event. The expected time of the a cycle from leaving the AOE to returning is $(R-a)/v + \mathbf{E}[T] + (R-a)/v$. We assume that the $m-1$ remaining AFPs are spread out among the three stages proportionally to these times. That is

stage 0	1 AFP:	just left AOE
stage 1	$\dfrac{(R-a)/v}{2(R-a)/v + \mathbf{E}[T]}(m-1)$ AFPs:	heading toward AO
stage 2	$\dfrac{\mathbf{E}[T]}{2(R-a)/v + \mathbf{E}[T]}(m-1)$ AFPs:	at AO
stage 3	$\dfrac{(R-a)/v}{2(R-a)/v + \mathbf{E}[T]}(m-1)$ AFPs:	heading back to AOE

For each stage we compute the probability that an AFP at that stage will have to wait for the AOE to return from port. The probability of waiting for stage 0 is $F(2a/u - 2(R-a)/v)$.

For stage 3 we assume the AFP is uniformly between the AO and a and thus

$$\mathbf{P}[\text{wait in stage 3}] = \min\left(1, \frac{2a/u}{(R-a)/v}\right)$$

For stage 2, the AFP is at AO waiting for next event. We assume the AFPs in stage 2 just arrived back to the AO, and thus an AFP will wait if $T + (R-a)/v < 2a/u$:

$$\mathbf{P}[\text{wait in stage 2}] = F(2a/u - (R-a)/v)$$

Finally for stage 1, we assume the AFP is uniformly between the a and the AO. If AFP is x time units from AO ($x \in [0, (R-a)/v]$), then it will have to wait if $x+T+(R-a)/v < 2a/u$:

$$\mathbf{P}[\text{wait in stage 1}] = \frac{1}{(R-a)/v} \int_0^{(R-a)/v} F(2a/u - (R-a)/v - x)dx$$

Combining everything we have our estimate for $\mathbf{E}[Z]$:

$$\begin{aligned}
\mathbf{E}[Z] =& F(2a/u - 2(R-a)/v) \\
&+ \frac{(R-a)/v}{2(R-a)/v + \mathbf{E}[T]}(m-1) \times \frac{1}{(R-a)/v} \int_0^{(R-a)/v} F(2a/u - (R-a)/v - x)dx \\
&+ \frac{\mathbf{E}[T]}{2(R-a)/v + \mathbf{E}[T]}(m-1) \times F(2a/u - (R-a)/v) \\
&+ \frac{(R-a)/v}{2(R-a)/v + \mathbf{E}[T]}(m-1) \times \min\left(1, \frac{2a/u}{(R-a)/v}\right)
\end{aligned}$$

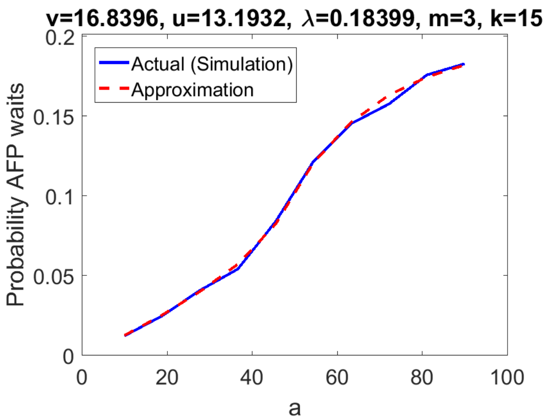
For the case where $T \sim \text{Exp}(\lambda)$ this becomes

$$\begin{aligned}
\mathbf{E}[Z] =& 1 - \exp\left(-\lambda(2a/u - 2(R-a)/v)^+\right) \\
&+ \frac{(R-a)/v}{2(R-a)/v + 1/\lambda}(m-1) \times \frac{\beta + \frac{1}{\lambda}\exp\left(-\lambda(2a/u - (R-a)/v)\right) - \frac{1}{\lambda}\exp\left(-\lambda(2a/u - (R-a)/v - \beta)\right)}{(R-a)/v} \\
&+ \frac{1/\lambda}{2(R-a)/v + 1/\lambda}(m-1) \times \left(1 - \exp\left(-\lambda(2a/u - (R-a)/v)^+\right)\right) \\
&+ \frac{(R-a)/v}{2(R-a)/v + 1/\lambda}(m-1) \times \min\left(1, \frac{2a/u}{(R-a)/v}\right)
\end{aligned}$$

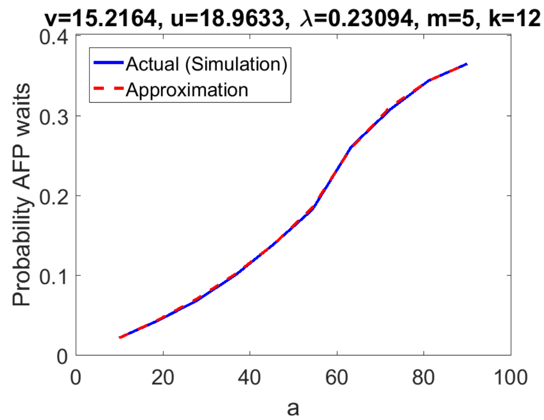
Where

$$\beta = \min\left((R-a)/v, (2a/u - (R-a)/v)^+\right)$$

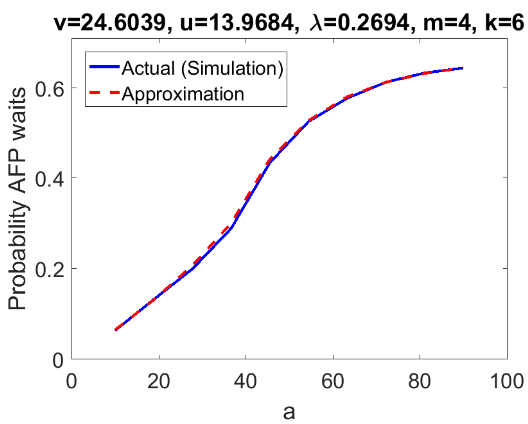
Figure 10 compares our analytic approximation with simulation for results for $R = 100$ and randomly chosen parameters. Our approximation is very accurate in most cases. Figure 10d illustrates when our approximation starts to break down: k and m are close and events occur frequently.



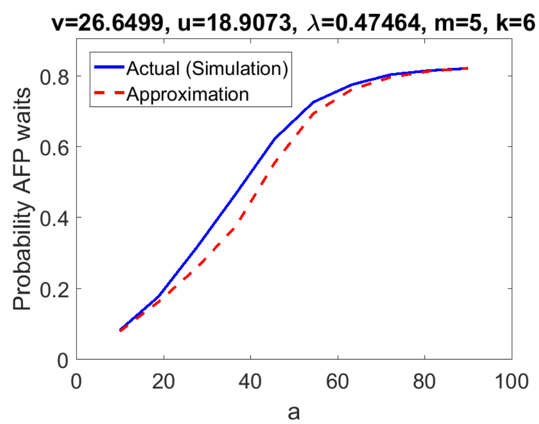
(a)



(b)



(c)



(d)

Figure 10: Probability AFP Waits, multiple AFPs

A.1.1 Expected Time Off-Station

To determine the utilization rate, we need to compute the time off-station. Each AFP has a minimum of $2(R-a)/v$ time off-station. Of the k refills at most m require waiting. This occurs when the AOE returns to a and all m AFPs are waiting. The other $k-m$ resupplies will occur with no additional waiting. Thus we can write

$$\mathbf{E}[\text{time off-station}] = \frac{2(R-a)}{v} + \frac{k-m}{k} \times 0 + \frac{1}{k}\sum_{i=1}^{m} \mathbf{E}[X_i]$$

Where X_i is the time the ith AFP has to wait on the first visit to a after the AOE has gone back to port. Rather than examining all m ships, we will assume each AFP falls into one of the four stages defined above for the probability, which yields

$$\begin{aligned}\mathbf{E}[\text{time off-station}] \approx\ & \frac{2(R-a)}{v} \\ + \ & \frac{1}{k} \times \Bigg(\mathbf{E}[Y_0] \\ + \ & \frac{(R-a)/v}{2(R-a)/v + \mathbf{E}[T]}(m-1)\mathbf{E}[Y_1] \\ + \ & \frac{\mathbf{E}[T]}{2(R-a)/v + \mathbf{E}[T]}(m-1)\mathbf{E}[Y_2] \\ + \ & \frac{(R-a)/v}{2(R-a)/v + \mathbf{E}[T]}(m-1)\mathbf{E}[Y_3] \Bigg)\end{aligned}$$

Where Y_i is the wait time given an AFP is in stage i when the AOE leaves to go back to port. If the AFP next reaches the a in x time units, the wait time is $(2a/u - x)^+$. The stage 0 AFP has to make the complete round trip:

$$\begin{aligned}\mathbf{E}[Y_0] &= \int_0^\infty \left[\frac{2a}{u} - \left(\frac{2(R-a)}{v} + t\right)\right]^+ f(t)dt \\ &= \left(\frac{2a}{u} - \frac{2(R-a)}{v}\right) F\left(\frac{2a}{u} - \frac{2(R-a)}{v}\right) - \int_0^{\left(\frac{2a}{u} - \frac{2(R-a)}{v}\right)^+} tf(t)dt\end{aligned}$$

The Stage 1 expression is the most complicated so we leave it for last. The Stage 3 AFP is heading back to gas station. Assuming a uniform location on the return leg yields

$$\begin{aligned}\mathbf{E}[Y_3] &= \frac{1}{\frac{R-a}{v}} \int_0^{\frac{R-a}{v}} \left[\frac{2a}{u} - x\right]^+ dx \\ &= \frac{\frac{2a}{u}\min\left(\frac{R-a}{v}, \frac{2a}{u}\right) - \frac{1}{2}\left(\min\left(\frac{R-a}{v}, \frac{2a}{u}\right)\right)^2}{\frac{R-a}{v}}\end{aligned}$$

The Stage 2 AFP just got back to the AO and is waiting for the event before returning to the gas station. The expression is very similar to $\mathbf{E}[Y_0]$

$$\mathbf{E}[Y_2] = \int_0^\infty \left[\frac{2a}{u} - \left(t + \frac{(R-a)}{v}\right)\right]^+ f(t)dt$$

$$= \left(\frac{2a}{u} - \frac{(R-a)}{v}\right) F\left(\frac{2a}{u} - \frac{(R-a)}{v}\right) - \int_0^{\left(\frac{2a}{u} - \frac{(R-a)}{v}\right)^+} tf(t)dt$$

Finally Stage 1 has a partial trip back to the AO, then waits for the event, and then a full return leg. This leads to two sources of randomness (location on leg back to AO and time until event). Assuming a uniform location we have

$$\mathbf{E}[Y_1] = \frac{1}{\frac{R-a}{v}} \int_0^{\frac{R-a}{v}} \int_0^\infty \left[\frac{2a}{u} - \left(x + t + \frac{(R-a)}{v}\right)\right]^+ f(t)dtdx$$

$$= \frac{1}{\frac{R-a}{v}} \int_0^{\min\left(\frac{R-a}{v}, \frac{2a}{u} - \frac{(R-a)}{v}\right)} \left(\frac{2a}{u} - x - \frac{(R-a)}{v}\right) F\left(\frac{2a}{u} - x - \frac{(R-a)}{v}\right) dx$$

$$- \frac{1}{\frac{R-a}{v}} \int_0^{\min\left(\frac{R-a}{v}, \frac{2a}{u} - \frac{(R-a)}{v}\right)} \int_0^{\left(\frac{2a}{u} - x - \frac{(R-a)}{v}\right)^+} tf(t)dtdx$$

To summarize:

$$\mathbf{E}[Y_0] = \left(\frac{2a}{u} - \frac{2(R-a)}{v}\right) F\left(\frac{2a}{u} - \frac{2(R-a)}{v}\right) - \int_0^{\left(\frac{2a}{u} - \frac{2(R-a)}{v}\right)^+} tf(t)dt$$

$$\mathbf{E}[Y_1] = \frac{1}{\frac{R-a}{v}} \int_0^{\min\left(\frac{R-a}{v}, \frac{2a}{u} - \frac{(R-a)}{v}\right)} \left(\frac{2a}{u} - x - \frac{(R-a)}{v}\right) F\left(\frac{2a}{u} - x - \frac{(R-a)}{v}\right) dx$$

$$- \frac{1}{\frac{R-a}{v}} \int_0^{\min\left(\frac{R-a}{v}, \frac{2a}{u} - \frac{(R-a)}{v}\right)} \int_0^{\left(\frac{2a}{u} - x - \frac{(R-a)}{v}\right)^+} tf(t)dtdx$$

$$\mathbf{E}[Y_2] = \left(\frac{2a}{u} - \frac{(R-a)}{v}\right) F\left(\frac{2a}{u} - \frac{(R-a)}{v}\right) - \int_0^{\left(\frac{2a}{u} - \frac{(R-a)}{v}\right)^+} tf(t)dt$$

$$\mathbf{E}[Y_3] = \frac{\frac{2a}{u}\min\left(\frac{R-a}{v}, \frac{2a}{u}\right) - \frac{1}{2}\left(\min\left(\frac{R-a}{v}, \frac{2a}{u}\right)\right)^2}{\frac{R-a}{v}}$$

Once we combine everything to compute $\mathbf{E}$[time off-station], we can calculate the utilization rate: $\frac{E[T]}{\mathbf{E}[\text{time off-station}] + E[T]}$. Figure 11 illustrates the performance of the approximation. As with Figure 10 the approximation is very accurate in most cases.

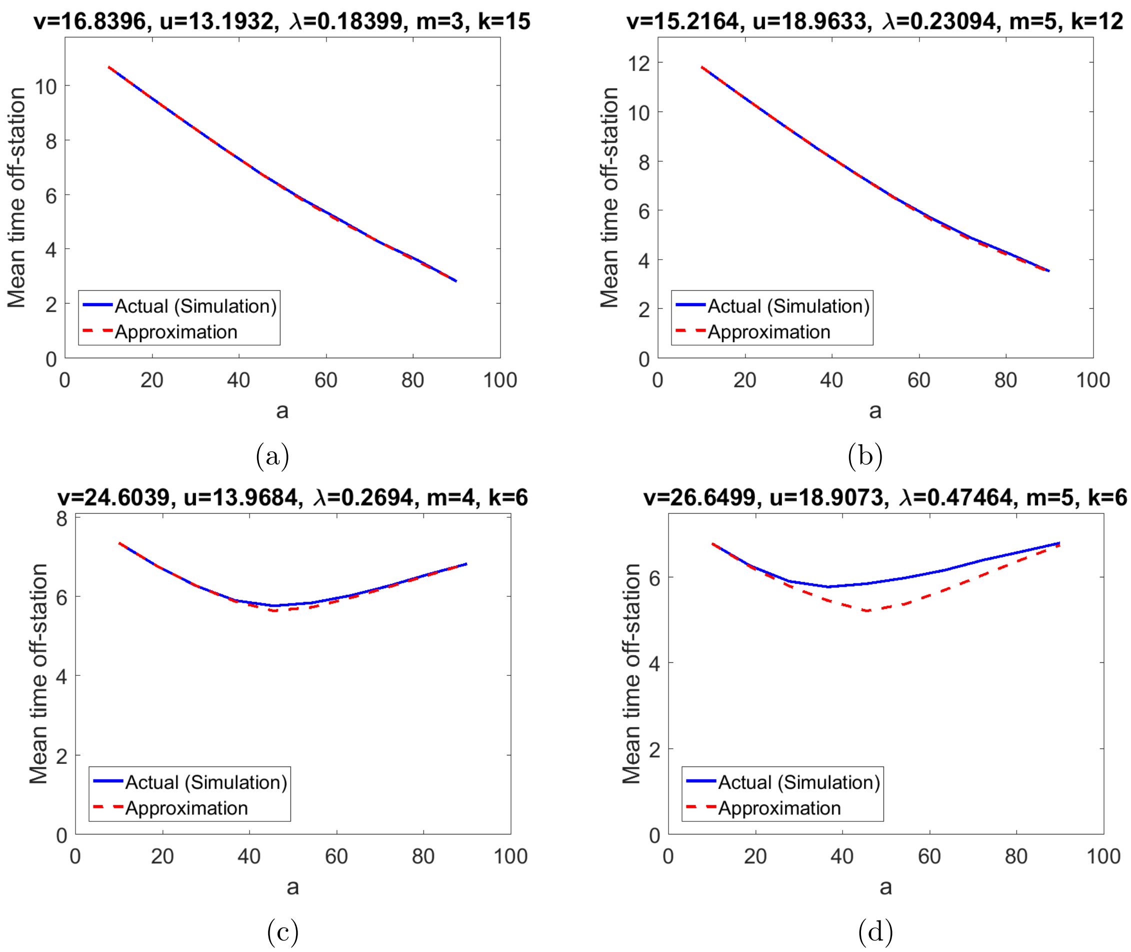


Figure 11: Expected time off-station, multiple AFPs