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A demonstration of ABM validation techniques by applying docking to the Epstein civil violence model

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Abstract

The increased focus of the United States Department of Defense (DoD) on irregular warfare and counterinsurgency has served to identify the lack of credible models and simulations to represent the relevant civilian populations – the centers of gravity of such operations. While agent-based models (ABMs) have enjoyed widespread use in the social science community, many senior DoD officials are skeptical that agent-based models can provide useful tools to underpin DoD analysis, training, and acquisition needs mainly because of validation concerns. This paper uses docking and other forms of alignment that enable the linking of the Epstein civil violence agent-based model results to other models. These examples of model-to-model analysis could serve to assist and encourage DoD ABM human domain model validation efforts.

Keywords

Agent-based model, validation, docking, alignment, civil violence, SIRS model, punctuated equilibrium, 'sparks and prairie fires', threshold distributions

1. Background

In 2006, as the campaigns in Iraq and Afghanistan wore on, the first codification of the doctrine that was being developed and applied was published in FM 3-24/MCWP 3-33.5, Counterinsurgency. This document made clear that the focus of a counterinsurgency campaign is on influencing local populations. Senior US Department of Defense (DoD) officials, who sought to better understand how their armed forces should be organized, trained and equipped for such operations, looked to their modeling and simulation community for help. That community, well-equipped to examine kinetic operations, had little to contribute in this domain. Modeling human and social behavior quickly became the focus of the DoD modeling and simulation (M&S) community, and agent-based models (ABMs) began to receive increased interest and scrutiny. ABMs have enjoyed popularity within several areas of the social sciences, and their use, coupled with the situations in Iraq and Afghanistan, has attracted the attention of DoD.

This increased DoD focus on irregular warfare (IW) and counterinsurgency has served to identify the lack of credible models and simulations to represent civilian populations in conflict environments. In addition, the lack of social science expertise to inform DoD M&S efforts and the lack of data to represent social science phenomenon have also been identified as critical gaps affecting DoD's ability to model IW-like scenarios. While agent-based models have enjoyed widespread use in the social science community, many senior DoD officials are skeptical that agent-based models can provide useful tools to underpin DoD analysis, training, and acquisition needs.

A key requirement for any M&S used by DoD is that it be validated.[1] DoD's definition of validation is "The process of determining the degree to which a model or simulation and its associated data are an accurate representation

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of the real world from the perspective of the intended uses of the model.'' Validation of DoD's kinetic-focused, physics-based combat models (PBCM) has been typically conducted by simulating a well-documented campaign or battle and then comparing outputs to battle outcome data. As DoD organizations contemplated developing ABMs for use in understanding IW and counterinsurgency campaigns, the task of validating such models was also being examined. Where cause-and-effect could be traced in the PBCM, the phenomenon of 'emergence' in ABMs has not yet been shown to be similarly auditable. Some even speculate that validation of human and social behavior models is an unattainable goal.

Recently completed research on DoD validation of IW models emphasized the need for DoD modelers to ensure their conceptual models had the appropriate referents,[2] that is, social science theories that had widespread acceptance in their communities of practice. Several DoD IW modeling efforts had been found either to have no social science underpinnings or to take form of proprietary software wherein the underlying conceptual models were not available for scrutiny. However, having the appropriately pedigreed referents only speaks to *conceptual validation*.[3]

In this paper, we seek to examine the possibility of augmenting the growing body of literature directed toward DoD modeling best practices. We explore docking and other alignment techniques as a step toward establishing the *operational validation* of conflict environment ABMs.[3] Sargent describes several techniques that might be used to establish a model's operational validity, including model-to-model output comparison and event validity, techniques that we employ in this paper, as well as many others (animation, degenerate tests, extreme condition, face validity, traces, etc.). In addition to a somewhat traditional docking practice,[4–16] we also explore the potential to establish operational validity by aligning ABM results with accepted social science theories as well as historical events and phenomenon.

Specifically, we use the Epstein civil violence model (ECVM) to demonstrate docking not only in the sense of the conceptual exercise of connecting two models and their theoretical foundations, but also in the sense of establishing model-to-model comparisons. In the more traditional sense of docking, such as that proposed by Axtell et al.,[4] we align ECVM results to those from another model, in this case the susceptible-infected-removed-susceptible (SIRS) differential equation model for the spread of rebellion in a contested population. In other instances the comparison is made between models from different fields (e.g. computational social sciences and theoretical social sciences); in the sense of Wilensky and Rand,[17] this type of model alignment from different fields and for different purposes constitutes an ambitious attempt at docking.

We replicate the ECVM ABM in NetLogo with several modifications,[18] and we dock the results to demonstrate a means of establishing the operational validity of an ABM. We employ docking in the sense of alignment of computational models, we align results of our implementation of ECVM with human domain models taken from the social sciences, and we align the ECVM results with analogous events and observed phenomenon. Our overarching goal is to demonstrate operational validation techniques. In so doing, we hope to show the usefulness of ABMs for simulating societies in a conflict environment to the DoD M&S user community. In addition, we hope to encourage DoD modelers to begin to pursue validation as a part of their modeling practice and to urge those in DoD commissioning new models to include prudent design requirements, such as conceptual validation, so that these new models have the potential to be validated for their intended uses.

2. The Epstein civil violence conceptual model

Epstein et al. presented an agent-based computational model of civil violence addressing a central authority's efforts to suppress insurrection in its population.[19] The simulation involves two principal actors, the state and its population. The first set of actors represents the central authority or government, which we will refer to as authority agents. The second set of actors represents members of the state's general population, which we simply refer to as agents. These agents are situated in a simulated society, and at any time they may be quiescent, actively rebellious, or incarcerated. Whether or not an agent is actively rebelling depends on its sense of its local environment through a vision radius, its threshold level for violence, its grievance and hardship levels, its arrest probability, and its perceived net risk. Our modifications to the Epstein implementation include variable vision lengths for each type agent, as well as the ability to randomly assign agent threshold levels for violence. We also have extended Epstein's work by incorporating the ability to adjust agent hardship and its perception of government legitimacy. None of the extensions affect the rules by which agents behave.

An agent's grievance (G) is the product of an individual's hardship (H) and its perception of the central authority's illegitimacy, $(1-L)$, and it is calculated based on the following relationship:

$$G = H(1-L)$$

It follows that an agent's grievance may be low due to a legitimate government (L approaching one), even while suffering hardship.

However any agent can become rebellious. This factor is captured by the agent's tolerance level and its inclination to undertake the risk of being noticed by the authority – to actively rebel. Tolerance, T, represents an agent's threshold level; if pushed beyond this value it will join the rebellion by becoming active. Agent tolerance is either fixed for the population, assigned from a uniform distribution centered about some value, or assigned from a normal distribution centered on some value and with some standard deviation. An agent's willingness to take action is based on three components referred to as risk aversion (R), chance of apprehension (P), and deterrence (J). Risk aversion is defined as an agent's willingness to take chances, and an agent's risk aversion is randomly assigned from the uniform distribution over the interval (0, 1), and it remains fixed during the course of each simulation run. The higher the value the more likely the agent is to take risk. The arrest probability, P, for an agent at a given time, is modeled by:

$$P = 1 - exp\left[-k(C/A)_v\right]$$

where C/A represents the ratio of authority agents, C, and agents, A, within the vision range v of the state agent, and k is fixed. An agent's vision of its environment is a Moore neighborhood of lattice positions in our implementation, and it is homogeneous among agent types. In our implementation, agents and state agents can have different vision radii.

Because we allow authority agents and agents to have different vision ranges, we can simulate dynamics in a society where the central authority might have varying degrees of understanding of its people. For instance, an insular central authority that has little understanding of its people would be represented in a simulation by authority agents having a short vision distance. A society where the people are well aware of government and the disposition of fellow citizens could be modeled by agents having a longer vision distance.

The arrest probability equation implies that as the number of authority agents, C, increases in an agent's neighborhood, the less likely it is that an individual will rebel against the central authority. Net risk, N, is calculated as the product of risk aversion, probability of arrest, and the deterrence of jail time if apprehended, given by

$$N = RPJ$$

This leads to the construction of the first agent behavioral rule.

Rule 1: If $G - N > \mathrm{T}$ *then the agent will rebel.*

In this artificial society, the authority agents are much simpler to describe since they possess just one attribute: they seek out and arrest agents that are actively rebelling. Authority agents have an assigned vision range, creating a neighborhood that they inspect during each time step. This leads to the authority agent's behavioral rule.

Rule 2: *Per iteration, each authority agent identifies all rebelling agents within its vision, randomly selects one, and then 'arrests' it*.

The final behavioral rule applies to both agent types; it addresses their movement.

Rule 3: *Move to a random position within vision range*.

Combined, these three simple behavioral rules govern the actions and interactions of all the agents in this artificial society. The society's environment is established on a 40-by-40 torus lattice grid (1600 cells) in NetLogo. Prior to each simulation, the user selects and sets parameters that include the initial number of agents (set through density), jail time, arrest probability parameter, agent rebellion tolerance (fixed or assigned from some distribution), and agents vision distance. For each turn, an agent may exist in one of three states; non-active (not rebelling but susceptible to becoming active), active (rebelling), or arrested.

Having reviewed the Epstein civil violence conceptual model, we note that the literature reflects that establishing both conceptual validity and operational validity are necessary to the process of validating an ABM.[3,20–22] Conceptual validity determines that the theories and assumptions underlying the conceptual model are correct and that the model's structure, logic, and causal relationships are ''reasonable'' for the intended purpose of the model.[3,23] For the sake of replication,[17] conceptual validity requires, at a minimum, a well-documented model.[24] In essence, conceptual validation ensures that the underlying intellectual foundations are sound.

Klemens et al. establish the conceptual validity of the Epstein model through an analysis of the model's underlying structure and an application of appropriate data to underpin the modeling effort that the conceptual model describes.[25] Klemens strongly suggests that hardship, regime legitimacy, and repressive capacity are powerful drivers of decentralized civil unrest.[25] In addition, the ECVM is ''found to be robust across a variety of statistical instruments for the theoretical independent variables.''

We also note that the Epstein model clearly states the rules by which the simulation's agents behave, an aspect of conceptual validation that is of concern to the M&S community, particularly within DoD. Establishing the rules by which agents behave affords replications and modifications of the model as we have done in this study.

We now turn to our purpose: demonstrating the means to establish the operational validity of an agent-based model. According to Sargent,[3] "Operational validation is defined as determining that the model's output behavior has sufficient accuracy for the model's intended purpose over the domain of the model's intended applicability." Operational validity – or external validity – refers to the accuracy and adequacy of the computational model in matching real world data.[5] Techniques for establishing operational validation will vary and depend on the system of interest and most importantly, for DoD purposes, the model's intended use. The following examples use the ECVM as an exemplar – we do not claim the ECVM is a validated model, nor are we claiming we have established the operational validity of any model as a result of docking or alignment. The first example fits the formal definition of docking, while the next three are examples of similar alignment techniques.

3. Docking examples

3.1. Docking example 1: aligning the Epstein model to the SIRS model for the spread of infectious diseases

We draw an analogy between the SIRS model from epidemiology and insurgency mobilization dynamics to obtain another theory for the spread of rebellion. The SIRS model is a refinement of the Kermack and McKendrick SIR epidemic model.[26]

Let $S(t)$ represent that portion of a population that is susceptible to joining a rebellion and thus becoming infected by a revolutionary idea; let $I(t)$ represent those from a population already infected with the revolutionary idea; and let $R(t)$ represent those incarcerated by the state's authority. Due to interactions between those in $S(t)$ and $I(t)$, we assume that $S(t)$ decreases at a rate proportional to the size of $S(t)$ and the size of $I(t)$. Furthermore, if we consider that freed individuals do not directly rejoin the rebellion, then $S(t)$ increases as individuals are freed from incarceration. We assume that losses from $S(t)$ are gains for $I(t)$. Because members of the rebellion can be captured and incarcerated – removed from the general population by the state's authority, then we assume that $I(t)$ decreases at a rate proportional to its size. We assume that those removed or incarcerated are freed at a rate proportional to the number incarcerated. We note, finally, that this description generally matches agent behavior in the Epstein model.

From the above descriptions, we obtain the following system of differential equations representing the SIRS model:

$$\frac{dS}{dt} = -\beta S I + \nu R$$

$$\frac{dI}{dt} = \beta S I - \gamma I$$

$$\frac{dR}{dt} = \gamma I - \nu R$$

where, β represents the rate at which susceptible are lost to the rebelling class, ν represents the rate at which prisoners are freed, and γ represents the rate at which rebels are removed to prison.

We assume that there are no gains or losses to the total population over the course of the rebellion, so

$$S(t) + I(t) + R(t) = N$$

for some constant, N. Finally, we assume that the rebellion begins with one individual while, concurrently, there is some number, S_0, in the susceptible class and none in the removed class. The initial conditions are thus $I(0) = 1, S(0) = S_0$, and $R(0) = 0$. Thus, $N = S_0 + 1$.

It is straightforward to find non-negative steady-state values for the system and that these equilibrium values are stable.[27] This means that once a revolutionary idea is introduced the revolutionary epidemic runs its course until the steady state condition (S_e, I_e, R_e) is achieved. It should be noted that the results of the ordinary differential equations (ODE) model imply that the revolutionary idea persists once it is introduced; that is, I_e is positive. Clearly, from the perspective of the state's authority, it is desirable to have as few rebels as possible. This corresponds to having I_e as small as possible. To achieve this requires an incarceration rate, γ, as large as possible, and/or prolonged incarceration periods, which would reduce ν. Furthermore, increasing S_e might be another objective of a central authority. Noting that increasing S_e correspondingly decreases I_e, then the state might attain an increased S_e by decreasing the infection rate, β. Thus, for the state to limit the extent of a nascent rebellion, decreasing β translates into having a general population with a strong resistance to the revolutionary narrative – in a sense, the population would be inoculated against the revolutionary idea. The state might achieve this through strengthening the population's allegiance to authority – perhaps accomplished through general well-being, or, in less benevolent circumstances, through threat, brutality, or increased indoctrination.

Figure 1 depicts the solution of the system of ODEs with the initial conditions $I(0) = 1, S(0) = S_0$, and $R(0) = 0$, alongside the NetLogo ABM implementation with vision radius set to two for all agents. When all agents are myopic or insular, the rules by which they operate cause them to mix in such a manner that rebellion is endemic at an essentially constant level. This is consistent with the ODE results, where it is assumed that non-rebels/susceptibles and rebels/infected mix continuously,

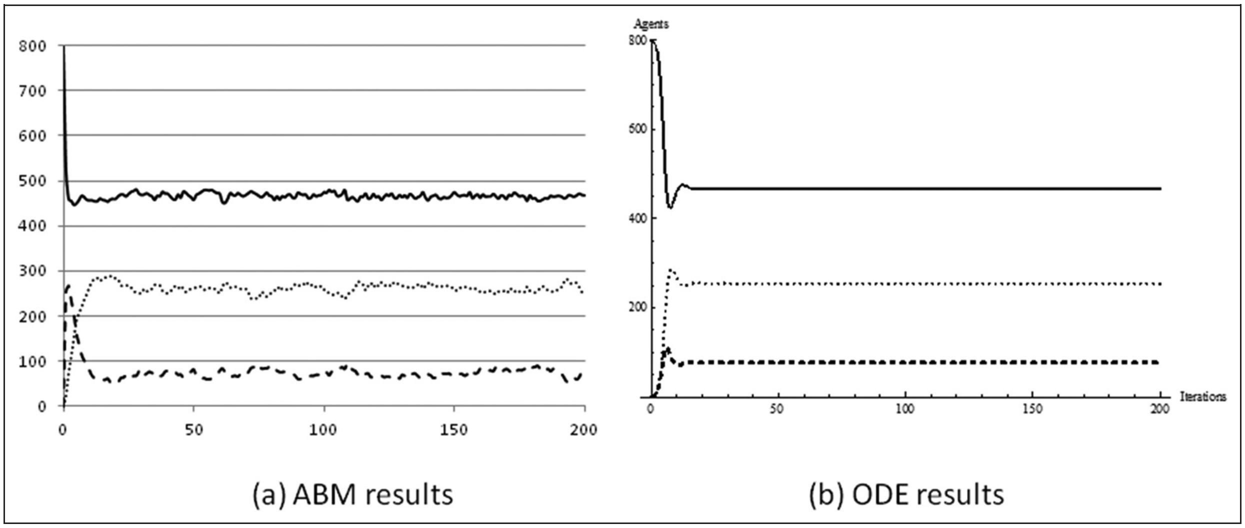

Figure 1. Solid curves denote the number of inactive (ABM) or susceptible (ODE) individuals, dotted curves denote rebelling (ABM) or infected (ODE) individuals, and dashed curves denote imprisoned (ABM) or removed (ODE) individuals as a function of time for (a) ABM civil violence NetLogo agents, and (b) SIRS ordinary differential equation model solution curves. Parameters used in the NetLogo civil violence implementation include authority agent density = 0.04, agent density = 0.5, max jail time = 30 time steps, k = 2.3, T = 0.1, L = 0.82, and both type agents vision = 2 units, where a unit is defined as a single lattice space on the NetLogo artificial landscape. SIRS ODE results from $S(0)$ = 799 and $I(0)$ = 1 with β = 0.00068, γ = 0.31875, and ν = 0.09625.

resulting in a constant level of rebellion. The results suggest that the two implementations capture, at the macroscopic level, the nature of the interactions between state and revolutionary actors in a contested population. This is an example of docking two computational models, and it serves to demonstrate how to establish the operational validity of two theories.[3]

3.2. Docking example 2: aligning the Epstein model to a model from the social sciences, in this case Kuran's 'sparks and prairie fires'

Among the benefits of ABM is that simulations can result in behaviors unattainable by more traditional models such as ODEs in example 1. For instance, ECVM results also align with observed phenomenon from socio-political systems. We obtain the interesting results by assigning various 'vision distances' to the ECVM agents.

Figure 2 depicts one such situation. The emergent behavior in this instance – a civil war – emerged from interactions among agents given a vision radius of ten lattice units while keeping authority agent vision radii at one unit. This might represent the situation alluded to earlier where the government is detached, unaware, or unconcerned with knowing about the population whereas the populace is very aware of its environment.

Prior to the spark that occurs near the 120th time step of the simulation shown in Figure 2, the originally established equilibrium condition finds approximately six agents rebelling at any time. However, at the 120th time step conditions are favorable for the ignition of a rebellion: agent dissatisfaction is sufficiently high and concentrated, and authority agent distribution is sufficiently sparse. The result: rebellious activity spreads in a flash. The myopic authority agents – or insular state agents – are unable to control the revolt, and a new equilibrium of approximately 130 actively rebelling agents persists. Rather than decaying to the original equilibrium position (as in the previous

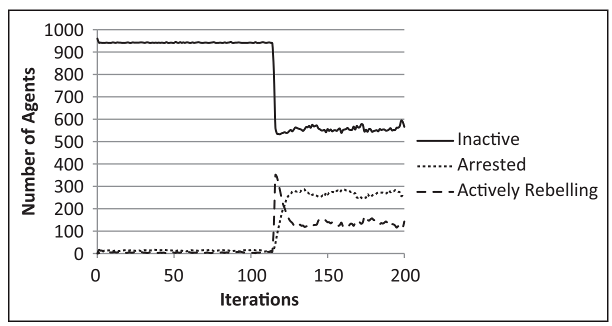

Figure 2. Bifurcated equilibrium, in this case an agent 'civil war' is obtained from the civil violence ABM. This situation matches Kuran's 'sparks and prairie fires', which is conjectured to be the cause of political revolutions in France, Russia, and Iran, among others. This bifurcated equilibrium was obtained when state agents had vision radius set to one lattice unit and agent vision radii set to ten lattice units.

example), a new level of actively rebelling agents is established, indicating a change from the original system order. In essence, a revolution, manifested as an agent 'civil war' has emerged. Similar to the previous example of punctuated equilibrium, this 'bifurcated equilibrium' can be likened to socio-political phenomena, in this case Kuran's 'sparks and prairie fires'.[28] According to Kuran, the French, Russian, and Iranian revolutions are examples of unanticipated events where corruption of public knowledge, modeled here by myopic state agents, led to unanticipated revolts. These revolutions, analogous to the ECVM's emergent civil war, serve to align the ABM to an hypothesis from the social sciences, as well as the aforementioned revolutions: Kuran's model for the cause of revolution is docked to the behavior of the ECVM. The simulated revolutions also align Kuran's theory and the ABM implementation through event validity: the historical record reflects at least three unanticipated revolutions. This provides another example of demonstrating the operational validity of an ABM.

3.3. Docking example 3: aligning the Epstein model to another theory from the social sciences, in this case Granovetter's 'threshold models of collective behavior'

Another result from the ABM implementation is shown in Figure 3, which depicts agent rebellion intensity for various uniformly distributed agent thresholds centered about mean threshold values. Here we define rebellion intensity as the total number of agents that have rebelled during a fixed simulation period. We assigned agent rebellion activity thresholds according to a uniform distribution centered at values shown along the horizontal axis when agent vision is set to ten lattice units and state agents had vision radii set to one lattice unit. Here an agent's riot threshold is a number carried throughout a simulation run, and rebellion results from the configuration of costs and benefits of riot/no riot behaviors in a particular simulation setting. Thresholds, movements, and initial agent distributions in the NetLogo environment vary from one simulation to another. Inevitably, some situations engage riot actors more than others. We note that riot intensity and variability decrease as the mean threshold distribution increases, which makes sense: the probability of an agent becoming active decreases with increased thresholds. The variability in riot intensity for the various distributions validate Granovetter's claim that collective behavior is sensitive to crowd threshold distributions.[29] We leave for future work investigation of thresholds that may change during the course of a simulation.

3.4. Docking example 4: aligning the Epstein model output to other observed phenomenon to establish event validity

As reported by Epstein,[19] and reproduced in our implementation, 'punctuated equilibrium' is shown in Figure 4. These are likened to riots, 'flash mobs', and other episodic rebellious outbreaks that have been observed throughout human history, particularly of late. The episodic disturbances emerge from our NetLogo implementation when

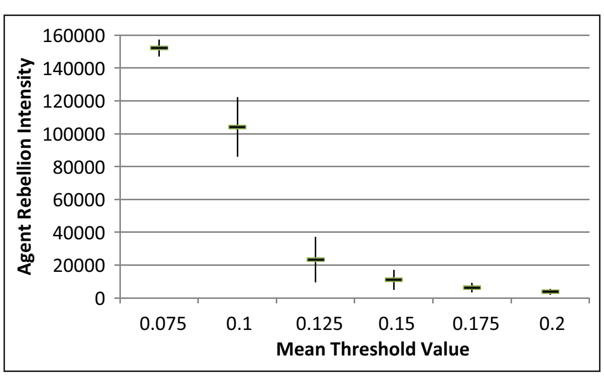

Figure 3. Agent rebellion intensity, defined to be the number of agents rebelling during a fixed period, versus uniform agent threshold distribution centered about the mean threshold value. In this case, agent vision is set to 10 lattice units, and state agent vision set to one unit. Vertical bars represent maximum and minimum intensity values over thirty runs. Results indicate that rebellion intensity depends on the initial agent threshold distribution.

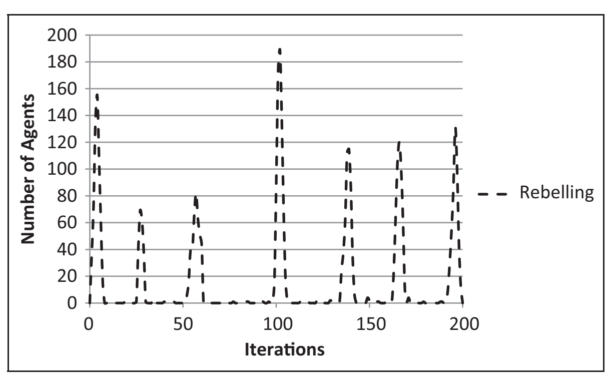

Figure 4. Episodic disturbances of quiescent state depicted by the number of rebelling agents (dashed curve) versus time. Parameters used to produce this result include authority agent density = 0.04, agent density = 0.5, max jail time = 30 time steps, k = 2.3, T = 0.1, L = 0.82, and both type agents vision = 7 units, where a unit is defined as a single lattice space on the NetLogo artificial landscape. Note that in each case, the authority agents succeed in quelling the rebellion.

both agent types are given a vision radius of 8 lattice units. Unlike the results in docking example 2, where the original quiescent equilibrium is never restored, in this example the authority agents are sufficiently numerous and aware of rebellious activity that the original quiescent equilibrium state is eventually restored. Likening ECVM results to such socio-political phenomena establishes the model's event validity.[3]

Having presented four examples that demonstrate how to establish the operational validity of an ABM, we note that we do not claim that the ECVM is itself a validated model. Nonetheless, the model provides insight into human behavior that we think should be of interest to DoD, the State Department, and/or other agencies interested in regime stability or instability. The ECVM incorporates grievance, hardship, regime legitimacy, repression, risk, and individual rebellion thresholds for violence as the primary factors affecting a population's propensity for violence. Thus, agencies responsible for enacting measures directed toward influencing these factors might better serve national interests with these factors and their effects in mind. For example, propaganda efforts could be directed toward moving threshold means in directions amenable to inducing either stability or instability, especially if planning and execution were to be coupled with other factors (e.g. male age composition).[30,31]

4. Conclusions

We have demonstrated potential ABM validation techniques by docking the Epstein civil violence model (ECVM) to several other models. We enhanced the ECVM represented in NetLogo with modifications that allow further exploration and investigation. In the traditional sense of aligning model behaviors, we dock results from the solution of the SIRS ODE system to those obtained from the ECVM implementation for certain agent parameters: the similarity of the ODE model solution to the results obtained from the ECVM implementation serves as a demonstration of cross-model validation. We also dock our implementation's results with several examples taken from the social sciences. A result obtained from the ABM, not attainable from the ODE model, matches not only an hypothesis from political science, in this case Kuran's 'sparks and prairie fires', but the ECVM results also match observed phenomenon. For ECVM agent vision settings that replicate a detached polity, unanticipated civil war can arise which serves to align the ECVM to Kuran's theory. In addition to the demonstration of validation through docking, historical events such as the French, Russian, and Iranian revolutions that fit Kuran's theory can also be demonstrated by the ECVM. This alignment with observed phenomenon demonstrates another form of model validation, in this case event validity. Then we showed that agent rebellion activity is sensitive to their rebellion threshold distributions. This we align with Granovetter's threshold theory of collective behavior, another model drawn from political science. Finally, we note that our implementation produces episodic 'punctuated equilibrium', an emergent feature of the Epstein implementation that resembles flash mobs or riots (distinct from revolution in that the original order is restored), another instance where the ECVM permits the demonstration of event validity. By docking the ECVM with several other models, we hope to demonstrate to the DoD M&S community (and others) techniques that can be employed to establish operational validation of ABMs.

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Declaration of conflicting interest

The author declares that there is no conflict of interest.

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