Calhoun: The NPS Institutional Archive

DSpace Repository

Theses and Dissertations | 1. Thesis and Dissertation Collection, all items

2018

EXPLORING CHARACTERISTICS OF AN EFFECTIVE MIX OF PRECISION AND VOLUME INDIRECT FIRE IN URBAN OPERATIONS USING AGENT-BASED SIMULATION

Cai, Licun Edwin

Monterey, CA; Naval Postgraduate School

http://hdl.handle.net/10945/60370

Copyright is reserved by the copyright owner.

Downloaded from NPS Archive: Calhoun

http://www.nps.edu/library

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

EXPLORING CHARACTERISTICS OF AN EFFECTIVE MIX OF PRECISION AND VOLUME INDIRECT FIRE IN URBAN OPERATIONS USING AGENT-BASED SIMULATION

by

Licun Edwin Cai

September 2018

Thesis Advisor: Susan M. Sanchez
Second Reader: Jeffrey A. Appleget

Approved for public release. Distribution is unlimited.

THIS PAGE INTENTIONALLY LEFT BLANK

REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC 20503.

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** September 2018	**3. REPORT TYPE AND DATES COVERED** Master's thesis

4. TITLE AND SUBTITLE EXPLORING CHARACTERISTICS OF AN EFFECTIVE MIX OF PRECISION AND VOLUME INDIRECT FIRE IN URBAN OPERATIONS USING AGENT-BASED SIMULATION	**5. FUNDING NUMBERS**
6. AUTHOR(S) Licun Edwin Cai	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A	**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release. Distribution is unlimited.	**12b. DISTRIBUTION CODE** A

13. ABSTRACT (maximum 200 words)

As the world continues its rapid rise in urbanization, the battlefield of the future will likely be urban. This thesis explores the effective mix of three types of 155mm artillery munitions with varying accuracy (M982 Excalibur, M1156 Precision Guided Kit, and M795 dumb bomb) in two urban battlefield scenarios. Mission success corresponds to maximizing effects on enemies with minimal fratricides and collateral damage. Pythagoras, an agent-based simulation, together with efficient design of experiments (DOE), is used to study the effects and interactions of controllable factors (system capabilities) and uncontrollable factors (environment and terrain) on the intended measures of effectiveness. Regression metamodels from the DOE are then used to project a Pareto optimal frontier (POF) of the mixture that satisfies multi-objective functions, which includes the cost of munitions expended. A multi-objective evolutionary algorithm is also employed to identify a set of POFs or near-optimal alternatives to the DOE results. Although results from both battle scenarios favor high allocation of M982 (10–15%) for operational effectiveness, the effectiveness of M1156 only proves to be significant in a smaller terrain. A trade-off analysis between operational and cost effectiveness also shows that a substantial amount of M795 (>60%) is needed in the mixes.

14. SUBJECT TERMS agent-based simulation, design of experiments, Pareto optimal frontier, multi-objectives evolutionary algorithm, circular error probable	**15. NUMBER OF PAGES** 131
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**19. SECURITY CLASSIFICATION OF ABSTRACT**	**20. LIMITATION OF ABSTRACT**
Unclassified	Unclassified	Unclassified	UU

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89) Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

Approved for public release. Distribution is unlimited.

EXPLORING CHARACTERISTICS OF AN EFFECTIVE MIX OF PRECISION AND VOLUME INDIRECT FIRE IN URBAN OPERATIONS USING AGENT-BASED SIMULATION

Licun Edwin Cai
Major, Singapore Army
BS, Nanyang Technological University, 2012

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 2018

Approved by: Susan M. Sanchez
Advisor

Jeffrey A. Appleget
Second Reader

W. Matthew Carlyle
Chair, Department of Operations Research

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

As the world continues its rapid rise in urbanization, the battlefield of the future will likely be urban. This thesis explores the effective mix of three types of 155mm artillery munitions with varying accuracy (M982 Excalibur, M1156 Precision Guided Kit, and M795 dumb bomb) in two urban battlefield scenarios. Mission success corresponds to maximizing effects on enemies with minimal fratricides and collateral damage. Pythagoras, an agent-based simulation, together with efficient design of experiments (DOE), is used to study the effects and interactions of controllable factors (system capabilities) and uncontrollable factors (environment and terrain) on the intended measures of effectiveness. Regression metamodels from the DOE are then used to project a Pareto optimal frontier (POF) of the mixture that satisfies multi-objective functions, which includes the cost of munitions expended. A multi-objective evolutionary algorithm is also employed to identify a set of POFs or near-optimal alternatives to the DOE results. Although results from both battle scenarios favor high allocation of M982 (10–15%) for operational effectiveness, the effectiveness of M1156 only proves to be significant in a smaller terrain. A trade-off analysis between operational and cost effectiveness also shows that a substantial amount of M795 (>60%) is needed in the mixes.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

I. INTRODUCTION....1
A. BACKGROUND....1
B. TYPES OF ARTILLERY FIRES....3
C. TYPES OF ARTILLERY MUNITIONS....4
1. M795 HE Projectile....5
2. M982 Excalibur GPS-Guided Projectile....6
3. M1156 PGK "Smart Precision Weapon"....6
D. DAMAGE FUNCTIONS....7
E. OBJECTIVES....9
F. RESEARCH QUESTIONS....9
1. Operational Effectiveness....9
2. Cost Effectiveness....10
G. BENEFIT OF THE STUDY....10
H. THESIS OUTLINE AND METHODOLOGY....10

II. SCENARIO AND MODEL DEVELOPMENT....13
A. SCENARIOS....13
1. Second Battle of Fallujah (November–December 2004)....13
2. Battle of Marawi (May–October 2017)....15
B. PYTHAGORAS CHARACTERISTICS....17
1. Overview....18
2. Terrain....18
3. Weapons....20
4. Sidedness....20
5. Sensors....20
6. Communication....21
7. Agents....21
8. Alternate Behaviors....21
9. Measures of Effectiveness....22
C. MODELING ASSUMPTIONS....24
1. Vulnerability of Blue Artillery and FO Agents....24
2. Engagement Desire of Blue Artillery....24
3. Ammunition Count for Artillery Munitions....25
4. Multiplier Effect of Sniper Weapon P_{Hit} and P_{Kill} against Other Direct Weapons....25
5. Multiplier Effect of Indirect Weapon CEP, M795 across Range Dependent CEP....25

6. Red Forces Positioning 26
7. Neutral Forces Positioning 26

III. EXPERIMENTAL DESIGN 27
A. VARIABLES OF INTEREST 27
1. Decision Factors 28
2. Noise Factors 31
B. EFFICIENT DESIGN OF EXPERIMENTS 32
C. EXPLORATORY DESIGN 33
D. FULL DESIGN 35
E. ROBUST DESIGN 38
F. EMPLOYING CLUSTERS OF HIGH-PERFORMANCE COMPUTERS 38

IV. DATA ANALYSIS 41
A. DATA FARMING AND PROCESSING 41
B. SIMULATION OUTPUT FOR MOE ANALYSIS 41
C. SCENARIO 1 (FALLUJAH)—INSIGHTS AND ANALYSIS 42
1. Factors Influencing MOEs 42
2. Effects of Munition Allocation against MOEs 48
3. Pareto Optimal Frontier Analysis 55
D. SCENARIO 2 (MARAWI)—INSIGHTS AND ANALYSIS 62
1. Factors Influencing MOEs 62
2. Effects of Munition Allocations against MOEs 67
3. Pareto Optimal Frontier Analysis 75

V. FURTHER EXPLORATION 83
A. MOTIVATION 83
B. CHARACTERISTICS OF MOEA 83
C. BASIC APPLICATION 84
D. EXPERIMENTAL SETUP 85
E. MOEA RESULT FOR SCENARIO 1 (FALLUJAH) 86
1. MOEA Run on Three Objectives Function 86
2. MOEA Run on Two Objectives Function 88
F. MOEA RESULT FOR SCENARIO 2 (MARAWI) 90
1. MOEA Run on Three Objectives Function 90
2. MOEA Run on Two Objectives Function 91
G. SUMMARY OF RESULTS 94

VI. CONCLUSIONS, RECOMMENDATIONS, AND FUTURE RESEARCH 97
A. INSIGHTS AND RECOMMENDATIONS 98
1. Recommendation on Factors Influencing FER 98
2. Recommendation on Factors Influencing Fratricide Rate 98
3. Recommendation on Factors Influencing Amount of Collateral Damage 98
4. Recommendation on Factors Influencing Artillery Effectiveness 99
5. Recommendation on Factors Influencing Time for Mission Success 99
6. Recommendation on Munition Allocations 99
B. FUTURE RESEARCH 100

LIST OF REFERENCES 101

INITIAL DISTRIBUTION LIST 105

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF FIGURES

Figure 1. Multi-dimensional Urban Battlefield. Source: HQDA (2006). 2

Figure 2. Damage Functions for Cookie-Cutter and Diffuse Gaussian Weapons, Each with Damage Area π. Source: Washburn and Kress (2009). 8

Figure 3. Battle Area of 2nd Battle of Fallujah (2004). Source: GlobalSecurity.org. (n.d.) 14

Figure 4. Heat Map of Affected Areas in Battle of Marawi. Source: Malicdem (2017). 16

Figure 5. Terrain Map of Scenario 1 (Fallujah) 19

Figure 6. Terrain Map of Scenario 2 (Marawi) 19

Figure 7. Historical Relationship between FER and Probability of Win. Source: Panel on Operational Test Design and Evaluation of the Interim Armored Vehicle (2003). 22

Figure 8. Scatterplot Matrix of Exploratory Design.....35

Figure 9. Regression Tree for Factors Influencing FER 43

Figure 10. Regression Tree for Factors Influencing Fratricide Rates 44

Figure 11. Regression Tree for Factors Influencing Collateral Damage 45

Figure 12. Regression Tree for Factors Influencing Artillery Effectiveness 46

Figure 13. Regression Tree for Factors Influencing Time for Mission Success.....47

Figure 14. Scatterplot Matrix of Response and Decision Variables 49

Figure 15. Correlation Values of Response and Decision Variables 49

Figure 16. Least Squares Regression of Mean and SD of FER 50

Figure 17. Least Squares Regression of Mean and SD of Fratricide 51

Figure 18. Least Squares Regression of Mean and SD of Collateral Damage.....52

Figure 19. Least Squares Regression of Mean and SD of Artillery Effectiveness 53

Figure 20. Least Squares Regression of Mean and SD of Time for Mission Success 54

Figure 21. Scatterplot Matrix of the Scaled Loss MOEs (All Five Scaled Loss MOEs) 57

Figure 22. Plot of M982 Allocation against M1156 Allocation 57

Figure 23. Histograms of the POF (All Five Scaled Loss MOEs) 58

Figure 24. Scatterplot Matrix of Three Scaled Loss MOEs 59

Figure 25. Histograms of the POF (Scaled Loss Total Collateral, Scaled Loss Artillery Effectiveness and Cost) 60

Figure 26. Plot of the Two Scaled Loss MOEs 61

Figure 27. Histograms of Characteristics of the POF (Scaled Loss Total Collateral and Artillery Effectiveness) 61

Figure 28. Regression Tree for Factors Influencing FER 62

Figure 29. Regression Tree for Factors Influencing Fratricide Rates 63

Figure 30. Regression Tree for Factors Influencing Collateral Damage 64

Figure 31. Regression Tree for Factors Influencing Artillery Effectiveness 65

Figure 32. Regression Tree for Factors Influencing Time for Mission Success 66

Figure 33. Scatterplot Matrix of Response and Decision Variables 68

Figure 34. Correlation Values of Response and Decision Variables 68

Figure 35. Least Squares Regression of Mean and SD of FER 69

Figure 36. Least Squares Regression of Mean and SD of Fratricide Rate 70

Figure 37. Least Squares Regression of Mean and SD of Collateral Damage 71

Figure 38. Least Squares Regression of Mean and SD of Artillery Effectiveness 72

Figure 39. Least Squares Regression of Mean and SD of Time for Mission Success 73

Figure 40. Scatterplot Matrix of the Scaled Loss MOEs 75

Figure 41. Plot of M982 Allocations against M1156 Allocations 76

Figure 42. Histograms of the POF (All Five Scaled Loss MOEs) 76

Figure 43. Scatterplot Matrix of the Three Scaled Loss MOEs 77

Figure 44. Histograms of the POF (Scaled Loss Total Collateral, Scaled Loss Artillery Effectiveness, and Cost) 78

Figure 45. Plot of Two Scaled Loss MOEs (Scaled Losses for Total Collateral Damage and Artillery Effectiveness) 79

Figure 46. Histograms of the POF (Scaled Losses for Total Collateral and Artillery Effectiveness) 79

Figure 47. "Best So Far" Curve. Source: Sanchez, Upton, McDonald, and Zabinski (2017). 85

Figure 48. 3D Scatterplot of MOEA-Determined POF (Three Objectives) 87

Figure 49. Histograms of Munitions Proportions for MOEA-Determined POF (Three Objectives) 87

Figure 50. Histograms of Munition Proportions for MOEA-Determined POF (Two Objectives) 88

Figure 51. 2D plot of Evolution of Population over Iterations 89

Figure 52. 2D plot of Evolution over POF over Iterations 89

Figure 53. 3D Scatterplot of MOEA-Determined POF (Three Objectives) 90

Figure 54. Histograms of Munition Proportions for MOEA-Determined POF (Three Objectives) 91

Figure 55. Histograms of Munition Proportions for MOEA-Determined POF (Two Objectives) 92

Figure 56. 2D Plot of Evolution of Population over Iterations 93

Figure 57. 2D Plot of Evolution over POF over Iterations 93

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

Table 1. Approximate CEP Values for Generic Conventional 155mm Projectile. Source: Hill (2007). 6

Table 2. Types of 155mm Artillery Munitions 7

Table 3. Factors of Interest 28

Table 4. Simulated Range Pairs for CEP of M795 29

Table 5. Factors for Exploratory Design....34

Table 6. Factors for Full Design (NOLH x NOB) 37

Table 7. Simulation Output for Desired MOEs 42

Table 8. Summary of Results for Scenario 1 Showing the Influence of Percent M982 and Percent M1156 on the MOEs, Based on Contributions to R^2....55

Table 9. Summary of Results for Scenario 2 Showing the Influence of Percent M982 and Percent M1156 on the MOEs, Based on Contributions to R^2....74

Table 10. Summary of Average Munition Allocations from POF....81

Table 11. Summary of Results for DOE and MOEA Approaches 95

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF ACRONYMS AND ABBREVIATIONS

ABS	Agent-Based Simulation
ARTeMIS	Automated Red Teaming Multi-objective Innovation Seeker
CEP	Circular Error Probable
CSV	Comma-Separated Values
DoD	Department of Defense
DOE	Design of Experiments
FER	Force Exchange Ratio
FO	Forward Observer
GPS	Global Positioning System
HE	High-Explosive
HPCC	High Performance Computing Cluster
HQDA	Headquarters, Department of the Army
MOE	Measure of Effectiveness
MOUT	Military Operations in Urbanized Terrain
NOB	Nearly Orthogonal-and-Balanced
NOLH	Nearly Orthogonal Latin Hypercube
MET	Meteorological
MOEA	Multi-Objectives Evolutionary Algorithm
MOUT	Military Operations on Urban Terrain
MPI	Mean Point Impact
PGK	Precision Guided Kit
POF	Pareto Optimal Frontier
PR	Precision Registration
RPG	Rocket Propelled Grenade Launcher
SD	Standard Deviation
TLE	Target Locating Error

THIS PAGE INTENTIONALLY LEFT BLANK

EXECUTIVE SUMMARY

This thesis explores the use of Agent-Based Simulation (ABS) to develop an effective mix of artillery precision and dumb munitions in urban warfare. Historically, artillery fire, also known as indirect fire, has paramount importance in conventional warfare; however, its effect on urban warfare is contentious due to the high risk of fratricide ("friendly fire") and collateral damage. The development of precision munition in recent years potentially reduces the likelihood of fratricide and collateral damage. These advantages, though, come with a trade-off between cost and operational effectiveness. Measures of effectiveness (MOE) consisting of 1) Force Exchange Ratio (FER), 2) fratricide rates, 3) collateral damage, 4) artillery effectiveness, and 5) time for mission success are used in this study for evaluating the effective allocation of munitions.

Two urban case studies in the 21st century, the Battle of Fallujah (2004) and the Battle of Marawi (2017), are used as notional and realistic scenarios implemented in the Pythagoras modeling platform. Pythagoras was originally developed by Northrop Grumman for the support of the U.S. Marine Corps' Project Albert initiative. It is an agent-based modeling platform that is stochastic and time-stepped, and it enables users to create intelligent agents by assigning them behaviors based on motivators. Three types of artillery munitions being explored and input into the agent-based models are the M982 Excalibur, M1156 Precision Guided Kit (PGK), and M795 dumb munition. The cost of the munitions is highly correlated with their precision level. The M982 is the most precise of these weapons with a Circular Error Probable (CEP) of 5–20 meters (m) (a common measure for a weapon system's precision) and has the highest cost of employment, followed by the M1156.

With the large number of factors such as system capabilities and terrain features that could affect urban battles, there is a large parameter space that can be explored in the models. An efficient Design of Experiments (DOE) approach allows a thorough exploration of input space in a computationally efficient manner by requiring many fewer runs than a brute-force exploration approach. Specifically, this research employs Nearly Orthogonal Latin Hypercube and Nearly Orthogonal-and-Balanced Mixed designs to

explore 24 continuous variables and one binary variable of interest that could affect the MOEs. The full design matrix of 2,176 design points is used with 30 replications at each point. This amounts to 65,280 simulated battles, which took the High Performance Computing Cluster (HPCC) about 50 hours to complete generating the results for each scenario. In contrast, a two-level full factorial design would require 33 million design points and about one billion simulated battles (after replications) for each scenario, which makes it 'computationally impossible' to complete in a lifetime even with the aid of the HPCC (~83 years).

This thesis utilizes various data analysis techniques, such as regression trees and standard least squares regression to gain initial insights on the factors that have a high influence on the MOEs. Commonly, these factors include the characteristics of buildings, such as their level of protection, concealment and mobility, and infantry sensor range, as well as percentage of precision munitions and their CEP values. While factors like building characteristics are considered 'noise variables' that could not be easily controlled in actual battles, meaningful insights can be gained on how best we can employ our forces and resources in urban battle from their interactions with other decision factors.

Other than using regression techniques to determine an optimal allocation of munitions for urban warfare, the thesis employs optimization techniques to find the Pareto Optimal Frontier (POF) of multiple objective functions using the regression metamodels from the DOEs. The concept of Pareto dominance enables us to evaluate the overall performance of a system with multiple objectives of interest such as MOEs and cost. The POF consists of decision points where it is impossible to improve one objective without making trade-offs (i.e., worsening at least one other objective), and hence these decision points are non-dominated. In this study, instead of using the raw values of the mean for each MOE, we employ the quadratic scaled loss function, which rewards good mean performance and penalizes high variability, to obtain robustness in the solutions.

Several combinations of objectives are explored in the study, such as 1) minimizing all five scaled loss MOEs and 2) minimizing the five scaled loss MOEs together with cost. While we are able to find a good set of POF for munition allocations, which minimizes all five scaled loss MOEs, we are unable to find a meaningful set of POF that minimize all

five scaled loss MOEs and cost together, where all the munition allocations being explored are non-dominated against each other. This is a case of the need for decision makers to specify additional constraints on objectives (such as budgetary constraints) or to reduce the number of objectives to obtain a smaller subset of POFs for munition allocations, which is more meaningful for decision making. From the POF analysis of the two battle scenarios, it is revealed that typically high allocations of M982 can reduce fratricides, collateral damage, and time for mission success while increasing the FER. However, its effect on artillery effectiveness is limited as compared to dumb munitions. The impact of the M1156 is limited in bigger and denser terrains such as the Fallujah scenario but holds relatively equal importance as the M982 in smaller terrains such as the Marawi scenario.

The final section of the thesis further explores the concept of Pareto dominance by using a Multi-Objective Evolutionary Algorithm (MOEA). The MOEA used in this thesis is a Naval Postgraduate School developed evolutionary algorithm named "ARTeMIS" (Automated Red Teaming Multiobjective Innovation Seeker), which is a stochastic, heuristic-based search algorithm where the best-performing solutions are chosen to pass on their traits or 'genes' to the next generation. Typically, the solutions get better as the POF converges with more generations. Due to the limited time available, 30 generations for each scenario are run on the HPCC with the objectives of minimizing the scaled loss of fratricide and collateral damage and minimizing the cost of munition allocations. While the final results from the ARTeMIS MOEA may not be conclusive due to the limited number of generations, the insights gained are similar to those gained from the DOE setup on the impact of M982 and M1156 allocations.

Comparing the two approaches, the ARTeMIS MOEA offers an alternative and more flexible way to search for the optimal munition allocations, by allowing the total munitions in each scenario to vary independently based on requirements. Nevertheless, this method is unable to account for other variabilities of the input spaces that the DOE can explore, making it somewhat sensitive to any changes in the input spaces. From the results of the two approaches, it is discovered that the sensitivity of cost impact on the proportion of M982 remains consistent, while there exists a wide variation in the proportion of M1156. This finding requires further study.

In summary, this thesis suggests that agent-based simulation has huge potential as a means and as an alternative to investigate the effectiveness of artillery munitions in urban battles. Just as a multi-objective view is essential for fighting and winning in urban warfare, so a multi-objective approach is essential for addressing this problem. With the incorporation of various methodologies, such as efficient DOEs for simulation, data farming and data mining techniques, and evolutionary algorithms, meaningful insights can be gained from a simple agent-based model. The analytical results provide insights for developing future concept of operations for the use of a mix of precision munitions and area indirect fire. The tools introduced in the thesis can be further expanded to suit the operational requirements of military decision makers for other multi-objective problems.

ACKNOWLEDGMENTS

I'm most thankful to my thesis advisor, Professor Susan Sanchez, who has been more than helpful and patient with me throughout the process of completing this thesis research. Her guidance in the field of simulation and modeling broadened my horizons in the view of the problem and enabled me to develop a different methodology to approach the same problem. I would also like to thank her for taking time to review my models and thesis progress each week despite her busy schedule.

I would also like to thank Dr. Jeffrey A. Appleget, my second reader, for his help and guidance in developing the operational concepts of employing indirect fire. His deep experience in the field of weaponeering and joint combat modeling helped me structure my thesis in a more complete and understandable manner.

I am very grateful to Mary McDonald, who had been providing me with continual technical support in running the simulations in the high performance computing cluster and offering valuable advice in the process of building the models for the thesis research. Your inputs and support were significant to the completion and success of this research.

Last, but not least, I would like to thank my loving wife for her unwavering support and encouragement throughout the preparation of the thesis and the completion of my master's program at NPS.

THIS PAGE INTENTIONALLY LEFT BLANK

I. INTRODUCTION

Precision indirect weapon systems have been the focus of modern-day battlefields due to their ability to fire from considerable distances and hit targets with only a minuscule margin of error. The proliferation of precision munitions for the artillery in recent decades has greatly enhanced an army's capability to conduct precision strikes on key enemy terrain with fewer instances of fratricides and collateral damage. As the reliance on precision munitions in modern warfare continues its upward trend, it is inevitable that non-precision area munitions will eventually be replaced by precision munitions or improvised with precision-guided kits (South, 2018). Due to budgetary constraints and the existing stockpiles of area munitions, however, it is unlikely that any military in the world would be able to pursue an all-precision capability for their army in the near future.

Until the day comes when area munitions become 'area precision munitions,' most armies will have to rely on an effective combination of area and precision munitions. Thus, the intent of this thesis is to explore an effective mix of area and precision munitions for use in urban warfare, which usually represents a highly dense and contested environment.

A. BACKGROUND

As the world continues its rapid rise in urbanization, the battlefield of the future will likely be located in a highly complex urban environment. According to the Headquarters, Department of the Army (HQDA) field manual on urban operations, an urban area is defined as "a topographical complex where man-made construction or high population density is the dominant feature" (HQDA, 2006, p. 13). The urban battlefield is also multi-dimensional with four main surfaces of consideration such as (1) Airspace External Space, (2) Surface, (3) Super-surface, and (4) Sub-surface, which further magnify its complexity. Figure 1 shows the multi-dimensional Urban Battlefield adapted from FM 3–06.

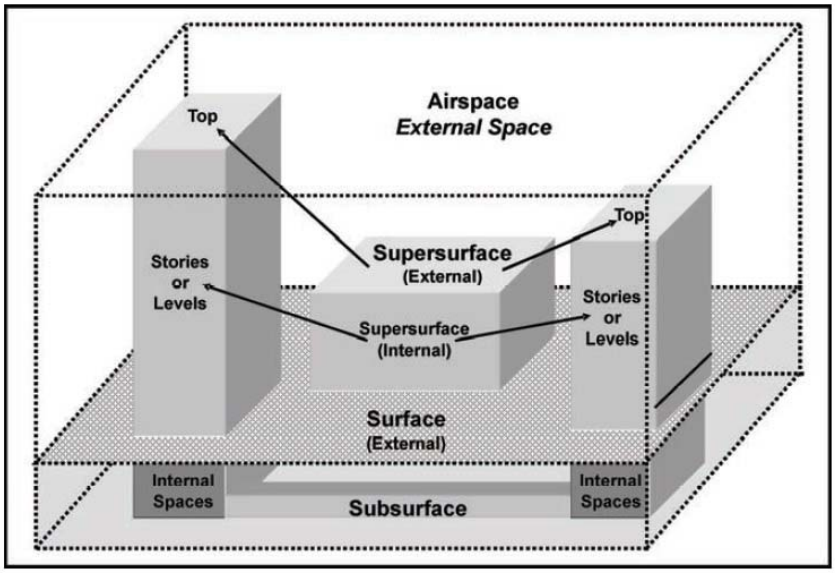


Figure 1. Multi-dimensional Urban Battlefield. Source: HQDA (2006).

Due to the multi-dimensional complexity, as well as the physical and human components of the urban battlefield, ground commanders often face tactical dilemmas in the employment of fire support weapons. With considerations such as fratricide to own forces, non-combatant casualties, and collateral damage, the potential use of massed non-precision area fires from the artillery to support urban operations is limited. Lessons learned from notable modern urban warfare such as the First Lebanon War in 1982 (also known as *Operation Peace for Galilee*) and the Second Lebanon War in 2006 (also known as *Israel-Hezbollah War*) have shown that both indirect and direct artillery strikes can be effective in maintaining fast operational tempo and crippling enemy strongholds. Nonetheless, these were often executed at the expense of collateral damage, fratricides, and high non-combatant casualties (Asymmetric Warfare Group, 2016).

The development of precision munitions for artillery in recent years, such as the M982 Excalibur and M1156 Precision Guidance Kit (PGK), provided much needed assurance in reducing battle risks by improving the accuracy of the weapon system. It is typically assumed that improvement in accuracy enables a reduction in the volume of fire while improving effectiveness in the urban battlefield. However, the high cost of employing precision munitions in an urban environment, and their ability to create area

suppression, remain contentious. Therefore, the key motivation of the research is to establish effective employment of fires in urban environments using a combination of precision and conventional area munitions to achieve the most desired effects determined by military commanders.

B. TYPES OF ARTILLERY FIRES

Artillery fire is indirect fire (without line-of-sight) delivered by large caliber guns that are able to amass destruction to enemy forces beyond the range of a small arms' weapon system. While the primary objective of artillery fire missions is to maximize effects on the enemy, it is imperative that its effects on friendlies and non-enemy elements are minimized as much as possible. Two commonly known artillery fire missions, Area/ Mass Fire and Precision Fire, are defined as follows:

Area/Mass Fire refers to

> fire missions that require numerous projectiles fired by multiple weapon system to achieve the effect of area suppression on an area target. A well-defined point at or near the center of the area to be attacked should be selected and used as an aiming point. (HQDA, 1991, p. 4-4)

Precision Fire refers to

> fire missions that require one or few projectiles fired by single weapon system either to obtain registration corrections or to accurately destroy a located point target. (HQDA, 1991, p. 4-3)

From the definitions of area and precision fires, it is clear that the key difference between them is the type of target and its associated mission profile for engagement. Area fire is particularly effective in pinning down immobile targets in an open environment with low risk of collateral damage and fratricide, while precision fire is effective against targets operating in highly cluttered environments where concerns of collateral damage and fratricide risks are high.

Conventional dumb munitions are largely used for area fire missions, where adjustments of fire are needed to account for inaccuracies of the munition caused either by meteorological (MET) factors or inaccurate target location. An observer conducts an "adjust fire mission" by locating the first round of the impact point and determining the

deviation of the round from the target before making the necessary adjustment to correct the error. The observer may need to make continual adjustment of rounds to achieve a small threshold of missed distance (typically within the damage radius of munition) around the target in order to destroy the target. This potentially increases the risks of the firing units being located by enemy target acquisition assets (HQDA, 1999).

A technique such as Precision Registration (PR) is often used to eliminate the inaccuracies of dumb munitions due to factors like meteorological conditions. The formal definition of PR is "a technique that requires an observer to adjust a group of rounds fired from the same howitzer so that their mean point of impact (MPI) occurs at a point of known location" (HQDA, 1999, p. 10-6). As compared to the standard adjust mission, PR can be conducted to the rear of a combat zone to avoid enemy target acquisition assets. Typically, dumb munitions can be accurate after PR, provided there is a continual update in MET data. Even after PR and the incorporation of current MET data into the firing solutions, however, the accuracy of dumb munitions can only be as good as about 50 meters (m), and it will begin to degrade as the atmospheric conditions change.

In contrast, precision munitions can easily achieve the desired effect of precision fire missions with fewer rounds without making multiple adjustments of fire or registrations. Precision munitions can also be employed for area fire missions by assigning multiple aim points to each firing unit in the target area to distribute the area suppression effects. Due to the high cost of each precision munitions, however, it may be unwise to deploy them for area fire missions where precision is not of critical need. A key consideration for using precision munitions is the need for accurate target location. A precision munition can be "precisely inaccurate" and deemed useless if the observer has a high Target Locating Error (TLE).

C. TYPES OF ARTILLERY MUNITIONS

Conventional unguided artillery munitions such as the 155mm M107/M795 High Explosive (HE) have been widely used since the early of 20th century and have been the weapon of choice for long-range area targets where precision is rarely a concern. From the late 20th century to the early 21st century, the need for precision has grown tremendously

due to rapid global urbanization, which has increased the risks of battle damage. In the past two decades, the development of precision guided munitions has rapidly picked up pace with the help of technological advancements such as the proliferation of Global Positioning Systems (GPS) that enable relatively cheap and accurate guidance for weapon systems. In the artillery domain alone, the development of precision guided munitions such as the 155mm M982 Excalibur, 227mm Guided Multiple Launch Rocket System and even the M1156 PGK have provided plenty of options for the military in conducting strategic strikes on accurately located high value targets in dense urban environments. Precision munitions have low Circular Error Probability (CEP), which is a common measure for a weapon system's precision. According to the *U.S. Department of Defense (DoD) Dictionary of Military and Associated Terms*, CEP is "the radius of a circle within which 50% of the missile's projectiles are expected to fall" (Joint Chiefs of Staff, 2001, p. 86). Hence, larger CEP implies higher uncertainty on the precision of the weapon system.

This study mainly focuses on comparing the effective mix of 155mm munitions such as the M795 HE, M982 Excalibur, and M1156 PGK in an urban environment. A brief summarization of these three types of munitions follows.

1. M795 HE Projectile

The 155mm M795 HE projectile was first introduced into service for use in the U.S. Army and U.S Marine Corps in the late 1990s to replace the ageing 155mm M107 HE. The M795 HE is classified as unguided conventional artillery munition as its accuracy is determined by interior, transitional, and exterior ballistics of the weapon system (HQDA, 1999). Hence, meteorological factors such as wind, temperature, and pressure will greatly affect the accuracy of the munition in its course of trajectory. The CEP of the M795 HE increases with firing distance and can reach as high as 275 m at its maximum engagement range (Hill, 2007) without registration. Table 1 presents approximate CEP values for a conventional 155mm projectile at various engagement ranges.

Table 1. Approximate CEP Values for Generic Conventional 155mm Projectile. Source: Hill (2007).

Range	CEP
15 km	95 m
20 km	115 m
25 km	140 m
30 km	275 m

2. M982 Excalibur GPS-Guided Projectile

The M982 Excalibur is a GPS-guided 155mm artillery projectile jointly developed by Raytheon and BAE Systems Bofors to improve the accuracy of artillery fire and address the growing concern of collateral damage in the modern urban battlefield. The M982 has a maximum effective range up to 57 km, depending on configuration, and is able to achieve a CEP of within 5 to 20 m depending on range (Kelly, 2018). Despite its enhanced capability, the steep cost of approximately US$68,000 (Freedberg, 2016) per round (equivalent to 68 times the cost of a conventional munition) is a major impediment for it to be extensively employed in any battle.

3. M1156 PGK "Smart Precision Weapon"

The M1156 PGK was developed by Orbital ATK in the early 2000s to address the U.S. Army's need for a cheaper precision munition alternative by turning a 155mm conventional munition into a 'smart precision weapon.' The PGK with its GPS guidance package and course correction control surface greatly improves the accuracy of conventional artillery and is able to reduce the CEP to less than 50 m regardless of the firing range. The key advantage of this development is the ability to convert existing inventory of conventional 155mm projectiles into affordable precision munitions simply by replacing the conventional fuze with a PGK (Storsved, 2009).

Table 2 details the specifications of the three types of 155mm munitions investigated in this study.

Table 2. Types of 155mm Artillery Munitions

Specification	M795 Conventional Munition	M1156 Precision Guided Kit	M982 Excalibur
Circular Error Probable (CEP) (m)	95–275	30–50	5–20
Max Effective Range (km)	30	30	57
Guidance Mechanism	Unguided Ballistic Trajectory	GPS guided	GPS Guided and Inertial Navigation
Damage Radius (m)	~50	~50	~50
Cost/unit (USD)	~1,000	~10,000 (on top of per unit cost of M795)	~68,000

Note: Data for CEP, Max Effective Range, Guidance Mechanism and Damage Radius of M795 and M1156 from Storsved (2009), for Excalibur from Raytheon (n.d.), for Cost/unit from Defense Industry Daily (2017).

Although this study mainly focuses on the effective mix of the three types of 155mm munitions in urban environments, the methodology and research objectives can be extended to include other caliber systems such as mortars, rocket artillery and missiles.

D. DAMAGE FUNCTIONS

Damage functions, denoted by $D(r)$, are commonly known as closed-form functions used to estimate the probability of damage by indirect area fire in a mathematical combat simulation (Klopcic, 1990). Miss distance, *r,* is defined as the distance between the impact point of the shot and the location of the target (Washburn & Kress, 2009). While there are numerous types of damage functions examined in the literature, the two most closely associated with modeling indirect artillery area fire are the Cookie-Cutter function and Carleton-von Neumann (Carleton) damage function (also known as the diffuse Gaussian damage function).

The Cookie-Cutter function is a commonly used and is a simplified damage function that assumes a fraction of the targets lies inside a weapon lethality area or damage radius, R, will be completely destroyed while those that lie beyond will receive no damage. The Carleton function, on the other hand, assumes a Gaussian distribution of Probability of Kill (P_{Kill}) value at any distance from the impact point. Unlike the Cookie-Cutter damage

function, the Carleton function always has a positive P_{Kill} no matter what the miss distance. While the resulting P_{Kill} of weapons modeled with the Carleton function is harder to predict, it is more realistic for modeling the fragmentation and blast overpressure effects caused by artillery munitions (Washburn & Kress, 2009). Figure 2 shows the comparison of the two damage functions for modeling the probability of damage against miss distance. The mathematical representations of the two damage functions are given as follows:

- Cookie-Cutter damage function, $D(r) = \begin{cases} 1, r \le R \\ 0, r \ge R \end{cases}$

- Carleton damage function, $D(r) = e^{\frac{-r}{2b^2}}$ for some scale factor *b*.

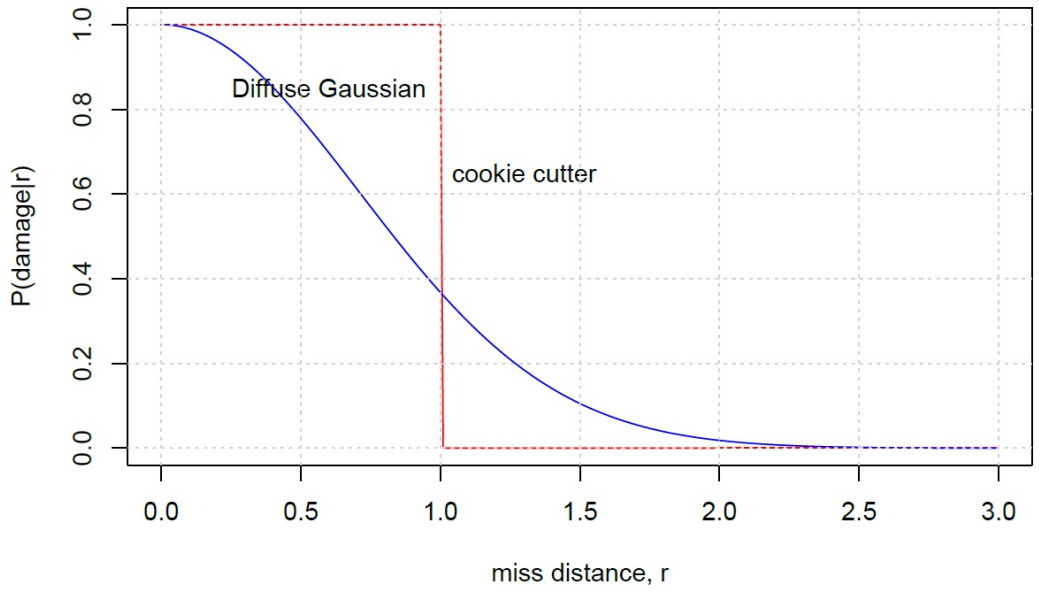


Figure 2. Damage Functions for Cookie-Cutter and Diffuse Gaussian Weapons, Each with Damage Area π. Source: Washburn and Kress (2009).

As illustrated by Professor Thomas Lucas, "the differences in estimating P_{Kill} can be drastic for large aim-point offsets, hence the form of damage function in combat models will affect the model's estimates of the number of fratricides and collateral damage in scenarios involving a non-linear battlefield" (Lucas, 2002, p.307). This may potentially affect the study on developing the effective mix of precision and area fires to reduce risks.

In this study, both the Cookie-Cutter and Carleton damage functions are incorporated into the experimental battle scenarios to represent the underlying P_{Kill} of both munitions to avoid the pitfall of overestimating or underestimating probability of killing or injuring friendly targets (i.e., fratricide, non-combatant causalities, or collateral damage).

E. OBJECTIVES

The purpose of this study is to establish a set of Pareto optimal solutions and a decision model matrix to guide tactical and strategic military planners in deciding an optimal mix of precision munitions (M982 and M1156) and conventional area munitions (M795) for use in urban battlefields. While it is obvious that precision munitions are the answer to maximize effects on enemy and minimize effects on collateral damage, the choice is not obvious when the cost factor comes into play. Hence, the decision model is a multi-objective tool that incorporates both operational and cost effectiveness considerations in the employment of fires.

F. RESEARCH QUESTIONS

The following questions under the consideration of operational and cost effectiveness guide the flow of the research objective.

1. Operational Effectiveness

- What are the key factors influencing the success of battle in the urban battlefield?
- How effective are precision fires relative to area fires in inflicting damage to enemy forces?
- How effective are precision fires relative to area fires in reducing collateral damage and fratricides?
- How effective are precision fires relative to area fires in increasing the probability of mission success and decreasing the time required for battle?

2. Cost Effectiveness

- What are the trade-offs involved between cost and operational effectiveness?
- How can the cost effectiveness of munition allocation alternatives be determined?

G. BENEFIT OF THE STUDY

The study highlights the potential challenges faced by combatants in urban environments, and provides a detailed analysis on the effective employment of precision and area indirect fires in urban battle to enhance the safety of friendly troops and non-combatants, while reducing collateral damage to infrastructures. In addition, this study provides insights into how a cost benefit analysis can be performed to decide on the best possible combination of munitions to achieve the desired effects given cost constraints.

H. THESIS OUTLINE AND METHODOLOGY

This study involves a combination of several models and techniques to answer the research questions. Chapter II outlines the experimental battle scenarios constructed using an agent-based simulation to determine the measures of effectiveness. Agent-based simulation (ABS) has the ability to model the salient features of real-world operations without considering every possible characteristic in a computer simulation environment (Cioppa, Lucas, & Sanchez, 2004). Due to the complexity of urban battles, where numerous controllable and uncontrollable factors are interacting non-linearly, any observations of emergent collective behaviors are particularly insightful in this study to determine the effective mix of munitions. The simulation tool used is the 'Pythagoras' ABS, which was originally developed by Northrop Grumman to support the U.S. Marine Corps Project Albert initiative (Northrop Grumman Space & Mission Systems Corp, 2008).

Chapter III discusses the conduct of efficient Design of Experiments (DOE) and data farming techniques using efficient, space filling designs such as the Nearly Orthogonal Latin Hypercubes (NOLH) and Nearly Orthogonal-and-Balanced Mixed designs (NOB) to

manipulate inputs to the system and to understand the effects of factors on the outputs of the system.

Chapter IV outlines the use of various statistical analysis methods such as multiple regression and partition trees to construct metamodels of the simulation's performance and identify the important interactions between variables and understanding of the system. A multi-objective optimization technique is also used to identify sets of Pareto optimal solutions. The results enable the creation of a decision model matrix to provide decision makers with the options of employment based on operational and cost considerations.

Chapter V further explores the model by using a multi-objective evolutionary algorithm to identity a set of Pareto optimal or near-optimal alternatives that complement the analytical results from the metamodels in Chapter IV.

Finally, Chapter VI highlights the key findings, conclusions from the study, and recommendations of future research areas.

THIS PAGE INTENTIONALLY LEFT BLANK

II. SCENARIO AND MODEL DEVELOPMENT

In this chapter, two case studies of Military Operations in Urbanized Terrain (MOUT) in the 21st century are examined and modeled as baseline scenarios for the simulation models. These two scenarios are inspired by actual events with the aim to capture the essence of conducting artillery operations in different urban environments. The force structures and the tactics, techniques, and procedures for employment of artillery munitions are fictitious to fulfill the research objectives. In addition, an overview of the Pythagoras simulation tool, the baseline model, and modeling assumptions developed for the combat scenarios are described at the end of this chapter.

A. SCENARIOS

Two actual urban battles that took place in the 21st century are described in this section.

1. Second Battle of Fallujah (November–December 2004)

The first scenario being modeled is known as the Second Battle of Fallujah (code-named *Operation Phantom Fury*), which was a joint coalition force offensive operation led by the U.S. Marine Corps against the Iraqi insurgents in the city of Fallujah during the peak of the Iraq war (Swift, 2017).

a. Terrain

Fallujah, which is located about 40 miles east of Baghdad, Iraq, was a densely populated city with an estimated population of 300,000 during the 2nd Battle of Fallujah in 2004. The main battle took place in the north of Fallujah with an operating terrain size of approximately 2 kilometers (km) by 2 km (4 square (sq) km). An interesting feature of the operating terrain is the presence of numerous mosques within, which greatly complicates the battle. An aerial view of the battle area, which is extracted from the Global Security organization website, is shown in Figure 3.

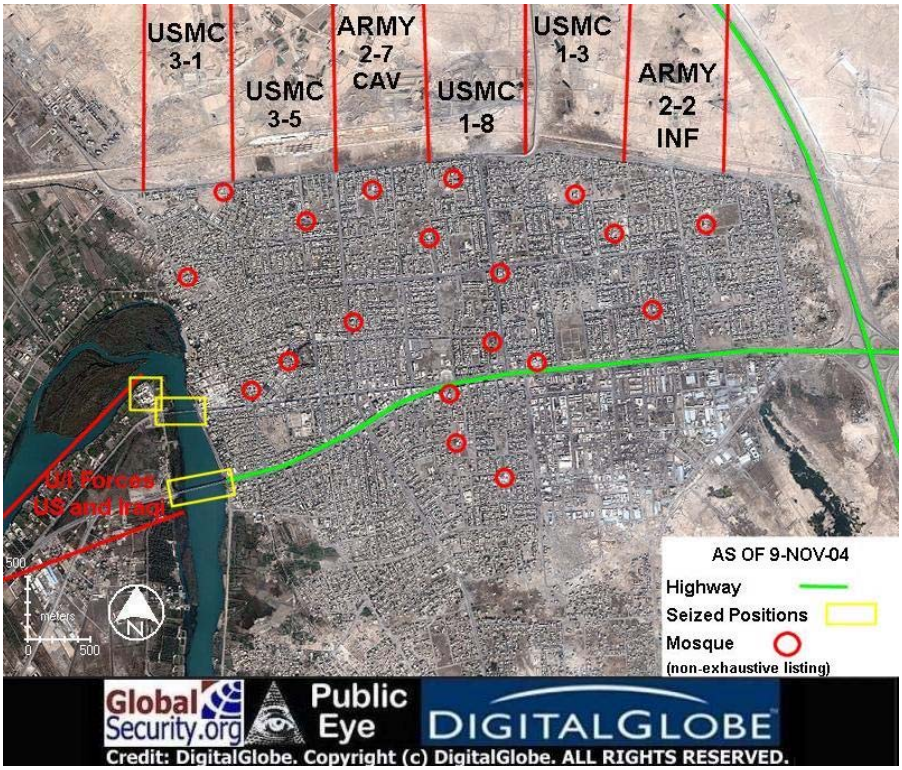


Figure 3. Battle Area of 2nd Battle of Fallujah (2004). Source: GlobalSecurity.org. (n.d.)

b. Battle Statistics

The 2nd Battle of Fallujah is widely considered as one of the largest engagements of the Iraq war. Based on estimates, the Coalition forces and U.S. forces suffered a total of more than 200 killed and 600 wounded, while insurgent casualties ranged from 1,500 to 2,000 (McWilliams & Schlosser, 2004). The Red Cross also estimated about 800 Iraqi civilians were killed during the offensive (Jamail, 2004). Due to the presence of densely urban infrastructures, over 10,000 buildings, including 60 mosques that were prevalently used by insurgents as their weapon strong points, were destroyed by artillery and air strikes (Tyson, 2005).

c. Key Operational Lessons Learned on Artillery Fire

Studies from the U.S. Army Asymmetric Warfare Group have shown that integration of artillery fire within the city of Fallujah during Operation Phantom Fury greatly enhanced the effectiveness of the assault in terms of the objectives. The battle was

supported with one battery of Howitzers and one battery of Paladins to provide artillery fire. While many had deemed the risks of using artillery in urban operations were too high, the Marine divisions demonstrated that with well-trained forces, good target identification, and well-controlled fires, the artillery was able to make a huge contribution to the fight with little or no friendly casualties (Asymmetric Warfare Group, 2016). Even without the use of precision artillery munition during the battle, the operation was a successful demonstration of effective employment of indirect fire in urban operations.

In this modeling scenario, we observe the impact of the battle by equipping the attacking forces with both precision and conventional munitions and determine an optimal mix that is both operational and cost effective.

2. Battle of Marawi (May–October 2017)

The second scenario being modeled is a more recent event known as the Battle of Marawi (also known as the *Marawi siege*), which was a five-months long urban battle between the Philippines security forces and militants affiliated with the Islamic State of Iraq and Levant (ISIL) in Marawi, Lanao del Sur (The South China Morning Post, 2017).

a. Terrain

Marawi is the largest city of Mindanao in Southern Philippines, with a population of about 200,000. The main battle took place on the east side of Marawi city with a terrain size of approximately 1.5 km by 1.5 km (2.25 sq km). The heat map of the battle area extracted from Schadow1 Expeditions (a travel and mapping resource for the Philippines) is shown in Figure 4.

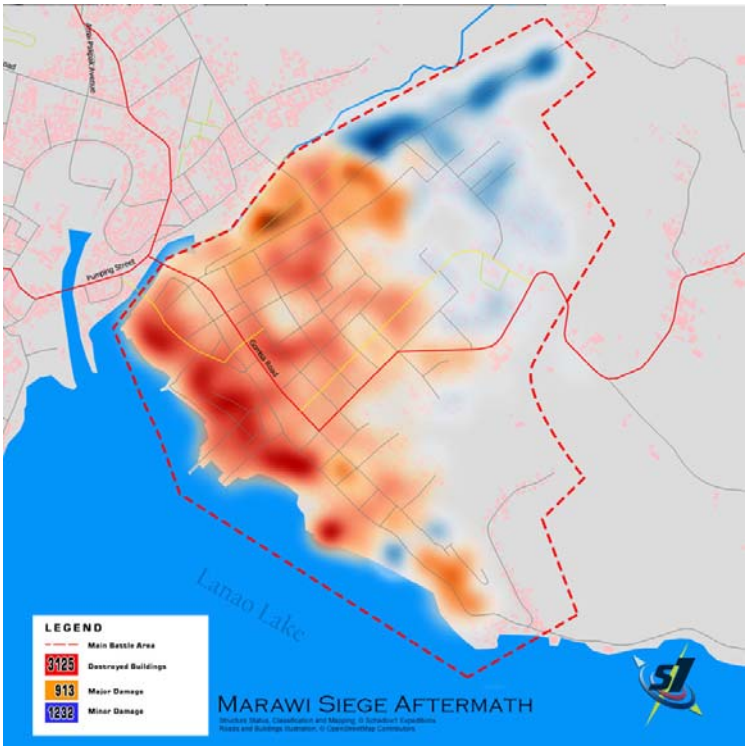


Figure 4. Heat Map of Affected Areas in Battle of Marawi. Source: Malicdem (2017).

b. Battle Statistics

According to official statistics released by the Philippines Government, the battle left more than 1,000 government forces wounded and at least 1,130 people dead, including 919 militants, 165 soldiers, 50 civilians, and hundreds of thousands of residents displaced (Gomez, 2017). In addition, more than 3,000 building structures were completely destroyed or partially damaged by artillery and airstrikes (Malicdem, 2017).

c. Key Operational Lessons Learnt on Artillery Fire

Studies from Amnesty International have shown that the series of artillery and airstrikes severely ravaged the infrastructures in Marawi and terrorized the civilians who were trapped or held hostage by the militants. There were also multiple accounts by eyewitnesses of civilians being killed by indirect fire and airstrikes, but these eyewitnesses were unable to report for fear of being accused as militants (Amnesty International, 2017). Due to security and access restrictions, a thorough assessment of the impact of indirect fire

causing death to civilians and damage to infrastructure is not possible. Nonetheless, from the statistics and heat map showing the damage to infrastructure, it can be inferred that inaccurate and ineffective employment of fire can cause tremendous damage in urban operations. From the length of battle (five months), we can also infer that indirect fire can be ineffective in supporting ground forces against well-protected enemies under concrete building structures.

In this modeling scenario, we assess the impact of the battle by equipping the attacking forces with both precision and conventional munitions and determine an optimal mix that is both operational and cost effective.

B. PYTHAGORAS CHARACTERISTICS

Pythagoras is an agent-based simulation environment that "enables users to create intelligent agents by assigning them behaviors based on motivators" (Henscheid, Middleton, & Bitinas, 2006, p. 41). Interaction among different agents can often create autonomous and emergent (unforeseen) behavior that is noteworthy. There are many agent characteristics available for selection and modification, which enables users to create scenarios that closely reflect real life events.

As the simulation is written in java, it can be run in a cluster computer environment, enabling thousands of runs and replications in a relatively short time, as well as in batch mode from a computer's command prompt (Henscheid, Middleton, & Bitinas, 2006). With a Graphical User Interface (GUI), it can also be run interactively for easy modification and verification of the model during model development and initial experimentation. The general areas modeled within Pythagoras in this study are as follows:

- Overview
- Terrain
- Weapons
- Sidedness
- Sensors

- Communications
- Agents
- Alternate Behavior
- Measure of Effectiveness

These characteristics are discussed further in the next section. For additional information on the characteristics, the reader should refer to the *Pythagoras User's Manual* (Northrop Grumman Space & Mission Systems Corp., 2008).

Although similar agent-based simulation modeling platforms are available, such as Map Aware Non-Uniform Automata (MANA), a product developed by New Zealand's Defense Technology Agency (McIntosh, Galligan, Anderson, & Lauren, 2007), Pythagoras is the better tool for modeling of indirect weapon systems. As compared to MANA, which uses basic range-pairs for modeling indirect fire probability of hit and kit, Pythagoras is able to model the characteristics of indirect fire weapon systems based on well-established damage functions such as Cookie-Cutter functions and Carleton functions. Pythagoras also enables options to include Target Locating Error for sensor systems, which directly affects the accuracy of fire.

1. Overview

Pythagoras is a time-stepped based, terminating simulation, where the simulation will end once the allocated number of time-steps has taken place. In this simulation, one time step represents 20 seconds in real time.

2. Terrain

The maximum terrain play box size is set to 4,000 by 4,000 pixels. The terrain size is set to 2,000 by 2,000 pixels and 1,500 by 1,500 pixels for Scenarios one (Fallujah) and two (Marawi) respectively, where one pixel represents 1 m in actual map size. The background maps of Scenario 1 and Scenario 2 are extracted from Google Earth. The basic deployment of forces and terrain features being modeled in the simulation are shown in

Figure 5 and Figure 6, respectively. The only terrain features being modeled are buildings and water bodies. The associated terrain attributes such as protection, concealment, and mobility are data farmable and are discussed in the Chapter III.

Figure 5. Terrain Map of Scenario 1 (Fallujah)

Figure 6. Terrain Map of Scenario 2 (Marawi)

3. Weapons

The types of weapons modeled consist of small arms direct weapon systems and high caliber indirect weapon systems. The small arms consist of the M16A4 infantry rifle, M24 sniper weapon, squad automatic weapon, general purpose machine gun, rocket propelled grenade (RPG) launcher, and AK47 rifle. The indirect weapons consist of the 155mm Artillery and 120mm Mortar. Three types of 155mm munitions, namely the M795, M1156, and M982, are created to model the effective mix of munitions for the scenarios. The basic properties such as the effective range and firing rate of the weapon systems are set according to open source data. Other properties such as the range- dependent P_{Kill} and P_{Hit} values of direct weapons, and range-dependent CEP and damage radius values of indirect weapons, are data-farmable and discussed in Chapter III.

4. Sidedness

Each agent may be assigned a value for each of the three color properties such as red, blue, and green to allow them to establish affiliation among each other. Each color ranges from a scale of 0 to 255, which defines how far or close in color space an agent must be to be considered friend, foe, or neutral. For the simulation models, the Blue agents are given a 'blueness' value of 255; Red agents are given a 'redness' value of 255; and Neutral agents are given a 'greenness' value of 255, 'redness' value of 175, and 'blueness' value of '175.' Color radius is an attribute used as a comparison with the color-distance between two agents to determine whether they are friends or enemies. In both scenarios' settings, Blue and Red agents classify each other as enemies but classify Neutral agents as friends, so as to prevent any deliberate firing at the Neutral agents.

5. Sensors

Each agent needs to have a sensor in order to navigate around the play board. The sensors have range-dependent probabilities of detection and can be affected by intervening terrain features. The sensors being modeled in the simulation models are Blue and Red infantry sensors, a Blue Forward Observer (for directing artillery fire), and Neutral sensors. The infantry sensors (classified as signature band A) refer to infantry soldiers with a 'naked eye view.' The infantry sensors are given a range-dependent probability of detection, P_d,

of 0.95 at 50 m and P_d of 0.50 at 300 m as a baseline. The Forward Observer (FO) sensor (classified as signature band B) refers to Artillery Liaison with enhanced field of view and detection range aided by electro-optics (EO) and infrared (IR) devices. The FO sensor has a maximum range of 2000 m with a set of range-dependent P_d. In addition, the FO sensor is set up to have a range-dependent Target Locating Error (TLE) attribute, which is used to define the amount of error involved in accurately locating other agents. This reflects the fact that the sensor is not perfect in locating a target at a longer range, which will affect the accuracy of precision munitions that need low TLE to destroy the target. The infantry sensor's range and TLE attribute are data farmable and discussed in Chapter III.

6. Communication

The agents in the model can communicate with friendly agents via their sensors or allocated communication devices. In this study, the communication attribute is not a variable of interest; hence, the agents are given the ability to communicate across the play board and are not restricted in range.

7. Agents

For both scenarios, the number of agents is fictitiously fixed with a 3:1 combat ratio between the Blue and Red agents. Although many attributes can be modeled for the agents' characteristics, this study focuses on only a few for data farming purposes. Attributes such as the vulnerability index (which determines the level of protection each agent possesses), speed of agents, and their movement desirability towards enemy forces are the few key attributes that are data farmable. They are discussed in Chapter III.

8. Alternate Behaviors

Alternate behaviors in the simulation are new behaviors that an agent will follow when a triggered event is activated. The alternate behavior trigger is only applicable to the artillery agent in the model where it is given trigger conditions to stop firing or resume firing. The alternate behaviors and their associated triggers are described in the section on modeling assumptions.

9. Measures of Effectiveness

Five key measures of effectiveness (MOE) for evaluating the effective mix of munitions and their associated simulation outputs are described in this section.

a. Force Exchange Ratio

The Force Exchange Ratio (FER) is the ratio of the percentage of Enemy casualties (Red agents) to the percentage of Friendly casualties (Blue agents) and is a commonly used MOE in quantifying the level of warfighting capability needed to achieve the attacker's objective of winning the battle (Panel on Operational Test Design and Evaluation of the Interim Armored Vehicle, 2003). FER is also a useful metric to predict the probability of mission success based on the historical relationship between FER and probability of win. From the historical results shown in Figure 7, a FER of 2.5 is associated with winning a battle 90 percent of the time. The desired MOE for the attackers is to achieve a high FER.

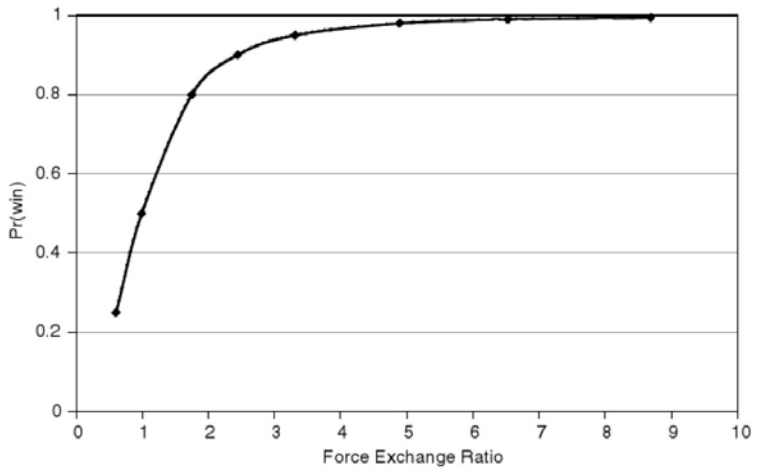


Figure 7. Historical Relationship between FER and Probability of Win. Source: Panel on Operational Test Design and Evaluation of the Interim Armored Vehicle (2003).

b. Fratricide Rates

Fratricide is defined as the accidental or unintentional act of killing one's own forces. In the model, this MOE can be calculated as the number of Blue agents killed by artillery fire divided by the total number of Blue agents killed in the battle. The desired MOE for the attacker is to minimize its fratricide rates.

c. Collateral Damage

According to the *DoD Dictionary of Military and Associated Terms*, collateral damage is "the unintentional or incidental injury or damage to persons or objects that are not lawful military targets in the circumstances of ruling during an attack on legitimate military targets" (Joint Chiefs of Staff, 2001, p. 93). While it is easy to model collateral damage inflicted on agents, the destruction of building structures can be difficult to model in an agent-based simulation. As such, we use Neutral agents as a proxy to model damages to building structures by placing them in the buildings adjacent to the Red agents. Here, the number of dead Neutral agents represents the collateral damage to both civilians and building structures. The desired MOE for the attacker is to minimize collateral damage to Neutral agents.

d. Effectiveness of Artillery Fire on Red Agents

The effectiveness of artillery fire is defined by the number/rate of kills and injuries it can inflict on enemies during a battle. In the model, this MOE can be calculated as the number of Red agents killed by artillery fire divided by the total number of Red agents killed in the battle. The desired MOE for the attacker is to maximize the effectiveness of artillery fire on Red agents.

e. Time to Achieve Mission Success

The condition for mission success in both scenarios is defined as the time when 90 percent or more of the Red agents are killed. This time-series MOE can be calculated using the time series output of "Survivorship – Dead agents" divided by the initial number for the Red agents. The desired MOE for the attacker is to achieve the conditions for mission

success in the earliest time, which indicates its superior combat effectiveness against the enemy.

C. MODELING ASSUMPTIONS

Modeling assumptions are integral to the successful development of the model and are needed to ensure that model captures the relevant information needed for data analysis. This section outlines a few key assumptions made when developing the model for both scenarios.

1. Vulnerability of Blue Artillery and FO Agents

The vulnerability index of Blue artillery and FO agents is set to zero to reflect that they will not be killed by any means of fire. This is realistic in the sense that Red insurgents do not have the equivalent long-range weapons and sensor systems to conduct counter-targeting of the Blue artillery, which is supposed to be located tens of miles away. This setting also allows the Blue artillery agent to continue firing on the enemies throughout the battle, enabling us to collect key data on the effects of area and precision munitions.

2. Engagement Desire of Blue Artillery

There are two trigger points for the artillery agent to exhibit an alternate behavior. The first trigger is activated when friendly casualties exceed 20 percent of the total initial count (including both Blue and Neutral agents) and its associated alternate behavior is to hold firing for 30 time steps. After 30 time steps, the artillery agent will resume firing. The second trigger is activated when friendly casualties exceed 50 percent, and its associated alternate behavior is to hold fire for another 30 time steps After this final alternate behavior is fulfilled (after 30 time steps), the artillery agent will continue to fire until all the Red agents are eliminated or until the agent runs out of ammunition (whichever happens first).

The initial weapon selection logic for Blue artillery is set to the one with the lowest P_{Kill} values, which is the M795 dumb munition. After the first trigger has taken place, the priority of weapon selection logic is subsequently changed to the weapon with the highest P_{Kill} in the sequence of M982, M1156, and M795, until all three ammunition types are depleted. This is to reflect the fact that attacking forces typically use area fire to suppress

the enemies in defense before launching ground troops to capture the terrain. However, when the battle intensifies and with more friendly casualties being inflicted in the close-in battle, it becomes necessary to launch precision fire to accurately target the enemy's stronghold and to reduce the risk of fratricide and collateral damage.

3. Ammunition Count for Artillery Munitions

The amount of ammunition allocated to each artillery munition type is based on the 'relative effectiveness' of the munitions. According to studies conducted by Orbital ATK (Storsved, 2009), the accuracy of each PGK round with a CEP of 20–50 m is able to reduce the required amount of dumb munition rounds by 55–68 percent. Another study published by the Project Manager of the Combat Ammunition Project Office compares the effectiveness of Excalibur and conventional dumb munition and finds that Excalibur is able to reduce the demand for dumb munitions by 85–90 percent (Milner, 2012). Hence, the maximum amount of M982 and M1156 allocated to both scenarios is set to 15 percent and 45 percent respectively to reflect their relative effectiveness against the dumb munition. The total amount of 155mm munition that can be carried by the artillery battery is assumed to be one basic load, which rounds up to about 400–500 rounds.

4. Multiplier Effect of Sniper Weapon P_{Hit} and P_{Kill} against Other Direct Weapons

To illustrate the fact that a sniper weapon is more effective in P_{Hit} and P_{Kill} against other direct weapon systems, a multiplier ratio of 1.0–1.30 (0 to 30 percent improvement) is applied for the sniper weapon across the range of data farmable P_{Hit} and P_{Kill} values of other direct weapon systems. This ensures that at each range of P_{Hit} and P_{Kill} for direct weapons, the sniper weapon is always more effective, with a magnitude of up to 30 percent.

5. Multiplier Effect of Indirect Weapon CEP, M795 across Range Dependent CEP

A multiplier ratio of 1.0–1.3 is applied across the range-dependent CEP of the M795 to reflect an improvement in accuracy of up to 30 percent with registration of fires.

6. Red Forces Positioning

The assumption made on the positioning of Red forces is that more than 70 percent of them will be tempted to reside in the buildings for cover and concealment, while about 15–30 percent of them would be conducting security patrol along the streets and open areas to detect any incoming Blue threats. The 60mm Red mortar is also assumed to be positioned out in the open area for providing indirect fire support for the Red agents. The Red mortar position is strategic, as there are surrounding buildings nearby where they can seek protection and be able to prevent line-of-sight targeting by Blue forces.

7. Neutral Forces Positioning

As described in Section II, Neutral agents are separately positioned inside building structures adjacent to the Red agents. This enables us to capture the artillery effect on collateral damage when the artillery is attempting to fire on the Red agents.

III. EXPERIMENTAL DESIGN

Complex simulation models often contain large numbers of potential factors or input variables that may have an impact on the desired MOEs. In order to understand the impact of these factors and their interactions, it is often necessary to use multi-dimensional DOE. In addition, the response surfaces of complex models are often non-linear, resulting in the need to sample across the entire space of each factor in order to produce meaningful results from the model. With efficient DOE, it is possible to reduce the size of the experiment at little cost, while still producing many meaningful insights in a short amount of time (Sanchez & Wan, 2015).

A. VARIABLES OF INTEREST

The two types of variables in this simulation experiment are controllable variables and uncontrollable or noise variables. Controllable variables, also known as decision factors, are things that are controllable by decision makers in real-world settings. Uncontrollable variables, often referred to as noise factors, are not easily controllable, or controllable only at great cost in real-world scenarios; such factors include environmental conditions and enemy capabilities (Sanchez, 2000). Note that variables and factors are used interchangeably in this context.

In this thesis, the decision factors studied are capability focused, such as system and equipment characteristics of the attackers (Blue force). The noise factors include the defender's (Red force) capabilities and the environmental factors, such as terrain and civilian (neutrals) population density. Table 3 lists the factors of interest in the simulation model.

Table 3. Factors of Interest

S/N	Factor	Remarks
1.	Proportion of Artillery Munitions (M982, M1156, and M795, respectively)	System Capabilities Factors (Continuous)
2.	CEP of Artillery Munitions (M982, M1156, and M795, respectively)	
3.	Probability of Hit (P_{Hit})– Small Arms and Sniper Weapon Systems	
4.	Probability of Kill (P_{Kill}) – Small Arms and Sniper Weapon Systems	
5.	Agents' Movement Speed	
6.	Agents' Sensors Range	
7.	Target Locating Error (TLE) of Forward Observer	
8.	Vulnerability Index of Agents	
9.	Agents Movement Desire towards Enemy	
10.	Damage Functions (Cookie-Cutter or Carleton)	Function for Modeling Indirect Fire P_{Kill} (Binary)
11.	Building Protection	Environmental Factors (Continuous)
12.	Building Concealment	
13.	Building Mobility	

1. Decision Factors

This section describes the various decision factors that are controllable in nature and are included for studying their effects on the respective MOEs.

a. Proportion of Artillery Munitions

The amount allocated to each type of artillery munition is the key factor for this study as we aim to find an optimal mix of munition in urban battle. The total amount of 155mm munition allocated for the artillery battery in Scenario 1 (Fallujah) and Scenario 2 (Marawi) are 500 and 400, respectively. This is because Scenario 1 is given an operating

terrain size about twice that of Scenario 2 and an overall combat size force that is 50 percent higher (while keeping the force ratio equal). The amount allocated for the M982 and the M1156 range from 5 to 15 percent and 5 to 45 percent of the total amount, respectively, to reflect their relative effectiveness against the M795. In this design, the amount of dumb munition allocated to the M795 will not fall below 40 percent of the total munition carried in the battle.

b. CEP of Artillery Munitions

The CEPs of the M982 and the M1156, which are not range dependent, varies from 5 to 20 m and from 20 to 50 m, respectively. The CEP of the M795, however, is range dependent, and is allocated with four range pairs as shown in Table 4 to simulate the fact that its CEP or inaccuracy increases with distance. Instead of varying the CEP of the M795 across the four range pairs, a multiplier ratio from 1 to 1.3 is applied across the default range pair settings in Table 4 to reflect improvement of accuracy up to 30 percent with registration of fires using the M795.

Table 4. Simulated Range Pairs for CEP of M795

Simulated Range in Pythagoras (pixels)	CEP
500	95 m
1,000	115 m
1,500	140 m
2,000	275 m

c. P_{Hit} (Small Arms Weapons Less Sniper)

P_{Hit} indicates the accuracy of weapon systems and is varied to explore the effects of their respective accuracy on the desired MOEs. The small arms modeled for Blue agents are the M16A4 rifle, squad automatic weapon, and general purpose machine gun. The small arms modeled for Red agents are the AK47 and RPG-7. The P_{Hit} factor being varied ranges from 0.1 to 0.7.

d. P_{Kill} (Small Arms Weapons Less Sniper)

P_{Kill} indicates the lethality of the weapon systems and is varied to explore the effects of their respective lethality on the desired MOEs. The P_{Kill} factor being varied ranges from 0.1 to 0.7.

e. P_{Hit} and P_{Kill} for Sniper Weapons

To illustrate the fact that sniper weapons are more effective in P_{Hit} and P_{Kill} against other direct weapon systems, a multiplier ratio from 1 to 1.30 (0 to 30 percent) is applied for the sniper weapons across the range of data farmable P_{Hit} and P_{Kill} values of other direct weapon systems. This ensures that the sniper weapon is always more effective than other direct weapons across the range of data farmable P_{Hit} and P_{Kill}. Furthermore, this applies to both Blue and Red agents in the simulation.

f. Movement Speed

Movement speed indicates the speed of agents moving to their objectives or enemies. Some agents in the model, such as Blue artillery, Blue FO, Neutrals, and Red 60mm mortar, remain static throughout the simulation; hence, it is not of interest to us to vary their movement speed. The movement speed of Blue and Red agents ranges from 20 to 30 pixels, which corresponds to 4 to 5km/hr in actual walking speed.

g. Sensor Range

As described in Chapter II, there are two types of sensors in the model, namely the normal infantry sensor and an advanced Blue FO sensor system. We intend to keep the capability of Blue infantry sensor constant at 300 m but vary the Red infantry sensor as a factor from 300 to 450 m to reflect that Red agents can have up to 1.5 times greater sensing capability due to their terrain advantages. The Blue FO sensor range remains constant at 2,000 m to ensure that they are able to observe and direct artillery fire in the entire range of the battlefield.

h. Target Locating Error (TLE) of Forward Observer

TLE indicates the accuracy of the Blue FO sensor in providing the location of the targets to the artillery agent, and ranges from 5 to 50 m.

i. Vulnerability Index

Vulnerability index, which has a data-farmable range from 0 to 100, indicates the level of vulnerability of the agent in receiving fire from the enemy. The vulnerability index is often used to represent the level of armor protection that agents may possess in the battle. The Blue infantry agents' vulnerability ranges from 5 to 100, while the artillery agent and FO agent vulnerability is set to a constant zero value (completely invulnerable) so that these agents can continue firing throughout the simulation based on their given triggers.

j. Movement Desire towards Enemy

Movement desire indicates the level of aggressiveness and desire agents have to approach their enemies depending on the agent's force ratio. A value of zero indicates that an agent is unwilling to approach the enemy if it has a lower force ratio. A value of 100 indicates the extreme aggressiveness of an agent and the agent's willingness to approach the enemy even with a lower force ratio. The movement desire for Blue and Red agents ranges from 0 to 100 to include all extreme conditions.

k. Damage Function

Damage function is a unique factor to model the probability of damage and to some extent the probability of kill of indirect weapon systems. The two types of damage functions available for selection in Pythagoras are the Cookie-Cutter and Carleton damage functions. In order to capture the essence of both damage functions, this factor is included in the DOE as a binary factor with 1 representing the Cookie-Cutter function and 0 representing the Carleton function.

2. Noise Factors

Noise factors describe the various factors that are uncontrollable or controllable only at great cost, such as the enemy characteristics and environmental conditions. The

noise factors for Red forces such as their system capabilities are discussed in conjunction with the decision factors in the previous section. The section describes the environmental factors such as building characteristics, which constitute a significant part in the study.

a. Building Protection

Building protection indicates the ability of the terrain to provide protection for agents residing in it. The level of protection ranges from 0 to 1, where 0 indicates no protection and 1 indicates complete protection or impenetrability of fire. The building protection ranges from 0.05 to 0.10. It is estimated that any protection level higher than 0.10 makes penetration by artillery fire difficult and hence is not useful for study.

b. Building Concealment

Building concealment indicates the ability of the terrain to provide concealment from the sensors so that agents residing within the terrain become less detectable. The level of concealment ranges from 0 to 1, where 0 indicates no concealment and 1 indicates complete concealment from any sensor system. The building concealment factor ranges from 0.05 to 0.10. It is estimated that any concealment level higher than 0.10 makes detection by sensors difficult and thus would severely slow down the battle and the simulation time of the model.

c. Building Mobility

Building mobility indicates how much mobility the terrain permits and how fast an agent can move within the terrain. The level of mobility ranges from 0 to 1, where 0 indicates immobility in the terrain and 1 indicates complete mobility, where agents can move freely in and out of the terrain feature. The building mobility ranges from 0.05 to 0.10. It is estimated that mobility above 0.1 will enable agents to move around and out of the buildings quickly and will artificially hasten the engagement of agents in the model.

B. EFFICIENT DESIGN OF EXPERIMENTS

An experimental design must allow a user to specify the levels of all decision factors and noise factors for each simulation run. There are many efficient DOEs in the

literature to consider. In this study, metamodels are developed using Nearly Orthogonal Latin Hypercubes and Nearly Orthogonal-and-Balanced Mixed designs. NOLH designs developed by Cioppa and Lucas (2007) have 'space-filling' properties that enable us to sample factors at higher combination levels. While NOLH designs are generally useful for constructing continuous-valued factors, their application to discrete-values factors is limited due to a rounding issue, which destroys the nearly orthogonal feature of the designs. NOBs designs are an extension of the same idea as NOLHs, but they are able to incorporate a mixture of factor types (categorical, discrete, and continuous) with low pairwise correlation, hence maintaining the nearly orthogonal feature of the design space (Vieira, Sanchez, Kienitz, & Belderrain, 2011).

In general, NOLH and NOB designs provide us the ability to discover different forms of non-linearity and interactions relationship among factors in the models. An Excel format of NOLH and NOB generation tools extracted from the Naval Postgraduate School (NPS) SEED Center for Data Farming is used in this thesis to generate the design points (Sanchez, 2011).

C. EXPLORATORY DESIGN

An initial exploratory design with a subset of factors discussed previously is explored to gain initial insights into the model. A 15-factor, 65 design points NOLH with 10 replications is carried out using the two experimental scenarios. Table 5 and Figure 8 present the subset of factors and the scatterplot matrix for the exploratory design, respectively. The scatterplot matrix shows a pairwise comparison of the factor levels that are space filling and nearly orthogonal with each other with small correlation values (< 0.05).

Table 5. Factors for Exploratory Design

S/N	Factors	Min value	Max value	Remarks
Decision Factors				
1.	Artillery CEP (M982 – Excalibur)	5m	20m	Blue Characteristics
2.	Artillery CEP (M1156 – PGK)	20m	50m	
3.	Proportion of Rounds (M982 – Excalibur)	0.05	0.15	
4.	Proportion of Rounds (M1156 – PGK)	0.05	0.45	
5.	Blue Small Arms Weapons P_{Hit}	0.1	0.7	
6.	Blue Small Arms Weapons P_{Kill}	0.1	0.7	
7.	Blue Agents Movement Speed	20	30	
8.	Blue Agents Vulnerability Index	0.5	1.0	
Noise Factors				
9.	Red Small Arms Weapons P_{Hit}	0.1	0.7	Red Characteristics
10.	Red Small Arms Weapons P_{Kill}	0.1	0.7	
11.	Red Agents Movement Speed	20	30	
12.	Red Agents Vulnerability Index	0.5	1.0	
13.	Building Protection	0.05	0.10	Environmental Characteristics
14.	Building Concealment	0.05	0.10	
15.	Building Mobility	0.05	0.10	

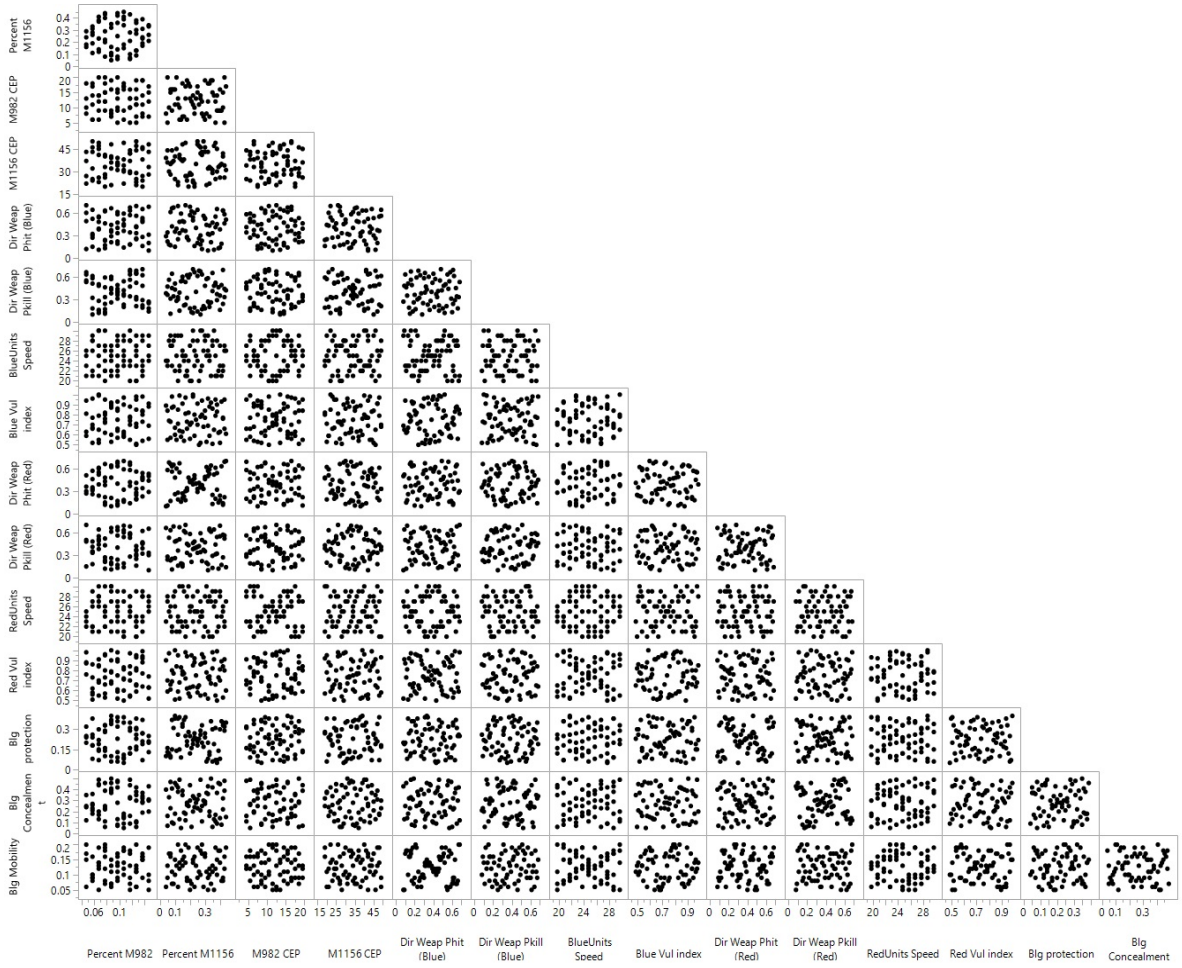


Figure 8. Scatterplot Matrix of Exploratory Design

D. FULL DESIGN

After exploring the initial characteristics of the model, we then explore a full experimental design consisting of a cross design composed of a two-factor, 17-design point NOLH and a 23-factor, 128-design point NOB. The two-factor NOLH design consists of the proportion of M982 and M1156 munition and the 23-factor NOB design consists of all other factors (controllable and uncontrollable) that are worth investigating. By crossing these two designs, 2,176 design points are produced, where every design point within the NOLH is simulated against each of the 128 design points in the NOB. Hence, crossing the two designs ensures that the system is relatively insensitive to the variability of the factors and enables us to objectively examine the 'best' allocation of munition that performs well

across the entire decision and noise space. The full design is replicated 30 times, which generates 65,280 simulated battles.

Table 6 presents the factors for the full design matrix, which consists of the cross design between the NOLH (17 design points (DP)) and NOB (128 DPs).

Table 6. Factors for Full Design (NOLH x NOB)

S/N	Factors	Min value	Max value	Remarks
Two Factors, NOLH (17 DPs)				
1.	Proportion of Rounds (M982 – Excalibur)	0.05	0.15	Key Decision Factors
2.	Proportion of Rounds (M1156 – PGK)	0.05	0.45	
22 Factors, 128 Design Points NOB (128 DPs)				
1.	Artillery CEP (M982 – Excalibur)	5m	20m	Blue Characteristics
2.	Artillery CEP (M1156 – PGK)	20m	50m	
3.	Artillery CEP (M795) range dependent multiplier ratio	1	1.3	
4.	Blue Small Arms Weapons P_{Hit}	0.1	0.7	
5.	Blue Small Arms Weapons P_{Kill}	0.1	0.7	
6.	Blue Sniper P_{Hit} multiplier ratio	1	1.3	
7.	Blue Sniper P_{Kill} multiplier ratio	1	1.3	
8.	Blue FO Target Locating Error	5m	50m	
9.	Blue Agents Movement Speed	20	30	
10.	Blue Agents Vulnerability Index	0.5	1.0	
11.	Blue Agents Movement Desire towards Nearest Enemy	0	100	
12.	Damage Functions (Binary) – Cookie Cutter (1), Carleton (0)	0	1	
13.	Red Small Arms Weapons P_{Hit}	0.1	0.7	Red Characteristics
14.	Red Small Arms Weapons P_{Kill}	0.1	0.7	
15.	Red Sniper P_{Hit} Multiplier Ratio	1	1.3	
16.	Red Sniper P_{Kill} Multiplier Ratio	1	1.3	
17.	Red Agents Movement Speed	20	30	
18.	Red Agents Sensor Range	300	450	
19.	Red Agents Vulnerability Index	0.5	1.0	
20.	Red Agents Movement Desire towards Nearest Enemy	0	100	
21.	Building Protection	0.05	0.10	Environmental Characteristics
22.	Building Concealment	0.05	0.10	
23.	Building Mobility	0.05	0.10	

E. ROBUST DESIGN

In addition to efficient DOEs, we apply the robust design methodology, which is defined as "a system optimization process that uses loss function which incorporates both the system mean and variability to evaluate system performance" (Sanchez, 2000, p. 73), rather than just plainly looking at its desired mean performance. Loss function or cost function is commonly used to combine the values of several variables of an event (such as the MOEs) onto a real number, which can be used to represent the cost associated with the event. Intuitively, a low-scale loss value derived from the loss function is desirable for any particular performance measure and the robust design is the one that minimizes expected loss.

Quadratic Loss Function

Quadratic loss function is used in studying the outputs/MOEs from the simulation results due to the function's mathematical tractability. Before computing the quadratic scaled loss, one must first specify the goal associated with the performance measure, *Y(x)* (e.g., high mean or low mean), and its performance target value, τ. The quadratic loss function can be expressed as follows: $l(Y(x)) = c[Y(x) - \tau]^2$ (Sanchez, 2000), where *c* is a scaling cost-conversion constant to facilitate comparisons of the system with different costs. The expected loss can then be expressed across the noise factor space, *Ω,* as follows: $E_{\Omega}[l(Y(x))] = c[\sigma^2_{Y(x)} + (\mu_{Y(x)} - \tau)^2]$ (Sanchez, 2000). The expected loss function is similar to mean squared error loss and it penalizes response variables with large variance and responses that deviate from its target value. A low loss would then represent a response mean that is near to the target value and has a small standard deviation, which is desirable to the decision makers in all aspects.

F. EMPLOYING CLUSTERS OF HIGH-PERFORMANCE COMPUTERS

High Performance Computing Clusters (HPCCs) enable users to complete the simulation time of experiments much faster and more efficiently due to the clusters' ability to utilize multiple processors in the cluster computers for parallel processing of data. The Pythagoras scenario files were sent to the NPS SEED Center HPCC managed by Mary

McDonald and Steve Upton to assist in the generation of results. The computer cluster, which has 192 processors, took approximately 50 hours to complete generating the results of an experimental scenario with 65,280 simulated battles. Comparatively, it would take 188 days to generate the results using a normal laptop with dual core processor.

Despite having the HPCC at our disposal to run big experiments, efficient DOE is still fundamental in constructing large-scale simulations as it allows us to explore the experiment to a greater degree with much fewer design points and with little cost involved. For instance, even by using a simple two-level, full factorial design with 25 factors, it would require 33 million designs points and about one billion simulated battles (after replications) for each scenario. This makes it "computationally impossible" to complete in a lifetime, even with the aid of the HPCC (~83 years).

THIS PAGE INTENTIONALLY LEFT BLANK

IV. DATA ANALYSIS

This chapter describes the process of collecting and processing data generated from the simulated battles for analysis. The analysis of the data generated from each scenario is discussed in three sections. We first look at the general effects of the factors and their interactions that are significant in each scenario. Next, using the metamodel generated from the crossed design, we examine the effects of munition allocation against each MOE. With multiple objectives, such as minimizing the quadratic scaled losses of the MOEs and minimizing cost of munitions allocation, the concept of Pareto dominance is then used to evaluate the overall performance of the munitions mix.

A. DATA FARMING AND PROCESSING

Data farming is described as the process of growing purposeful data from simulation models through efficient and large scale DOEs, which allows us to explore massive input spaces and uncover useful insights from complex response surfaces (Lucas, Kelton, Sánchez, Sanchez, & Anderson, 2015). The data and output generated from using the DOEs described in Chapter III are presented in the form of a comma separated values (CSV) file. Each design point from the DOEs is appended with a summary of outputs necessary for studying the required MOEs of the model. The analysis of the data is performed using JMP Pro 13, a statistical analysis software (SAS Institute, 2013).

B. SIMULATION OUTPUT FOR MOE ANALYSIS

Operational effectiveness of the mix of artillery munitions is determined by the five MOEs described in Chapter II. Table 7 summarizes the simulation outputs required for each of the MOEs. Stepwise regression analysis and regression trees are some statistical methods used to provide an intuitive summary for each MOE.

Table 7. Simulation Output for Desired MOEs

S/N	Simulation Outputs	MOEs
1.	Total No. of Blue Killed / Total No. of Red Killed	$FER = \frac{\frac{Total\ No.of\ Red\ Killed}{No.of\ Red\ Initial}}{\frac{Total\ No.of\ Blue\ Killed}{No.of\ Blue\ Initial}}$
2.	No. of Blue Killed by Artillery / Total No. of Blue Killed	$Fratricide\ Rates = \frac{No.of\ Blue\ Killed\ by\ Artillery}{Total\ No.of\ Blue\ Killed}$
3.	No. of Neutral Killed by Artillery	Collateral damage
4.	No. of Red Killed by Artillery / Total No. of Red Killed	$Artillery\ Effectiveness = \frac{No.of\ Red\ Killed\ by\ Artillery}{Total\ No.of\ Red\ Killed}$
5.	Time when >= 90% Red Killed	Time for mission success

C. SCENARIO 1 (FALLUJAH)—INSIGHTS AND ANALYSIS

This section discusses the insights gained from the results generated by Scenario 1.

1. Factors Influencing MOEs

General insights of the main factors and their interactions that affect each MOE can be gained using regression tree analysis. Regression trees are generally useful in identifying factors that are impactful to the response variables, especially in a domain that involves a large number of factors. For continuous response variables, each split of the regression tree corresponds to a factor and factor level that minimizes the residual sum of squares (RSS) and separates the data into two distinct groups. To get a general sense of the data, we chose to terminate the splitting when R^2 is above 0.5 and there are at least four splits.

a. Factors Influencing FER

Figure 9 shows the regression tree analysis on factors influencing the FER. The R^2 for this tree is 0.583, which indicates that 58.3 percent of the variance is accounted for after four splits of the data. From Figure 10, we see that Red infantry sensor range is highly

dominant and has the greatest impact on the FER. For instance, the mean value of the FER drops from 1.95 to 1.45 as the Red infantry sensor range increases from below 341 m to at least 341 m. Since the Blue infantry sensor capability is set to 300 m, this implies that if the Red infantry sensors are 13 percent or more capable in range, it can greatly reduce the FER, which is undesirable for the Blue force. There are also potential interactions of Red infantry sensor range with Building concealment and the Red vulnerability index that can influence FER. Interactions are evident when different branches of the tree have different splits. After ten splits, the R^2 increases to 0.718, but the factors of Red infantry sensor range, building concealment, and Red vulnerability index remain the most important.

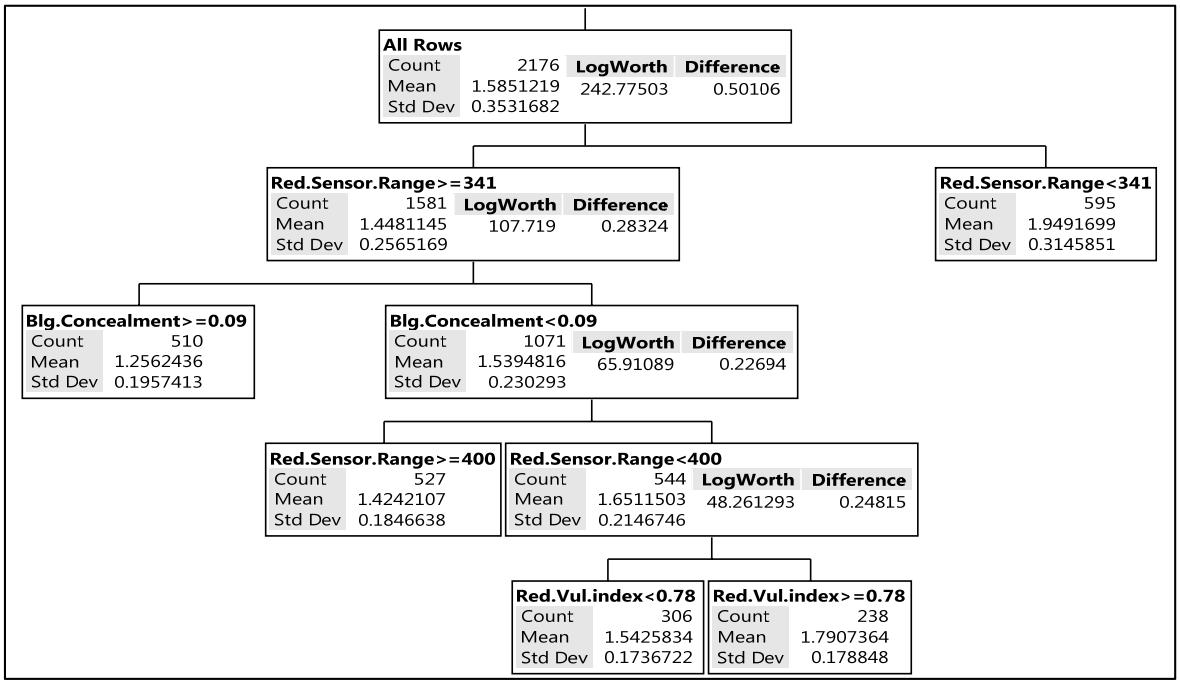


Figure 9. Regression Tree for Factors Influencing FER

b. Factors Influencing Fratricide Rates

Figure 10 shows the regression tree analysis on factors influencing the fratricide rates. The R^2 for this tree after four splits is 0.605. From Figure 10, we see that building concealment is highly dominant and has the greatest impact on fratricide rates. Thus, it can be inferred that with better building concealment, the Blue forces face greater difficulty in locating the Red forces and hence get caught by own artillery fire in the midst of the search, which results in a higher fratricide rate. There are also potential interactions between building concealment and Red infantry sensor range that can influence fratricide rates. After ten splits, the R^2 increases to 0.684. While building concealment and Red infantry sensor range remain the most important factors, other factors such as the CEP of the M982 are also included.

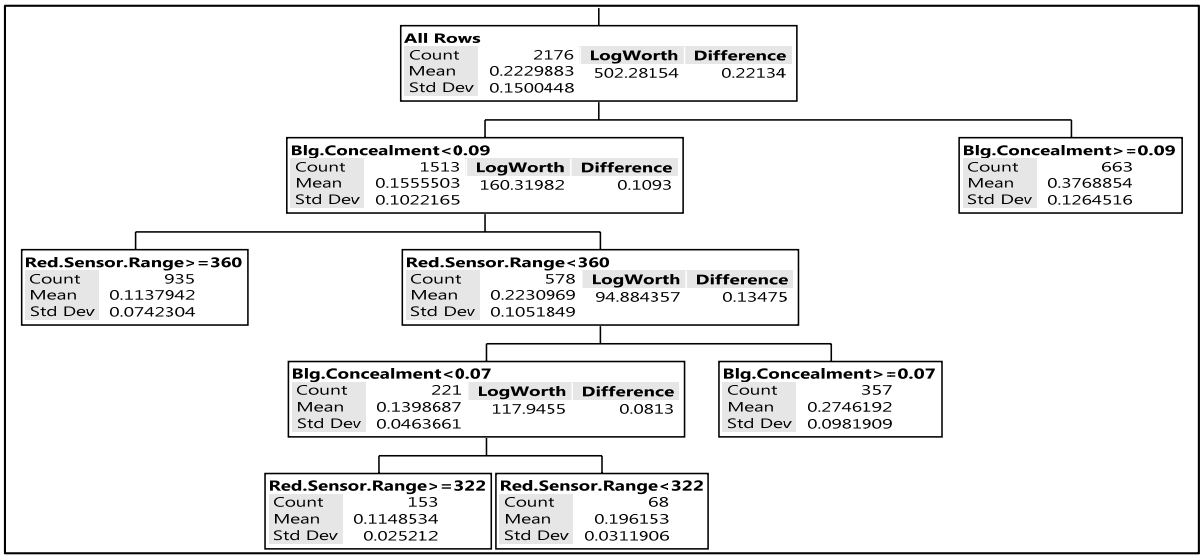


Figure 10. Regression Tree for Factors Influencing Fratricide Rates

c. Factors Influencing Collateral Damage

Figure 11 shows the regression tree analysis on factors influencing collateral damage. The R^2 for this tree after five splits is 0.510. From Figure 11, we see that building protection is highly dominant and has the greatest impact on the amount of collateral damage. This is hardly surprising as the Neutral agents are placed in the building structure, and higher building protection levels will keep them from being killed by artillery fire. There are also interactions between building protection and building concealment, the P_{Hit} ratio of the Red sniper weapon, and the Red vulnerability index that can influence collateral damage. After ten splits, the R^2 increases to 0.597, but the factors of building protection, building concealment and Red vulnerability index remain the most important.

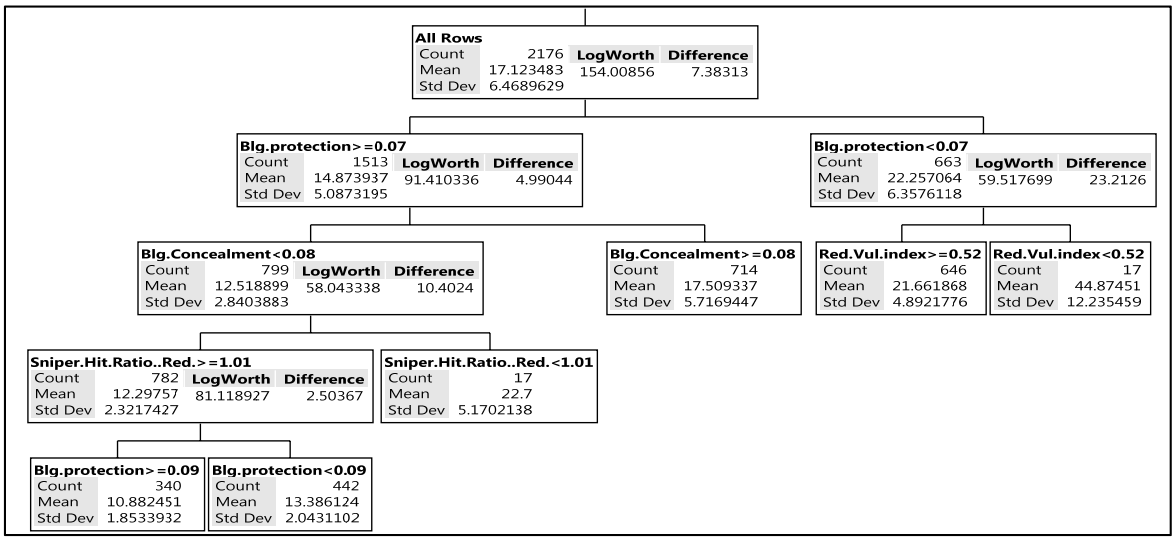


Figure 11. Regression Tree for Factors Influencing Collateral Damage

d. Factors Influencing Artillery Effectiveness

Figure 12 shows the regression tree analysis on factors influencing artillery effectiveness. The R^2 for this tree after four splits is 0.562. From Figure 12, we see that building concealment is highly dominant and has the greatest impact on artillery effectiveness. Similar to the explanation about the effect of fratricide rates, high concealment levels prevent the Red forces from being located by Blue forces and hence reduces the Blue infantry rate of kill. Consequently, this increases the artillery rate of kill or the artillery effectiveness. There are also interactions between building concealment and the Blue direct weapons P_{Kill}, building protection, and Red vulnerability index that can influence artillery effectiveness. After ten splits, the R^2 increases to 0.794, but the factors of building concealment, Blue direct weapon P_{Kill}, building protection, and Red vulnerability index remain the most important.

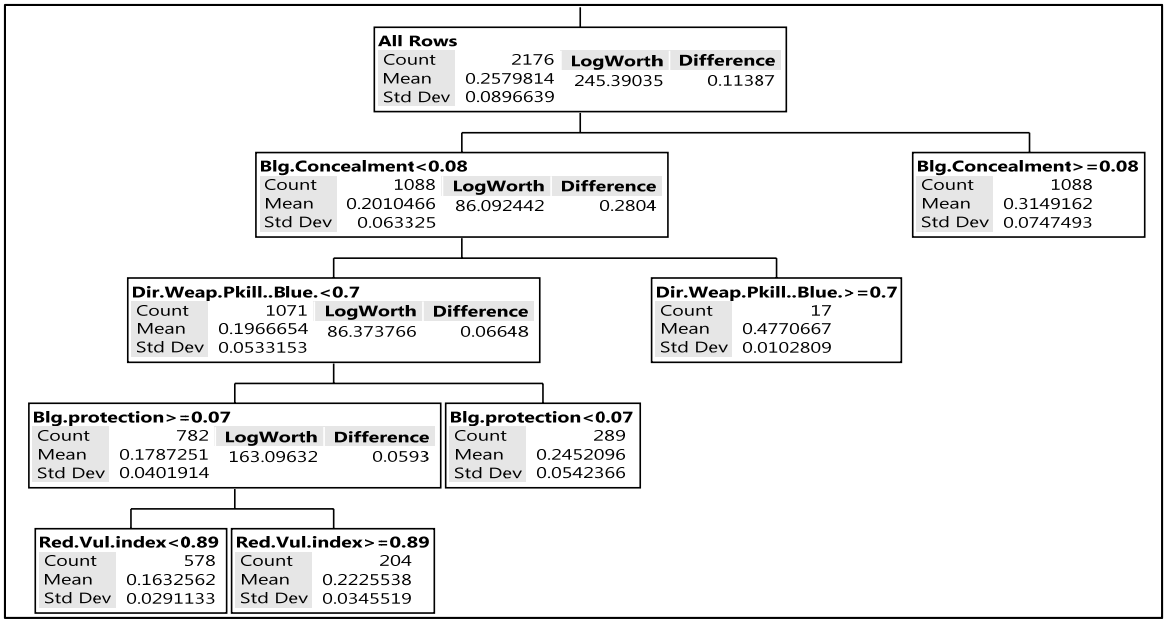


Figure 12. Regression Tree for Factors Influencing Artillery Effectiveness

e. Factors Influencing Time for Mission Success

Figure 13 shows the regression tree analysis on factors influencing time for mission success. The R^2 for this tree after four splits is 0.639. From Figure 13, we can see that building concealment is highly dominant and has the greatest impact on the time for mission success. Similar to the explanation about the effect of fratricide rates, high concealment levels prevent the Blue infantry from finding the Red forces and hence reduces the Blue infantry rate of kill. The lower Blue infantry kill rate consequently prolongs the time required for mission completion. There are also interactions between building concealment and Blue movement desirability as well as Red sniper weapon P_{Kill} ratio that can influence time for mission success. After ten splits, the R^2 increases to 0.771. While building concealment, Blue movement desirability, and Red sniper P_{Kill} ratio remain the most important factors, other factors such as the CEP of the M1156 are also included.

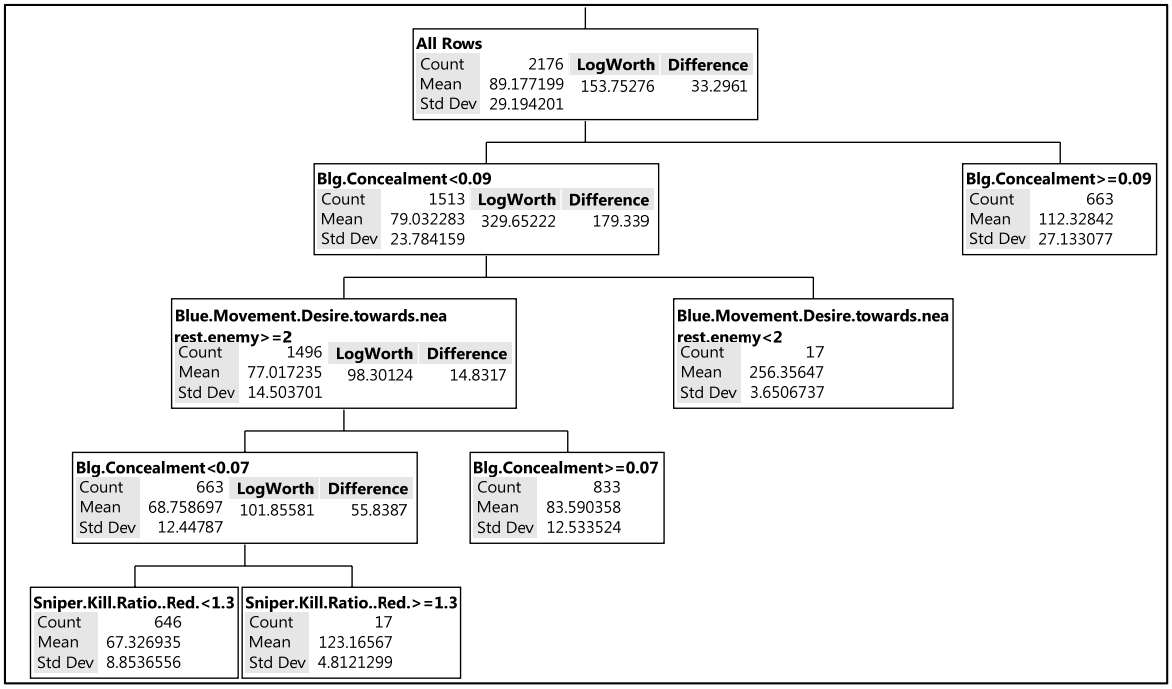


Figure 13. Regression Tree for Factors Influencing Time for Mission Success

2. Effects of Munition Allocation against MOEs

To investigate the effects of munition allocation (Percent M982 and Percent M1156) against the MOEs, the crossed designs are collapsed over the entire noise space to construct regression metamodels for the mean and standard deviation (SD) of the five MOEs as functions of the decision factors alone. This is a step toward exploring the robustness of different munition allocation decisions against a variety of Red threats and environmental conditions. A summary of the results from the individual metamodels is provided in Section IV.2.g.

a. Data Exploration

Prior to fitting the regression models of each MOE for analysis, we conduct data exploration to screen the relationship between the response variables (means of each MOE) and the decision factors (Percent M982 and Percent M1156) in the metamodels. Figure 14 and Figure 15 show the scatterplot matrix and correlation values between the response variables and decision factors, respectively.

From Figures 14 and 15, we can see a high positive correlation between Percent M982 and FER, indicating that a higher proportion of M982 improves the FER, which increases the number of Red agents killed relative to Blue killed. There are also high negative correlations of percent of M982 against (1) time for mission success, (2) amount of collateral damage, (3) fratricide rate, and (4) artillery effects. This indicates that a higher proportion of M982 reduces the time needed for mission success, as well as lowers the rates of collateral damage, fratricide, and artillery effects on enemies. On the other hand, the Percent M1156 has some positive correlation with artillery effectiveness, low negative correlation with collateral damage, and negligible correlation values against the other three MOEs.

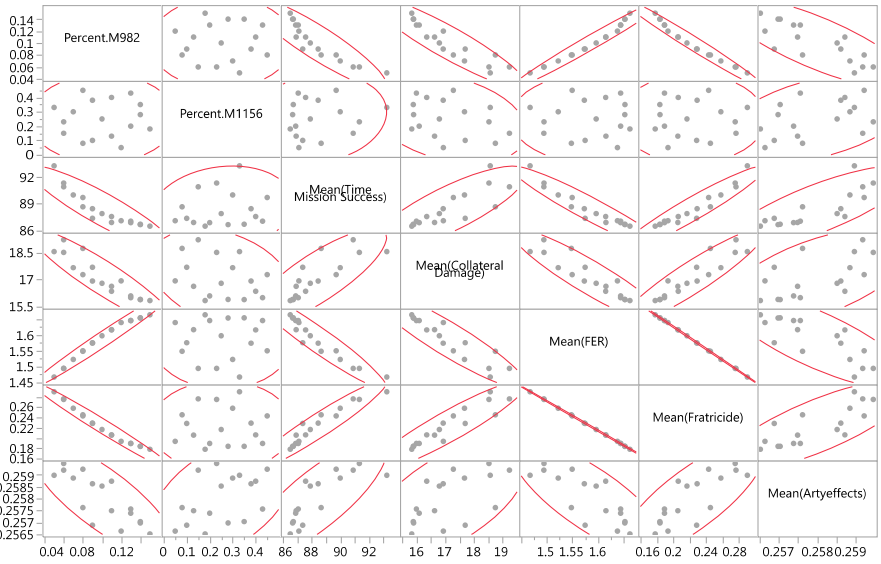


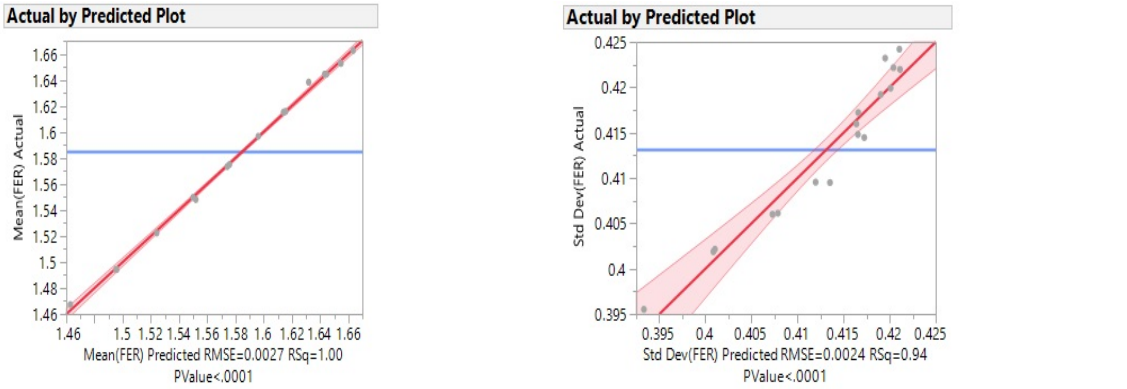
Figure 14. Scatterplot Matrix of Response and Decision Variables

	Percent.M982	Percent.M1156	Mean(Time Mission Success)	Mean(Collateral Damage)	Mean(FER)	Mean(Fratricide)	Mean(Artyeffects)
Percent.M982	1.0000	0.0123	-0.9073	-0.9188	0.9838	-0.9846	-0.7988
Percent.M1156	0.0123	1.0000	0.1504	-0.3072	-0.0090	0.0008	0.4442
Mean(Time Mission Success)	-0.9073	0.1504	1.0000	0.8205	-0.9535	0.9528	0.8030
Mean(Collateral Damage)	-0.9188	-0.3072	0.8205	1.0000	-0.9212	0.9261	0.5874
Mean(FER)	0.9838	-0.0090	-0.9535	-0.9212	1.0000	-0.9998	-0.7916
Mean(Fratricide)	-0.9846	0.0008	0.9528	0.9261	-0.9998	1.0000	0.7913
Mean(Artyeffects)	-0.7988	0.4442	0.8030	0.5874	-0.7916	0.7913	1.0000

Figure 15. Correlation Values of Response and Decision Variables

b. Regression Metamodels of FER

Figure 16 shows the standard least squares regression of the mean and SD of the FER using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and SD are well fitted with an R^2 of 1.0 and 0.94, respectively. It can be observed that the main effect of Percent M982 and its quadratic effects are significant on both metamodels, while the main effect of Percent M1156 is insignificant (p-value > 0.05). The interaction effect and quadratic effect of Percent M1156 are insignificant in both metamodels; hence, they are excluded.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.3993042	0.00268	522.04	<.0001*
Percent M982	1.9820981	0.021261	93.23	<.0001*
Percent M1156	-0.005454	0.005281	-1.03	0.3205
(Percent M982-0.10059)*(Percent M982-0.10059)	-13.19536	0.797522	-16.55	<.0001*

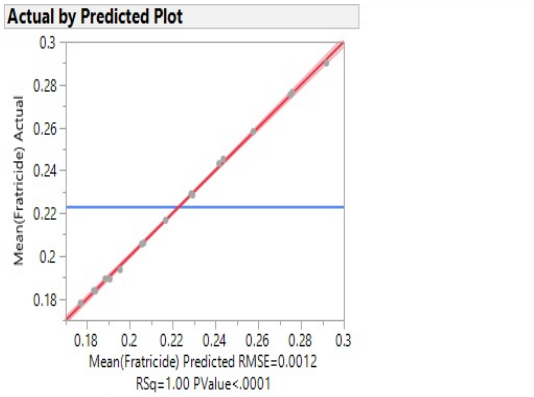
Parameter Estimates

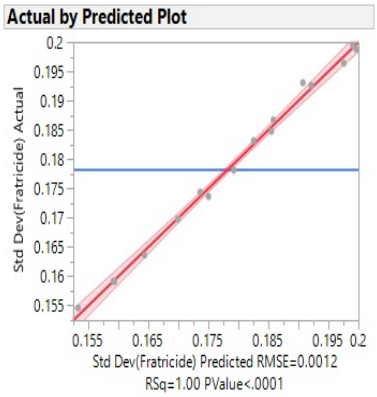
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.4399682	0.00238	184.88	<.0001*
Percent M982	-0.19814	0.018875	-10.50	<.0001*
Percent M1156	-0.002553	0.004689	-0.54	0.5953
(Percent M982-0.10059)*(Percent M982-0.10059)	-6.741231	0.708043	-9.52	<.0001*

Figure 16. Least Squares Regression of Mean and SD of FER

c. Regression Metamodels of Fratricide Rates

Figure 17 shows the standard least squares regression of the mean and the SD of fratricide using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 1.0. It can also be observed that the main effect of M1156 is significant on the mean metamodel, but it is insignificant on the SD metamodel. Interaction effects between Percent M982 and Percent M1156 are significant in both metamodels. A quadratic effect for Percent M982 is significant in the mean metamodel but insignificant in the SD metamodel. The quadratic effect of Percent M1156 is insignificant in both metamodels, and hence it is excluded.





Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.3306591	0.001205	274.32	<.0001*
Percent.M982	-1.141149	0.009572	-119.2	<.0001*
Percent.M1156	0.0006319	0.002374	0.27	0.7946
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	0.2955381	0.09949	2.97	0.0117*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	7.5201219	0.359585	20.91	<.0001*

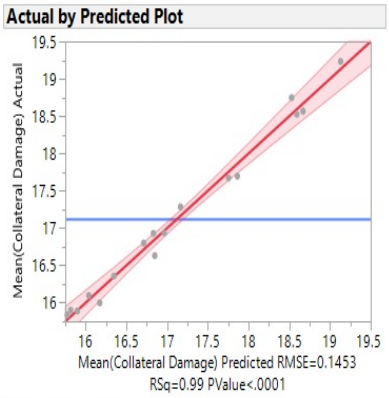
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2288146	0.001162	196.93	<.0001*
Percent.M982	-0.47659	0.009227	-51.65	<.0001*
Percent.M1156	-0.007724	0.002289	-3.38	0.0055*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	0.2705995	0.095903	2.82	0.0154*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	-0.683258	0.34662	-1.97	0.0722

Figure 17. Least Squares Regression of Mean and SD of Fratricide

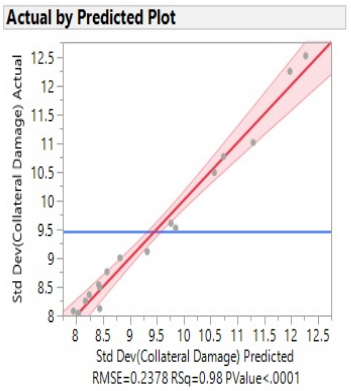
d. Regression Metamodels of Collateral Damage

Figure 18 shows the standard least squares regression of the mean and the SD of collateral damage using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 0.99 and 0.98, respectively. It can be observed that the main effect of Percent M982, Percent M1156, their quadratic effects, and their interaction effects are significant in both metamodels.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	20.681107	0.159696	129.50	<.0001*
Percent.M982	-32.84953	1.162512	-28.26	<.0001*
Percent.M1156	-2.738851	0.288366	-9.50	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	60.88116	12.09534	5.03	0.0004*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	273.19405	49.95642	5.47	0.0002*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	12.182972	3.018421	4.04	0.0020*



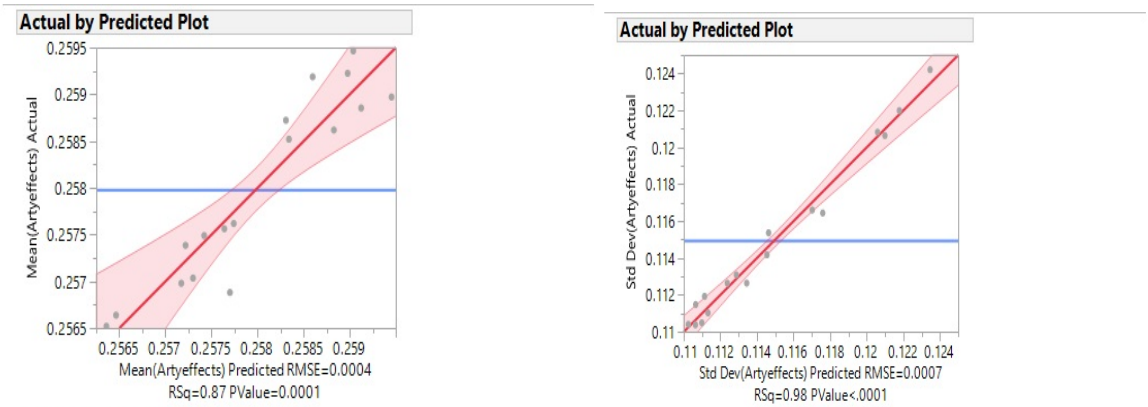
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	13.623902	0.26139	52.12	<.0001*
Percent.M982	-27.9684	1.902795	-14.70	<.0001*
Percent.M1156	-8.047426	0.471996	-17.05	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	134.93959	19.79761	6.82	<.0001*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	206.40828	81.76849	2.52	0.0283*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	32.508749	4.94054	6.58	<.0001*

Figure 18. Least Squares Regression of Mean and SD of Collateral Damage

e. Regression Metamodels of Artillery Effectiveness

Figure 19 shows the standard least squares regression of the mean and the SD of artillery effectiveness using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are relatively well fitted with R^2 values of 0.87 and 0.98, respectively. It can be observed that the main effects of both Percent M982 and Percent M1156 are significant in both metamodels. The interaction effect and quadratic effects of both Percent M982 and Percent M1156 are significant in the SD metamodel but insignificant in the mean metamodel.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2599991	0.000473	549.95	<.0001*
Percent M982	-0.025736	0.003442	-7.48	<.0001*
Percent M1156	0.0036602	0.000854	4.29	0.0013*
(Percent M982-0.10059)*(Percent M1156-0.25235)	-0.001865	0.035807	-0.05	0.9594
(Percent M982-0.10059)*(Percent M982-0.10059)	-0.151688	0.147893	-1.03	0.3271
(Percent M1156-0.25235)*(Percent M1156-0.25235)	-0.014162	0.008936	-1.58	0.1413

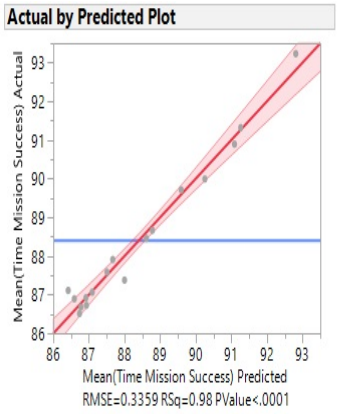
Parameter Estimates

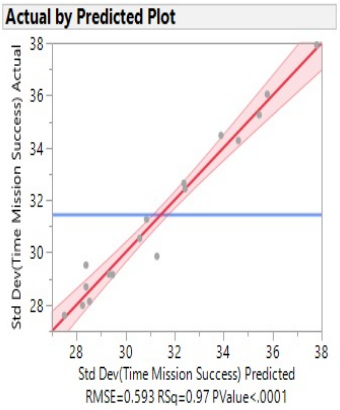
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1280204	0.000773	165.72	<.0001*
Percent M982	-0.125015	0.005623	-22.23	<.0001*
Percent M1156	-0.012558	0.001395	-9.00	<.0001*
(Percent M982-0.10059)*(Percent M1156-0.25235)	0.2601791	0.05851	4.45	0.0010*
(Percent M982-0.10059)*(Percent M982-0.10059)	1.8761937	0.241657	7.76	<.0001*
(Percent M1156-0.25235)*(Percent M1156-0.25235)	0.062501	0.014601	4.28	0.0013*

Figure 19. Least Squares Regression of Mean and SD of Artillery Effectiveness

f. Regression Metamodels of Time for Mission Success

Figure 20 shows the standard least squares regression of the mean and SD of time for mission success using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 0.98 and 0.97, respectively. It can be observed that both the main effect and quadratic effect of Percent M982, and main effect of Percent M1156 are significant in both metamodels. The interaction effect and quadratic effect of M1156 are insignificant in both metamodels, and hence they are excluded.





Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	92.76904	0.338144	274.35	<.0001*
Percent.M982	-56.46867	2.682045	-21.05	<.0001*
Percent.M1156	2.1862044	0.666216	3.28	0.0060*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	837.1388	100.6083	8.32	<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	39.427515	0.597012	66.04	<.0001*
Percent.M982	-95.93383	4.735309	-20.26	<.0001*
Percent.M1156	4.0977728	1.176243	3.48	0.0040*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	709.63466	177.6298	4.00	0.0015*

Figure 20. Least Squares Regression of Mean and SD of Time for Mission Success

g. Summary of Results for Regression Metamodels

The results from all the regression metamodels can be used for determining the level of influence that the munition proportion factors have on the respective MOEs. Table 8 summarizes the results of the respective factors' influence on the MOEs based on their contribution to the R^2 of the regression models. It is evident that Percent M982 has a much greater influence than Percent M1156 on the MOEs in Scenario 1.

Table 8. Summary of Results for Scenario 1 Showing the Influence of Percent M982 and Percent M1156 on the MOEs, Based on Contributions to R^2

Variables	Percent M982	Percent M1156	Interaction Effect
Mean FER	VS	VW	VW
SD FER	VS	VW	VW
Mean Fratricide Rates	VS	VW	W
SD Fratricide Rates	VS	W	W
Mean Collateral Damage	VS	S	M
SD Collateral Damage	VS	S	M
Mean Artillery Effectiveness	VS	W	VW
SD Artillery Effectiveness	VS	S	M
Mean Time for Mission Success	VS	W	VW
SD Time for Mission Success	VS	W	VW

VS denotes very strong influence (>0.25), S denotes strong influence (>0.125), M denotes moderate influence (>0.0625), W denotes weak influence (>0.03125), and VW denotes very weak influence (<0.03125)

3. Pareto Optimal Frontier Analysis

The concept of Pareto dominance is commonly used to evaluate the overall performance of a system with multiple objectives. The Pareto Optimal Frontier (POF) is a set of decision points where it is impossible to improve any objective without making trade-offs with at least one other. Based on the metamodels from the preceding section, a total of 451 points are populated using the predicted quadratic scaled loss function of all five MOEs for all munitions allocations of the M982 (0.05 – 0.15) and the M1156 (0.05 – 0.45) that were being explored. A POF of the desired objectives is then identified from the 451 points. The generic quadratic loss function for each MOE can be expressed as $loss(MOE) = \sigma^2_{MOE} + (\mu_{MOE} - \tau)^2$, where τ is the target value of the MOE. The loss

function ensures robustness in decision making as it rewards good mean performance and penalizes large variability over replications.

JMP's built-in function can be used to obtain the Pareto optimal points by clicking on ***Rows*** > ***Row Selection*** > ***Select Dominant***. From the ***select columns for (Pareto) dominant points*** window that appears next, we can select one or more columns that we want to analyze. In the following ***select dominant high values*** window, we can then select the check boxes for the MOE columns where high values are desirable, and uncheck the boxes for MOE columns where low values are desirable (SAS Institute Inc, 2013). We can then obtain the Pareto optimal points from the rows that are highlighted. Interested readers should refer to the JMP user guide (SAS Institute, 2013) for additional information.

a. Objective: Minimizing Scaled Loss MOEs

With the objective of minimizing all five scaled losses of the MOEs, a POF with 88 out of 451 munition allocations is identified. Figure 21 shows the scatterplot matrix of the five scaled losses of the MOEs, with the red points representing the identified 88 munition allocations. Figure 22 shows a plot of the M982 allocations against the M1156 allocations, with the red points representing the non-dominated set among all allocation configurations being explored (88/451) by the DOE. The concentration of red points along the top and right side of the plot show that high allocations for the M982 and M1156 are usually preferred. Figure 23 shows the histograms of the POF for this multi-objective function. Figure 23 shows another way of observing that high allocations for the M982 and M1156 are usually preferred, as indicated by the concentration of points near the upper end of the explored ranges. Although the average proportions across different points on the POF are not in themselves particularly meaningful, the mean proportion of M982 is 13.2 percent (95% CI [12.7, 13.8]) and the mean proportion of M1156 is 33.5 percent (95% CI of [31.0,36.0]).

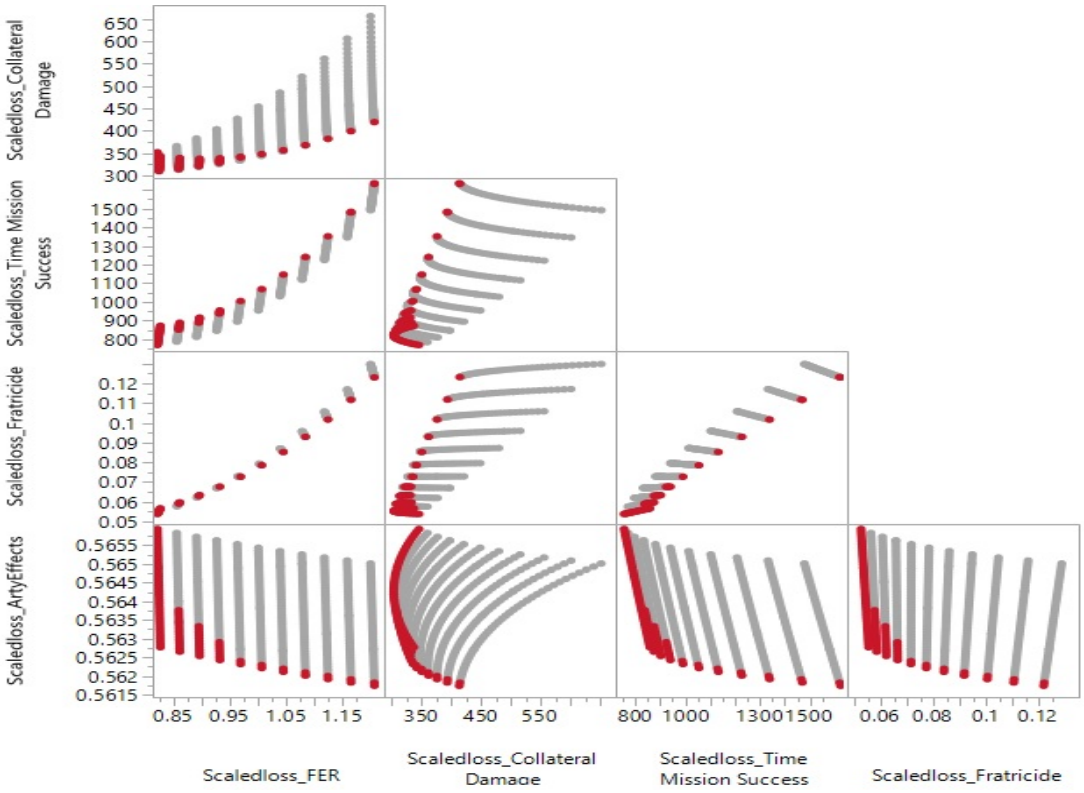


Figure 21. Scatterplot Matrix of the Scaled Loss MOEs (All Five Scaled Loss MOEs)

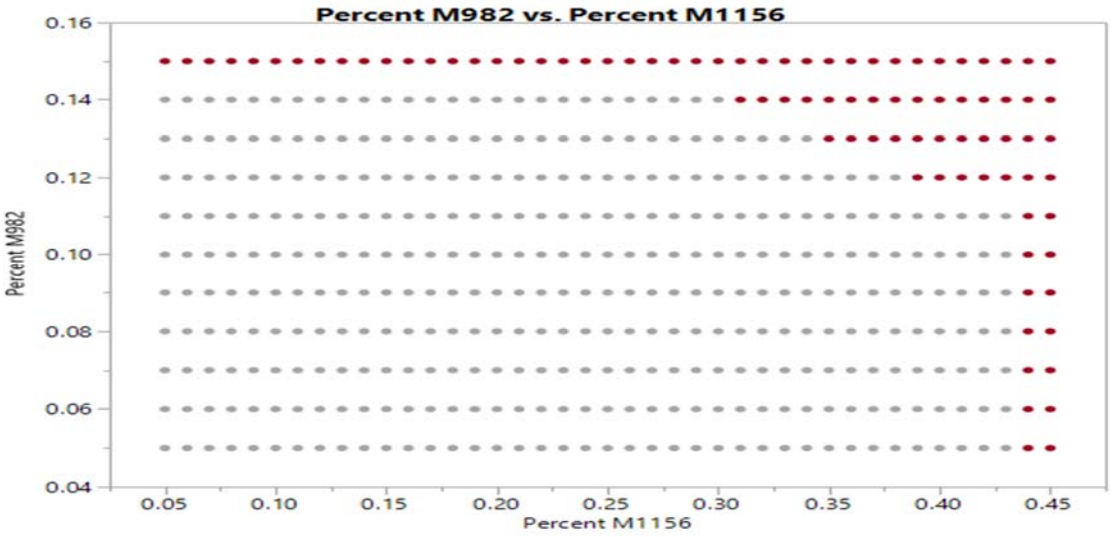


Figure 22. Plot of M982 Allocation against M1156 Allocation

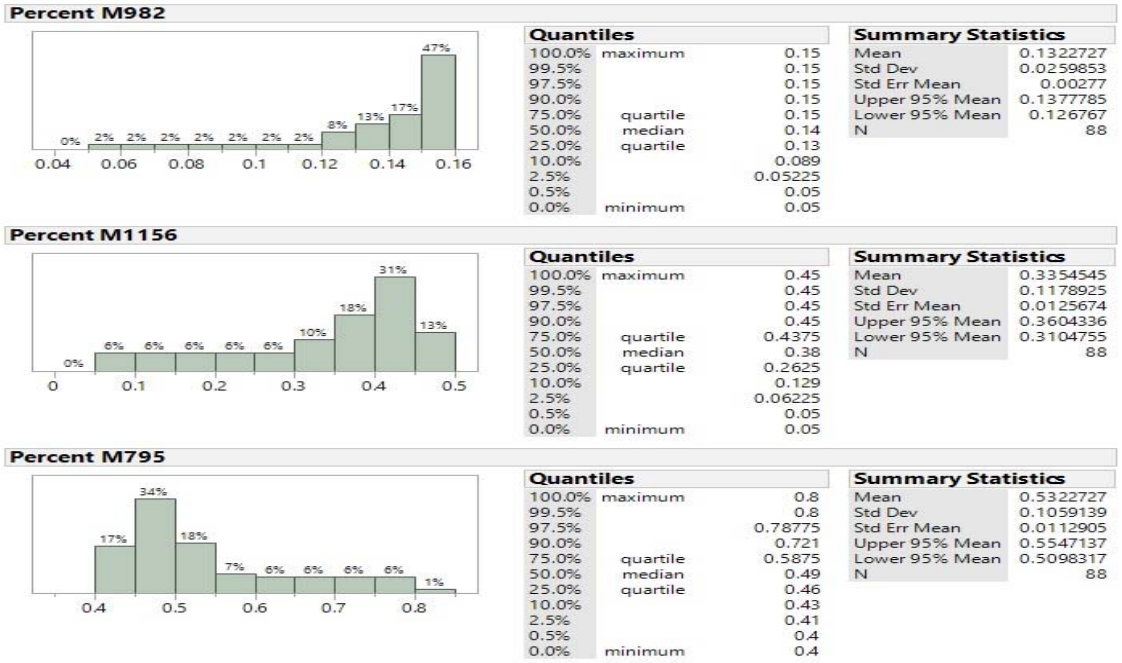


Figure 23. Histograms of the POF (All Five Scaled Loss MOEs)

b. Objective: Minimizing Scaled Loss MOEs and Cost

When all five scaled loss MOEs are applied against cost, all of the 451 munition allocations are non-dominant against each other; hence, it does not provide any meaningful resolution on cost versus operational effectiveness. However, it is possible to obtain a smaller subset of points for the POF in certain circumstances. The number of objectives could be reduced, either be eliminating some entirely or by weighting and combining some of the scaled loss calculations. Alternatively, if the decision makers are able to set additional constraints on cost or other MOEs, that may reduce the number of POF points.

c. Objective: Minimizing Scaled Loss Total Collateral, Scaled Loss Artillery Effectiveness and Cost

In this section, it is assumed that the decision makers have decided on three key objectives, namely to minimize (1) scaled loss of total collateral, (2) scaled loss of artillery effectiveness, and (3) expenditure cost of the munitions. Total collateral for this case is defined as the total number of friendlies agents (Blue and Neutral) being killed by artillery. A POF with 183 out of 451 munition allocations is identified from this multi-objectives

function. Figure 24 shows the scatterplot matrix of the three scaled loss objectives, with the red points representing the 183 munition allocations identified. Figure 25 shows the histograms of the POF. As before, the average proportions across different points on the POF are not in themselves particularly meaningful, but the shape of the histograms show that the proportion of M982 is usually near the high end of its allowable range. The mean proportions are 11.6 percent (95% CI [11.1,12.1]) for the M982 and 26.7 percent (95% CI [24.9,28.4]) for the M1156, respectively.

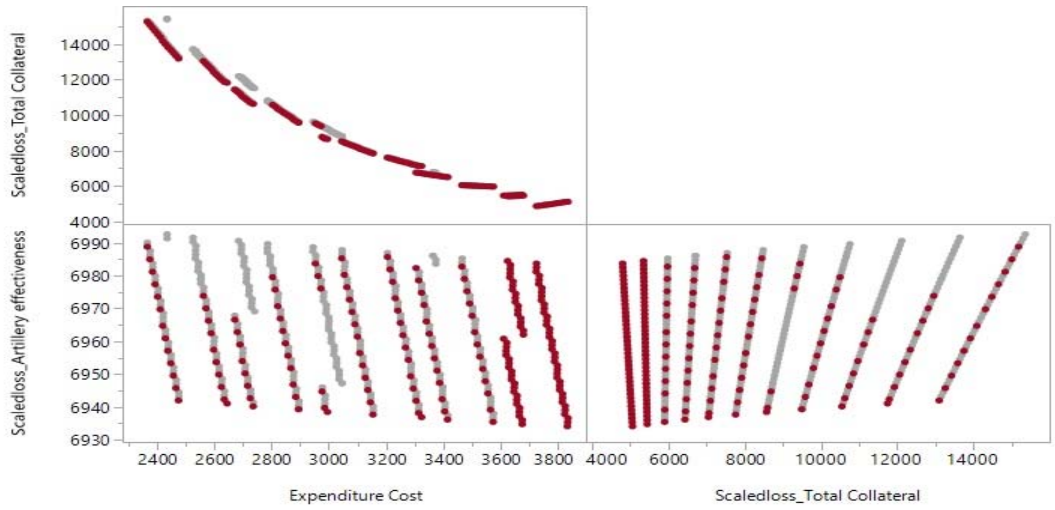


Figure 24. Scatterplot Matrix of Three Scaled Loss MOEs

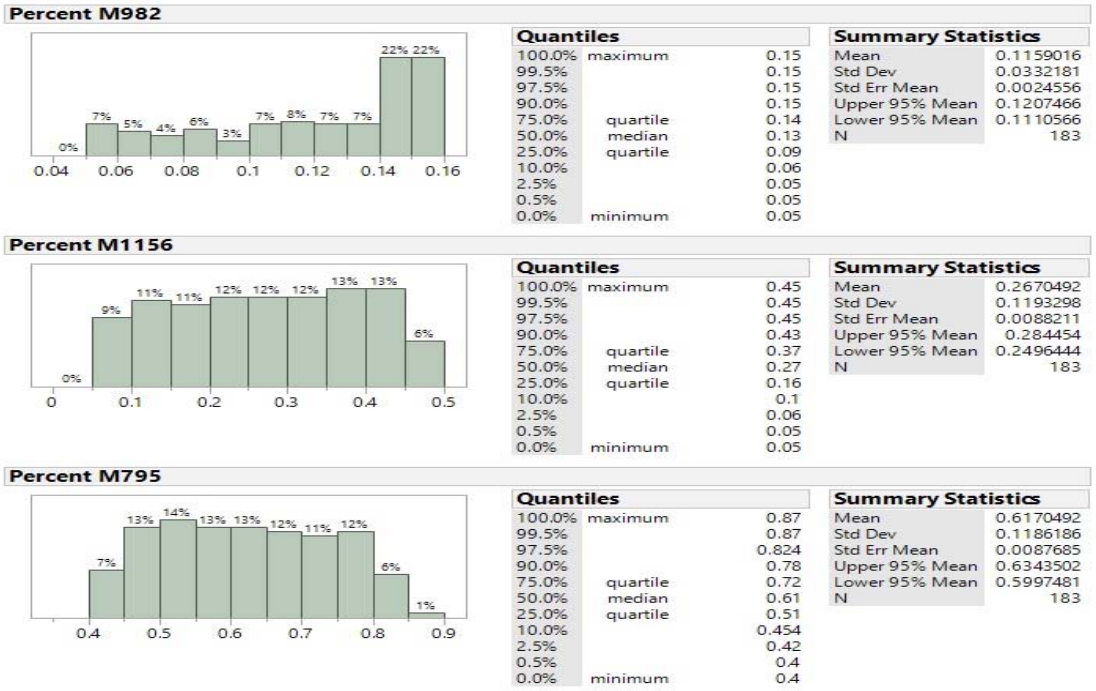


Figure 25. Histograms of the POF (Scaled Loss Total Collateral, Scaled Loss Artillery Effectiveness and Cost)

Comparatively, if the main objectives are only to minimize (1) scaled loss of total collateral and (2) scaled loss of artillery effectiveness, a POF with 41 out of 451 munition allocations is identified. Figure 26 shows a 2D plot of the scaled losses for the two objectives, with the red points representing the 41 munition allocations identified on the POF. Figure 27 shows the histograms of the POF with a constant proportion of 15 percent for the M982 and 25 percent (95% CI [21.2,28.8]) for the M1156, respectively. It is observed that without cost as an objective, the proportion of M982 increases significantly from 11.6 percent (on average) to its upper bound of 15 percent, while the change in proportion for the M1156 is negligible. This provides confirmation that there is indeed a trade-off between cost and operational effectiveness. We can also conclude that the effect of the M982 is dominant against the M1156 in terms of operational effectiveness for this scenario.

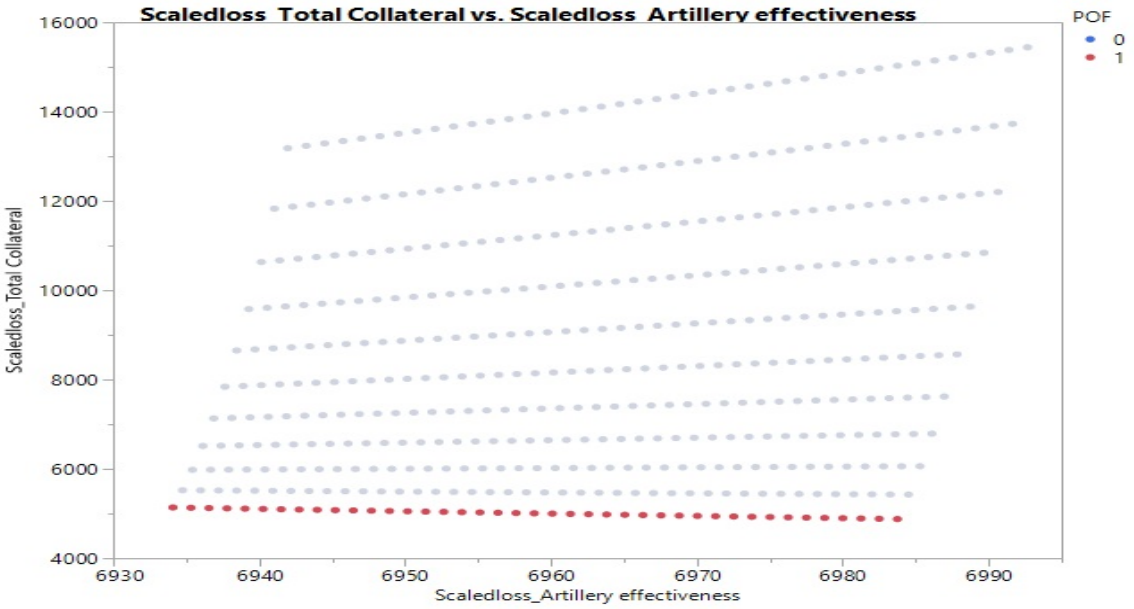


Figure 26. Plot of the Two Scaled Loss MOEs

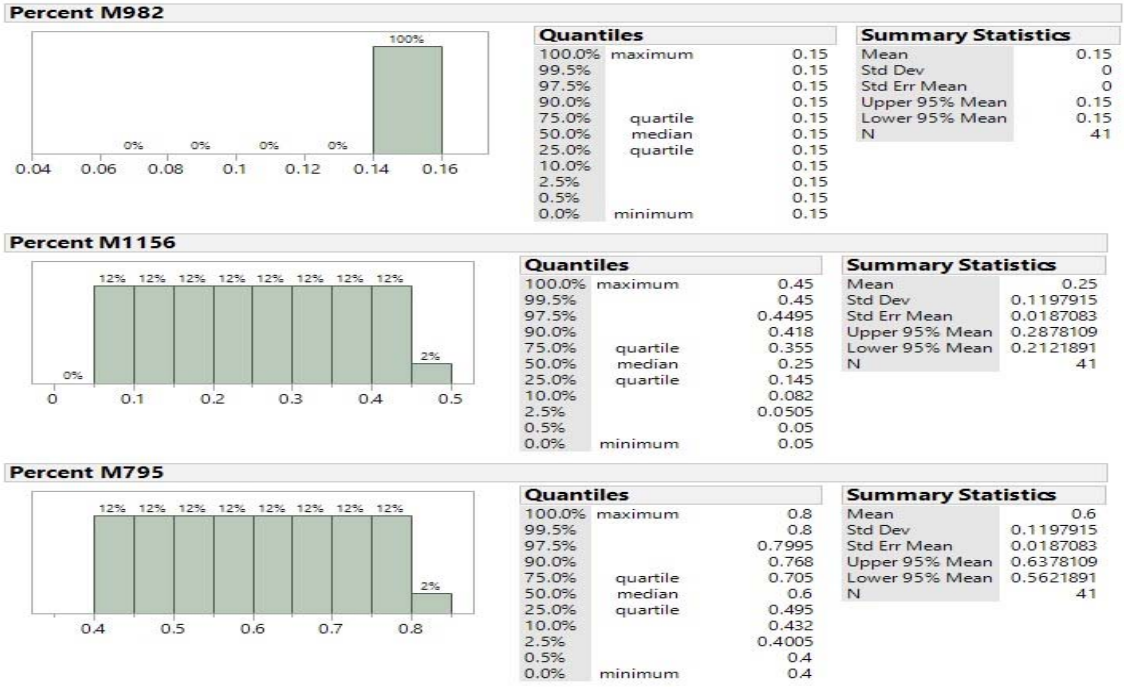


Figure 27. Histograms of Characteristics of the POF (Scaled Loss Total Collateral and Artillery Effectiveness)

D. SCENARIO 2 (MARAWI)—INSIGHTS AND ANALYSIS

This section discusses the insights gained from the results generated from Scenario 2.

1. Factors Influencing MOEs

The factors influencing MOEs in Scenario 2 are discussed in this section.

a. Factors influencing FER

Figure 28 shows the regression tree analysis on factors influencing the FER. It takes seven splits for the R^2 for this tree to reach 0.523 (above 0.5 as required). Similar to Scenario 1, we can see that the Red infantry sensor range is highly dominant and has the greatest impact on the FER. Scenario 2 has two other factors, the M982 CEP and the M1156 CEP, that are significant as compared to Scenario 1. There are also potential interaction effects between the Red infantry sensor range with the M1156 CEP, the M982 CEP, Percent M1156, and the Red vulnerability index that can influence FER. After ten splits, the R^2 increases to 0.584, but the factors of Red infantry sensor range, M1156 CEP, Red vulnerability index, M982 CEP, and Percent M1156 remain the most important.

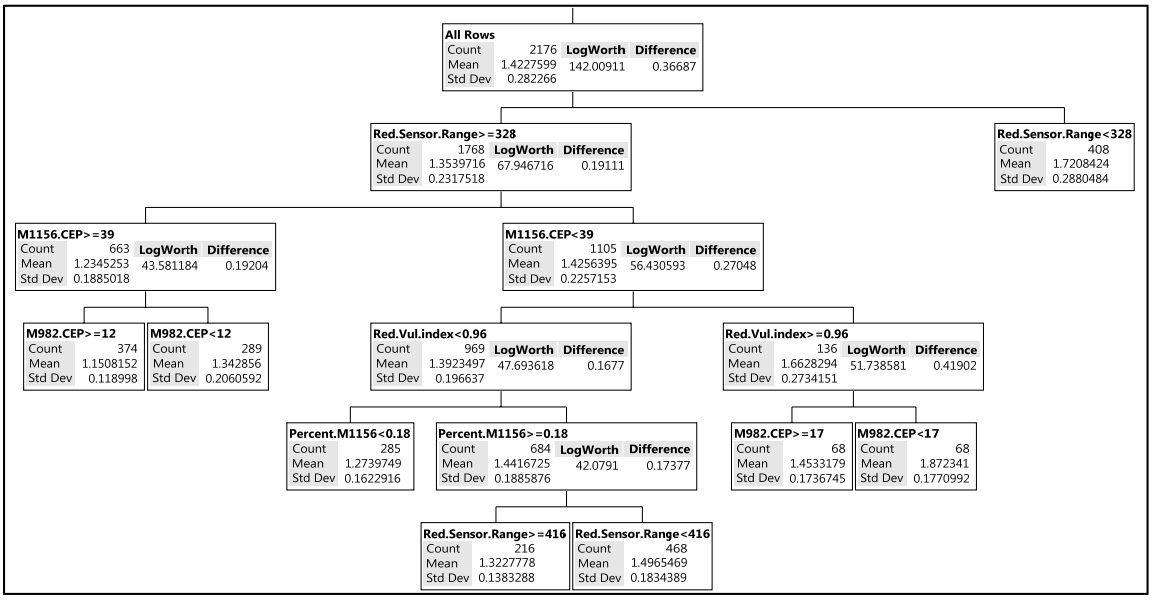


Figure 28. Regression Tree for Factors Influencing FER

b. Factors Influencing Fratricide Rates

Figure 29 shows the regression tree analysis on factors influencing the fratricide rates. It takes seven splits for the R^2 for this tree to reach 0.535 (above 0.5 as required). Similar to Scenario 1, we can see that building concealment is highly dominant and has the greatest impact on fratricide rates. There are also potential interactions effects of building concealment with Red Infantry sensor range, building mobility, M1156 CEP, Percent M1156, and M795 CEP that can influence fratricide rates. After ten splits, the R^2 increases to 0.597, but the factors of building concealment, building mobility, Red infantry sensor range, M1156 CEP, and Percent M1156 and M795 CEP remain the most important.

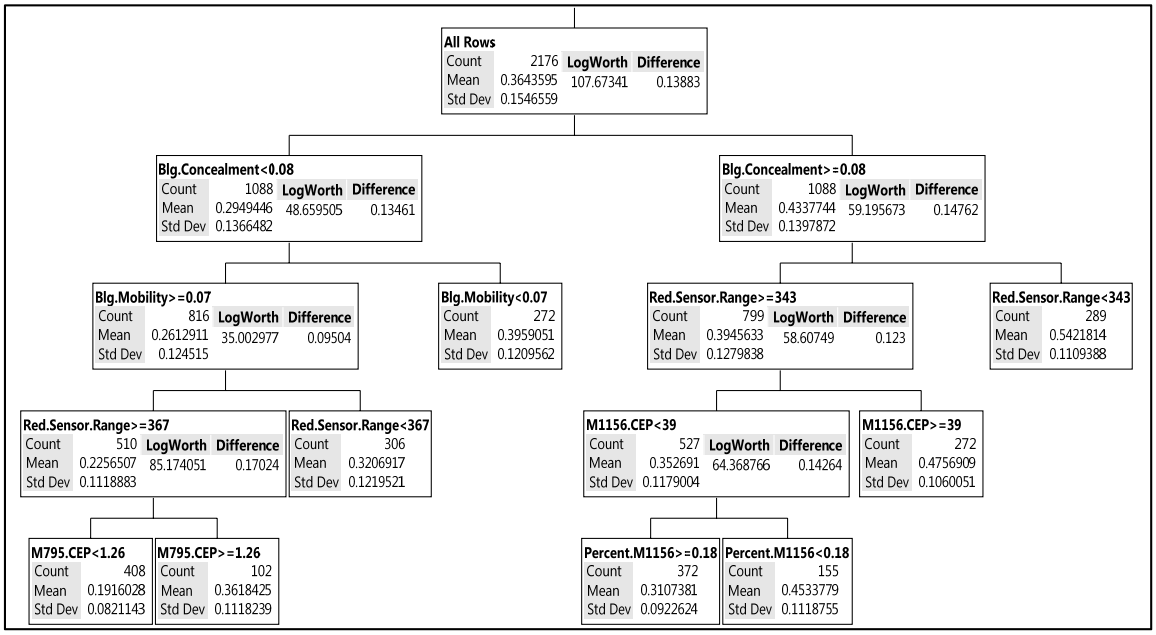


Figure 29. Regression Tree for Factors Influencing Fratricide Rates

c. Factors Influencing Collateral Damage

Figure 30 shows the regression tree analysis on factors influencing collateral damage. The R^2 for this tree after five splits is 0.534. From Figure 30, we see that Percent M1156 is highly dominant and has the greatest impact on collateral damage. As compared to Scenario 1, it is interesting to observe that building protection is not significant, while a Percent M1156 of 18 and more can significantly lower collateral damage. There are also potential interactions of Percent M1156 with building mobility, M1156 CEP, and Red vulnerability index that can influence collateral damage. After ten splits, the R^2 increases to 0.635. Although Percent M1156, building mobility, M1156 CEP, and Red vulnerability remain the most important factors, other factors such as building protection and Red infantry sensors are also included.

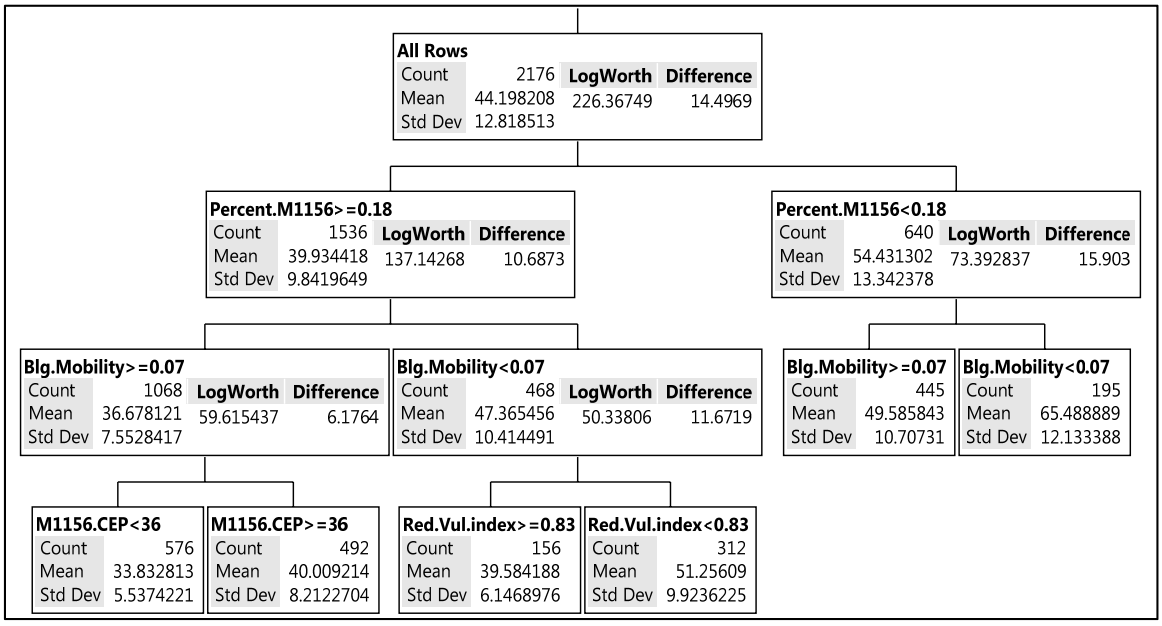


Figure 30. Regression Tree for Factors Influencing Collateral Damage

d. Factors Influencing Artillery Effectiveness

Figure 31 shows the regression tree analysis on factors influencing artillery effectiveness. The R^2 for this tree after four splits is 0.542. From Figure 31, we see that building mobility is highly dominant and has the greatest impact on artillery effectiveness. This is different from Scenario 1, where building concealment is most dominant. We can infer that higher building mobility (>=0.07) enables more random movement of Red forces, leading to a higher target locating error for the artillery to conduct firing, hence reducing the artillery effectiveness. There are also potential interactions of building mobility with building concealment, Red vulnerability index, and building protection that can influence artillery effectiveness. After ten splits, the R^2 increases to 0.728. Although building mobility, building concealment, Red vulnerability index, and building protection remain the most important factors, other factors such as Red infantry sensor range and M795 CEP are also included.

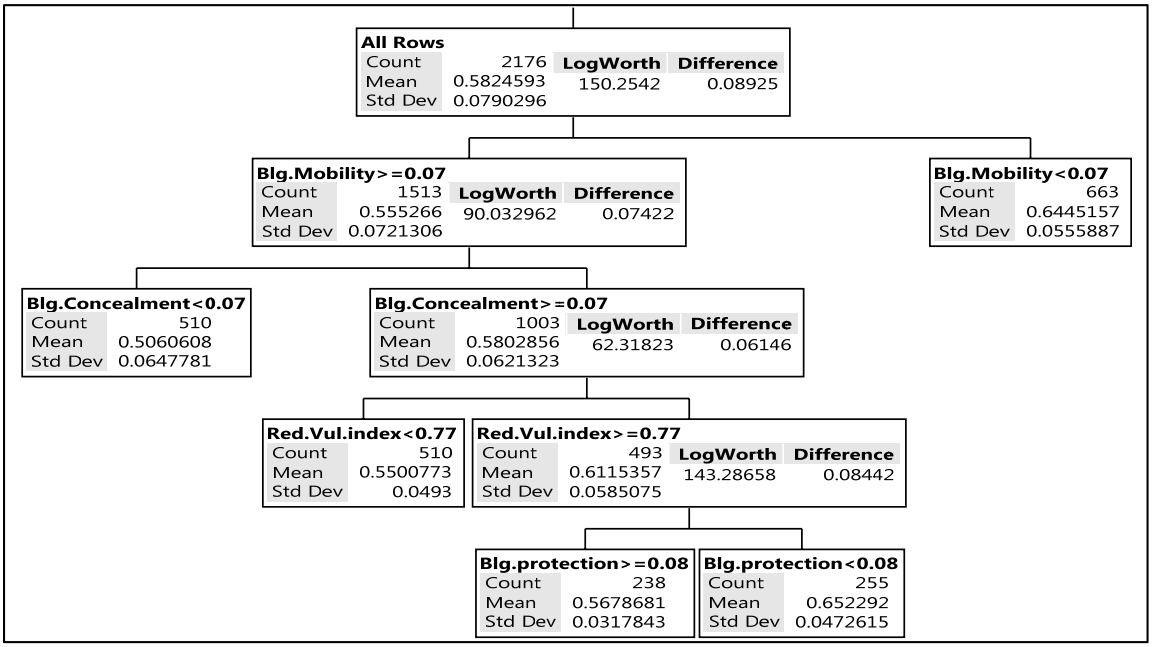


Figure 31. Regression Tree for Factors Influencing Artillery Effectiveness

e. Factors Influencing Time for Mission Success

Figure 32 shows the regression tree analysis on factors influencing time for mission success. The R^2 for this tree after four splits is 0.595. From Figure 32, we see that building mobility is highly dominant and has the greatest impact on time for mission success. This is different from Scenario 1, where building concealment is most dominant. We can infer that higher building mobility (>=0.07) enables higher frequency of engagement between both infantry forces, which reduces the time required for mission success. There is also the potential interaction of building mobility with the Red movement desire, Red vulnerability index, and M1156 CEP that can influence time for mission success. After ten splits, the R^2 increases to 0.748. Although building mobility, Red movement desire, and Red vulnerability index remain the most important factors, other factors such as building protection, M1156 CEP, and Red infantry sensor range are also included.

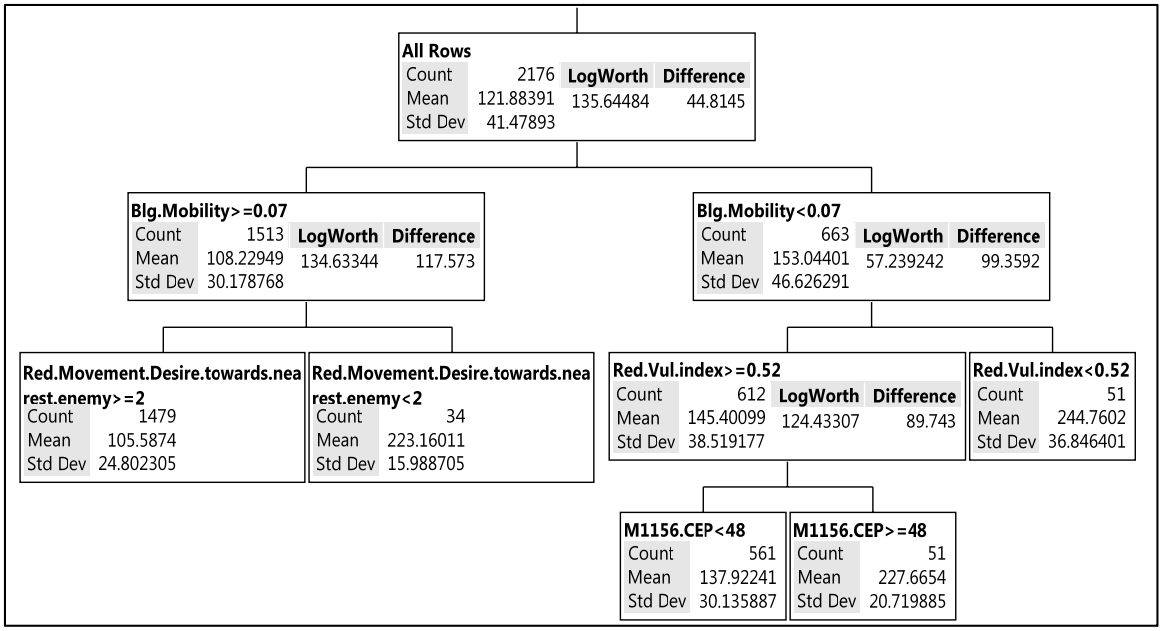


Figure 32. Regression Tree for Factors Influencing Time for Mission Success

2. Effects of Munition Allocations against MOEs

The effects of munition allocation (Percent M982 and Percent M1156) against the MOEs for Scenario 2 are discussed in this section. A summary of the regression metamodel results is provided in Section IV.D.2.g.

a. Data Exploration

Figure 33 and Figure 34 show the scatterplot matrix and correlation values between the response variables and decision factors, respectively, for Scenario 2. From Figure 34, we can see a high positive correlation between Percent M982 and FER, high negative correlation between Percent M982 versus Time for Mission Success and Fratricide, and low negative correlation between Percent M982 versus Collateral Damage and Artillery Effectiveness. On the other hand, there is some positive correlation between Percent M1156 and FER, high negative correlation between Percent M1156 versus Collateral Damage, Fratricide Rate, and Artillery Effectiveness, and a low negative correlation between M1156 and Time for Mission Success. As such, Percent M1156 in the Marawi scenario is observed to have a more salient effect against the respective MOEs as compared to its effect in the Fallujah scenario.

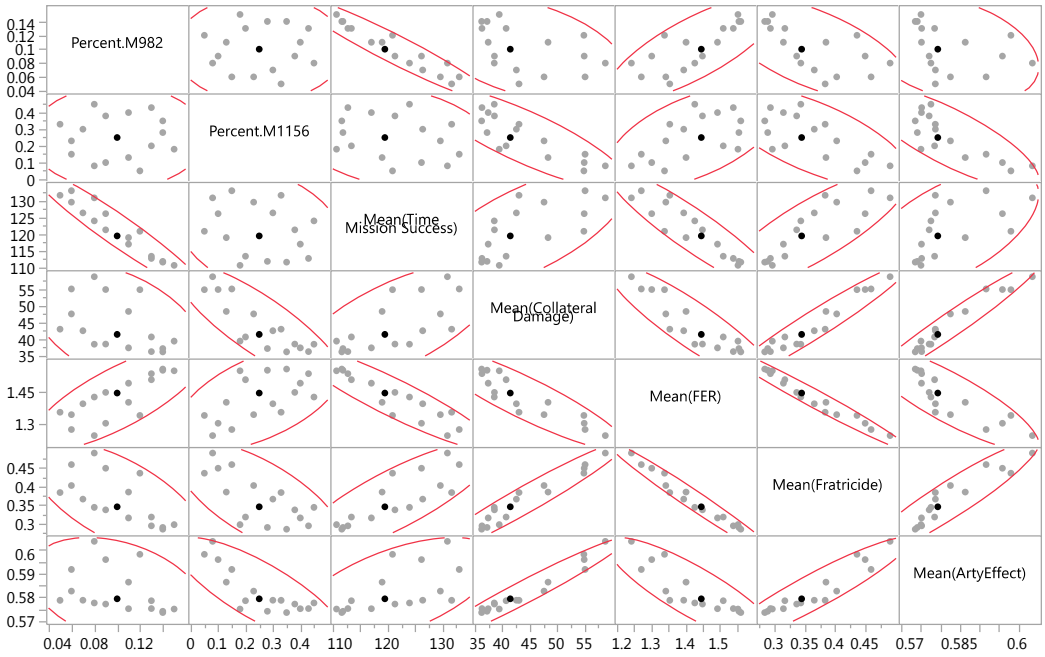


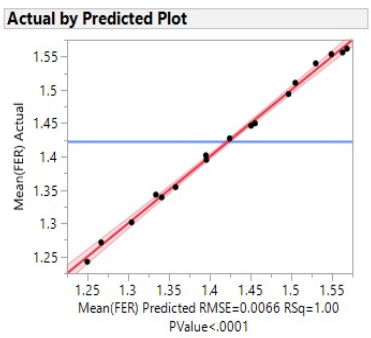
Figure 33. Scatterplot Matrix of Response and Decision Variables

	Percent.M982	Percent.M1156	Mean(Time Mission Success)	Mean(Collateral Damage)	Mean(FER)	Mean(Fratricide)	Mean(ArtyEffect)
Percent.M982	1.0000	0.0123	-0.9546	-0.4434	0.7783	-0.6455	-0.3430
Percent.M1156	0.0123	1.0000	-0.2613	-0.8392	0.5666	-0.6917	-0.8028
Mean(Time Mission Success)	-0.9546	-0.2613	1.0000	0.6750	-0.9193	0.8320	0.5884
Mean(Collateral Damage)	-0.4434	-0.8392	0.6750	1.0000	-0.9018	0.9664	0.9720
Mean(FER)	0.7783	0.5666	-0.9193	-0.9018	1.0000	-0.9814	-0.8532
Mean(Fratricide)	-0.6455	-0.6917	0.8320	0.9664	-0.9814	1.0000	0.9331
Mean(ArtyEffect)	-0.3430	-0.8028	0.5884	0.9720	-0.8532	0.9331	1.0000

Figure 34. Correlation Values of Response and Decision Variables

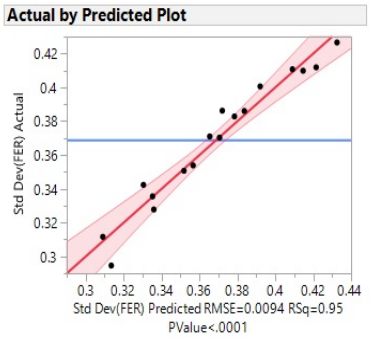
b. Regression Metamodels of FER

Figure 35 shows the standard least squares regression of the mean and SD of the FER using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 1.0 and 0.95, respectively. It can be observed that the main effect of Percent M982 and Percent M1156, as well as their interaction effect, are significant for both metamodels. The quadratic effect of Percent M1156 is also significant in both the mean and the SD metamodels. The quadratic effect of Percent M982 is insignificant in both metamodels, and hence it is excluded.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.0772824	0.006686	161.13	<.0001*
Percent.M982	2.5910899	0.053138	48.76	<.0001*
Percent.M1156	0.4568918	0.013154	34.74	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	-3.226027	0.550337	-5.86	<.0001*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	-2.019583	0.120592	-16.75	<.0001*



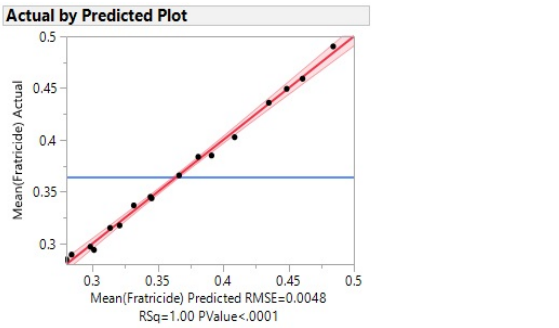
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2481362	0.00949	26.15	<.0001*
Percent.M982	1.0946832	0.075425	14.51	<.0001*
Percent.M1156	0.0833957	0.018671	4.47	0.0008*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	-2.451771	0.781159	-3.14	0.0086*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	-0.680754	0.171171	-3.98	0.0018*

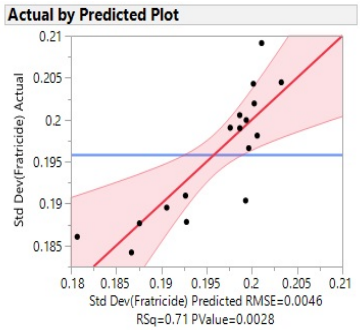
Figure 35. Least Squares Regression of Mean and SD of FER

c. Regression Metamodels of Fratricide Rate

Figure 36 shows the standard least squares regression of the mean and SD of the fratricide rate using the two decision variables, Percent M982 and Percent M1156. The model for the mean is well fitted with an R^2 of 1.0, while the model for the SD is relatively well fitted with an R^2 of 0.71. It can also be observed that the main effect of Percent M982 is significant on the mean metamodel but does not have a significant effect on the SD metamodel. By contrast, the main effect of Percent M1156 is significant on both metamodels. There is also the presence of interaction effects of Percent M982 and Percent M1156 that are significant in both metamodels. The quadratic effect of Percent M1156 is significant in the mean metamodel but insignificant in the SD metamodel. The quadratic effect of Percent M982 is insignificant in both metamodels, and hence it is excluded.



Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.5672891	0.004783	118.62	<.0001*
Percent.M982	-1.351385	0.038011	-35.55	<.0001*
Percent.M1156	-0.351486	0.009409	-37.36	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	2.9316676	0.393676	7.45	<.0001*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	1.4372929	0.086264	16.66	<.0001*

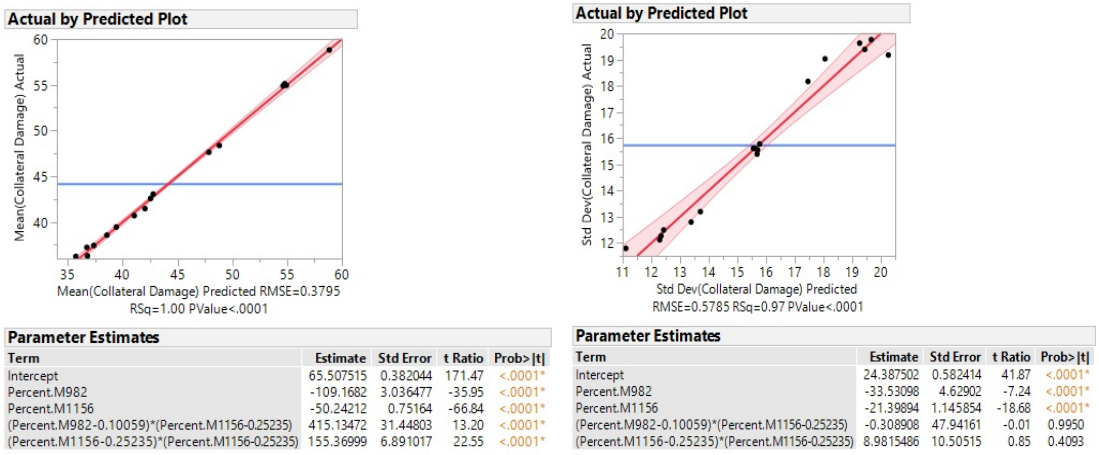


Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2139335	0.004605	46.46	<.0001*
Percent.M982	-0.065646	0.036597	-1.79	0.0981
Percent.M1156	-0.039034	0.009059	-4.31	0.0010*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	-0.87981	0.379025	-2.32	0.0387*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	-0.105225	0.083054	-1.27	0.2292

Figure 36. Least Squares Regression of Mean and SD of Fratricide Rate

d. Regression Metamodels of Collateral Damage

Figure 37 shows the standard least squares regression of the mean and SD of Collateral Damage using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 1.0 and 0.97, respectively. It can be observed that the main effect of Percent M982 and Percent M1156 is significant for both metamodels. The interaction effect and quadratic effect of Percent M1156 are significant for the mean metamodel but insignificant for the SD metamodel. The quadratic effect of Percent M982 is insignificant in both metamodels, and hence it is excluded.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.507515	0.382044	171.47	<.0001*
Percent.M982	-109.1682	3.036477	-35.95	<.0001*
Percent.M1156	-50.24212	0.75164	-66.84	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	415.13472	31.44803	13.20	<.0001*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	155.36999	6.891017	22.55	<.0001*

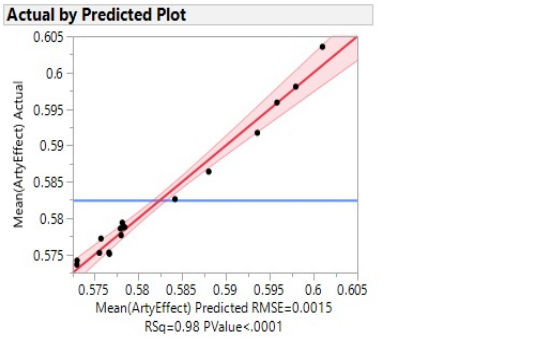
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	24.387502	0.582414	41.87	<.0001*
Percent.M982	-33.53098	4.62902	-7.24	<.0001*
Percent.M1156	-21.39894	1.145854	-18.68	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	-0.308908	47.94161	-0.01	0.9950
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	8.9815486	10.50515	0.85	0.4093

Figure 37. Least Squares Regression of Mean and SD of Collateral Damage

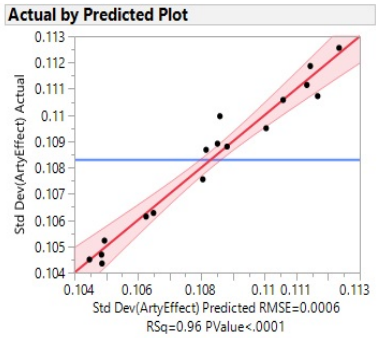
e. Regression Metamodels of Artillery Effectiveness

Figure 38 shows the standard least squares regression of the mean and SD of Artillery effectiveness using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 0.98 and 0.96, respectively. It can be observed that the main effect of Percent M982 and Percent M1156 is significant for both metamodels. The interaction effect and quadratic effect of Percent M1156 are significant for the mean metamodel but insignificant for the SD metamodel. The quadratic effect of Percent M982 is insignificant in both metamodels, and hence it is excluded.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.6033655	0.001476	408.72	<.0001*
Percent.M982	-0.105178	0.011733	-8.96	<.0001*
Percent.M1156	-0.058794	0.002904	-20.24	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	0.5309645	0.121517	4.37	0.0009*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	0.2989443	0.026627	11.23	<.0001*



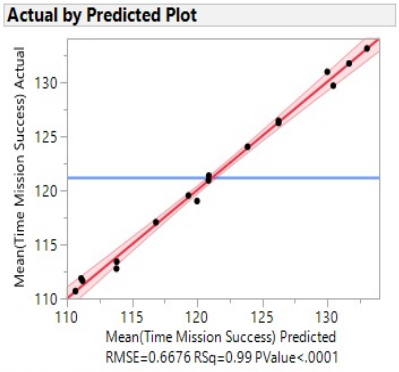
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1179664	0.000613	192.29	<.0001*
Percent.M982	-0.077163	0.004876	-15.83	<.0001*
Percent.M1156	-0.00839	0.001207	-6.95	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	0.0260482	0.050499	0.52	0.6154
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	0.0153402	0.011066	1.39	0.1909

Figure 38. Least Squares Regression of Mean and SD of Artillery Effectiveness

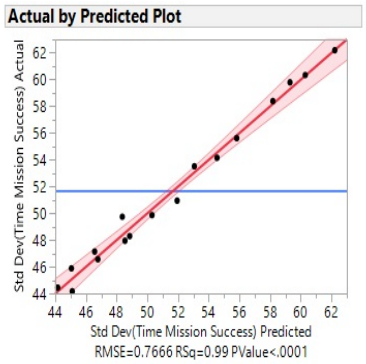
f. Regression Metamodels of Time for Mission Success

Figure 39 shows the standard least squares regression of the mean and SD of Time for Mission Success using the two decision variables, Percent M982 and Percent M1156. The models for both the mean and the SD are well fitted with an R^2 of 0.99. It can be observed that the main effect of Percent M982 and Percent M1156, their interaction effects, and the quadratic effect of M1156 are significant for both metamodels. The quadratic effect of Percent M982 is significant in the mean metamodel but insignificant in the SD metamodel.



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	146.39856	0.733804	199.51	<.0001*
Percent.M982	-232.2692	5.341739	-43.48	<.0001*
Percent.M1156	-15.3565	1.325039	-11.59	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	161.38696	55.57807	2.90	0.0143*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	816.72176	229.5497	3.56	0.0045*
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	84.110379	13.86964	6.06	<.0001*



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	70.254874	0.842648	83.37	<.0001*
Percent.M982	-114.108	6.134068	-18.60	<.0001*
Percent.M1156	-34.18374	1.52158	-22.47	<.0001*
(Percent.M982-0.10059)*(Percent.M1156-0.25235)	219.49611	63.82186	3.44	0.0055*
(Percent.M982-0.10059)*(Percent.M982-0.10059)	14.390251	263.5983	0.05	0.9574
(Percent.M1156-0.25235)*(Percent.M1156-0.25235)	102.09944	15.9269	6.41	<.0001*

Figure 39. Least Squares Regression of Mean and SD of Time for Mission Success

g. Summary of Results for Regression Metamodels

Table 9 summarizes the results for the respective factors' influence on the MOEs based on their contribution to the R^2 of the regression models. The results show that Percent M982 and Percent M1156 are almost equally influential on the MOEs in Scenario 2. We can thus infer that Percent M1156 has an obvious edge in influencing the battle in Scenario 2 as compared to Scenario 1.

Table 9. Summary of Results for Scenario 2 Showing the Influence of Percent M982 and Percent M1156 on the MOEs, Based on Contributions to R^2

Variables	Percent M982	Percent M1156	Interaction Effect
Mean FER	VS	VS	S
SD FER	VS	VS	M
Mean Fratricide rates	VS	VS	S
SD Fratricide rates	VW	S	VW
Mean Collateral damage	VS	VS	S
SD Collateral damage	VS	VS	VW
Mean Artillery effectiveness	S	VS	M
SD Artillery effectiveness	VS	VS	VW
Mean Time for mission success	VS	VS	W
SD time for mission success	VS	VS	W

VS denotes very strong influence (>0.25), S denotes strong influence (>0.125), M denotes moderate influence (>0.0625), W denotes weak influence (>0.03125), and VW denotes very weak influence (<0.03125)

3. Pareto Optimal Frontier Analysis

The POF analysis for Scenario 2 (Marawi) is discussed in this section.

a. Objective: Minimizing Scaled Loss MOEs

With the objectives of minimizing scaled losses for all five of the MOEs, a POF with 49 out of 451 munition allocations is identified. Figure 40 shows the scatterplot matrix of the five scaled losses of the MOEs, with the red points representing the 49 munition allocations identified. Figure 41 shows a plot of M982 allocations against M1156 allocations, with the red points representing the non-dominated set among all allocation configurations being explored (49/451) by the DOE. Figure 42 shows the histograms of the POF for this multi-objective function. From Figure 42, the mean proportion of M982 is 12.4 percent (95% CI [11.5,13.3]) and the mean proportion of M1156 is 15.1 percent (95% CI [11.0,19.2]. As compared to Scenario 1, high allocations of M982 are still often needed, but low allocations for M1156 are usually preferred.

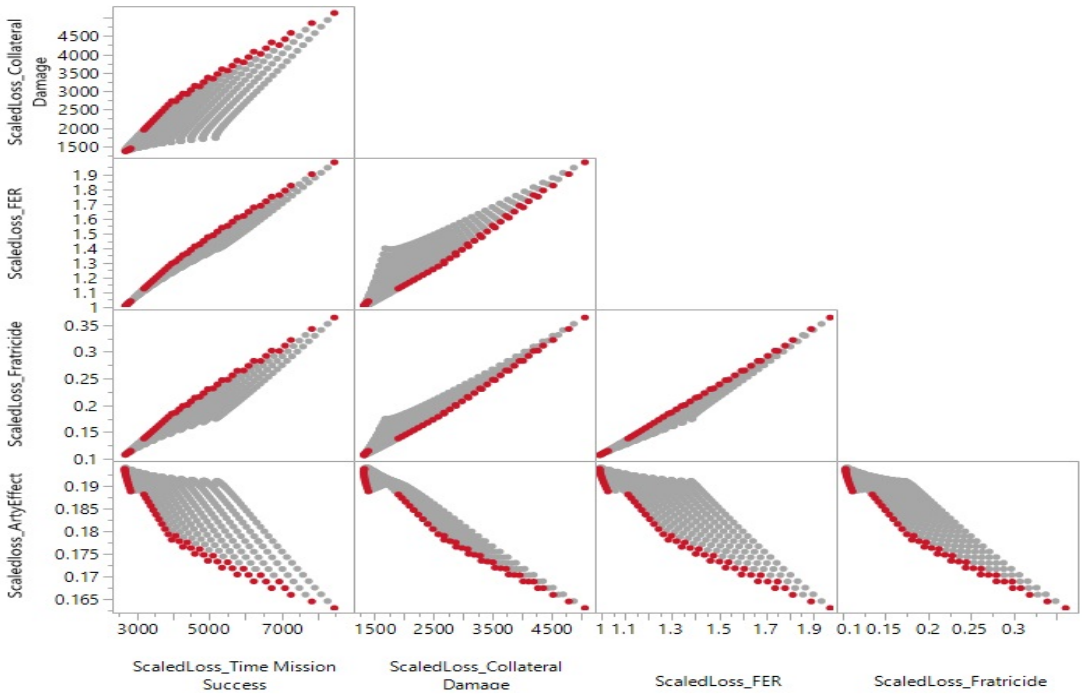


Figure 40. Scatterplot Matrix of the Scaled Loss MOEs

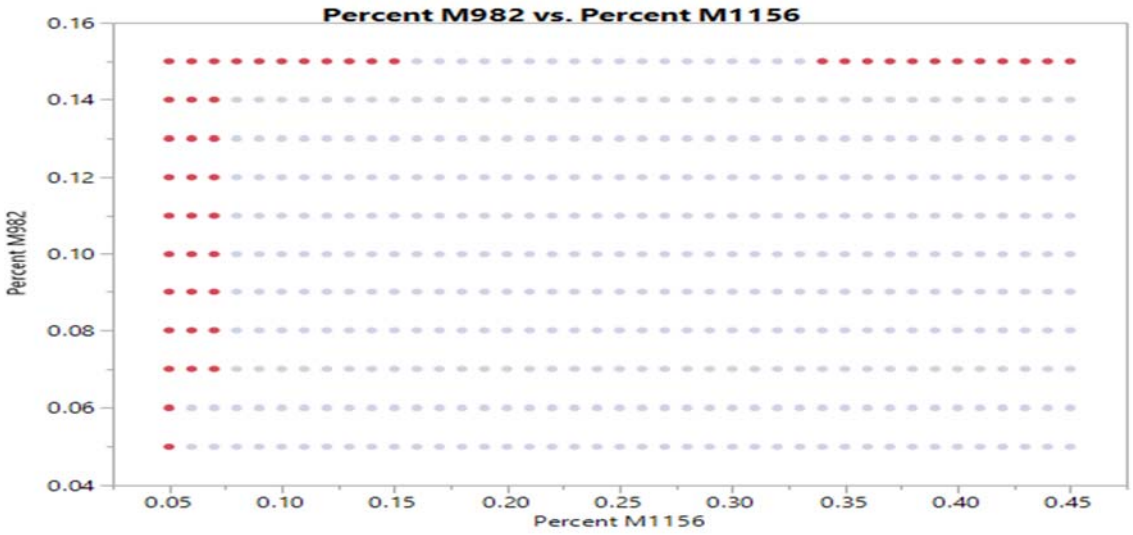


Figure 41. Plot of M982 Allocations against M1156 Allocations

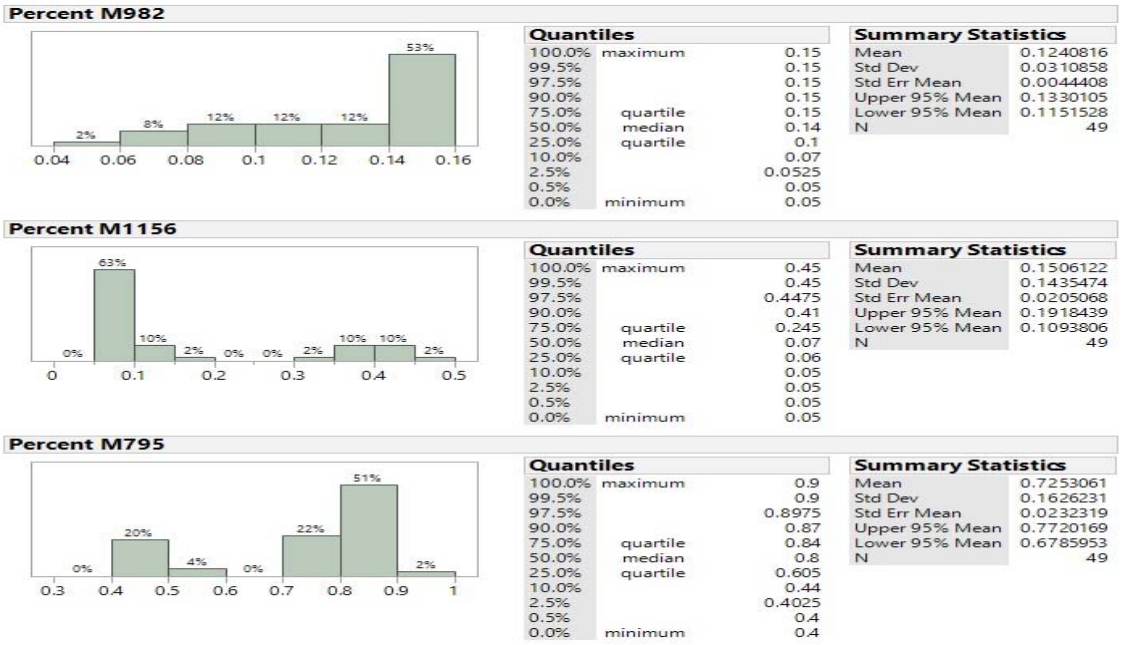


Figure 42. Histograms of the POF (All Five Scaled Loss MOEs)

b. Objective: Minimizing Scaled Loss MOEs and Cost

Similar to Scenario 1, when all five scaled loss MOEs are applied against cost, all of the 451 munition allocations are non-dominated against each other.

c. Objective: Minimizing Scaled Loss Total Collateral, Scaled Loss Artillery Effectiveness, and Cost

With the objectives of minimizing (1) scaled loss of total collateral, (2) scaled loss of artillery effectiveness, and (3) expenditure cost of the munitions, a POF with 303 out of 451 munition allocations can be identified. Figure 43 shows the scatterplot matrix of the three scaled loss objectives, with the red points representing the 303 munition allocations identified. Figure 44 shows the histograms of the POF with a mean proportion of 9.5 percent (95% CI [9.2, 9.9]) for the M982 and 22.5 percent (95% CI [21.0,23.9]) for the M1156, respectively.

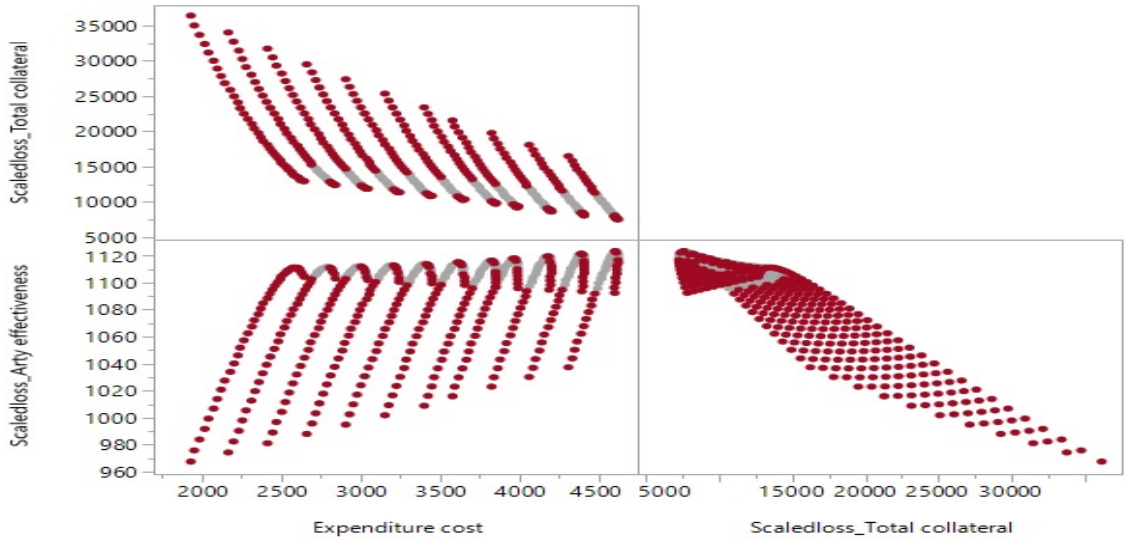


Figure 43. Scatterplot Matrix of the Three Scaled Loss MOEs

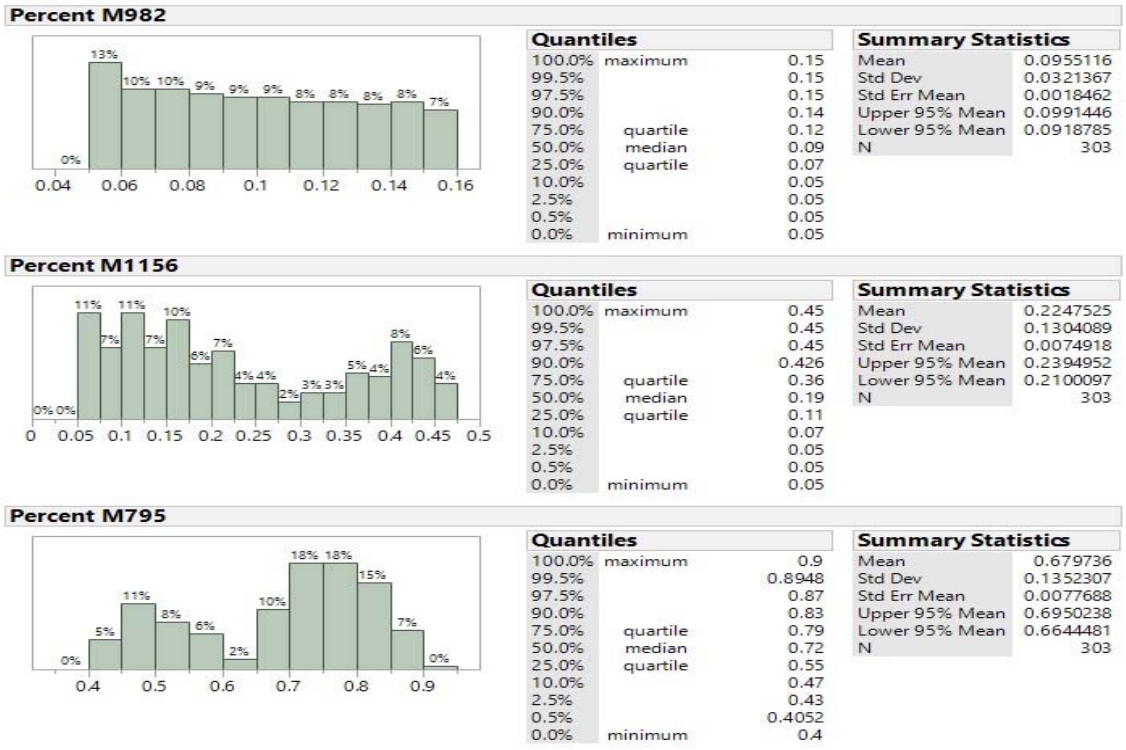


Figure 44. Histograms of the POF (Scaled Loss Total Collateral, Scaled Loss Artillery Effectiveness, and Cost)

With the objectives of minimizing only (1) scaled loss of total collateral and (2) scaled loss of artillery effectiveness, a POF with 32 out of 451 munition allocations is identified. Figure 45 shows a 2D plot of the two scaled loss of the objectives, with the red points representing the 32 munition allocations identified. Figure 46 shows the histograms of the POF with a mean proportion of 13.2 percent (95% CI [12.1,14.3]) for the M982 and 16.9 percent (95% CI [11.3,22.5]) for the M1156, respectively. It is observed that without cost as an objective, the proportion of M982 increases significantly from 9.5 to 13.2 percent, while the proportion for the M1156 decreases from 22.5 to 16.9 percent.

Similar to Scenario 1, we can observe a trade-off between cost and operational effectiveness as the proportion of M982 is reduced when cost is introduced as one of the objectives. By contrast, the proportion of M1156 increases consequently with the decrease of M982 when the cost factor is considered. This shows that the impact of the M1156 is more prominent in Scenario 2 in terms of operational effectiveness.

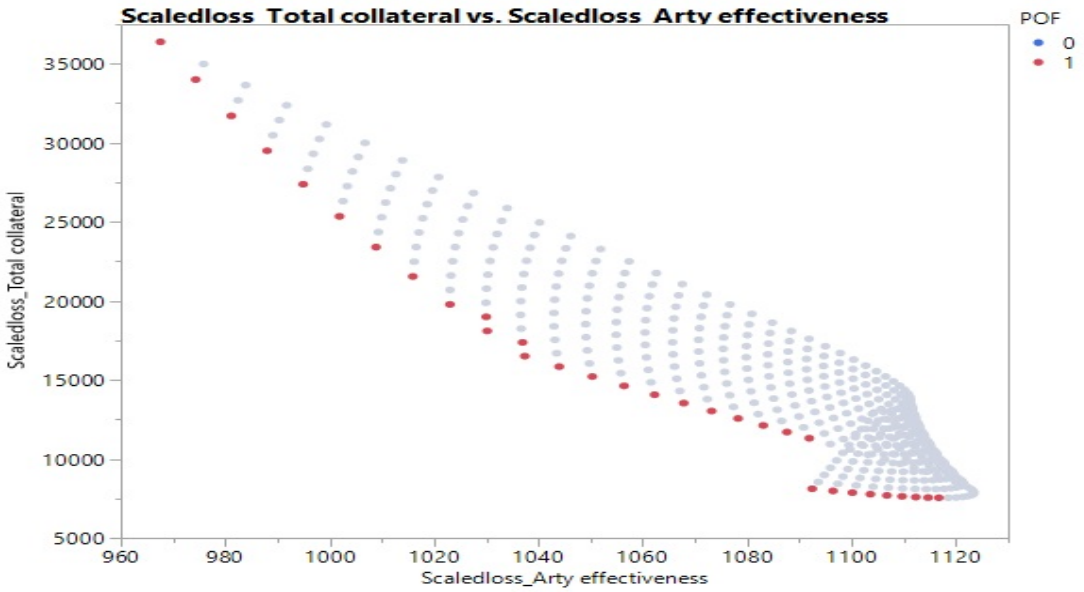


Figure 45. Plot of Two Scaled Loss MOEs (Scaled Losses for Total Collateral Damage and Artillery Effectiveness)

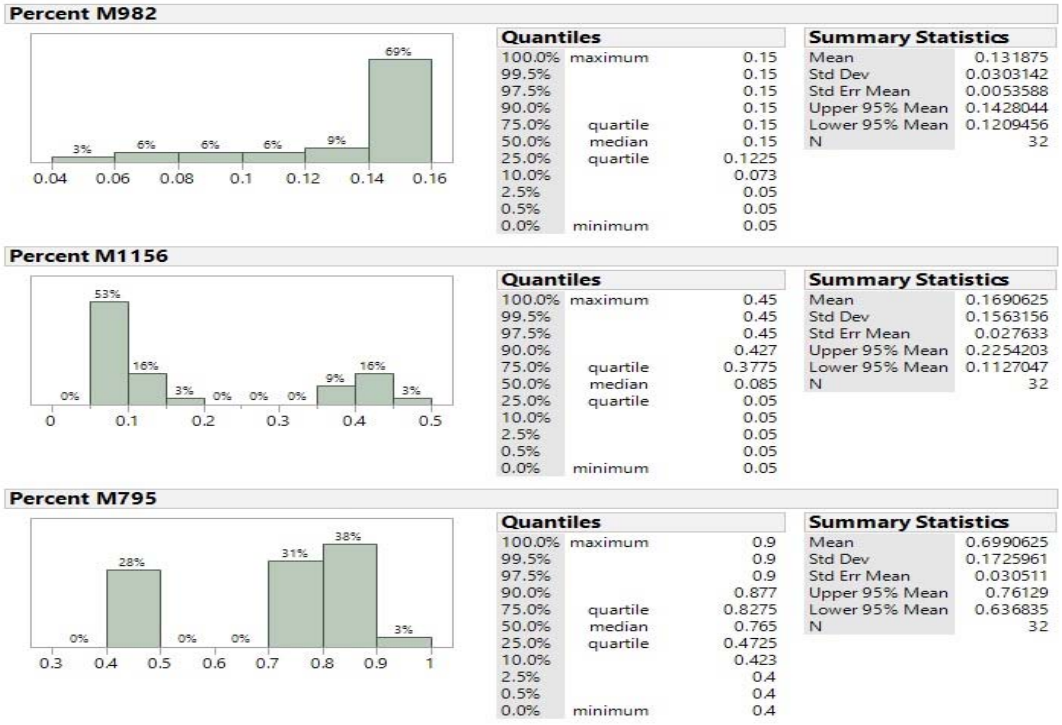


Figure 46. Histograms of the POF (Scaled Losses for Total Collateral and Artillery Effectiveness)

d. Summary of Findings for POF

Table 10 presents a compilation of munition allocations, averaged over the POF, for the two scenarios with different multiple objective functions. Recall that these average proportions are not in themselves particularly meaningful, but do provide some guidance about the differences in composition of the POFs. It is interesting to observe that both scenarios are fairly consistent in the change in proportion of M982 with different objective functions, but the change in proportion for the M1156 varies greatly. From the regression analysis of the effects of munition allocations, it is observed that the M982 has great impact on operational effectiveness for both scenarios, while the M1156 has a great impact on Scenario 2 but mild impact on Scenario 1. As such, in Scenario 1, the Percent of M1156 is less sensitive to the cost metric, where there is little change in proportion when cost is added to the two objectives function (minimize scaled loss of total collateral and minimize scaled loss of artillery effectiveness).

Comparing the two scenarios, we note that both have similar setups and agent characteristics, but Scenario 1 has a terrain size that is about 1.75 times bigger with buildings located more densely together. The first scenario also has an overall force size 1.5 times than that of Scenario 2. As such, it is reasonable to conclude that the M982, which has the lowest CEP, is operationally effective when deployed in an urban terrain of different sizes and densities. Nonetheless, although the impact of the M1156 in smaller terrain is substantial, its impact on operations in bigger and denser terrains does not appear to be significant.

Table 10. Summary of Average Munition Allocations from POF

Scenario	Objectives	Percent M982 [95% CI]	Percent M1156 [95% CI]	Percent M795 [95% CI]
1 – Battle of Fallujah	1. Min Scaled Loss FER 2. Min Scaled Loss Fratricide 3. Min Scaled Loss Collateral Damage 4. Min Scaled Loss Artillery Effectiveness 5. Min Scaled Loss Time for Mission Success	13.3% [12.7,13.8]	33.5% [31.0,36]	53.2% [50.9,55.5]
	1. Min Scaled Loss Total Collateral 2. Min Scaled Loss Artillery Effectiveness 3. Min Cost	11.6% [11.1,12.1]	26.7% [24.9,28.4]	61.7% [60.0,63.4]
	1. Min Scaled Loss Total Collateral 2. Min Scaled Loss Artillery Effectiveness	15.0%	25.0% [21.2,28.8]	60.0% [56.2,63.8]
2 – Battle of Marawi	1. Min Scaled Loss FER 2. Min Scaled Loss Fratricide 3. Min Scaled Loss Collateral Damage 4. Min Scaled Loss Artillery Effectiveness 5. Min Scaled Loss Time for Mission Success	12.4% [11.5,13.3]	15.1% [11.0,19.2]	72.5% [68.0,77.0]
	1. Min Scaled Loss Total Collateral 2. Min Scaled Loss Artillery Effectiveness 3. Min Cost	9.5% [9.2, 9.9]	22.5% [21.0,23.9]	68.0% [66.0,70.0]
	1. Min Scaled Loss Total Collateral 2. Min Scaled Loss Artillery Effectiveness	13.2% [12.1,14.3]	16.9% [11.3,22.5]	69.9% [63.7,76.1]

THIS PAGE INTENTIONALLY LEFT BLANK

V. FURTHER EXPLORATION

> To raise new questions, new possibilities, to regard old problems from a new angle requires creative imagination and makes real advances in science.
>
> —*Albert Einstein (1879–1955), Physicist & Nobel Laureate*

In this chapter, we further explore the data by using a Multi-Objective Evolutionary Algorithm (MOEA) to conduct a search for an alternative set of robust solutions in this multi-objective problem space. The final POF from the MOEA search, together with the findings from the fixed DOE approach in Chapter IV, provide decision options for selecting the optimal mix of munitions.

A. MOTIVATION

The main aim of this study is to solve a multi-objective problem that involves maximizing effects on enemies, minimizing effects on non-enemies, and minimizing cost of employment. These objectives are conflicting in nature but by formulating the decision as a multi-objective optimization problem, we are able to obtain a set of Pareto optimal solutions. MOEA is field of optimization that makes use of heuristic, stochastic algorithms to perform searches through large possible space of alternatives while considering all the multiple objectives to find a set of robust solutions (Sanchez, Upton, McDonald, & Zabinski, 2017). It is worth noting that like many integer or non-linear programming methods, MOEAs do not guarantee optimal solutions, but they can often find attractive alternative solutions in a complicated environment (Michalewicz & Fogel, 2010). The key advantage of using the MOEA is its speed of execution and its ability to seek a diverse set of solutions instead of a single best solution (Zitzler, 1991), which enables decision makers to adjust their preferences before making a final decision.

B. CHARACTERISTICS OF MOEA

The MOEA used in this study is an NPS–developed evolutionary algorithm named "ARTeMIS" (Automated Red Teaming Multi-Objective Innovation Seeker), after Artemis, the mythological Greek goddess of hunting (Sanchez, Upton, McDonald, & Zabinski,

2017). Like other MOEAs, the ARTeMIS algorithm is self-adaptive; its exploration decision is dynamically based on the characteristics of the set of candidates evaluated. ARTeMIS is also elitist and diversity preserving, where the user-specified percentage of both the best and diverse solutions are retained and passed onto the next generation of candidate solutions. Here, diversity refers to how far apart the candidate solutions are in terms of their input values: in this study, these are the numbers of each type of munitions. This limits the number of potential points on the POF that will be identified, to avoid having two (or more) solutions that are essentially the same. Another unique aspect of ARTeMIS is that it uses scaled losses for the objective functions. This means that as generations progress, the POF solutions that it generates are robust and so are likely to perform well for a wide variety of enemy and environmental conditions.

C. BASIC APPLICATION

In general, a MOEA starts by selecting a set of randomly generated candidate solutions as a guide to select better performing solutions from the population. In a large solution space, a designed experiment can be used to select an initial set of "known" good solutions. The algorithm iteratively generates new populations, using attractive characteristics of a solution from the previous generation, similar to selecting the best performing "parents" (initial solutions) to produce "children" (new candidate solutions) with good genetics (MacCalman, 2013). Some random mutation is then applied to the children, which constitutes the next generation. The intention of the process is to produce better solutions of the predetermined "fitness value" or performance metric with each new generation. This process is repeated until a user-determined stopping criterion is met, such as the number of generations required. A typical way to track the progress is with a "Best so Far" curve, as shown in Figure 47, which tracks the best fitness over time. The rate of diminishing returns could be used to determine the terminating condition. With many fitness measures, however, the terminating criteria can be difficult to determine as the rate of diminishing returns cannot easily be tracked.

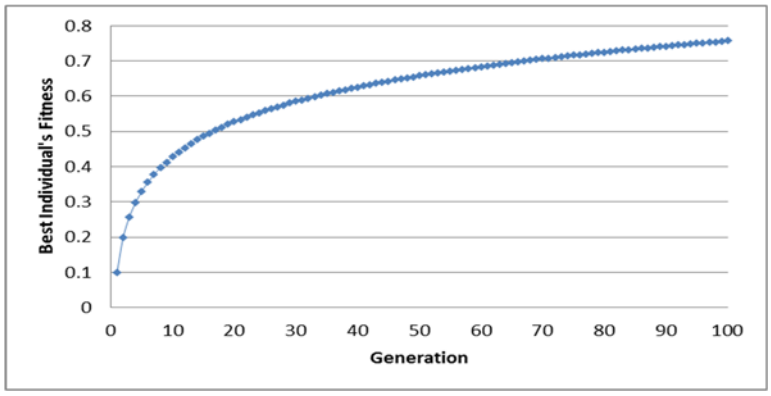


Figure 47. "Best So Far" Curve. Source: Sanchez, Upton, McDonald, and Zabinski (2017).

D. EXPERIMENTAL SETUP

The objectives (fitness measures) of the MOEA run for the two simulated scenarios are to (1) minimize the scaled loss of total collateral, (2) minimize the scaled loss of artillery effectiveness, and (3) minimize the cost of munition expenditure. The input variables are (1) the amount of M982, (2) the amount of M1156, and (3) the amount of M795.

The initial population of solutions uses the range of min – max values of each of the munition proportions from the fixed DOE setup. Instead of setting a total allocation limit as per the DOE, however, the algorithm allows the proportion of the three types of munitions to vary independently with no maximum capacity for total number of munitions used. This is an added flexibility of using the MOEA over the DOE setup, as we do not know for sure how many rounds are needed in each simulated battle. Also, in the DOE setup, the allocated amount is often different from the expenditure amount, which makes it difficult to determine the optimal proportion needed while considering cost. Here, the MOEA has the flexibility to vary the numbers needed according to the actual expended munition cost and search for the Pareto Front, which consists of the set of non-dominated solutions at the end of each generation run. Each scenario's MOEA is run for 30 generations with 80 individuals per generation, and it took the HPCC approximately 60 hours to complete a full run.

E. MOEA RESULT FOR SCENARIO 1 (FALLUJAH)

This section discusses the results of the MOEA run for Scenario 1. We first look at the results for the three objectives function and then compare the results with those for the two objectives function (without cost).

1. MOEA Run on Three Objectives Function

For the three objectives function, a POF with 104 out of 2480 alternatives is identified. Figure 48 shows the three-dimensional scatterplot of the three objectives function, where the POF are indicated by the red points. Figure 49 shows the histograms of these munitions proportion that are non-dominated. Although the average proportions across different points on the POF are not in themselves particularly meaningful, they can be used to compare the characteristics of the POFs based on different objective functions. It is observed that relatively high proportions of M982 with a mean of 11 percent (95% CI [10.2,11.8]) and low proportions of M1156 with a mean of 11.6 percent (95% CI [10.5,12.8]) are preferred. The mean proportion of M795 dumb munition is relatively high at 77.3 percent. As per the DOE setup, the number of non-dominated points can always be reduced further by introducing additional constraints on any of the objectives.

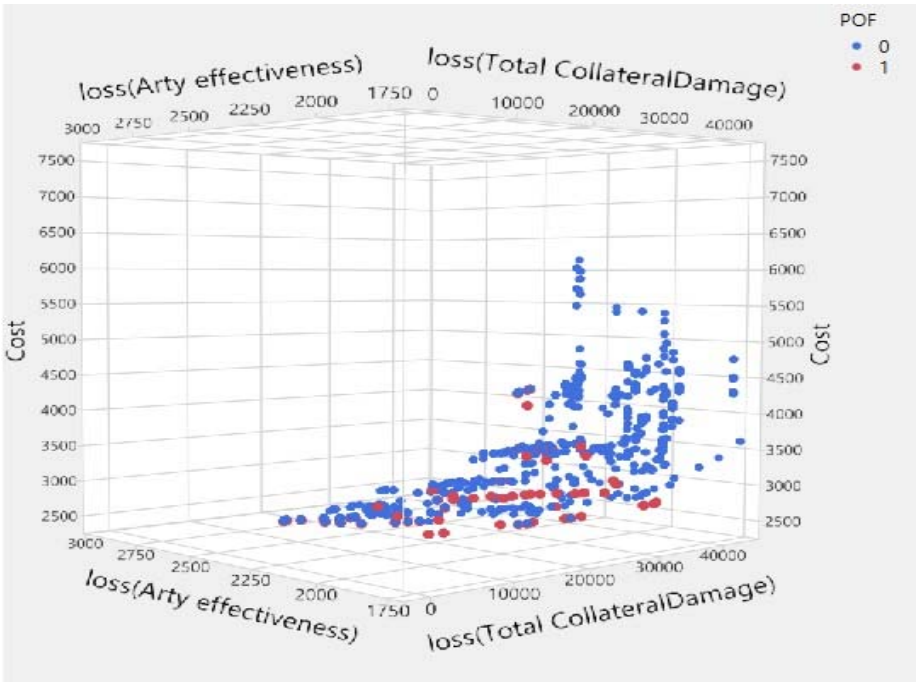


Figure 48. 3D Scatterplot of MOEA-Determined POF (Three Objectives)

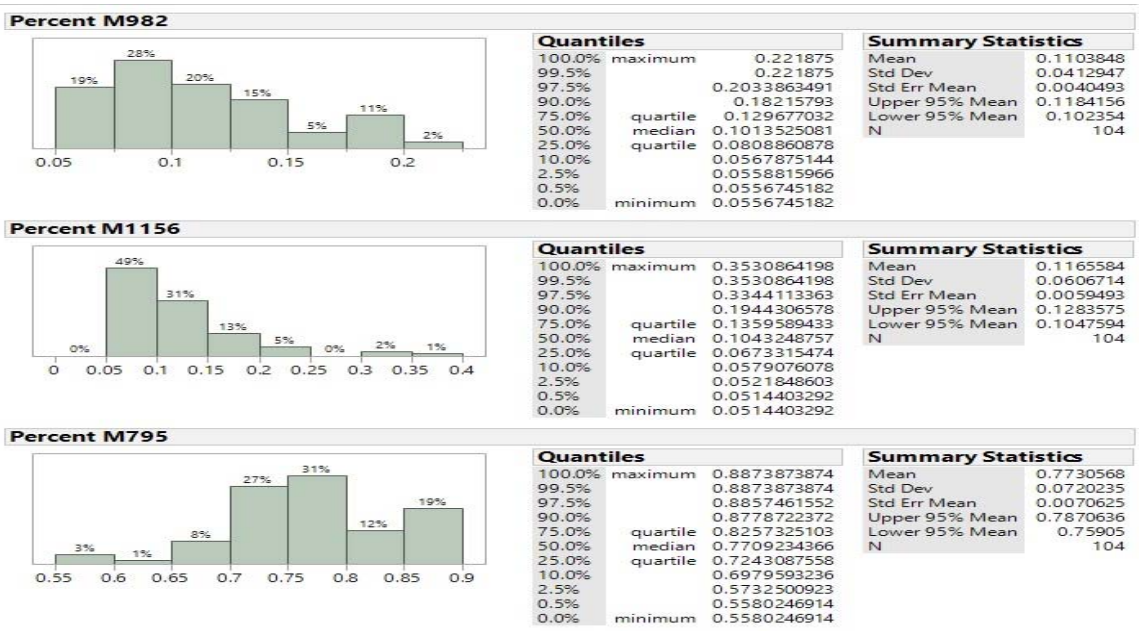


Figure 49. Histograms of Munitions Proportions for MOEA-Determined POF (Three Objectives)

2. MOEA Run on Two Objectives Function

A POF with 599 out of 2,480 alternatives is identified for the two objectives function MOEA run without cost consideration. Figure 50 shows the histograms of the munition proportions that are non-dominated. Compared to the three objectives function, the proportions of M982 and M1156 are both higher with means of 13.5 percent (95% CI [13.4,13.6]) and 17.2 percent (95% CI [15.9,18.5]), respectively. The mean proportion of M795 dumb munitions is still high at 69.3 percent, which indicates that area munitions are relatively useful in inflicting damage on enemy forces, and the model is not too sensitive on the cost of munition allocations.

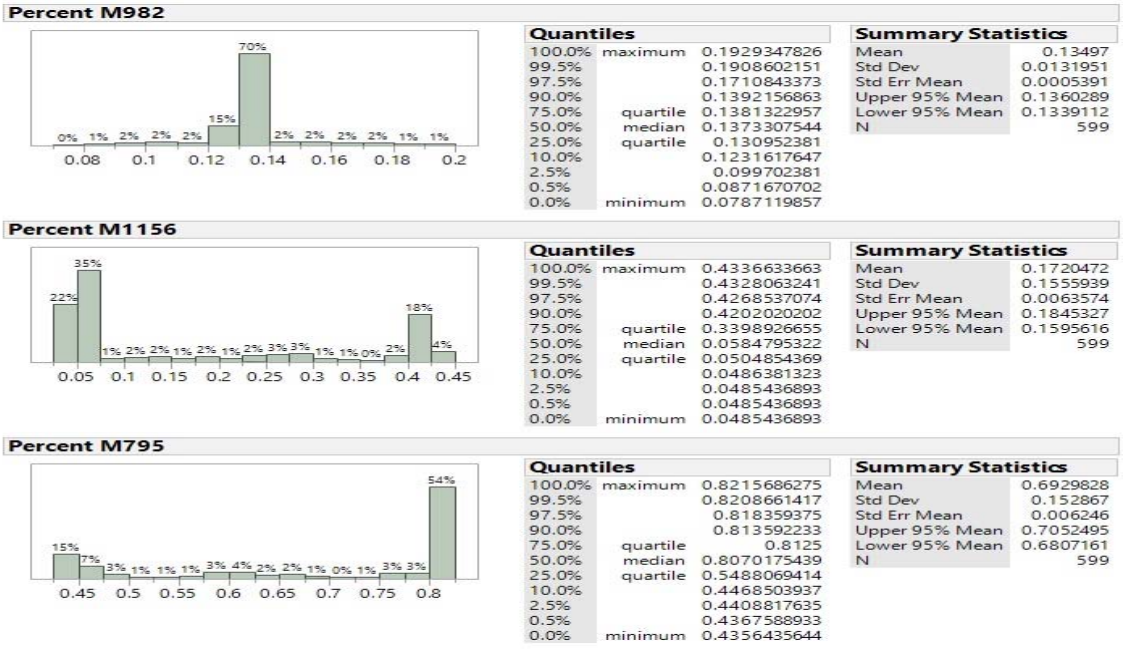


Figure 50. Histograms of Munition Proportions for MOEA-Determined POF (Two Objectives)

a. Convrgence of POF

A 2D plot of the evolution of population over iterations on the two objectives function is shown in Figure 51, where the colors represent the explored regions generated over iterations (generations). Another 2D plot of evolution over POF over iterations is shown in Figure 52, where the enlarged dots represent the POF, and its colors show the

POF that evolves over time. Comparing the two plots, we observe a general convergence of POF with higher iterations as shown by the red dots at the bottom left of the plot. However, due to the diversity-seeking nature of the MOEA, where new regions are still being explored at high number of iterations, some of the red dots are observed to be far from the optimal frontier. This shows that better and more robust solutions may be generated with higher number of iterations over time.

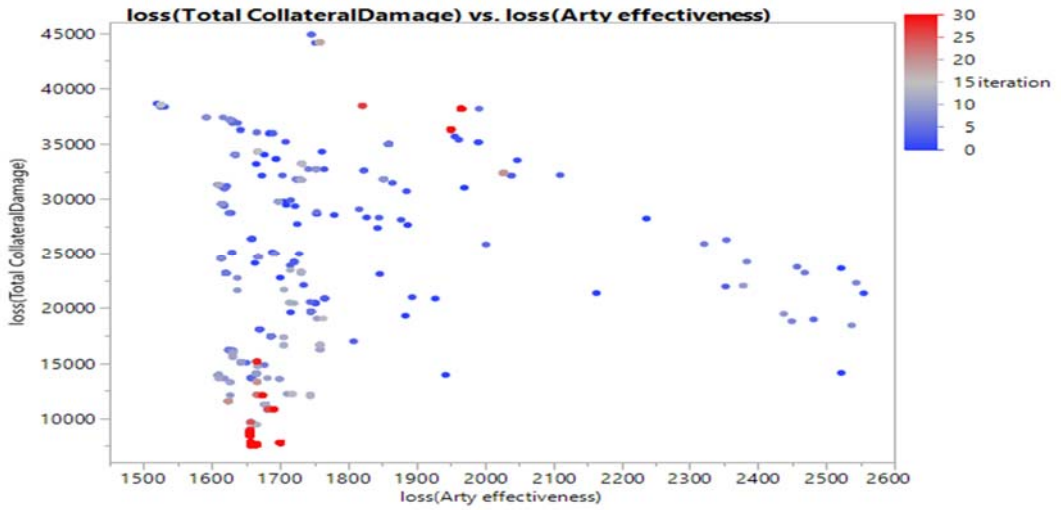


Figure 51. 2D plot of Evolution of Population over Iterations

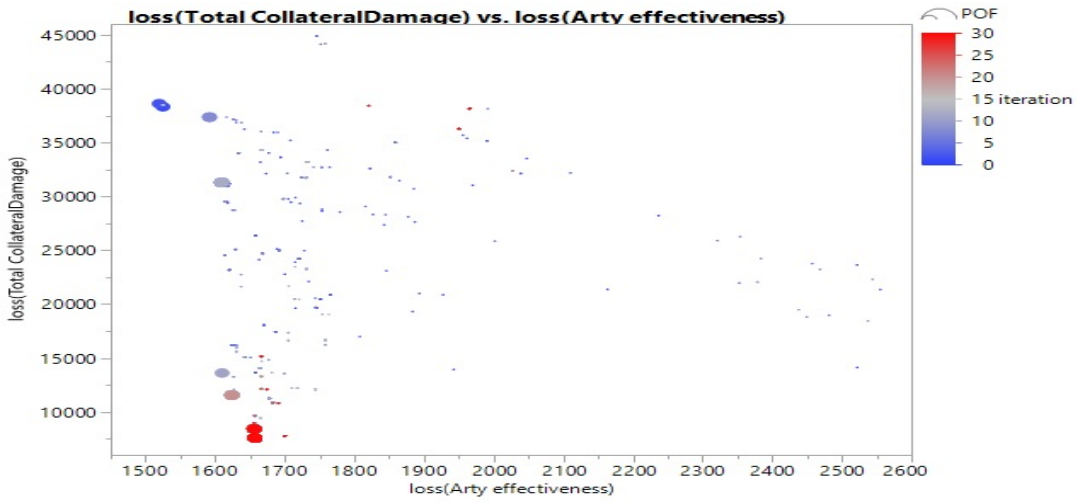


Figure 52. 2D plot of Evolution over POF over Iterations

F. MOEA RESULT FOR SCENARIO 2 (MARAWI)

This section discusses the MOEA result for Scenario 2.

1. MOEA Run on Three Objectives Function

A POF with 324 out of 2,480 alternatives is identified for the three objectives function MOEA run for Scenario 2. Figure 53 shows the three-dimensional scatterplot of the three objectives function, where the POF are indicated by the red points. Figure 54 shows the histograms of these munition proportions that are non-dominated. As compared to Scenario 1, the proportions of M982 and M1156 are lower with means of 8.2 percent (95% CI [7.8,8.6]) and 11.6 percent (95% CI [10.9,12.7]), respectively. Consequently, the mean proportion of M795 dumb munition is also higher at 80.2 percent.

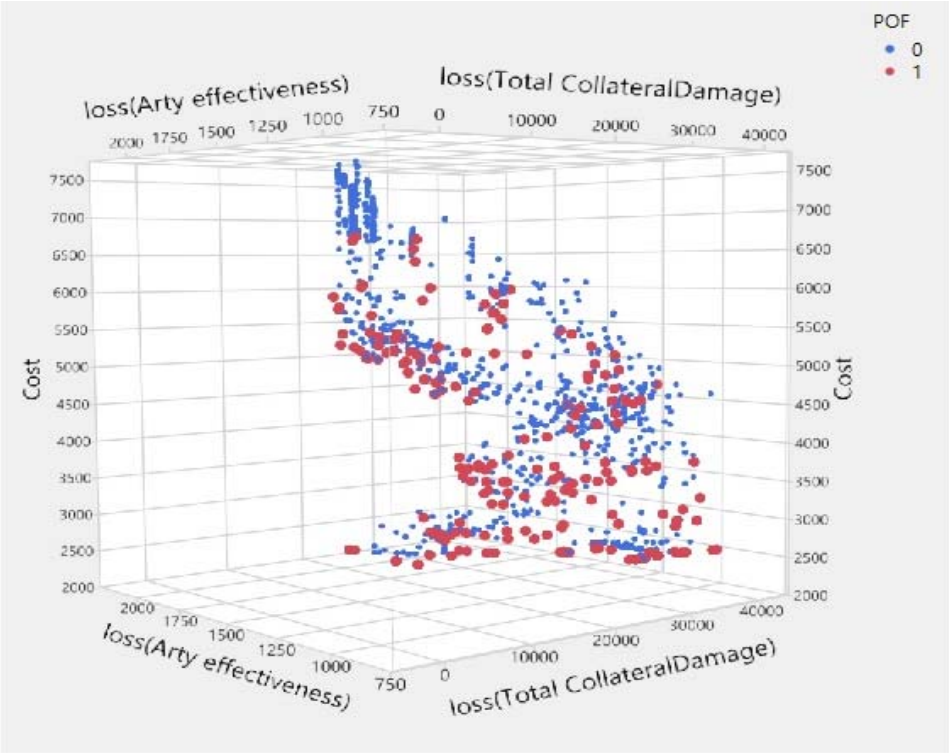


Figure 53. 3D Scatterplot of MOEA-Determined POF (Three Objectives)

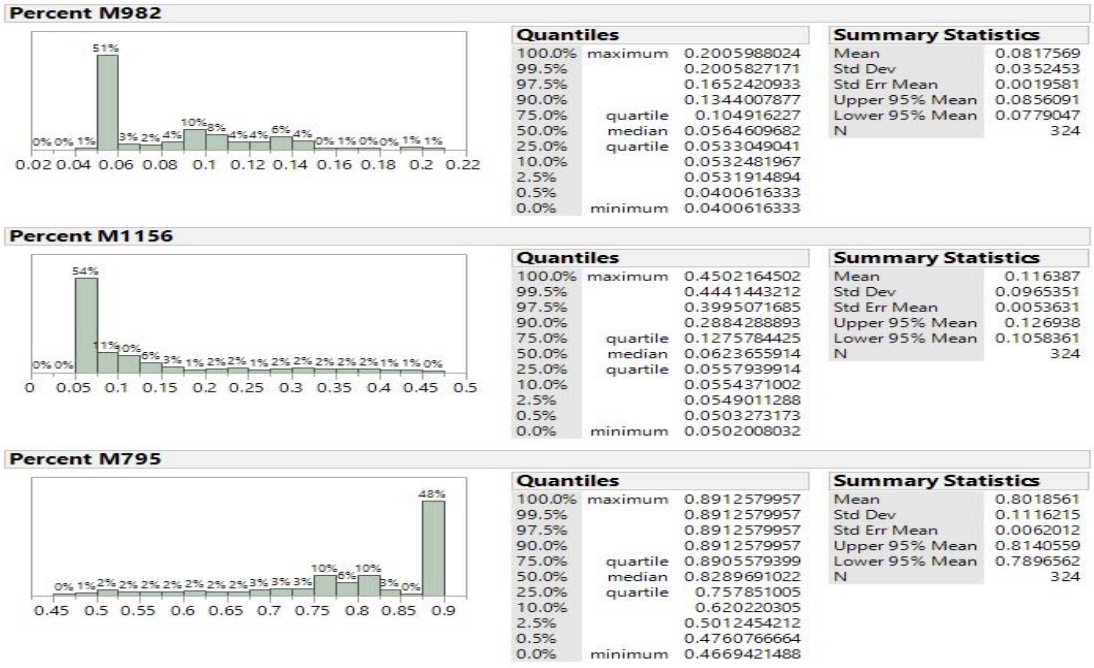


Figure 54. Histograms of Munition Proportions for MOEA-Determined POF (Three Objectives)

2. MOEA Run on Two Objectives Function

A POF with 339 out of 2,480 alternatives is identified for the two objectives function MOEA run without cost consideration. Figure 55 shows the histograms of these munition proportions that are non-dominated. Compared with the three objectives function, both proportions of M982 and M1156 are higher with means of 10.1 percent (95% CI [9.9,10.4]) and 38.6 percent (95% CI [37.9,39.2]), respectively. The huge jump in the proportion for M1156 indicates its importance in operational effectiveness in this scenario. Consequently, the mean proportion of M795 dumb munition dropped significantly to 51.3 percent, which implies that the dumb munition in Scenario 2 is not as effective as in Scenario 1. We can also conclude that the model for Scenario 2 is highly sensitive to the cost of munition allocations.

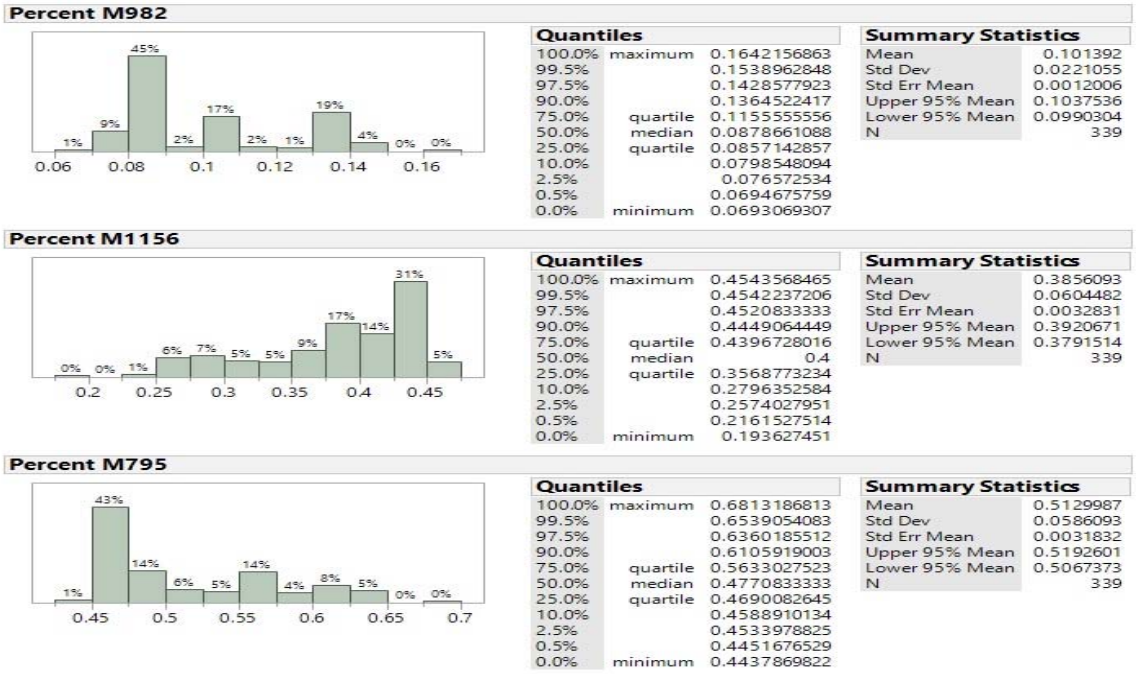


Figure 55. Histograms of Munition Proportions for MOEA-Determined POF (Two Objectives)

a. Convergence of POF

A 2D plot of the evolution of population over iterations on the two objectives function is shown in Figure 56 where the colors represent the explored regions generated over iterations (generations). Another 2D plot of evolution over POF over iterations is shown in Figure 57, where the enlarged dots represent the POF, and its colors show the POF that evolves over time. Comparing the two plots, we still observe a general convergence of POF with higher iterations, however there are large number of red dots that are observed to be far from the optimal frontier. This could mean that more iterations are needed in this scenario to generate better and more robust solutions where the convergence of POF is clearer.

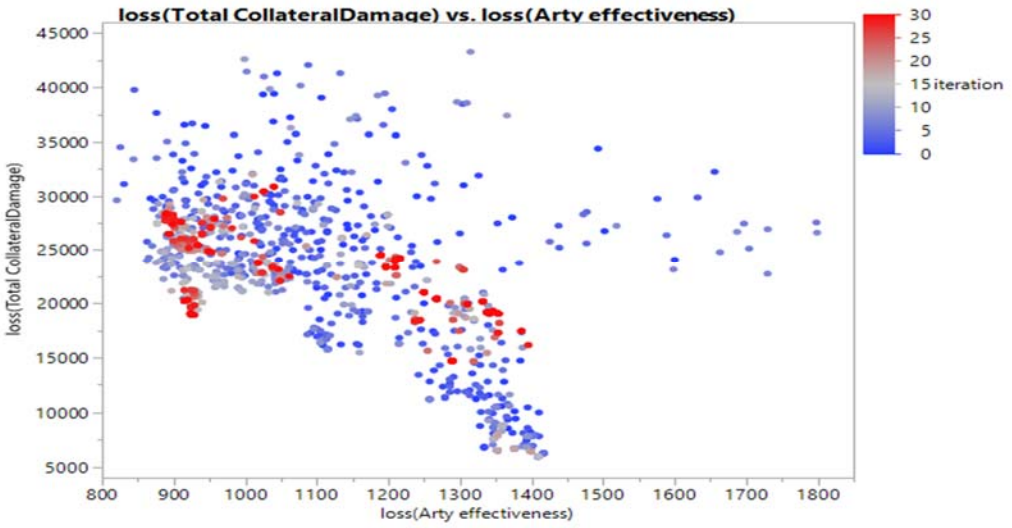


Figure 56. 2D Plot of Evolution of Population over Iterations

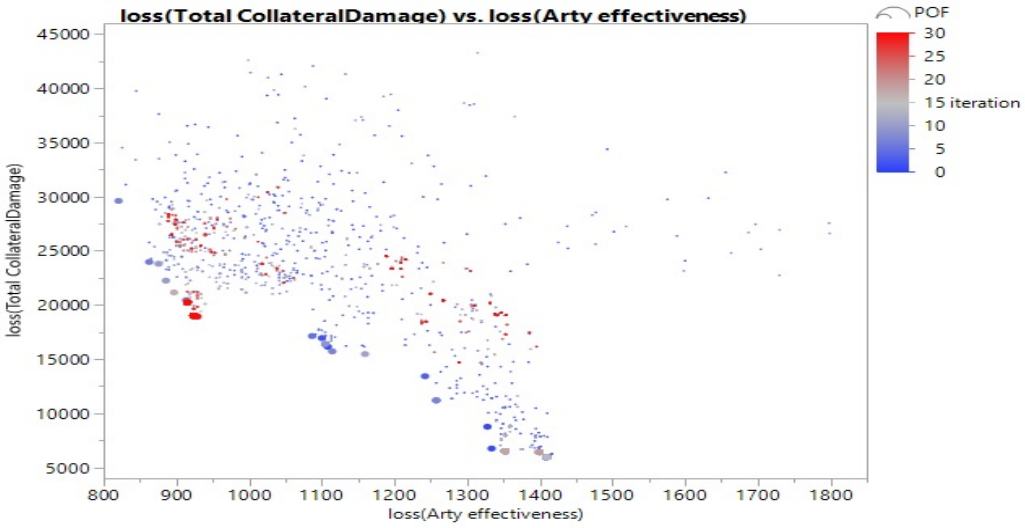


Figure 57. 2D Plot of Evolution over POF over Iterations

G. SUMMARY OF RESULTS

Table 11 presents a compilation of the proposed munition allocations for both scenarios using the DOE and MOEA approaches with the objectives of minimizing (1) the scaled loss of total collateral, (2) the scaled loss of artillery effectiveness, and (3) the cost of munition expenditures. Both approaches are generally consistent in the average (across the POF) allocation of M982, where the allocation for the M982 increases when the cost factor is disregarded. Due to the differences in the fundamental characteristics of both approaches, however, the allocations for the M1156 are fairly different and have high variability.

Although the DOE approach is systematic and is evaluated over wide coverage of the noise space, the fixed input space that was set initially—such as the total number of munitions and min-max proportion of each munition in the experiment prevent us from exploring other possible mixes. In contrast, the MOEA allows us to vary the total munitions required, enabling the exploration of an effective munition mix in a larger input space. On the other hand, the MOEA approach is unable to account for other variability of the input spaces that the DOE explored, making it somewhat sensitive to any changes in the input spaces. The MOEA is also a flexible search tool that produced the best possible results based on the number of generations run thus far. Hence, the result may not be conclusive considering the number of generations conducted in this study.

In a nutshell, both approaches have their strengths and limitations, and it is not the purpose of the study to determine which method is better or more accurate. Instead, having an alternate method offers a different perspective for looking at the same problem and for making further improvements.

Table 11. Summary of Results for DOE and MOEA Approaches

Scenario	Experiment	Percent M982 [95% CI]	Percent M1156 [95% CI]	Percent M795 [95% CI]
1 – Battle of Fallujah	DOE with 3 objectives	11.6% [11.1,12.1]	26.7% [24.9,28.4]	61.7% [60.0,63.4]
	DOE with 2 objectives (w/ o cost)	15%	25% [21.2,28.8]	60.0% [56.2,63.8]
	MOEA with 3 objectives	11.0% [10.2,11.8]	11.6% [10.5,12.8]	77.3% [75.9,78.7]
	MOEA with 2 objectives (w/o cost)	13.5% [13.4,13.6]	17.2% [15.9,18.5]	69.3% [68.1,70.5]
2 – Battle of Marawi	DOE with 3 objectives	9.5% [9.2, 9.9]	22.5% [21.0,23.9]	68.0% [66.0,70.0]
	DOE with 2 objectives (w/ o cost)	13.2% [12.1,14.3]	16.9% [11.3,22.5]	69.9% [63.7,76.1]
	MOEA with 3 objectives	8.2% [7.8,8.6]	11.6% [10.9,12.7]	80.2% [79.0,81.4]
	MOEA with 2 objectives (w/o cost)	10.1% [9.9,10.4]	38.6% [37.9,39.2]	51.3% [50.7,51.9]

Note: 95% CIs are computed across the POF, and intended for comparison purposes rather than as specific recommended allocations.

THIS PAGE INTENTIONALLY LEFT BLANK

VI. CONCLUSIONS, RECOMMENDATIONS, AND FUTURE RESEARCH

The objectives of this research are to explore the characteristics of an effective mix of precision and volume indirect fires in urban operations using an agent-based simulation approach, and to generate insights on the operational and cost effectiveness of the proposed munitions mix by the concept of Pareto dominance.

Efficient DOEs were used to generate data from the agent-based models where various decision and noise factor effects were considered. It is concluded that generally high proportions of precision munitions, such as the M982 (Excalibur) and M1156 (Precision Guided Kit), can significantly improve the Force Exchange Ratio and effectively reduce fratricide rates, amount of collateral damage, and time for mission success. In particular, the artillery kill rate against the enemy did not improve with a high proportion of precision munitions, which indicates that substantial amounts of dumb munitions are needed to maintain high overall operational effectiveness of artillery munitions. In identifying robust solutions to maintain the balance between operational and cost effectiveness, Pareto frontier analysis combined with efficient DOEs enables decision makers to find a set of non-dominated decision points based on the decision makers' key objectives and constraints.

The concept of Pareto dominance was further demonstrated with the use of a MOEA in searching for optimal munition allocations. The MOEA approach offers a flexible and alternative way to search for optimal munition allocations, by allowing the total munitions required in each scenario to vary independently. While the results from the MOEA may not be conclusive due to the limited number of generations used in the experiment (due to time constraints), it offers good initial insights on the required allocation of munitions based on the multi-objective function.

In comparing the two approaches, it is discovered that the impact of cost sensitivity on the proportion of M982 remains generally consistent, while there exists a wide variation in the proportion of M1156, which requires further study.

A. INSIGHTS AND RECOMMENDATIONS

The goals of this thesis are to provide general insight on the key factors influencing mission success in an urban battlefield from regression analysis and the effective allocation of precision and dumb munitions from POF analysis. Success in battle is measured by FER, fratricide rates, amount of collateral damage, artillery effectiveness, and time for mission success. While uncontrollable factors such as terrain characteristics and the Red agent's system capabilities have great impact on the MOEs, useful insights can still be gleaned on areas where the Blue agent (attacker) can improve.

1. Recommendation on Factors Influencing FER

FER is heavily influenced by Red sensor range, indicating that a higher superiority in Red sensor capability is able to significantly reduce FER, which translates to higher Blue casualty rates. Hence, it is recommended that the attacker invest in better sensor systems in order to increase probability of mission success.

2. Recommendation on Factors Influencing Fratricide Rate

The fratricide rate is heavily influenced by the level of concealment that a building can provide as well as the percentage of precision munitions and their CEP. While it is unlikely for one to determine the building characteristics that one is going to operate in, it is recommended that sufficient precision munitions be allocated to reduce fratricide rates in urban operations.

3. Recommendation on Factors Influencing Amount of Collateral Damage

The amount of collateral damage is heavily influenced by the level of concealment and protection provided by buildings, the percentage of precision munitions, and their CEP. Hence, it is also recommended that sufficient precision munitions be allocated to reduce collateral damage in urban operations.

4. Recommendation on Factors Influencing Artillery Effectiveness

Artillery effectiveness is heavily influenced by the levels of concealment and mobility provided by buildings, as well as Blue direct weapons, $P_{kill.}$ This suggests that the characteristics of precision munitions do not have much impact on artillery kill rates and that we do not need a high proportion of precision munitions to improve the artillery kill rates in urban battle.

5. Recommendation on Factors Influencing Time for Mission Success

Time for mission success is heavily influenced by the levels of concealment and mobility provided by the buildings and the desire of Blue and Red agents to move toward each other. While there is no recommendation to improve on system capability, it is suggested that the motivation for engagement is important to reduce the time needed for mission success.

6. Recommendation on Munition Allocations

Without knowing the specific key objectives that decision makers might wish to achieve or their perceived relative importance for weighting each of the objectives, we are unable to make any fair recommendation for munition allocations. Nevertheless, if the assumption is equal weight given to each objective, the following recommendations can be made for munition allocations in an urban operation:

- Maximize the allocation of M982, while maintaining a fair balance of M1156 and M795 if minimizing cost is not one of the objectives.
- Maximize the allocation for M1156 and minimize the allocation of M982 if minimizing cost is one of the objectives, provided the operating terrain is within 2.25 km^2.

B. FUTURE RESEARCH

The following are areas that might warrant further research.

- Explore the MOEA methodology further by increasing the number of individuals for each generation. Determine the point of diminishing returns by increasing the number of generations where better and more robust solutions may be found.
- Collect and further explore on open source/classified data of each munition type's characteristics such as their suppression effect, as well as blast and fragmentation radius, and include them in the model.
- Explore other tactical employments of artillery fire by adjusting the fuze setting (impact, ambient, time based, proximity) of munitions for different dimensional surfaces of the urban battlefield.
- Conduct in-depth sensitivity analyses for cost of munition allocations against operational effectiveness.
- Extend the scenario to include mortars, rocket artillery, and missiles for effective integration of artillery fire in the urban battlefield.

In summary, this thesis suggests that agent-based simulation has huge potential as a means and as an alternative to investigate the effectiveness of artillery munitions in urban battles. With the incorporation of various methodologies, such as efficient DOEs for simulation, data farming and data mining techniques, and evolutionary algorithms, meaningful insights can be gained from a simple agent-based model.

LIST OF REFERENCES

Amnesty International. (2017). *'The Battle of Marawi': Death and destruction in the Philippines.* London, UK: Amnesty International Ltd.

Asymmetric Warfare Group. (2016, November). *Modern urban operations: Lessons learned from urban operations from 1980 to present*. Retrieved from Modern War Institute website: https://mwi.usma.edu/wp-content/uploads/2017/03/ModernUrbanWarfareLessonsLearned_U_AWG_20161123.pdf

Cioppa, T. M., & Lucas, T. W. (2007). Efficient nearly orthogonal and space-filling Latin hypercubes. *Technometrics*, *49*(1), 45–55.

Cioppa, T. M., Lucas, T. W., & Sanchez, S. M. (2004). Military applications of agent-based simulations. *Proceedings of the 2004 Winter Simulation Conference,* 180–190. doi: 10.1109/WSC.2004.1371314

Defense Industry Daily. (2017, September 20). GPS-guided shells: The new Excaliburs. Retrieved from https://www.defenseindustrydaily.com/us-begins-ordering-excalibur-ib-gps-guided-sheels-08943/

Freedberg, S. J. (2016, January 12). Excalibur goes to sea: Raytheon smart artillery shoots back. *The Breaking Defense*. Retrieved from https://breakingdefense.com/2016/01/excalibur-goes-to-sea-raytheon-smart-artillery-shoots-back/

GlobalSecurity.org. (n.d.). Fallujah, Iraq: Satellite imagery. Retrieved May 8, 2018, from https://www.globalsecurity.org/military/world/iraq/fallujah-imagery.htm

Gomez, J. (2017, October 23). Philippines declares end to 5-month militant siege in Marawi. *The AP news*. Retrieved from https://apnews.com/78d9ed99e45540fb9be359d828bf4881

Headquarters, Department of the Army. (1991, July 16). *Tactics, techniques, and procedures for observed fires* (FM 6–30). Washington, DC: Author. Retrieved from https://www.marines.mil/Portals/59/Publications/FM%206-30.pdf

Headquarters, Department of the Army. (2006). *Urban operations* (FM 3–06). Washington, DC: Author. Retrieved from https://fas.org/irp/doddir/army/fm3-06.pdf

Headquarters, Department of the Army, U.S Marine Corps. (1999). *Tactics, techniques, and procedures for field artillery manual cannon gunnery* (FM 6–40). Washington, DC: Author. Retrieved from https://www.marines.mil/Portals/59/Publications/mcwp3_16_4.pdf

Henscheid, Z., Middleton, D., & Bitinas, E. (2006). Pythagoras: An agent-based simulation environment. *The Scythe – Pythagoras (Issue I),* 40–44. Retrieved from https://my.nps.edu/documents/106696734/108129278/IDFW13-Scythe-Pythagoras.pdf/181c3cfe-e77b-43c8-964f-982ce8cc5559?version=1.0

Hill, R. (2007, April 24). *XM1156 Precision guidance kit (PGK) program overview.* Presented at Picatinny Arsenal, NJ. Retrieved from https://ndiastorage.blob.core.usgovcloudapi.net/ndia/2007/psa_apr/RussellHill.pdf

Jamail, D. (2004, November 17). Red Cross estimates 800 Iraqi civilians killed in Fallujah. Retrieved from https://www.democracynow.org/2004/11/17/red_cross_estimates_800_iraqi_civilians

Joint Chiefs of Staff. (2001, April 12). *Department of Defense dictionary of military and associated terms* (Joint Publication 1-02). Washington, DC: Author.

Kelly, F. (2018, January 30). Raytheon awarded $95 million contract for Excalibur precision artillery munition engineering services. *The Defense Post*. Retrieved from https://thedefensepost.com/2018/01/30/raytheon-contract-excalibur-precision-artillery/

Klopcic, J. T. (1990). *A comparison of damage functions for use in artillery effectiveness codes* (Report BRL-MR-3823). Retrieved from http://www.dtic.mil/dtic/tr/fulltext/u2/a222589.pdf

Lucas, T. W. (2002). Damage functions and estimates of fratricide and collateral damage. *Naval Research Logistics, 50(3),*306–321. https://doi.org/10.1002/nav.10057

Lucas, T. W., Kelton, W. D., Sánchez, P. J., Sanchez, S. M., & Anderson, B. L. (2015). Changing the paradigm: simulation, now a method of first resort. *Naval Research Logistics, 62*, 293–303. DOI 10.1002/nav.21628.

MacCalman, A. D. (2013). *Flexible space-filling designs for complex system simulations* (Doctoral dissertation). Retrieved from http://hdl.handle.net/10945/34701.

Malicdem, E. (2017, November 30). Aftermath of the battle of Marawi. Retrieved from http://www.s1expeditions.com/2017/11/223-marawi-battle-structures.html

McIntosh, G. C., Galligan, D. P., Anderson, M. A., & Lauren, M.K. (2007). MANA (Map Aware Non-uniform Automata) version 4.0 user manual. Auckland, N.Z.: Defence Technology Agency.

McWilliams, T. S., & Schlosser, N. J. (2004, November/December). *The battle of Fallujah. U.S Marines in Battle Fallujah*. Quantico, VA: History Division, United States Marine Corps, 2014.

Michalewicz, Z., & Fogel, D. B. (2010). *How to solve it: Modern heuristics.* Berlin Heidelberg, NY: Springer.

Milner, M. (2012). *Precision strike association: Excalibur overview*. Presented at Picatinny Arsenal, NJ. Retrieved from https://ndiastorage.blob.core.usgovcloudapi.net/ndia/2012/annual_psr/Milner.pdf

Northrop Grumman Space & Mission Systems Corp. (2008). *Pythagoras users manual version 2.1.* Cleveland, OH: Author.

Panel on Operational Test Design and Evaluation of the Interim Armored Vehicle. (2003, June 11). *Improved operational testing and evaluation: Better measurement and test design for the interim brigade combat team with Stryker vehicles, phase I report.* Retrieved from: https://ebookcentral.proquest.com/lib/ebook-nps/reader.action?docID=3375803&query=

Raytheon. (n.d.). Excalibur projectile: Revolutionary extended range precision projectile. Retrieved May 5, 2018, from https://www.raytheon.com/capabilities/products/excalibur/

Sanchez, S. M. (2000). Robust design: Seeking the best of all possible worlds. *Proceedings of the 2000 Winter Simulation Conference,* 69–76. DOI: 10.1109/WSC.2000.899700

Sanchez, S. M. (2011). NOLHdesigns spreadsheet. Retrieved May 8, 2018, from http://harvest.nps.edu/

Sanchez, S. M., & Wan, H. (2015). Work smarter, not harder: a tutorial on designing and conducting simulation experiments. *Proceedings of the 2015 Winter Simulation Conference,* 1795–1809. DOI: 10.1109/WSC.2015.7408296

Sanchez, S.M., Upton, S. C., McDonald, M. L. & Zabinski, Holly. (2017, December). Invoking Artemis: The multi-objective hunt for diverse and robust alternative solutions. Paper presented at 2017 Winter Simulation Conference, Las Vegas, NV.

Sanchez, S.M., Upton, S. C., McDonald, M. L. & Zabinski, Holly. (2017). Invoking ARTeMIS – The multi-objective hunt for diverse and robust alternative solutions. *Proceedings of the 2017 Winter Simulation Conference,* 4501–4502. Retrieved from https://www.informs-sim.org/wsc17papers/includes/files/409.pdf

SAS Institute Inc. (2013). *Using JMP 11.* Cary, NC: Author. Retrieved from http://www.jmp.com/support/downloads/pdf/jmp11/Using_JMP.pdf

South, T. (2018, February 6). Precision kits will help Army artillery rounds hit targets without GPS. *Army Times*. Retrieved from https://www.armytimes.com/news/your-army/2018/02/06/precision-kits-will-help-army-artillery-rounds-hit-targets-without-gps/

Storsved, D. (2009). PGK Precision guidance kit - affordable precision for future artillery. Presented at 44th Annual Gun & Missile Systems Conference & Exhibition, Kansas City, MO. Retrieved from https://ndiastorage.blob.core.usgovcloudapi.net/ndia/2009/gunmissile/7876storsved.pdf

Swift, J. (2017). Second battle of Fallujah. Retrieved from https://www.britannica.com/event/Second-Battle-of-Fallujah

The South China Morning Post. (2017, October 23). Battle over: Philippines declares end of Marawi siege after dozens of militants die in final showdown. Retrieved from http://www.scmp.com/news/asia/southeast-asia/article/2116564/battle-over-philippines-declares-end-marawi-siege-after

Tyson, A. S. (2005, April 19). Increased security in Fallujah slows efforts to rebuild. *The Washington Post*. Retrieved from http://www.washingtonpost.com/wp-dyn/articles/A64292-2005Apr18.html

Vieira, H., Sanchez, S. M., Kienitz, K. H., & Belderrain, M. C. (2011). Improved efficient, nearly orthogonal, nearly balanced mixed designs. *Proceedings of the 2011 Winter Simulation Conference,* 3605–3616. Retrieved from https://www.informs-sim.org/wsc11papers/320.pdf

Washburn, A., & Kress, M. (2009). *Combat modeling.* New York, NY: Springer Science + Business Media.

Zitzler, E. (1991). *Evolutionary algorithms for multiobjectives optimization: Methods and applications.* (Doctoral dissertation). Retrieved from https://pdfs.semanticscholar.org/0aa8/6c5d58b77415364622ce56646ba89c30cb63.pdf

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California