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A note on indirect adaptive control with stabilizable plant estimates

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(7):

$$\bar{P}^*(2,2) = F_2^*[\bar{P}^*(2,2) - P^*(2,1)H_1^*\Sigma^{-1}H_1^{*\prime}P^*(1,2)]F_2^{*\prime} + Q^*(2,2) \quad (19)$$

then if (5) and (6) are in steady state, but [7] is not, we have

$$P_{t+1}^*(2,2) - \bar{P}^*(2,2) = F_2^*[P_t^*(2,2) - \bar{P}^*(2,2)]F_2^{*\prime}. \quad (20)$$

Now if $\|F_2^*\|_s \geqslant 1$, (20) does not converge. In fact, the fixed point, $\bar{P}^*(2, 2)$ cannot even be positive semidefinite, *in general,* if (F^*, H^*) is not detectable. We cannot make a stronger statement than this, because there *are* nondetectable systems in which the filter covariance converges, such as the following:

$$x_{1,t} = x_{1,t-1} + w_{1,t}$$
$$x_{2,t} = x_{2,t-1}$$
$$y_t = x_{1,t} + x_{2,t} + v_t. \quad (21)$$

In (21) we have $F = I$, so any two-vector is an eigenvector of F, and in particular, $[1 \; -1]'$, which is orthogonal to H. It is trivial to show that any matrix of the form

$$P = \begin{bmatrix} P_{11} & -P_{22} \\ -P_{22} & P_{22} \end{bmatrix}$$

where

$$P_{11} = P_{22} + 1/2(Q_{11} + \{Q_{11}^2 + 4RQ_{11}\}^{1/2})$$

is a steady-state of the Riccati difference equation for (21).

The argument in Section IV is, essentially, that if $P_t^*(1, 1)$ converges to a point at which $\|\bar{F}_1^*\|_s < 1$, then $\|\bar{F}_{1,t}^*\|_s < 1$, for all $t \geqslant \tau$, for some finite τ. This is, of course, saying no more than that convergence of $P_t^*(1, 1)$ implies convergence of $\bar{F}_{1,t}^*$. The key feature is then that $\|F_2^*\|_s \leqslant 1$; if this condition fails, then $P_t^*(2, 1)$ will not, in general, converge except in special cases. More generally, we note that since it is possible for P_tH to converge without P_t doing so, we cannot decide whether or not K_t will *always* converge by using the approach we have adopted here. Thus, further work remains to be done.

The argument we have used can obviously be applied whenever the canonical form of the system can be arranged so that (F_1^*, H_1^*) is observable, $(F_1^*, Q^*(1, 1))$ is stabilizable, $\|F_2^*\|_s \leqslant 1$, and $H^* = [H_1^{*\prime}, 0, \cdots, 0]$.

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A Note on Indirect Adaptive Control with Stabilizable Plant Estimates

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Abstract—**Recent on-line estimation schemes presented in [1] and [2] suitable for indirect adaptive control applications, are extended to general estimation procedures, and can be fully justified on the basis of simple arguments.**

I. INTRODUCTION

Most indirect adaptive control schemes presented in the literature are based on the certainty equivalence principle, and the compensator parameters are computed on-line from the estimated model of the plant. It turns out that standard recursive estimation techniques do not guarantee the estimated plant to be stabilizable, and therefore the compensator problem at times might not be solvable. In order to guarantee that the adaptive system can recover from these singular situations two approaches have been taken by researchers: 1) ensure convergence of the estimated plant to the true one (assumed to be stabilizable), by means of proper excitation [3]-[5]; or 2) modify the plant estimation procedure in order to guarantee stabilizability at each step [1], [2].

It is this second approach which is addressed in this note. In [1] and [2] two on-line identification schemes are presented which guarantee stabilizability of the estimated plant; these results are obtained as modifications of recursive least-squares algorithms. In this note it is shown that these techniques are not only generally extendable to other identification schemes (such as the projection algorithm), but they can also be explained with a minimum of mathematical technicalities.

The next section gives the details of the argument and presents an identification scheme with the stabilizability property.

II. PARAMETER ESTIMATION WITH STABILIZABILITY CONSTRAINTS

A linear SISO system with $u(\cdot)$ and $y(\cdot)$ as input and output sequences, respectively, can be described in a regression form as

$$y(t) = \theta^{*T}\phi(t-1) \quad (2.1)$$

with

$$\phi(t-1) = [y(t-1), \cdots, y(t-n), u(t-1), \cdots, u(t-n)]^T$$

$t \in Z$ and θ^* an array of unknown parameters. In this formulation the order n and the array of parameters θ^* fully characterize the plant.

In indirect adaptive control the linear controller assumes the form

$$R_t(D)u(t) = S_t(D)y(t) + v(t) \quad (2.2)$$

where R_t, S_t are polynomials with time-varying coefficients in the time delay operator D as $Dx(t) = x(t-1)$, and v is an external input. An alternative form to (2.1) is obtained by writing

$$u(t) = K_t^T\phi(t-1) + v(t) \quad (2.3)$$

in state feedback form. At each step the compensator parameters K_t (of dimension $2n$) are determined on the basis of the recursive estimates of θ^*, say $\hat{\theta}(t)$, computed according to a variety of identification algorithms, as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(\phi(t-2), e(t-1)) \quad (2.4)$$

with $e(t) = y(t) - \hat{\theta}(t)^T\phi(t-1)$, and P being a function determined by

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the particular recursive scheme (mostly projection, recursive least-squares with all their variations [6]). The compensator gains are computed on the basis of the estimated plant, say

$$K_{t+1}=C(\hat{\theta}(t)) \tag{2.5}$$

where usually at least one (or more) sampling interval is allowed for computational purposes.

The mapping $C:R^{2n} \rightarrow R^{2n}$ in (2.5) determines the compensator parameters for the plant $\hat{\theta}(t)$ as a solution of the pole placement problem. Necessary requirement for the solvability of (2.5) is that the estimated plant $\hat{\theta}(t)$ should not only be stabilizable, but also that the gain vector K_t contains entries within limits of saturation.

Therefore, there are areas (such as "stripes") in the parameter space $\Theta \subset R^{2n}$ for which (2.5) does not have a physically implementable solution. In our case we assume Θ to be a rectangular region whose sides are the intervals $[-M, +M]$, for some M positive. Fig. 1 shows an example for a first-order system.

The difficulty encountered in the analysis of many indirect adaptive control algorithms has been the need to guarantee the estimated plant to be stabilizable infinitely often, so that necessary corrective action can be taken.

The algorithm proposed here is based on the following.

Definition: Given the sequences $\phi = \{\phi(t)\}$ and $y = \{y(t)\}$, and a vector $a \in R^{2n}$, define the sequence $\hat{\theta}_a(t)$ as

$$\hat{\theta}_a(t)=\hat{\theta}_a(t-1)+P(\phi(t-2),\, e_a(t-1)) \tag{2.6}$$

with

$$\hat{\theta}_a(0)=a \tag{2.7}$$

$$e_a(t)=y(t)-\hat{\theta}_a(t)^T\phi(t-1).$$

If the sequences ϕ and y are related as in (2.1), then it is a standard result [6] that

$$\lim_{t\to\infty} \frac{e_a(t)^2}{1+\|\phi(t)\|^2}=0$$

$$\lim_{t\to\infty} \|\hat{\theta}_a(t)-\hat{\theta}_a(t-1)\|=0$$

$$\|\hat{\theta}_a(t)-\theta^*\| \leqslant \|\hat{\theta}_a(t-1)-\theta^*\| \leqslant \|a\|. \tag{2.8}$$

In the following we show that the initial condition (2.7) is not constraining the adaptive identification scheme, in the sense that at any instant of time we can easily see "what would have happened if" the initial conditions were different. In fact, both projection and recursive least-squares algorithms with all their variations satisfy a relation like

$$\tilde{\theta}(t)=F_{t-1}\tilde{\theta}(0) \tag{2.9}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta^*$, and $F_{t-1} \in R^{2n\times 2n}$ depends on the sequence ϕ only. In particular,

$$F_t=(I+\phi(t-1)\phi(t-1)^T)^{-1} \cdots (I+\phi(0)\phi(0)^T)^{-1} \tag{2.10}$$

for the projection algorithm, and

$$F_t=[I+P(-1)^{-1}\sum_{\tau=0}^{\tau=t}\phi(\tau)\phi(\tau)^T]^{-1} \tag{2.10}$$

for standard recursive least-squares, with $P(-1)$ the initial covariance matrix.

From (2.9) and (2.7) we can see that

$$\tilde{\theta}_a(t)=F_{t-1}(a-\theta^*)$$

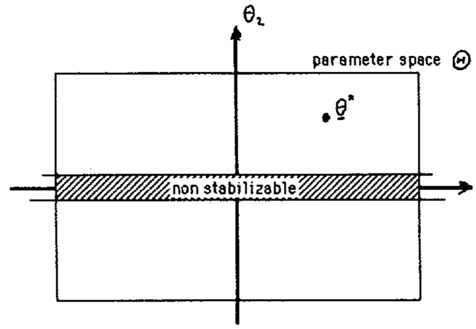

Fig. 1.

and therefore, given any vectors $a_1, a_2 \in R^{2n}$ it follows that

$$\hat{\theta}_{a_1}(t)=\hat{\theta}_{a_2}(t)+F_{t-1}(a_1-a_2). \tag{2.11}$$

This relation just shows that if $\hat{\theta}_{a_1}(t)$ is the estimate due to initial condition a_1, then the parameter estimate due to initial condition a_2 can be readily obtained by (2.11). From this fact the following on-line estimation scheme guarantees asymptotic stabilizability of the estimated model with probability one.

Initialize: $\hat{\theta}_{a_0}(0) = a_0$, with a_0 arbitrary in Θ.

Repeat at each $t \in Z$:

Parameter Update: $\hat{\theta}_{a_{t-1}}(t) = \hat{\theta}_{a_{t-1}}(t-1) + P(\phi(t-2), e_{a_{t-1}}(t-1))$;

Compensator Update:

if $\hat{\theta}_{a_{t-1}}(t)$ is stabilizable **then** $K_{t+1} = C(\hat{\theta}_{a_{t-1}}(t))$; $a_t = a_{t-1}$;

else $K_{t+1} = K_t$; a_t = random uniformly distributed vector in Θ;

end_ if

$\hat{\theta}_{a_t}(t) = \hat{\theta}_{a_{t-1}}(t) + F_{t-1}(a_t - a_{t-1})$

end_ repeat

For the above algorithm we can prove the following.

Main Result: A time instant $\tau \in Z$ exists such that the estimated plant $\hat{\theta}_{a_{t-1}}(t)$ is stabilizable for each $t > \tau$, with probability one.

Proof: Define $\rho \in R$ to be the largest real for which $\|\theta - \theta^*\| < \rho$ implies the model θ is stabilizable. Clearly, $\rho > 0$ by assumption of the plant itself being stabilizable.

Therefore, by the above algorithm and (2.8) if $\|a_\tau - \theta^*\| < \rho$, then $\hat{\theta}_{a_{t-1}}$ is stabilizable for all $t > \tau$. This implies that, if the result does not hold (i.e., the estimated model is unstabilizable infinitely often) the random vector a_t never belongs to the disk centered on θ^* with radius ρ, contradicting the fact that a_t is uniformly distributed in Θ.

III. Conclusions

An on-line parameter estimation scheme for linear discrete-time SISO systems has been presented. It is intended to be an extension of algorithms that recently appeared in the literature [1], [2], to a general class of recursive estimation techniques. Similar considerations can be made in a continuous-time framework.

An interesting problem of current research is a further extension to cases with modeling errors.

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