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# Multiple Tone Interference of Frequency-Hopped Noncoherent MFSK Signals Transmitted Over Ricean Fading Channels

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***Abstract*—This paper investigates the performance degradation resulting from multitone interference of orthogonal, frequency-hopped, noncoherent $M$-ary frequency-shift keyed receivers (FH/MFSK) where the effect of thermal and other wideband noise is not neglected. The multiple, equal power jamming tones are assumed to correspond to some or all of the possible FH $M$-ary orthogonal signaling tones. Furthermore, the channel is modeled as a Ricean fading channel; and both the signaling tones and the multiple interference tones are assumed to be affected by channel fading. It is also assumed that channel fading need not necessarily affect the signaling tones and the interference tones in the same way. When the information signal power exceeds the power of the individual interference tones, poorer overall system performance is obtained when the multiple interfering tones experience fading. This trend is accentuated as $M$ increases. When the information signal experiences fading, the effect of fading multiple interference tones on overall system performance lessens, and for a Rayleigh-faded information signal, fading of the multiple interference tones has no effect on overall system performance regardless of $M$.**

## I. Introduction

FREQUENCY-HOPPING (FH) communication systems are a type of spread spectrum communications that have become an important component of military communications strategy. FH systems offer an improvement in performance when the communication system is attacked by hostile interference and reduce the ability of a hostile observer to receive and demodulate the communication signal. An FH communication system is implemented by changing, or hopping, the carrier frequency of the transmitted signal in an apparently random fashion over a very broad bandwidth as compared to the signal bandwidth without FH. The apparently random sequence of hopping frequencies is presumably known only by the transmitter and the intended receiver or receivers, thus precluding reception by unintended receivers and making intentional interference more difficult. If the hop rate is assumed fast enough to preclude a follower jammer, then the principal means of intentionally interfering with a FH communication system are partial-band noise jamming, barrage noise jamming, and multitone interference. Partial-band noise jamming implies a noise-like signal, that is, a signal with the spectral characteristics of bandlimited additive white Gaussian noise (AWGN), where the bandwidth of the interference signal covers only a portion of the total bandwidth of the FH system. Barrage noise jamming is a special case of partial-band noise jamming where the bandwidth of the interference signal covers the entire spread spectrum bandwidth of the FH system. Multitone interference implies that the jamming signal consists of one or more tones transmitted within the total bandwidth of the FH system.

The effect of barrage and partial-band noise jamming on frequency-hopped, $M$-ary frequency-shift keyed (FH/MFSK) noncoherent receivers, when one or more symbols per hop are transmitted, has been examined both for channels with no fading and for Rayleigh fading channels in [1] and [2], respectively. In the latter reference, the fading channel is assumed to affect the partial-band noise signal as well as the information signal in addition to modeling channel fading as affecting only the information signal. The effect of partial-band noise jamming on fast frequency-hopped (FFH) binary frequency-shift keyed (BFSK) noncoherent receivers with diversity has been examined for channels with no fading [3], as has the effect of partial-band noise jamming on FFH/MFSK for Ricean fading channels [4]. The performance degradation resulting from both band and independent multitone jamming of FH/MFSK, where the jamming tones are assumed to correspond to some or all of the possible FH $M$-ary signaling tones and when thermal and other wideband noise is negligible, is examined in [5]–[7]. Band multitone jamming implies that the multiple, equal power jamming tones are distributed randomly across the entire FH bandwidth with a specific number of jamming tones placed in each jammed FH band. Independent multitone jamming implies that the multiple, equal power jamming tones are distributed randomly across the entire FH bandwidth. In other words, for band multitone jamming, a jammed hop band is always jammed with the same number of tones, while for independent multitone jamming, a jammed hop may be jammed with as few as one tone and as many as $M$ tones. The effect of tone interference on noncoherent MFSK when AWGN is not neglected is examined for channels with no fading in [8], and the effect of independent multitone jamming on noncoherent FH/BFSK when AWGN is not neglected is examined for channels with no fading in [9].

This paper investigates the performance degradation resulting from multitone interference of orthogonal, noncoherent

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FH/MFSK where the effect of thermal and other wideband noise is not neglected. The multiple, equal power jamming tones are assumed to correspond to some or all of the possible FH $M$-ary orthogonal signaling tones. Furthermore, the channel for each hop band is modeled as an independent, frequency-nonselective, slowly fading Ricean process [10]. Both the signaling tones and the multiple interference tones are assumed to be affected by channel fading. It is also assumed that channel fading need not necessarily affect the signaling tones and the interference tones in the same way. By modeling the channel as a Ricean fading channel, a general result is obtained that is valid in the limit of large direct-to-diffuse signal power ratios for channels with no fading and in the limit of small direct-to-diffuse signal power ratios for Rayleigh fading channels as well as for the general case where neither the direct nor the diffuse components of the signal are negligible. In this paper, it is assumed that for band multitone interference that jammed hop bands contain a single interfering tone at one of the $M$ possible signaling frequencies. It has been shown that this results in the poorest system performance for band multitone interference when thermal noise is negligible [6]. This intuitive result does not seem likely to be affected by the addition of the effects of either AWGN or channel fading. For independent multitone interference, which is shown to have an analogous, but not quite as severe, effect on performance when thermal noise is negligible [6], only FH/BFSK is considered in this paper.

## II. FH/MFSK System and Channel Model

The FH/MFSK communication system examined in this paper is assumed to have $N$ nonoverlapping FH bands, each with bandwidth $B_h$ where $B_h$ is the bandwidth required to transmit an MFSK signal in the absence of FH. Each FH band contains $M = 2^k$ orthogonal signaling tone locations, where $M$ is the order of the MFSK modulation and $k$ represents the number of bits per symbol transmitted. The $M$ possible signal tones in any given FH band are also assumed to be orthogonal to the possible signal tones in all other FH bands. Hence, there are $M \times N$ possible signal tones that the FH/MFSK communication system can transmit within the total spread spectrum bandwidth $B_T \geq NB_h$ where the equality holds for contiguous FH bands. The symbol rate is $R_s = 1/T_s = R_b/k$, where $T_s$ is the symbol duration, and $R_b$ is the bit rate. The symbol duration is related to the hop duration $T_h = 1/R_h$ by $T_h = KT_s$ where $K = 1, 2, 3, \cdots$ and $R_h$ is the hop rate. The receiver is assumed to dehop the signal with perfect timing, and the dehopped signal is demodulated with a bank of $M$ quadrature detectors, one for each of the $M$ possible signal tones, where the integrator time constants are normalized to the symbol duration for notational convenience. The outputs of the $M$ quadratic detectors provide the decision statistics for the MFSK demodulator.

Thermal noise and other wideband noise that corrupt the channel are modeled as AWGN, and the power spectral density (PSD) of this wideband noise is defined as $N_0/2$. The multitone interference is assumed to have a total power of $P_{J_T}$ which is transmitted in a total of $q$ equal power interfering tones spread randomly over the spread spectrum bandwidth of the FH/MFSK system. Hence, the power of a single interfering tone is $P_J = P_{J_T}/q$. The multiple interfering tones are all assumed to be transmitted at frequencies exactly corresponding to the various $M \times N$ FH/MFSK signaling tones, and none of the multiple interfering tones are transmitted at the same frequency.

## III. FH/MFSK Receiver Performance with Band Multitone Interference

In this section, the probability of bit error for orthogonal, noncoherent FH/MFSK communication systems when band multitone interference is present is obtained. As previously mentioned, for band multitone interference only the case of at most a single interfering tone at one of the $M$ possible orthogonal signaling frequencies in a specific FH band is considered since this provides worst case performance as compared with band multitone interference when more than one interference tone per FH band is allowed. Consequently, the maximum number of interfering tones corresponds to the total number of FH bands, and $N \geq q \geq 1$.

Equally likely $M$-ary symbols are assumed, and for FH/MFSK the $j$th transmitted symbol may be represented as

$$s_m(t) = \sqrt{2}a_c \cos\{2\pi[f_{h_i} + f_1 + (m-1)\Delta f]t + \theta_{mj}\} \quad (1)$$

for $jT_s \geq t \geq (j-1)T_s$ where $i = 1, 2, \cdots, N$ designates the FH band, $m$ is determined by the data and is chosen from $m = 1, 2, \cdots, M$, $f_{h_i}$ and $f_1$ are both integer multiples of the symbol rate and $\Delta f = p/T_s$ with $p$ an integer for orthogonal signaling, and $\theta_{mj}$ is the unknown phase. The amplitude $a_c$ is modeled as a random variable (rv) since the channel is modeled as a fading channel.

For band multitone interference with at most one interference tone per FH band, the probability that a FH band contains an interfering tone is $q/N$, and the probability that a FH band does not contain an interfering tone is $(N-q)/N$. Hence, the probability of symbol error for band multitone interference with at most one interference tone per FH band is

$$P_s = \frac{q}{N}P_s(\text{hop jammed}) + \frac{N-q}{N}P_s(\text{hop not jammed}). \quad (2)$$

For orthogonal signaling, the average probability of bit error is related to the average probability of symbol error by [10]

$$P_b = \frac{M/2}{M-1}P_s \quad (3)$$

where the average energy per bit $E_b$ is related to the average energy per symbol by

$$E_s = (\log_2 M)E_b = kE_b \quad (4)$$

and $E_b = P_cT_b$ where $P_c$ is the average information signal power and $T_b$ is the average bit duration.

Given (2)–(4), in order to obtain the average probability of bit error for FH/MFSK with band multitone interference with at most one interfering tone per FH band, the conditional probabilities of symbol error given that the FH band contains no interference and that the FH band contains a single interfering tone, respectively, must be found.

### *A. Probability of Symbol Error with No Tone Interference*

The conditional probability of symbol error when a FH band contains no interfering tones is the standard probability of symbol error found in the literature for noncoherent, orthogonal MFSK for channels with no fading when AWGN is present. It is given by [10]

$$P_s(\text{hop not jammed}|a_c) = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp\left[\frac{-na_c^2}{(n+1)\sigma^2}\right] \quad (5)$$

where $\sigma^2 = N_0/T_s$ is the power due to AWGN at the integrator outputs and $a_c^2$ is the signal power at the output of the detector branch corresponding to the signal. Equation (5) is conditioned on the absence of interference tones in the FH band and, since the channel is modeled as a Ricean fading channel, on the amplitude of the information signal.

Since the Ricean fading channel is assumed to affect both the information signal and the multitone interfering signal, the amplitudes of both the signal tone and the interference tones are modeled as Ricean rv's. Hence, [11]

$$f_{A_i}(a_i) = \frac{a_i}{\sigma_i^2} \exp\left(-\frac{a_i^2+\alpha_i^2}{2\sigma_i^2}\right) I_0\left(\frac{a_i\alpha_i}{\sigma_i^2}\right) u(a_i) \quad (6)$$

where $u(\bullet)$ is the unit step function, $i = c, J$ represents the information signal and the jamming tone, respectively, $\alpha_i^2$ is the power of the direct signal component and $2\sigma_i^2$ is the power of the diffuse signal component of the respective tone. The total average signal power of a tone is $P_i = \alpha_i^2 + 2\sigma_i^2$ and in this paper is assumed to remain constant from hop to hop. We will find it convenient to define the signal-to-noise power ratios of the direct and diffuse signal components as

$$\rho_i = \frac{\alpha_i^2}{\sigma^2} \quad (7)$$

and

$$\xi_i = \frac{2\sigma_i^2}{\sigma^2} \quad (8)$$

respectively.

Integrating the product of (5) and (6) over all possible values of $a_c$ and using the notation introduced in the previous paragraph, we get the unconditional probability of symbol error for a hop with no tone jamming as [12]

$$P_s(\text{hop not jammed}) = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{1+n(1+\xi_c)} \binom{M-1}{n} \cdot \exp\left[\frac{-n\rho_c}{1+n(1+\xi_c)}\right]. \quad (9)$$

### *B. Probability of Symbol Error with Tone Interference in the Signal Branch*

The multiple tone interfering signals are represented by

$$s_{J_m}(t) = \sqrt{2}a_J \cos\{2\pi[f_{h_i} + f_1 + (m-1)\Delta f]t + \theta_J\} \quad (10)$$

for $jT_s \geq t \geq (j-1)T_s$ where $m$ is selected randomly and is chosen from $m = 1, 2, \cdots, M$, the phase $\theta_J$ is modeled as a uniform rv and, as previously mentioned, the amplitude $a_J$ is modeled as a Ricean rv. When a FH band contains an interfering tone, the probability that the interfering tone corresponds to the signal branch is $1/M$, while the probability that the interfering tone corresponds to one of the $M-1$ branches other than the signal branch is $(M-1)/M$. Hence, the conditional probability of symbol error when the FH band contains a single interfering tone is

$$P_s(\text{hop jammed}) = \frac{1}{M}P_s(\text{hop jammed}|\text{signal branch jammed}) + \frac{M-1}{M}P_s(\text{hop jammed}|\text{signal branch not jammed}). \quad (11)$$

In this subsection, the conditional probability of symbol error when the interfering tone corresponds to the signal branch is obtained. The conditional probability of symbol error when the interfering tone does not correspond to the signal branch is obtained in the following subsection.

When the frequency of the interference tone is the same as the frequency of the signaling tone, it is straightforward to show that the output signal power of the detector branch corresponding to the signal is modified such that

$$a_c^2 \Rightarrow a_c^2 + a_J^2 + 2a_c a_J \cos\theta \quad (12)$$

where $\theta$ represents the phase difference between the signal tone and the interference tone and is modeled as a uniform rv over $[0, 2\pi]$. Making this substitution in (5), and removing the conditioning on $\theta$, we obtain the conditional probability of symbol error [8]

$$P_s(\text{hop jammed}|\text{signal branch jammed}, a_c, a_J) = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp\left[\frac{-n(a_c^2+a_J^2)}{(n+1)\sigma^2}\right] \cdot I_0\left[\frac{2na_ca_J}{(n+1)\sigma^2}\right] \quad (13)$$

where $I_0(\bullet)$ is the modified Bessel function of the first kind and order zero, and the integral representation of $I_0(\bullet)$ is used to obtain (13). Now, in order to remove the conditioning on $a_c$ and $a_J$, we evaluate

$$P_s(\text{hop jammed}|\text{signal branch jammed}) = \int_0^\infty f_{A_J}(a_J) \int_0^\infty f_{A_c}(a_c) P_s(\text{hop jammed}|\text{signal branch jammed}, a_c, a_J)\, da_c\, da_J. \quad (14)$$

This double integral can be evaluated with the aid of (6.633.4) on p. 718 of [13] to obtain

$$P_s(\text{hop jammed}|\text{signal branch jammed}) = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{1+n(1+\xi_c+\xi_J)} \binom{M-1}{n} \cdot \exp\left[\frac{-n(\rho_c+\rho_J)}{1+n(1+\xi_c+\xi_J)}\right] \cdot I_0\left[\frac{2n\sqrt{\rho_c\rho_J}}{1+n(1+\xi_c+\xi_J)}\right]. \tag{15}$$

### C. *Probability of Symbol Error with Tone Interference in a Nonsignal Branch*

In this subsection, the probability of symbol error is obtained for the case of a single interfering tone different from the signal tone present in the FH band. The outputs of each of the $M$ branches of the FH/MFSK receiver are represented by the independent rv $X_i, i = 1, 2, \cdots, M$. Since, except for the interference tones, all other noise is modeled as additive white Gaussian noise, the signal at the output of one of the $M$ receiver branches when either a signal tone or an interference tone is present in that branch can be modeled as a rv having the conditional probability density function (pdf) [11]

$$f_{X_m}(x_m|a_i) = \frac{1}{2\sigma^2} \exp\left(-\frac{x_m + 2a_i^2}{2\sigma^2}\right) \cdot I_0\left(\frac{a_i\sqrt{2x_m}}{\sigma^2}\right) u(x_m) \tag{16}$$

where $i = c, J$ represents the signal and the interference tone, respectively. Since a Ricean fading channel is assumed, $a_i$ is a Ricean rv, and the unconditional pdf for $X_m$ is found by evaluating

$$f_{X_m}(x_m) = \int_0^\infty f_{X_m}(x_m|a_i) f_{A_i}(a_i)\, da_i. \tag{17}$$

Substituting (6) and (16) into (17) and integrating, we get

$$f_{X_m}(x_m) = \frac{1}{2(\sigma^2+2\sigma_i^2)} \exp\left[-\frac{1}{2}\left(\frac{x_m+2\alpha_i^2}{\sigma^2+2\sigma_i^2}\right)\right] \cdot I_0\left(\frac{\alpha_i\sqrt{2x_m}}{\sigma^2+2\sigma_i^2}\right) u(x_m). \tag{18}$$

The signal at the output of one of the $M$ receiver branches when neither a signal tone nor an interference tone is present in that channel can be modeled as a rv having the pdf obtained from (16) when $a_i \to 0$. Hence

$$f_{X_m}(x_m) = \frac{1}{2\sigma^2} \exp\left[-\frac{x_m}{2\sigma^2}\right] u(x_m). \tag{19}$$

Without loss of generality, due to the symmetry of the receiver, we can assume that the signal is present in branch 1. Hence,

$$P_s(\text{hop jammed}|\text{signal branch not jammed}) = 1 - \Pr\left[X_1 > X_2 \bigcap X_1 > X_3 \bigcap \cdots \bigcap X_1 > X_M\right]. \tag{20}$$

Also, without loss of generality due to the symmetry of the receiver, we can assume that the interference tone is present in branch 2. Since the outputs of each branch of the FH/MFSK demodulator are assumed to be independent, and since the rv's that represent the output of each channel that do not contain either the signal or an interference tone are identical, (20) can be expressed as

$$P_s(\text{hop jammed}|\text{signal branch not jammed}) = 1 - \int_0^\infty f_{X_1}(x_1|s_1, s_{J_2}) \cdot \left\{\int_0^{x_1} f_{X_2}(x_2|s_1, s_{J_2})\, dx_2 \cdot \left[\int_0^{x_1} f_{X_3}(x_3|s_1, s_{J_2})\, dx_3\right]^{M-2}\right\} dx_1. \tag{21}$$

Using (19), we can evaluate the square bracketed term in (21) to obtain

$$\left[\int_0^{x_1} f_{X_3}(x_3|s_1, s_{J_2})\, dx_3\right]^{M-2} = \sum_{n=0}^{M-2} (-1)^n \binom{M-2}{n} \exp\left[\frac{-nx_1}{2\sigma^2}\right]. \tag{22}$$

Now using (18) with $i = J$ and $m = 2$, we can evaluate the integral over $x_2$ in (21) to obtain

$$\int_0^{x_1} f_{X_2}(x_2|s_1, s_{J_2})\, dx_2 = 1 - \mathcal{Q}\left(\sqrt{\frac{2\alpha_J^2}{\sigma^2+2\sigma_J^2}}, \sqrt{\frac{x_1}{\sigma^2+2\sigma_J^2}}\right) \tag{23}$$

where $\mathcal{Q}(a, b)$ is Marcum's $Q$-function [10], [11], [14]. Finally, using (18) with $i = c$ and $m = 1$, (22), and (23), we can evaluate (21) with the aid of (F.2.4) on p. 395 of [14] to obtain

$$P_s(\text{hop jammed}|\text{signal branch not jammed}) = 1 - \sum_{n=0}^{M-2} \frac{(-1)^n}{1+n(1+\xi_c)} \binom{M-2}{n} \cdot \exp\left[\frac{-n\rho_c}{1+n(1+\xi_c)}\right] \mathcal{G}_n(\rho_c, \xi_c, \rho_J, \xi_J) \tag{24}$$

where

$$\mathcal{G}_n(\rho_c, \xi_c, \rho_J, \xi_J) = 1 - \frac{1+\xi_c}{\beta_n}\left[1 - \mathcal{Q}\left(\sqrt{\frac{2\rho_c}{[1+n(1+\xi_c)]\beta_n}}, \sqrt{\frac{2\rho_J[1+n(1+\xi_c)]}{\beta_n}}\right)\right] + \frac{(1+\xi_J)[1+n(1+\xi_c)]}{\beta_n} \mathcal{Q}\left(\sqrt{\frac{2\rho_J[1+n(1+\xi_c)]}{\beta_n}}, \sqrt{\frac{2\rho_c}{[1+n(1+\xi_c)]\beta_n}}\right) \tag{25}$$

and

$$\beta_n = 2 + \xi_c + \xi_J + n(1+\xi_c)(1+\xi_J). \tag{26}$$

Using the identity [14]

$$\mathcal{Q}(a,b) = 1 - \mathcal{Q}(b,a) + \exp\left[\frac{-(a^2+b^2)}{2}\right] I_0(ab) \tag{27}$$

in (25), we get

$$\begin{aligned}\mathcal{G}_n(\rho_c,\xi_c,\rho_J,\xi_J) &= \mathcal{Q}\left(\sqrt{\frac{2\rho_c}{\beta_n[1+n(1+\xi_c)]}}, \sqrt{\frac{2\rho_J[1+n(1+\xi_c)]}{\beta_n}}\right) \\ &\quad - \frac{(1+\xi_J)[1+n(1+\xi_c)]}{\beta_n} \\ &\quad \cdot \exp\left\{-\frac{\rho_c + \rho_J[1+n(1+\xi_c)]^2}{\beta_n[1+n(1+\xi_c)]}\right\} I_0\left(\frac{2\sqrt{\rho_c\rho_J}}{\beta_n}\right)\end{aligned} \tag{28}$$

which is computationally more efficient than (25) since Marcum's $Q$-function only appears once. Furthermore, using (27) on $\mathcal{G}_0(\rho_c,\xi_c,\rho_J,\xi_J)$, we can express (24) as

$$\begin{aligned}&P_s(\text{hop jammed}|\text{signal branch not jammed}) \\ &= \mathcal{Q}\left(\sqrt{\frac{2\rho_J}{2+\xi_c+\xi_J}}, \sqrt{\frac{2\rho_c}{2+\xi_c+\xi_J}}\right) \\ &\quad - \frac{1+\xi_c}{2+\xi_c+\xi_J}\exp\left[-\frac{\rho_c+\rho_J}{2+\xi_c+\xi_J}\right] \\ &\quad \cdot I_0\left(\frac{2\sqrt{\rho_c\rho_J}}{2+\xi_c+\xi_J}\right) \\ &\quad + \sum_{n=1}^{M-2} \frac{(-1)^n}{1+n(1+\xi_c)}\binom{M-2}{n} \\ &\quad \cdot \exp\left[\frac{-n\rho_c}{1+n(1+\xi_c)}\right]\mathcal{G}_n(\rho_c,\xi_c,\rho_J,\xi_J)\end{aligned} \tag{29}$$

where the two leading terms represent the result for the binary case, and it is understood that the summation is zero when $M = 2$.

This completes the analysis of the probability of bit error for FH/MFSK communication systems with AWGN and band multitone jamming when there is at most one interfering tone per FH band. Equations (15) and (29) are used in (11). Then, (9) and (11) are used in (2)–(3).

## IV. FH/BFSK Receiver Performance with Independent Multitone Interference

In this section, the performance of noncoherent, orthogonal FH/BFSK communication systems over Ricean fading channels with independent multitone interference is examined. For binary systems, independent multitone interference implies that a FH band may contain either one or two equal power interference tones at the frequencies of the orthogonal signaling frequencies. Hence, there are a maximum of $2 \times N$ equal power interfering tones, and $2N \geq q \geq 1$.

There are two ways in which a FH bin can contain only one interfering tone, and the probability of either of these possibilities is

$$\begin{aligned}&\Pr(s_{J_1} \textstyle\bigcap \text{branch 2 not jammed}|s_{J_1}) \\ &= \Pr(s_{J_2} \textstyle\bigcap \text{branch 1 not jammed}|s_{J_2}) \qquad (30) \\ &= \frac{q}{2N}\left(1 - \frac{q-1}{2N-1}\right). \qquad (31)\end{aligned}$$

The probability that a FH bin contains two interfering tones is

$$\Pr(s_{J_1} \textstyle\bigcap s_{J_2}|s_{J_1}) = \frac{q}{2N}\left(\frac{q-1}{2N-1}\right). \tag{32}$$

Finally, the probability that a FH bin contains no interfering tones is

$$\Pr(\text{hop not jammed}) = \left(1 - \frac{q}{2N}\right)\left(1 - \frac{q}{2N-1}\right). \tag{33}$$

Hence, using (30)–(33), we obtain the probability of bit error for FH/BFSK with independent multitone interference as [9]

$$\begin{aligned}P_b &= \frac{q}{2N}\left(1 - \frac{q-1}{2N-1}\right)P_b(\text{hop jammed}|1 \text{ jamming tone}) \\ &\quad + \frac{q}{2N}\left(\frac{q-1}{2N-1}\right)P_b(\text{hop jammed}|2 \text{ jamming tones}) \\ &\quad + \left(1 - \frac{q}{2N}\right)\left(1 - \frac{q}{2N-1}\right)P_b(\text{hop not jammed}).\end{aligned} \tag{34}$$

The probabilities of bit error conditioned on the FH band containing no tone interferers and conditioned on the FH band containing a single tone interferer are identical to those found for band multitone interference in the last section with $M = 2$. Hence, $P_b$(hop not jammed) is obtained from (9) with $M = 2$, while $P_b$(hop jammed|1 jamming tone) is obtained from (11), (15), and (29) with $M = 2$.

For a binary system, two tone interferers in a single FH band implies that both the signal branch and the nonsignal branch contain an interfering tone. Hence, rather than find the probability of bit error conditioned on the FH band containing two tone interferers from first principles, we recognize that $P_b$(hop jammed|2 jamming tones) can be obtained from the result for the probability of symbol error with tone interference in a nonsignal branch previously obtained in Section III-C if the output signal power of the detector branch corresponding to the signal is modified according to (12). Consequently, in (29) with $M = 2$, we first let $\xi_c = \xi_J \to 0, \rho_c \to a_c^2/\sigma^2$, and $\rho_J \to a_J^2/\sigma^2$ to obtain the conditional probability of bit error for noncoherent, orthogonal FH/BFSK with tone interference in the nonsignal branch and no channel fading. Then, in order to obtain the probability of bit error conditioned on the FH band containing two tone interferers, we make the substitution for $a_c^2$ indicated by (12). The result of these manipulations is

$$\begin{aligned}&P_b(\text{hop jammed}|2 \text{ jamming tones}, a_c, a_J, \theta) \\ &= \mathcal{Q}\left(\frac{a_J}{\sigma}, \sqrt{\frac{a_c^2 + a_J^2 + 2a_ca_J\cos\theta}{\sigma^2}}\right)\end{aligned}$$

$$- \frac{1}{2} \exp\left[\frac{-(a_c^2 + 2a_J^2 + 2a_c a_J \cos\theta)}{2\sigma^2}\right] \cdot I_0\left(\frac{a_J}{\sigma}\sqrt{\frac{a_c^2 + a_J^2 + 2a_c a_J \cos\theta}{\sigma^2}}\right) \tag{35}$$

which is conditional on $a_c, a_J$, and $\theta$. In order to remove the conditioning on $\theta, a_c$, and $a_J$, we must evaluate

$$P_b(\text{hop jammed}|2\text{ jamming tones}) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} f_{A_J}(a_J)\int_0^{\infty} f_{A_c}(a_c)P_b(\text{hop jammed}|2\text{ jamming tones}, a_c, a_J, \theta)\, da_c\, da_J\, d\theta. \tag{36}$$

Equation (36) must be evaluated numerically where the numerical evaluation of (36) is somewhat simplified by taking advantage of the symmetry of the integrand with respect to $\theta$ which allows the range of integration over $\theta$ to be halved while dividing by $\pi$ rather than $2\pi$.

## V. Numerical Results

The performance of the FH/MFSK noncoherent receiver is investigated for both worst case multitone interference and for a fixed number of interfering tones as a function of the ratio of the information signal power to the total interference power $P_c/P_{J_T}$. Worst case performance is approximated by choosing $q_{wc}$ to be either the integer portion of $P_{J_T}/P_c$ or 1, whichever is larger. This makes the power of a single interfering tone $P_J \geq P_c$ ($P_J \approx P_c$ for $q \gg 1$) and is the condition that is used to compute worst case performance when thermal noise is zero. It is found that this condition remains essentially worst case for all reasonable levels of thermal noise. In addition, the effect of a broad range of channel fading on system performance is examined, from near the limit of essentially no fading where $\alpha_i^2/2\sigma_i^2$ is very large down to the Rayleigh limit where $\alpha_i^2/2\sigma_i^2 = 0$. For purposes of numerical computation, all results are obtained with $E_b/N_0 = 13.35$ dB and $N = 1000$. The results obtained here are also compared with the performance obtained when thermal noise is zero. For band multitone interference with a single interfering tone per hop and zero thermal noise, the probability of bit error is independent of $M$ and is given by [1], [5]

$$P_b = \frac{q}{N}\left[\frac{1}{2}u\left(\frac{P_{J_T}}{q} - P_c\right)\right] \tag{37}$$

while for independent multitone interference with zero thermal noise, the probability of bit error for FH/BFSK is [6]

$$P_b = \frac{q}{2N}\left(1 - \frac{q-1}{2N-1}\right)\frac{1}{2}u\left(\frac{P_{J_T}}{q} - P_c\right) + \frac{q}{2N}\left(\frac{q-1}{2N-1}\right)\frac{1}{\pi}\cos^{-1}\left(\sqrt{\frac{qP_c}{4P_{J_T}}}\right) \cdot u\left(\frac{P_{J_T}}{q} - \frac{P_c}{4}\right). \tag{38}$$

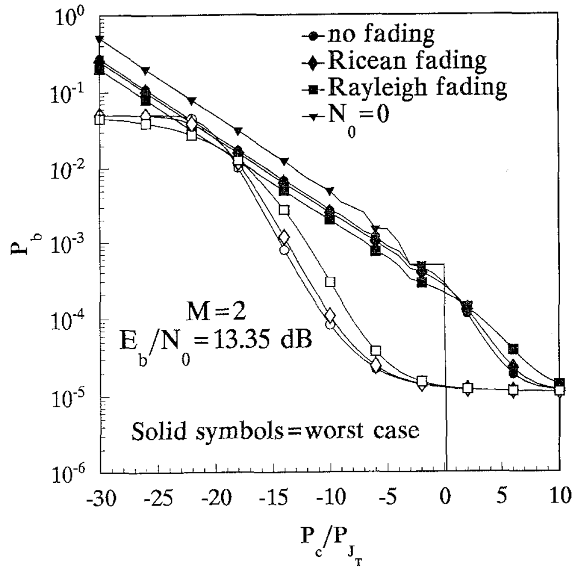


Fig. 1. Performance of a FH/BFSK noncoherent receiver with essentially no information signal fading for band multitone interference with various conditions of fading of the multiple interference tones. The open symbols represent performance obtained with a fixed number of interference tones ($q = 100$).

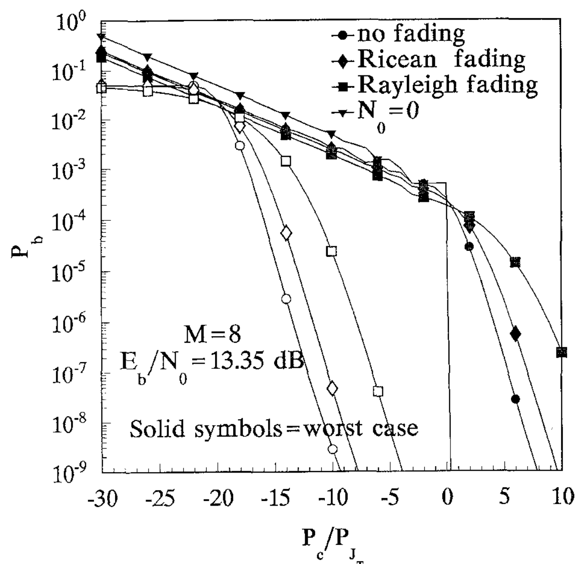


Fig. 2. Performance of a FH/8FSK noncoherent receiver with essentially no information signal fading for band multitone interference with various conditions of fading of the multiple interference tones. The open symbols represent performance obtained with a fixed number of interference tones ($q = 100$).

### A. Band Multitone Interference

The performance of FH/MFSK with band multitone interference when the information signal is essentially unaffected by channel fading ($\alpha_c^2/2\sigma_c^2 = 1000$) with $M = 2$ and $M = 8$ is illustrated in Figs. 1 and 2, respectively. In each case, performance is computed both for $q = q_{wc}$, which implies $P_J \approx P_c$ for all $P_c/P_{J_T} \leq 0$ dB, and $q = 100$, which implies $P_J > P_c$ for all $P_c/P_{J_T} < -20$ dB, as well as for essentially no fading of the interfering

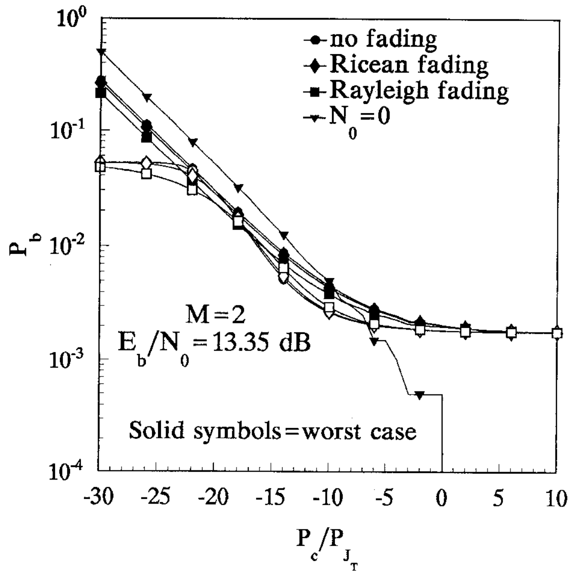


Fig. 3. Performance of a FH/BFSK noncoherent receiver with Ricean fading of the information signal ($\alpha_c/2\sigma_c^2 = 10$) for band multitone interference with various conditions of fading of the multiple interference tones. The open symbols represent performance obtained with a fixed number of interference tones ($q = 100$).

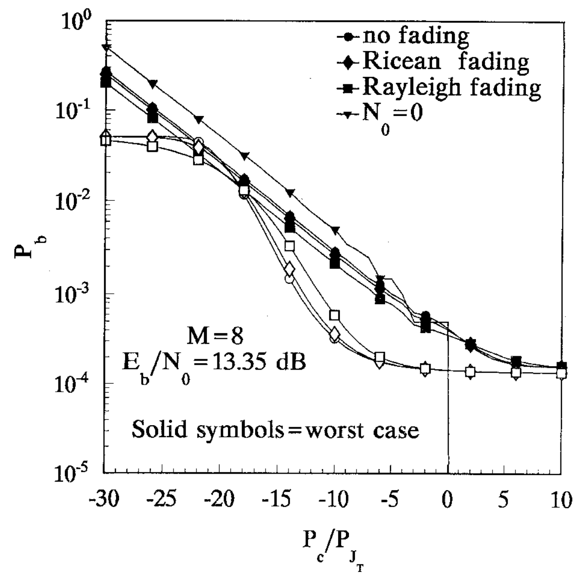


Fig. 4. Performance of a FH/8FSK noncoherent receiver with Ricean fading of the information signal ($\alpha_c/2\sigma_c^2 = 10$) for band multitone interference with various conditions of fading of the multiple interference tones. The open symbols represent performance obtained with a fixed number of interference tones ($q = 100$).

tones ($\alpha_J^2/2\sigma_J^2 = 1000$), Ricean fading of the interfering tones ($\alpha_J^2/2\sigma_J^2 = 10$), and Rayleigh fading of the interfering tones ($\alpha_J^2/2\sigma_J^2 = 0$). Also shown, is worst case performance with zero thermal noise as computed from (37). As can be seen, when $P_J \geq P_c$, there is virtually no difference in the performance obtained with no fading and that obtained with Ricean fading of the interference tones. Indeed, there is very little effect on performance for the extreme case of Rayleigh fading of the interference tones. This is true regardless of $M$. On the other hand, when $P_c > P_J$, channel fading has a much more significant effect, and performance is poorest when the interfering tones experience Rayleigh fading. This phenomenon becomes more pronounced as $M$ increases, and for $M = 8$ a channel with no fading of the interference tones provides as much as a 5 dB advantage over one with Rayleigh fading. The foregoing may initially seem counterintuitive; however, it seems reasonable when we consider that a small increase in the amplitude of the interference tones will significantly increase the probability of bit error when $P_c > P_J$. On the other hand, $P_b$ is already maximized when $P_J \geq P_c$. As might be expected, worst case performance obtained with zero thermal noise is only slightly pessimistic as long as $P_J \geq P_c$ but deviates significantly from actual performance when $P_c > P_J$.

The performance of FH/MFSK with band multitone interference when the information signal experiences Ricean fading ($\alpha_c^2/2\sigma_c^2 = 10$) with $M = 2$ and $M = 8$ is illustrated in Figs. 3 and 4, respectively. As before, in each case performance is computed both for $q = q_{wc}$ and $q = 100$ as well as for essentially no fading, Ricean fading, and Rayleigh fading of the interfering tones. The trends in receiver performance observed when the information signal does not experience fading are similar to those observed in Figs. 3 and 4 except that the degradation in performance obtained for $P_c > P_J$ when the interfering tones experience Rayleigh fading is much less pronounced. As in the case of essentially no fading of the information signal, when $P_J \geq P_c$, there is virtually no difference between performance obtained when the interfering tones experience no fading and when they experience Ricean fading regardless of $M$; and there is only a minor effect on performance for the extreme case of Rayleigh fading of the interference tones.

The performance of FH/MFSK with band multitone interference when the information signal experiences Rayleigh fading with $M = 2$ and $M = 8$ is illustrated in Fig. 5. In this case, only worst case performance is computed for essentially no fading, Ricean fading, and Rayleigh fading of the interfering tones. The trends in receiver performance observed when the information signal does not experience fading as compared to when the information signal experiences Ricean fading continue in the case of Rayleigh fading; and as can be seen, there is virtually no difference between the performance obtained when the interfering tones experience no fading and when they experience either Ricean or Rayleigh fading regardless of $M$ and regardless of whether $P_c > P_J$ or $P_J \geq P_c$.

### *B. Independent Multitone Interference*

The performance of FH/BFSK with independent multitone interference when the information signal is essentially unaffected by channel fading ($\alpha_c^2/2\sigma_c^2 = 1000$) is illustrated in Fig. 6. Receiver performance is computed both for $q = q_{wc}$ and $q = 100$ as well as for essentially no fading of the interfering tones ($\alpha_J^2/2\sigma_J^2 = 1000$) and Rayleigh fading of the interfering tones. Also shown, is worst case performance with zero thermal noise as computed from (38). As can be seen by

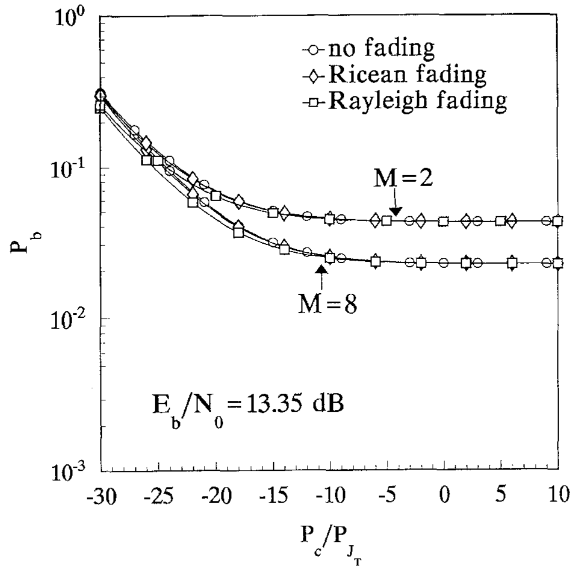


Fig. 5. Worst case performance of a FH/MFSK noncoherent receiver with Rayleigh fading of the information signal for band multitone interference with various conditions of fading of the multiple interference tones.

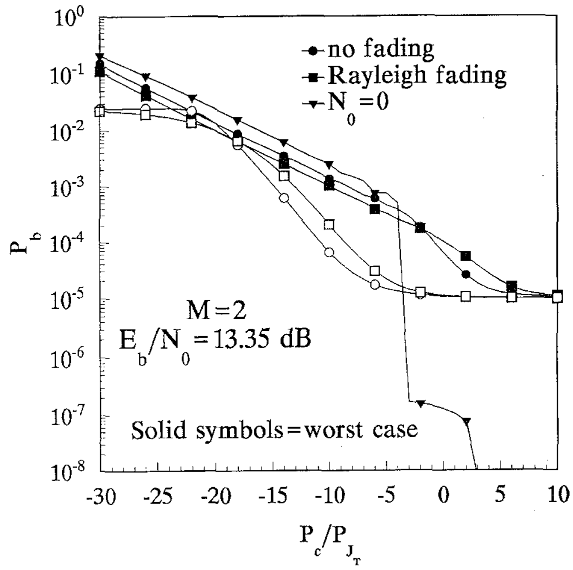


Fig. 6. Performance of a FH/BFSK noncoherent receiver with essentially no information signal fading for independent multitone interference with various conditions of fading of the multiple interference tones. The open symbols represent performance obtained with a fixed number of interference tones ($q = 100$).

comparing Fig. 1 with Fig. 6, both the worst case performance and the performance for a fixed number of interfering tones obtained with band multitone interference of FH/BFSK are only marginally poorer, roughly about a factor of two, than that obtained with independent multitone interference; and the effect of fading of the interference tones on overall receiver performance is virtually identical in the two cases. The latter observation is not surprising given that, when $P_J \geq P_c, P_b$ (hop jammed |2 jamming tones) and $P_b$(hop jammed |1 jamming tone) in (34) are the same order of magnitude; while when $P_c > P_J, P_b$(hop jammed |2 jamming tones) is at most an order of magnitude larger than $P_b$(hop jammed |1 jamming tone). Hence, the contribution to the overall probability of bit error from $P_b$(hop jammed |2 jamming tones) that arises with independent multitone interference cannot dominate overall performance when either $2N \gg q$ and/or $P_J \geq P_c$ as is the case for worst case independent multitone interference. In Fig. 6, the condition $2N \gg q$ corresponds to $P_c/P_{J_T} \geq -23$ dB and the condition $P_J \geq P_c$ corresponds to $P_c/P_{J_T} \leq 0$ dB for worst case performance. For the fixed $q$ case illustrated in Fig. 6, $2N \gg q$ is always true, while $P_J \geq P_c$ corresponds to $P_c/P_{J_T} \leq -20$ dB.

## VI. Conclusion

When the information signal is not affected by fading but the multiple interference tones are, the effect on the overall performance of the FH/MFSK system is very small when the power of the individual multiple interference tones is greater than or equal to the information signal power ($P_J \geq P_c$), a condition typical of worst case multitone interference. When the information signal power exceeds the power of the individual multiple interference tones ($P_c > P_J$), a condition typical of a fixed number of interfering tones with insufficient total interference power or an underestimation of the information signal power, poorer overall system performance is obtained when the multiple interference tones experience fading. This trend is accentuated as $M$ increases. When the information signal experiences fading, the effect of fading multiple interference tones on overall system performance lessens, and for a Rayleigh faded information signal, fading of the multiple interference tones has no effect on overall system performance regardless of $M$.

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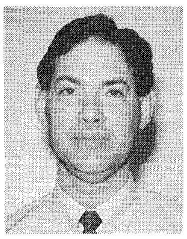

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