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Generalized Kirchhoff approach to the ocean surface-scatter communication channel. Part II: Second-order functions[a]

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Three second-order functions which characterize the ocean surface-scatter communication channel are derived from the transfer function of the ocean surface. These second-order functions include the two-frequency correlation function or the two-frequency mutual coherence function, the scattering function, and the power spectral density function of the scattered acoustic pressure field. These functions are shown to be dependent upon the general form of the directional wavenumber spectrum. Both the slightly rough and very rough surface cases are included. The interrelationships which exist amongst these functions are demonstrated. As an example, the power spectral density function is computed for the very rough surface case using the Neumann–Pierson directional wavenumber spectrum.

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INTRODUCTION

In part I of this two-part paper (see Ziomek[1]), the following expression for the random, time-varying transfer function of the ocean surface was obtained by using a *generalized* Kirchhoff approach:

$$H(f,t)=\frac{\exp[-jk_{\mathrm{EFF}}(R_1+R_2)]}{R_1R_2}\int_x\int_y Z(k,x,y)\times\exp\{-jk_{\mathrm{EFF}}[lx+my+n\xi(x,y,t')]\}\times\exp\{-jk_{\mathrm{EFF}}[(l_f/2)x^2+(m_f/2)y^2]\}\,dx\,dy\,, \tag{1}$$

where

$$Z(k,x,y)\triangleq D_T(k,x,y)K(x,y)D_R(k,x,y)\,, \tag{2}$$

$$l=\sin\theta_1-\sin\theta_2\cos\psi_2\,, \tag{3}$$

$$m=-\sin\theta_2\sin\psi_2\,, \tag{4}$$

$$n=-(\cos\theta_1+\cos\theta_2)\,, \tag{5}$$

$$l_f=\cos^2\theta_1/R_1+[(1-\sin^2\theta_2\cos^2\psi_2)]/R_2\,, \tag{6}$$

$$m_f=1/R_1+[(1-\sin^2\theta_2\sin^2\psi_2)]/R_2\,, \tag{7}$$

and

$$k_{\mathrm{EFF}}\triangleq k-j[\alpha(f)+\rho_V E\{\sigma_t\}]\,, \tag{8}$$

where $k=2\pi f/c$ is the wavenumber, $\alpha(f)$ is the frequency dependent pressure amplitude attenuation coefficient (in Np/m) due to sound absorption, ρ_V is the volume density function of any randomly distributed discrete point scatterers in the medium (e.g., fish or air bubbles), and $E\{\sigma_t\}$ is the average total cross section of the particles. The generalized Kirchhoff approach uses a Fresnel corrected Kirchhoff integral, *no* small slope approximation, and the Rayleigh hypothesis that the scattered acoustic pressure field can be represented as a superposition of plane waves traveling in many directions.[1] The spherical angles θ_1, θ_2, and ψ_2 and the ranges R_1 and R_2 are defined in Fig. 1. Since *no* small slope approximation was made in deriving Eq. (1), $K(x,y)$ can be thought of as a slope correction factor. The factor $K(x,y)$ is defined in Ziomek.[1] The expressions $D_T(k,x,y)$ and $D_R(k,x,y)$ are the frequency dependent transmit and receive directivity patterns, respectively, projected onto the XY plane. Any vertical deviation of the randomly rough, time-varying ocean surface from the XY plane is represented by the random process $\xi(x,y,t')$ where t' is the retarded time.

Since the transfer function given by Eq. (1) is random, it is more appropriate to characterize surface scatter with second-order functions which can be derived from $H(f,t)$. More specifically, the two-frequency correlation function, the scattering function, and the power

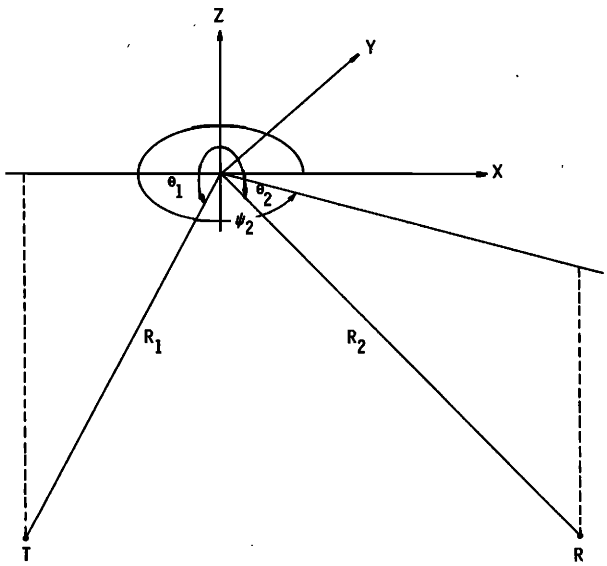

FIG. 1. Spherical angles θ_1, θ_2, and ψ_2. Range R_1 is the distance from the origin ($x=y=z=0$) to the center of the transmit array T, and R_2 is the distance from the origin to the center of the receive array R.

[a] This paper is based, in part, on Chap. 5 of the author's Ph.D. dissertation, "A Scattering Function Approach to Underwater Acoustic Detection and Signal Design," The Pennsylvania State University (1981).

[b] This work was done while the author was at the Applied Research Laboratory, The Pennsylvania State University, State College, PA.

spectral density function are useful second-order functions and, hence, will be derived in this paper. Other investigators, using a variety of different simplifying assumptions, have also derived expressions for some of these functions. However, all three functions have never been considered together in one single paper on surface scatter to the author's knowledge. It is the purpose of this paper, therefore, to derive *general* expressions for all three of these second-order functions using a *consistent* notation, to discuss the significance of these functions, and most important, to demonstrate the interrelationships which exist amongst them. As an example, the power spectral density function is computed for the very rough surface case using the Neumann–Pierson directional wavenumber spectrum.

I. SECOND-ORDER FUNCTIONS

A. Two-frequency correlation function

Ishimaru[2] defines the two-frequency correlation function or the two-frequency mutual coherence function as

$$R_H(f_1, f_2, t_1, t_2) \triangleq E\{H(f_1, t_1)H^*(f_2, t_2)\}, \tag{9}$$

where $E\{\cdot\}$ is the expectation operator and the asterisk denotes complex conjugation. Upon substituting Eq. (1) into the right-hand side of Eq. (9) and replacing f_1 with (f_1+f_c) and f_2 with (f_2+f_c), the following expression for the two-frequency correlation function of the ocean surface-scatter communication channel is obtained:

$$\begin{aligned} R_H(f_1+f_c, f_2+f_c, t_1, t_2) = {} & \frac{\exp[-j(k_{\mathrm{EFF}_1}-k^*_{\mathrm{EFF}_2})(R_1+R_2)]}{(R_1R_2)^2}\int_{x_1}\int_{y_1}\int_{x_2}\int_{y_2} Z\left(\frac{2\pi(f_1+f_c)}{c}, x_1, y_1\right)Z^*\left(\frac{2\pi(f_2+f_c)}{c}, x_2, y_2\right) \\ & \times \exp[-jl(k_{\mathrm{EFF}_1}x_1 - k^*_{\mathrm{EFF}_2}x_2)]\exp[-jm(k_{\mathrm{EFF}_1}y_1 - k^*_{\mathrm{EFF}_2}y_2)]\,\Phi_{t_1,t_2}(\nu_1, \nu_2) \\ & \times \exp\left[-j\left(\frac{l_f}{2}\right)(k_{\mathrm{EFF}_1}x_1^2 - k^*_{\mathrm{EFF}_2}x_2^2)\right]\exp\left[-j\left(\frac{m_f}{2}\right)(k_{\mathrm{EFF}_1}y_1^2 - k^*_{\mathrm{EFF}_2}y_2^2)\right]dx_2\,dy_2\,dx_1\,dy_1, \end{aligned} \tag{10}$$

where f_c is the center or carrier frequency of the bandpass transmit signal, f_1 and f_2 correspond to frequency deviations from f_c, and the characteristic function $\Phi_{t_1,t_2}(\nu_1, \nu_2)$ is given by

$$\Phi_{t_1,t_2}(\nu_1, \nu_2) = E(\exp\{j[\nu_1\xi(x_1, y_1, t_1') + \nu_2\xi(x_2, y_2, t_2')]\}), \tag{11}$$

where

$$\nu_1 = -nk_{\mathrm{EFF}_1}, \tag{12}$$

$$\nu_2 = +nk^*_{\mathrm{EFF}_2}, \tag{13}$$

$$k_{\mathrm{EFF}_1} = [2\pi(f_1+f_c)/c] - j\alpha'(f_1+f_c), \tag{14}$$

$$k^*_{\mathrm{EFF}_2} = [2\pi(f_2+f_c)/c] + j\alpha'(f_2+f_c), \tag{15}$$

and

$$\alpha'(f) = \alpha(f) + \rho_V E\{\sigma_t\}. \tag{16}$$

The importance of the two-frequency correlation function is that both the coherence time and the coherence bandwidth, and hence, the frequency and time spreading associated with surface scatter, can be computed from Eq. (10). If two time-harmonic waves are transmitted at the same frequency f and the resulting output fields are observed at two different times t_1 and t_2, the correlation between the output fields *decreases* as the time difference $\Delta t = t_1 - t_2$ *increases*.[2] The value of the time difference Δt at which the correlation function $R_H(f, f, t_1, t_2) = E[H(f, t_1)H^*(f, t_2)]$ is approximately equal to zero or decreases to a specified level is called the *coherence time*.[2] The reciprocal of the coherence time is equal to the frequency spreading a wave will undergo as it propagates in a random, time-varying medium.[2]

Similarly, if two time-harmonic waves are transmitted at two different frequencies f_1 and f_2 and the resulting output fields are observed at the same time t, the correlation between the two output fields *decreases* as the frequency difference $\Delta f = f_1 - f_2$ *increases*.[2] The value of the frequency difference Δf at which the correlation function $R_H(f_1, f_2, t, t) = E[H(f_1, t)H^*(f_2, t)]$ is approximately equal to zero or decreases to a specified level is called the *coherence bandwidth*.[2] The reciprocal of the coherence bandwidth is equal to the spread in round-trip time delay.

Equation (10) is applicable to a general bistatic geometry and to both broadband and narrow-band bandpass transmit signals. The distribution of the random process $\xi(x, y, t)$ has not yet been specified. McDonald and Spindel[3] obtained analytical expressions for $R_H(f, f, t_1, t_2)$ for a specular ocean surface-scatter geometry only, assuming both non-Gaussian and Gaussian distributions for $\xi(x, y, t)$. With this correlation function, they were able to derive expressions for the bifrequency correlation function which is used to determine the power spectral density of the scattered acoustic pressure field. In addition, McDonald[4] and Zornig and McDonald[5] studied the second-order function $R_H(f_1, f_2, t, t)$ for a specular ocean surface-scatter geometry only, assuming that $\xi(x, y, t)$ was Gaussian. Besides a specular geometry versus a general bistatic geometry, the major differences between the expressions obtained by McDonald and Spindel[3] and McDonald[4] when compared to Eq. (10) are that their equations do not include the expression $K(x, y)$ as defined in Ziomek,[1] and they assumed a Gaussian functional form for the projected transmit directivity pattern while the transmit and receive directivity patterns in Eq. (10) can be projected exactly onto the XY plane (see Appendix A in Ziomek[1]).

For a specular geometry,

$$l = m = 0,$$

$$n = -2\cos\theta, \tag{17}$$

$$l_f = [(R_1+R_2)/(R_1R_2)]\cos^2\theta,$$

and (18)

$$m_f = [(R_1+R_2)/(R_1R_2)],$$

since $\theta_1=\theta_2=\theta$ and $\psi_2=0$.

For a backscatter geometry,

$$l = 2\sin\theta,$$

$$m = 0, \quad (19)$$

$$n = -2\cos\theta,$$

$$l_f = [(R_1+R_2)/(R_1R_2)]\cos^2\theta,$$

and (20)

$$m_f = [(R_1+R_2)/(R_1R_2)],$$

since $\theta_1=\theta_2=\theta$ and $\psi_2=\pi$.

Simplified versions of the characteristic function given by Eq. (11) will be provided in Sec. IB of this paper.

Another second-order function of importance to underwater acoustics is the scattering function. The scattering function determines how a narrow-band bandpass transmit signal's power will be spread in both round-trip time delay and frequency after propagating through a wide-sense stationary uncorrelated spreading (WSSUS) communication channel.[6] The term round-trip time delay is used in this paper to refer to the time delay associated with either monostatic (backscatter) or bistatic geometries. If Eq. (10) can be reduced to a function of $\Delta f = f_1 - f_2$ and $\Delta t = t_1 - t_2$; i.e., if $H(f,t)$ can be shown to be wide-sense stationary in both frequency and time, then the surface scattering function can be obtained from $R_H(\Delta f, \Delta t)$ via a two-dimensional Fourier transformation.[6]

B. Scattering function

Equation (10) can be simplified by assuming that the bandpass transmit signal is *narrow band* so that $|f_c| \gg |f_1|$ and $|f_c| \gg |f_2|$. As a result

$$Z\{[2\pi(f_1+f_c)/c], x_1, y_1\} \approx Z(k, x_1, y_1), \quad (21)$$

$$Z^*\{[2\pi(f_2+f_c)/c], x_2, y_2\} \approx Z^*(k, x_2, y_2), \quad (22)$$

$$\alpha'(f_1+f_c) \approx \alpha'(f_c), \quad (23)$$

and

$$\alpha'(f_2+f_c) \approx \alpha'(f_c), \quad (24)$$

where

$$k \triangleq 2\pi f_c/c. \quad (25)$$

By substituting Eqs. (14), (15), and (21)–(25) into Eq. (10) and letting $x_1 = x_2 + \Delta x$ and $y_1 = y_2 + \Delta y$, one obtains

$$R_H(f_1+f_c, f_2+f_c, t_1, t_2) = \frac{\exp[-j(2\pi\Delta f/c)(R_1+R_2)]}{(R_1R_2)^2}\exp[-2\alpha'(f_c)(R_1+R_2)]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J(k,\Delta x,\Delta y)\exp\left\{-jk\left[l\Delta x + m\Delta y + \left(\frac{l_f}{2}\right)(\Delta x)^2 + \left(\frac{m_f}{2}\right)(\Delta y)^2\right]\right\}\exp\left\{-\alpha'(f_c)\left[l\Delta x + m\Delta y + \left(\frac{l_f}{2}\right)(\Delta x)^2 + \left(\frac{m_f}{2}\right)(\Delta y)^2\right]\right\}d\Delta x\, d\Delta y, \quad (26)$$

where

$$J(k,\Delta x,\Delta y) = \int_x\int_y Z(k, x+\Delta x, y+\Delta y)Z^*(k,x,y)\Phi_{\xi_1,\xi_2}(\nu_1,\nu_2)\exp\left\{-j\frac{2\pi\Delta f}{c}\left[lx + my + \left(\frac{l_f}{2}\right)x^2 + \left(\frac{m_f}{2}\right)y^2\right]\right\}\exp\left\{-2\alpha'(f_c)\times\left[lx+my+\left(\frac{l_f}{2}\right)x^2+\left(\frac{l_f}{2}\right)x\Delta x+\left(\frac{m_f}{2}\right)y^2+\left(\frac{m_f}{2}\right)y\Delta y\right]\right\}\exp\left\{-j\frac{2\pi}{c}(f_1+f_c)\left[\left(\frac{l_f}{2}\right)2x\Delta x+\left(\frac{m_f}{2}\right)2y\Delta y\right]\right\}dx\,dy, \quad (27)$$

and $\Delta f = f_1 - f_2$.

Now, let us turn our attention to the characteristic function appearing in Eq. (27) as given by Eq. (11). Since the attenuation due to the random deviations of $\xi(x,y,t)$ from the XY plane is negligible in comparison with the other terms contributing to attenuation, one can set $\alpha'(f_c)=0$ in Eq. (11). If it is also assumed that $\xi(x,y,t)$ is Gaussian, zero mean, and wide-sense stationary in both space and time, then it can be shown[7] that Eq. (11) becomes

$$\Phi_{\xi_1,\xi_2}(\Delta x,\Delta y,\Delta t') = \exp\left[-\left(\frac{2\pi(f_1+f_c)}{c}\sigma_\xi n\right)^2\times\left(1-\frac{R_\xi(\Delta x,\Delta y,\Delta t')}{\sigma_\xi^2}\right)\left(1-\frac{\Delta f}{f_1+f_c}\right)\right]\times\exp\left[-\frac{1}{2}\left(\frac{2\pi(f_1+f_c)}{c}\sigma_\xi n\right)^2\left(\frac{\Delta f}{f_1+f_c}\right)^2\right], \quad (28)$$

where

$$R_\xi(\Delta x,\Delta y,\Delta t') = E[\xi(x+\Delta x, y+\Delta y, t_1')\xi(x,y,t_2')] \quad (29)$$

and

$$E[\xi^2(x,y,t)] = \sigma_\xi^2, \quad (30)$$

where σ_ξ^2 is a constant and $\Delta t' = t_1' - t_2'$. Using the narrow-band assumption, i.e., $f_1+f_c \approx f_c$ and $f_1+f_c \gg \Delta f$, Eq. (28) reduces to

$$\Phi_{\xi_1,\xi_2}(\Delta x,\Delta y,\Delta t') = \exp(-(k\sigma_\xi n)^2\times\{1-[R_\xi(\Delta x,\Delta y,\Delta t')/\sigma_\xi^2]\})\times\exp[-\tfrac{1}{2}(k\sigma_\xi n)^2(\Delta f/f_c)^2], \quad (31)$$

where k is defined by Eq. (25). If Eq. (29) can be shown to be a function of Δt, then Eq. (26) becomes a function of Δf and Δt which is our desired result. Indeed, in the Appendix it is shown that $R_\xi(\Delta x,\Delta y,\Delta t') \equiv R_\xi(\Delta x,\Delta y,\Delta t)$, where $R_\xi(\Delta x,\Delta y,\Delta t)$ is given by Eq.

(A10). Equation (A10) expresses $R_\zeta(\Delta x, \Delta y, \Delta t)$ in terms of the directional wavenumber spectrum of the ocean surface. Therefore Eq. (31) can be written as

$$\Phi_{\zeta_1,\zeta_2}(\Delta x, \Delta y, \Delta t') \equiv \Phi_{\zeta_1,\zeta_2}(\Delta x, \Delta y, \Delta t) = \exp(-(k\sigma_\zeta n)^2 \times \{1 - [R_\zeta(\Delta x, \Delta y, \Delta t)/\sigma_\zeta^2]\}) \times \exp[-\tfrac{1}{2}(k\sigma_\zeta n)^2(\Delta f/f_c)^2] . \quad (32)$$

Thus the characteristic function originally given by Eq. (11) has been reduced to the expression given by Eq. (32) where $R_\zeta(\Delta x, \Delta y, \Delta t)$ is given by Eq. (A10).

With the use of Eq. (32), and assuming that $f_1 + f_c \approx f_c$ in the last complex exponential term in Eq. (27), the two-frequency correlation function given by Eq. (26) can be written as a function of Δf and Δt, i.e.,

$$R_H(\Delta f, \Delta t) = \frac{\exp[-2\alpha'(f_c)(R_1+R_2)]}{(R_1R_2)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J(k, \Delta x, \Delta y, \Delta f) \exp\{-(k\sigma_\zeta n)^2[1-\rho_\zeta(\Delta x, \Delta y, \Delta t)]\} \times \exp\left\{-jk\left[l\Delta x + m\Delta y + \left(\frac{l_f}{2}\right)(\Delta x)^2 + \left(\frac{m_f}{2}\right)(\Delta y)^2\right]\right\} \exp\left\{-\alpha'(f_c)\left[l\Delta x + m\Delta y + \left(\frac{l_f}{2}\right)(\Delta x)^2 + \left(\frac{m_f}{2}\right)(\Delta y)^2\right]\right\} d\Delta x\, d\Delta y , \quad (33)$$

where

$$J(k, \Delta x, \Delta y, \Delta f) = \int_x\int_y Z(k, x+\Delta x, y+\Delta y) Z^*(k, x, y) \exp\left[-\frac{1}{2}(k\sigma_\zeta n)^2\left(\frac{\Delta f}{f_c}\right)^2\right] \exp\left\{-j\frac{2\pi\Delta f}{c} \times \left[lx + my + \left(\frac{l_f}{2}\right)x^2 + \left(\frac{m_f}{2}\right)y^2 + (R_1+R_2)\right]\right\} \exp\left\{-2\alpha'(f_c)\left[lx + my + \left(\frac{l_f}{2}\right)x^2 + \left(\frac{l_f}{2}\right)x\Delta x + \left(\frac{m_f}{2}\right)y^2 + \left(\frac{m_f}{2}\right)y\Delta y\right]\right\} \exp\left\{-jk\left[\left(\frac{l_f}{2}\right)2x\Delta x + \left(\frac{m_f}{2}\right)2y\Delta y\right]\right\} dx\, dy , \quad (34)$$

and

$$\rho_\zeta(\Delta x, \Delta y, \Delta t) \triangleq R_\zeta(\Delta x, \Delta y, \Delta t)/\sigma_\zeta^2 . \quad (35)$$

The scattering function $R_S(\tau, \phi)$ can be obtained from $R_H(\Delta f, \Delta t)$ by evaluating the following two-dimensional Fourier transform[6]:

$$R_S(\tau, \phi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_H(\Delta f, \Delta t) \exp[+j2\pi(\Delta f\tau - \phi\Delta t)]\, d\Delta f\, d\Delta t . \quad (36)$$

Substituting Eq. (33) into Eq. (36) yields

$$R_S(\tau, \phi) = \frac{\exp[-2\alpha'(f_c)(R_1+R_2)]}{(R_1R_2)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J(k, \Delta x, \Delta y, \tau)\Gamma(\Delta x, \Delta y, \phi) \exp[-jk(l\Delta x + m\Delta y)] \exp\left\{-jk\left[\left(\frac{l_f}{2}\right)(\Delta x)^2 + \left(\frac{m_f}{2}\right)(\Delta y)^2\right]\right\} \exp[-\alpha'(f_c)(l\Delta x + m\Delta y)] \exp\left\{-\alpha'(f_c)\left[\left(\frac{l_f}{2}\right)(\Delta x)^2 + \left(\frac{m_f}{2}\right)(\Delta y)^2\right]\right\} d\Delta x\, d\Delta y , \quad (37)$$

where

$$J(k, \Delta x, \Delta y, \tau) = |b| \int_x\int_y Z(k, x+\Delta x, y+\Delta y) Z^*(k, x, y) \exp\{-\pi b^2[\tau - \tau_0(x,y)]^2\} \exp\left\{-jk\left[\left(\frac{l_f}{2}\right)2x\Delta x + \left(\frac{m_f}{2}\right)2y\Delta y\right]\right\} \times \exp\left\{-2\alpha'(f_c)\left[lx + my + \left(\frac{l_f}{2}\right)x^2 + \left(\frac{l_f}{2}\right)x\Delta x + \left(\frac{m_f}{2}\right)y^2 + \left(\frac{m_f}{2}\right)y\Delta y\right]\right\} dx\, dy , \quad (38)$$

$$\tau_0(x, y) = \left(\frac{1}{c}\right)\left[lx + my + \left(\frac{l_f}{2}\right)x^2 + \left(\frac{m_f}{2}\right)y^2 + (R_1+R_2)\right] , \quad (39)$$

$$b = c/[n\sigma_\zeta(2\pi)^{1/2}] , \quad (40)$$

and

$$\Gamma(\Delta x, \Delta y, \phi) = \int_{-\infty}^{\infty} \exp\{-(k\sigma_\zeta n)^2[1-\rho_\zeta(\Delta x, \Delta y, \Delta t)]\} \exp(-j2\pi\phi\Delta t)\, d\Delta t . \quad (41)$$

Equation (37) is the scattering function of the ocean surface. It determines how the transmitted signal's power will be spread in round-trip time delay τ (s) and frequency ϕ (Hz) as a result of being scattered by the ocean surface.

The spread in round-trip time delay is due to the presence of the Gaussian function in τ appearing in the integrand of Eq. (38) *and* to the variety of possible propagation paths which exist between the transmit and receive arrays as specified by Eq. (39). These paths are associated with different portions of the insonified area of the surface. The most obvious propagation path is, of course, (R_1+R_2).

From Eq. (41) it can be seen that the frequency spread is due to the time variations or motion of the ocean surface itself as characterized by $\rho_\zeta(\Delta x, \Delta y, \Delta t)$.

McDonald and Tuteur,[7] and Tuteur *et al.*,[8] also derived expressions for the ocean surface-scattering function. Their scattering functions were based upon a Fresnel corrected Kirchhoff integral and a small slope approximation and pertain *only* to a specular geometry. In addition, they did not include a receive directivity function and they assumed a Gaussian functional form for the projected transmit beam pattern.

In contrast, the surface-scattering function given by Eq. (37) is applicable to a general bistatic geometry. As a result, expressions for both the specular and backscatter geometries can easily be obtained from it. Equation (37) is based upon a *generalized* Kirchhoff approach. In addition, the transmit and receive directivity functions are general, frequency dependent expressions. Gaussian functional forms have not been assumed. The necessary transformation equations which will project both the transmit and receive directivity functions exactly onto the XY plane are provided in Appendix A in Ziomek.[1] And finally, the scattering function given by Eq. (37) is dependent upon the general form of the directional wavenumber spectrum.

The expression $\Gamma(\Delta x, \Delta y, \phi)$ as given by Eq. (41) can be simplified by noting that surface scatter problems are generally divided into two categories. The first category is characterized by

$$(k\sigma_\zeta n)^2 \ll 1, \tag{42}$$

and is referred to as the slightly rough surface case (low-frequency or low sea state case).[9] The second category is characterized by

$$(k\sigma_\zeta n)^2 \gg 1, \tag{43}$$

and is referred to as the very rough surface case (high-frequency or high sea state case).[9] The value used for n in Eqs. (42) and (43) is either its specular or backscatter value which happen to be identical [see Eqs. (17) and (19)]. The expression $k\sigma_\zeta n$ is referred to as the Rayleigh parameter.[7]

1. Slightly rough surface

Consider the following expression which appears in the integrand of Eq. (41):

$$\exp\{-(k\sigma_\zeta n)^2[1-\rho_\zeta(\Delta x, \Delta y, \Delta t)]\}. \tag{44}$$

Since $R_\zeta(\Delta x, \Delta y, \Delta t)$ is an autocorrelation function, its maximum value is attained at the origin, i.e., $R_\zeta(0,0,0) = \sigma_\zeta^2$. As a result, $\rho_\zeta(0,0,0)=1$ and $|\rho_\zeta(\Delta x, \Delta y, \Delta t)| \le 1$. Since $|\rho_\zeta(\Delta x, \Delta y, \Delta t)| \le 1$ and Eq. (42) is applicable to the slightly rough surface case, Eq. (44) can be approximated by using only the first two terms in the following expansion:

$$\exp(x) = 1 + x + \tfrac{1}{2}x^2 + (1/3!)x^3 + \cdots, \tag{45}$$

which can be used for all real values of x. Thus Eq. (44) can be expressed as

$$\exp\{-(k\sigma_\zeta n)^2[1-\rho_\zeta(\Delta x, \Delta y, \Delta t)]\} \approx 1 - (k\sigma_\zeta n)^2 + (k\sigma_\zeta n)^2\rho_\zeta(\Delta x, \Delta y, \Delta t) \tag{46}$$

or

$$\exp\{-(k\sigma_\zeta n)^2[1-\rho_\zeta(\Delta x, \Delta y, \Delta t)]\} \approx 1 + (k\sigma_\zeta n)^2\rho_\zeta(\Delta x, \Delta y, \Delta t). \tag{47}$$

Upon substituting Eq. (47) into Eq. (41), one obtains

$$\Gamma(\Delta x, \Delta y, \phi) \approx \delta(\phi) + (k\sigma_\zeta n)^2 F_{\Delta t}\{\rho_\zeta(\Delta x, \Delta y, \Delta t)\}, \tag{48}$$

where $\delta(\cdot)$ is the Dirac delta function and $F_{\Delta t}\{\cdot\}$ indicates a forward Fourier transformation with respect to Δt. The first term in Eq. (48) indicates *no* frequency spreading. It is referred to as the specular reflection term and characterizes reradiation from a smooth surface (i.e., $\sigma_\zeta = 0$).[9] However, the second term in Eq. (48) does indicate frequency spreading due to the random motion of the time-varying surface.

2. Very rough surface

Since Eq. (43) is applicable to the very rough surface case, expressing Eq. (44) in terms of the expansion given by Eq. (45) is not advantageous. Because $\rho_\zeta(0,0,0) = 1$ and $|\rho_\zeta(\Delta x, \Delta y, \Delta t)| \le 1$, the magnitude of the integrand of $\Gamma(\Delta x, \Delta y, \phi)$ is significant only near the origin, i.e., near $\Delta x = \Delta y = \Delta t = 0$. Therefore, in order to handle the very rough surface case, expand $\rho_\zeta(\Delta x, \Delta y, \Delta t)$ in a truncated Maclaurin series as follows[9-11]:

$$\rho_\zeta(\Delta x, \Delta y, \Delta t) \approx 1 + \left(\Delta x \frac{\partial}{\partial \Delta x} + \Delta y \frac{\partial}{\partial \Delta y} + \Delta t \frac{\partial}{\partial \Delta t}\right) \times \rho_\zeta(\Delta x, \Delta y, \Delta t)\Big|_0 + \frac{1}{2}\left(\Delta x \frac{\partial}{\partial \Delta x} + \Delta y \frac{\partial}{\partial \Delta y} + \Delta t \frac{\partial}{\partial \Delta t}\right)^2 \rho_\zeta(\Delta x, \Delta y, \Delta t)\Big|_0, \tag{49}$$

where the partial derivatives are evaluated at $\Delta x = \Delta y = \Delta t = 0$. Upon substituting Eq. (49) into Eq. (44) and expanding, one obtains

$$\exp\{-(k\sigma_\zeta n)^2[1-\rho_\zeta(\Delta x, \Delta y, \Delta t)]\} \approx \exp\{(k\sigma_\zeta n)^2[DX\Delta x + DY\Delta y + DT\Delta t + DXY\Delta x\Delta y + DXT\Delta x\Delta t + DYT\Delta y\Delta t + (DXX/2)(\Delta x)^2 + (DYY/2)(\Delta y)^2 + (DTT/2)(\Delta t)^2]\}, \tag{50}$$

where

$$DX \triangleq \frac{\partial}{\partial \Delta x}\rho_\zeta(\Delta x, \Delta y, \Delta t)\Big|_0, \tag{51}$$

$$DXX \triangleq \frac{\partial^2}{\partial (\Delta x)^2}\rho_\zeta(\Delta x, \Delta y, \Delta t)\Big|_0, \tag{52}$$

$$DXY \triangleq \frac{\partial^2}{\partial \Delta x \partial \Delta y}\rho_\zeta(\Delta x, \Delta y, \Delta t)\Big|_0, \quad \text{etc.} \tag{53}$$

Substituting Eq. (50) into Eq. (41) yields

$$\Gamma(\Delta x, \Delta y, \phi) \approx \exp\left[(k\sigma_\xi n)^2\left(DX\Delta x + DY\Delta y + \frac{DXX}{2}(\Delta x)^2 + DXY\Delta x\Delta y + \frac{DYY}{2}(\Delta y)^2\right)\right] \int_{-\infty}^{\infty} \exp\left[(k\sigma_\xi n)^2 \times \left(DT\Delta t + DXT\Delta x\Delta t + DYT\Delta y\Delta t + \frac{DTT}{2}(\Delta t)^2\right)\right] \times \exp(-j2\pi\phi\Delta t)d\Delta t. \quad (54)$$

One can evaluate the coefficients DX, DXX, DXY, etc. by either working with a functional form for the normalized autocorrelation function $\rho_\xi(\Delta x, \Delta y, \Delta t)$ or with a functional form for the directional wavenumber spectrum since $\rho_\xi(\Delta x, \Delta y, \Delta t)$ and the wavenumber spectrum are related by a two-dimensional Fourier transformation. For example, Parkins[9] obtained values for the coefficients DX, DXX, DXY, etc. by using a Neumann–Pierson spectrum in his study of the power spectral density function. Swarts and Eggen[11] also computed values for DX, DXX, DXY, etc. by using the Neumann–Pierson spectrum, the Pierson–Moskowitz spectrum, and others. Swarts and Eggen[11] used a Fraunhofer version of the Kirchhoff integral, based upon Parkins'[9] work, to study the power spectrum of the acoustic pressure field scattered by the ocean surface.

C. Power spectral density

Since the underwater acoustic propagation path between transmit and receive arrays via the surface of the ocean is being treated as a linear, time-varying random filter, the scattered acoustic pressure field can be regarded as the random output $y(t)$ of the filter for a particular input (transmit) signal $x(t)$. One can therefore express $y(t)$ as

$$y(t) = \int_{-\infty}^{\infty} X(f)H(f,t)\exp(+j2\pi ft)df, \quad (55)$$

where $X(f)$ is the Fourier transform of $x(t)$ and $H(f,t)$ is the random, time-varying transfer function of the ocean surface given by Eq. (1). For the derivation which follows, it will be assumed that $x(t)$ is a wide-sense stationary random process which is uncorrelated with $H(f,t)$.

In order to find the power spectral density of $y(t)$, the autocorrelation function

$$R_y(t_1, t_2) = E[y(t_1)y^*(t_2)] \quad (56)$$

will be computed first. Substituting Eq. (55) into Eq. (56) yields

$$R_y(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[X(f_1)X^*(f_2)]R_H(f_1,f_2,t_1,t_2) \times \exp[+j2\pi(f_1t_1 - f_2t_2)]df_1\,df_2, \quad (57)$$

where $R_H(f_1,f_2,t_1,t_2)$ is the two-frequency correlation function. Now, if one substitutes the following relationship[12]:

$$E[X(f_1)X^*(f_2)] = S_x(f_1)\delta(f_1 - f_2), \quad (58)$$

into Eq. (57), where $S_x(\cdot)$ is the power spectral density of the transmit signal, one obtains

$$R_y(\Delta t) = \int_{-\infty}^{\infty} S_x(f)R_H(f,f,\Delta t)\exp(+j2\pi f\Delta t)df, \quad (59)$$

where it has been assumed that $H(f,t)$ is wide-sense stationary in time and where $\Delta t = t_1 - t_2$. The output power spectral density is now obtained by Fourier transforming both sides of Eq. (59) with respect to Δt. Doing so yields

$$S_y(\eta) = \int_{-\infty}^{\infty} S_x(f)R_B(f,f,\eta - f)df, \quad (60)$$

where

$$R_B(f,f,\eta-f) = \int_{-\infty}^{\infty} R_H(f,f,\Delta t)\exp[-j2\pi(\eta - f)\Delta t]d\Delta t, \quad (61)$$

and is referred to as the bifrequency correlation function.[12] It is not to be confused with the two-frequency correlation function. If one makes the additional assumption that $H(f,t)$, is also wide-sense stationary in frequency, then the power spectral density of the scattered acoustic pressure field is given by

$$S_y(\eta) = \int_{-\infty}^{\infty} S_x(f)R_B(0,\eta - f)df, \quad (62)$$

where

$$R_B(0,\eta - f) = \int_{-\infty}^{\infty} R_H(0,\Delta t)\exp[-j2\pi(\eta - f)\Delta t]d\Delta t, \quad (63)$$

and $R_H(0,\Delta t)$ can be obtained from Eq. (33) by setting $\Delta f = 0$. Note that Eq. (62) is in the form of a convolution integral which accounts for the frequency spreading of the input power spectral density.

1. Example calculation

As an example, let us try to match the expression for $S_y(\eta)$ as given in Parkins[9] for the very rough surface case. We begin by setting $\Delta f = 0$ in both Eqs. (33) and (34). In addition, since Parkins[9] used a Fraunhofer version of the Kirchhoff integral and did not include attenuation, the Fresnel coefficients l_f and m_f, and $\alpha'(\cdot)$ are also set equal to zero. Thus Eqs. (33) and (34) reduce to

$$R_H(0,\Delta t) = \frac{1}{(R_1R_2)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J(k,\Delta x,\Delta y,0) \times \exp\{-(k\sigma_\xi n)^2[1 - \rho_\xi(\Delta x,\Delta y,\Delta t)]\} \times \exp[-jk(l\Delta x + m\Delta y)]d\Delta x\,d\Delta y, \quad (64)$$

and

$$J(k,\Delta x,\Delta y,0) = \int_x\int_y Z(k, x+\Delta x, y+\Delta y)Z^*(k,x,y)dx\,dy, \quad (65)$$

respectively, where $Z(k,x,y)$ is defined by Eq. (2). If we now use the classical Kirchhoff approach as Parkins[9] did, it can be shown that $K(x,y)$ appearing in Eq. (2) equals a *constant*; namely, $F(\theta_1,\theta_2,\psi_2)C_{REF}$ where $C_{REF} = -1$ (refer to Sec. III of Ziomek[1]). Parkins[9] also did not include transmit and receive directivity functions. This is equivalent to assuming that they were both omnidirectional. In addition, he assumed that the insonified area of the surface formed a square with sides of length $2L$, i.e., from $-L$ to $+L$. With the use of these additional assumptions, Eq. (65) further reduces to

$$J(k, \Delta x, \Delta y, 0) = F^2(\theta_1, \theta_2, \psi_2) \int_{-L}^{L} \int_{-L}^{L} dx\, dy \tag{66}$$

or

$$J(k, \Delta x, \Delta y, 0) = 4L^2 F^2(\theta_1, \theta_2, \psi_2)\,. \tag{67}$$

Parkins[9] found that for the Neumann–Pierson spectrum,

$$DX = DY = DT = DXY = DYT = 0\,, \tag{68}$$

$$DXX = -2E/\sigma_\xi^2\,, \tag{69}$$

$$DYY = -2E/(3\sigma_\xi^2)\,, \tag{70}$$

$$DTT = -2N/\sigma_\xi^2\,, \tag{71}$$

and

$$DXT = +H/\sigma_\xi^2\,, \tag{72}$$

where E, N, H, and σ_ξ^2 are related to the parameters in the spectrum (e.g., wind speed and acceleration due to gravity). Substituting Eq. (68) into Eq. (50) yields

$$\exp\{-(k\sigma_\xi n)^2[1-\rho_\xi(\Delta x, \Delta y, \Delta t)]\}$$
$$\approx \exp\left[(k\sigma_\xi n)^2\left(\frac{DXX}{2}(\Delta x)^2 + DXT\,\Delta x\Delta t + \frac{DYY}{2}(\Delta y)^2 + \frac{DTT}{2}(\Delta t)^2\right)\right], \tag{73}$$

and upon substituting Eqs. (67) and (73) into Eq. (64), one obtains

$$R_H(0, \Delta t) = 4L^2 B \exp\left((k\sigma_\xi n)^2 \frac{DTT}{2}(\Delta t)^2\right) \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[(k\sigma_\xi n)^2\left(\frac{DXX}{2}(\Delta x)^2 + DXT\,\Delta x\Delta t + \frac{DYY}{2}(\Delta y)^2\right)\right] \exp[-jk(l\Delta x + m\Delta y)]\, d\Delta x\, d\Delta y\,, \tag{74}$$

where

$$B \triangleq [F(\theta_1, \theta_2, \psi_2)/(R_1 R_2)]^2\,. \tag{75}$$

Equation (74) can be integrated by first completing the square of $(\Delta x)^2$, i.e.,

$$\frac{DXX}{2}(\Delta x)^2 + DXT\,\Delta x\Delta t = \frac{DXX}{2}\left(\Delta x + \frac{DXT}{DXX}\Delta t\right)^2 - \frac{(DXT)^2}{2DXX}(\Delta t)^2\,. \tag{76}$$

Substituting Eq. (76) into Eq. (74) and integrating yields

$$R_H(0, \Delta t) = 4L^2 B((2\pi)/\{(kn\sigma_\xi)^2[(DXX)(DYY)]^{1/2}\}) \exp\left[(k\sigma_\xi n)^2\left(\frac{DTT}{2} - \frac{(DXT)^2}{2DXX}\right)(\Delta t)^2\right] \exp\left[-jkl\left(\frac{DXT}{-DXX}\right)\Delta t\right] \times \exp\{-(kl)^2/[2\sigma_\xi^2(-DXX)(kn)^2]\} \exp\{-(km)^2/[2\sigma_\xi^2(-DYY)(kn)^2]\}\,. \tag{77}$$

Upon substituting Eq. (77) into Eq. (63) and integrating, one obtains the following expression for the bifrequency correlation function:

$$R_B(0, \eta - f) = 4L^2 B((2\pi)/\{(kn\sigma_\xi)^2[(DXX)(DYY)]^{1/2}\})\{(2\pi)/[kn\sigma_\xi(2\pi\{-DTT - [(DXT)^2/(-DXX)]\})^{1/2}]\} \times \exp\{-(kl)^2/[2\sigma_\xi^2(-DXX)(kn)^2]\} \exp\{-(km)^2/[2\sigma_\xi^2(-DYY)(kn)^2]\} \times \exp\{-[2\pi^2/((kn)^2\sigma_\xi^2\{-DTT - [(DXT)^2/(-DXX)]\})]\{\eta - f + (klDXT)/[2\pi(-DXX)]\}^2\}, \tag{78}$$

or, with the use of Eqs. (69)–(72),

$$R_B(0, \eta - f) = [[4L^2 B(\pi^{3/2})\sqrt{3}]/((kn)^3 E\{N - [H^2/(4E)]\}^{1/2})] \exp\{-(kl)^2/[4E(kn)^2]\} \times \exp\{-[3(km)^2]/[4E(kn)^2]\} \exp\{-[\pi^2/((kn)^2\{N - [H^2/(4E)]\})]\{\eta - f + [(kl)/(2\pi)][H/(2E)]\}^2\}. \tag{79}$$

Since Parkins[9] assumed that a time-harmonic signal was transmitted, the power spectrum $S_x(f)$ of the real input signal $x(t)$ is then proportional to $\delta(f+f_c) + \delta(f-f_c)$. Considering only the positive frequency spectrum, i.e., substituting $S_x(f) = \delta(f-f_c)$ into Eq. (62), yields $S_y(\eta) = R_B(0, \eta - f_c)$ which is consistent with the interpretation of the bifrequency correlation function; namely, that it measures the degree to which input energy at frequency f_c is converted to output energy at frequency η by the time variation of the scatter channel.[3] Therefore, by replacing f with f_c in Eq. (79), it is then in the same form as Parkins'[9] Eq. (32) for the spectral density of the scattered acoustic pressure field. Note that Parkins'[9] Eq. (32) contains several typographic errors. The factor $(\Omega - \omega - K_x L)$ appearing in the last exponential function in Parkins'[9] Eq. (32) should be raised to the power two (2), i.e., this last exponential should be a Gaussian function in Ω as follows from his Eq. (31). In addition, a multiplicative term $(1/E)$ is missing in Parkins'[9] Eq. (32), although it is present in his Eq. (31); and the multiplicative term $(1/K_z^2)$ appearing in Parkins'[9] Eq. (32) should be $(1/K_z^3)$ which also follows from his Eq. (31). Also note that η and $f(f_c)$ in our Eq. (79) is in Hertz while Ω and ω in Parkins'[9] Eq. (32) is in rad/s.

II. SUMMARY

Three second-order functions which characterize the ocean surface-scatter communication channel were derived from the transfer function of the ocean surface. The transfer function used was obtained from a *gen-*

eralized Kirchhoff approach. The second-order functions derived were the two-frequency correlation function, the scattering function, and the power spectral density function of the scattered acoustic pressure field. These functions were shown to be dependent upon the general form of the directional wavenumber spectrum of the ocean surface. Both the slightly rough and very rough surface cases were included.

It was noted that an estimate of the spread in both round-trip time delay and frequency due to surface scatter can be obtained from the two-frequency correlation function for both narrow-band and broadband bandpass transmit signals. Although the scattering function can also provide estimates of both round-trip time delay and frequency spread, its applicability is limited to narrow-band bandpass transmit signals and WSSUS communication channels.

The interrelationships which exist amongst these functions were also demonstrated. For example, the two-frequency correlation function and the scattering function form a two-dimensional Fourier transform pair *if* the two-frequency correlation function is wide-sense stationary in both frequency and time. And if the two-frequency correlation function is at least wide-sense stationary in time, then the bifrequency correlation function can be obtained from it via a Fourier transformation. Once the bifrequency correlation function is known, the output power spectral density function can be determined.

As an example, the power spectral density function of the scattered acoustic pressure field was computed for the very rough surface case using the Neumann–Pierson directional wavenumber spectrum.

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APPENDIX

Let us examine the retarded time

$$t_1' = t_1 - \frac{R_2}{c}\left[1 - \left(\frac{\sin\theta_2\cos\psi_2}{R_2}x + \frac{\sin\theta_2\sin\psi_2}{R_2}y + \frac{\cos\theta_2}{R_2}\xi(x,y,t_1')\right) + \frac{(1-\sin^2\theta_2\cos^2\psi_2)}{2R_2^2}x^2 + \frac{(1-\sin^2\theta_2\sin^2\psi_2)}{2R_2^2}y^2\right], \quad \text{(A1)}$$

where Eq. (30) and (32) of Ziomek[1] were substituted into Eq. (7) of Ziomek[1] to obtain Eq. (A1). Note that t_1', as given by Eq. (A1), is a random variable since $\xi(x,y,t_1')$ is random. Since

$$|(\cos\theta_2/R_2)\sigma_\xi| \ll 1, \quad \text{(A2)}$$

and assuming that

$$|x/R_2| < 1 \quad \text{and} \quad |y/R_2| < 1, \quad \text{(A3)}$$

Eq. (A1) can be approximated by

$$t_1' = t_1 - \frac{R_2}{c}\left[1 - \left(\frac{\sin\theta_2\cos\psi_2}{R_2}x + \frac{\sin\theta_2\sin\psi_2}{R_2}y\right)\right], \quad \text{(A4)}$$

which is now a deterministic quantity.

Referring back to Eq. (29), it can be seen that t_1' is associated with the spatial coordinates $(x+\Delta x)$ and $(y+\Delta y)$, and that t_2' is associated with x and y. Therefore, using the form of Eq. (A4), t_1' and t_2' can be written as

$$t_1' = t_1 - \frac{R_2}{c}\left[1 - \left(\frac{\sin\theta_2\cos\psi_2}{R_2}(x+\Delta x) + \frac{\sin\theta_2\sin\psi_2}{R_2}(y+\Delta y)\right)\right] \quad \text{(A5)}$$

and

$$t_2' = t_2 - \frac{R_2}{c}\left[1 - \left(\frac{\sin\theta_2\cos\psi_2}{R_2}x + \frac{\sin\theta_2\sin\psi_2}{R_2}y\right)\right]. \quad \text{(A6)}$$

By subtracting Eq. (A6) from Eq. (A5), one obtains

$$\Delta t' = \Delta t + (1/c)(\sin\theta_2\cos\psi_2\Delta x + \sin\theta_2\sin\psi_2\Delta y), \quad \text{(A7)}$$

where $\Delta t = t_1 - t_2$. Thus from Eq. (A7), it can be seen that $\Delta t'$ is a function of Δx, Δy, and Δt. Using the two-dimensional Fourier transform relationship[13]

$$R_\xi(\Delta x, \Delta y, \Delta t') = \left(\frac{1}{4}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(p,q)\exp[-jp\Delta x - jq\Delta y + j\,\omega(p,q)\Delta t']\,dp\,dq \quad \text{(A8)}$$

and upon substituting Eq. (A7) into Eq. (A8), one obtains

$$R_\xi(\Delta x, \Delta y, \Delta t') \equiv R_\xi(\Delta x, \Delta y, \Delta t), \quad \text{(A9)}$$

where

$$R_\xi(\Delta x, \Delta y, \Delta t) = \left(\frac{1}{4}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(p,q)\exp[+j\omega(p,q)\Delta t] \times \exp\left[-j\left(p - \omega(p,q)\frac{\sin\theta_2\cos\psi_2}{c}\right)\Delta x\right] \times \exp\left[-j\left(q - \omega(p,q)\frac{\sin\theta_2\sin\psi_2}{c}\right)\Delta y\right]dp\,dq. \quad \text{(A10)}$$

The expression $W(p,q)$ is the directional wavenumber spectrum of the ocean surface, where $W(p,q)dp\,dq$ is the amount of the component of the rough surface having the spatial wavenumber between p and $p+dp$ in the x direction and between q and $q+dq$ in the y direction. The corresponding angular frequency (in rad/s) is given by

$$\omega(p,q) = \pm[g(p^2+q^2)^{1/2}]^{1/2}, \quad \text{(A11)}$$

where g is the acceleration due to gravity.[13] The spectrum $W(p,q)$ is a real positive function of p and q, and since $\xi(x,y,t)$ is real, $W(p,q)$ is an even function of p and q, i.e., $W(-p,-q) = W(p,q)$. Equation (A11) is applicable to deep-water ocean surface waves and it is an odd function of p and q, i.e., $\omega(-p,-q) = -\omega(p,q)$. The choice of sign in Eq. (A11) depends upon the motion of the surface.[13]

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