2018

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Meteorological Institute

M.T. Montgomery and Roger K. Smith. "Towards understanding the dynamics of spin up in Emanuels tropical cyclone model." Tropical Cyclone Research Report TCRR 1: 16 (2018)

https://hdl.handle.net/10945/60907

TROPICAL CYCLONE RESEARCH REPORT
TCRR **1**: 1–6 (2018)
Meteorological Institute
Ludwig Maximilians University of Munich

Towards understanding the dynamics of spin up in Emanuel's tropical cyclone model

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Abstract:
We seek to understand the mechanisms of vortex spin up in Emanuel's 2012 axisymmetric theory for tropical-cyclone intensification in physical coordinates, starting from first principles. It is noted that, while spin up must occur in the friction layer, this spin up is unconstrained by a radial momentum equation in this layer. Rather, the spin up in this layer is constrained by a parameterization of turbulent mixing in the upper troposphere. It is shown that the inclusion of a nonlinear radial momentum equation into the theory using the assumed geometry of the Emanuel model (i.e. the assumption of a well-mixed boundary layer in terms of specific entropy and absolute angular momentum to leading order, together with the assumption that the absolute angular momentum is continuous at the top of this layer) strongly constrains the vortex evolution. If, for example, the initial radial inflow and its radial derivative are not too large in magnitude, then the boundary layer dynamics would damp this inflow and would lead to outflow just above it so that the vortex would ultimately spin down. The physics of how upper-tropospheric mixing leads to spin up in the boundary layer are unclear and, as discussed, may be irrelevant to spin up in Emanuel's model.

KEY WORDS Hurricane; tropical cyclone; typhoon; boundary layer; vortex intensification

Date: November 27, 2018; Revised ; Accepted

1 Introduction

In a highly influential paper, Emanuel (1986) presented a closed analytical theory for the structure and intensity of an axisymmetric, steady-state, tropical cyclone. A key feature of the theory was the assumption that surfaces of saturation moist entropy (s^*) and absolute angular momentum (M) are congruent. Both of these surfaces emanate from the boundary layer and flare outwards with height, becoming nearly horizontal in the upper troposphere (his Figure 1). Inspired by the pioneering tropical cyclone model of Ooyama (1969), the theory incorporates a simple slab-like boundary layer in which the entropy and M surfaces are assumed to be effectively well mixed in the vertical at leading order[1]. Emanuel showed that the maintenance of a tropical cyclone depends exclusively on self-induced latent and sensible heat transfer from the ocean in the form of moist enthalpy fluxes in contrast to ambient conditional instability. An appraisal of the theory was presented by Montgomery and Smith (2017). Limitations relating to unbalanced aspects of the theory were discussed by Smith et al. (2008) and Bryan and Rotunno (2009).

Over the years, the theory has been further developed[2] and extended to account for storm intensification (Emanuel 1997, henceforth E97, Emanuel 2012, henceforth E12, Emanuel et al. 2004). These time dependent theories incorporate the same basic geometry as the steady-state theory including, in particular, the effective slab boundary layer. Deficiencies of the E97 formulation were noted in E12 and Montgomery and Smith (2014). The most recent time-dependent theory presented in E12 takes a fundamentally different approach to that of E97 in which it is postulated that small-scale turbulent mixing in the upper troposphere plays a crucial role in determining the spatial distribution of outflow temperature and the stratification thereof. The E12 time-dependent theory has recently been invoked to suggest that tropical cyclones will be more prone to rapid intensification in a warmer climate, with the rate of storm intensification scaling as the square of the potential intensity Emanuel (2017). The basic argument is that since the square of the potential intensity is a more sensitive metric than the potential intensity, itself, the increase in rapid intensification rate would be a more detectable signal of global warming than the potential intensity used previously in climate change assessments.

Despite the inclusion of time dependence, questions remain concerning the unbalanced dynamics as well as the key physical mechanisms of vortex spin up, even in the limiting case of strictly axisymmetric balance dynamics. In particular, the precise role of small-scale turbulent mixing in the upper troposphere begs a physical interpretation. In fact, recent work presented evidence supporting the view that this turbulent mixing is incidental to the spin-up

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[1]See Emanuel (1986), p593, and Emanuel and Rotunno (2011), top line, left column on p2240.
[2]For a brief review see Montgomery and Smith (2017), section 5.

process in three dimensions (Montgomery et al. 2018).

A recent paper by Peng et al. (2018) has appraised the validity of some key assumptions of the E12 theory, based mainly on simulations using an axisymmetric non-hydrostatic numerical model designed to mimic as close as possible the E12 model. Starting from an initial vortex with no secondary circulation and a saturated initial sounding, they identified two phases of evolution. In Phase I, the M and s* surfaces evolve from nearly orthogonal to almost congruent, while in Phase II, these surfaces remain approximately congruent satisfying a key assumption of the E12 model.

Peng et al. (2018) argue that compared with their non-hydrostatic cloud model, the E12 model "possesses the chief virtue of transparency". However, the physics of spin up in the E12 model remain unclear. As noted above, it is unexplained how small-scale turbulent mixing in the upper troposphere "drives" an amplification of the system-scale maximum tangential velocity at the top of the boundary layer as encapsulated in Equation (16) of E12 (see also Equation (19) in the appendix of Montgomery et al. (2018) where the effects of upper-level mixing appear as an azimuthal force per unit mass acting on the gradient wind at the top of the friction layer). It is further unclear how the spin-up process relates to the classical intensification paradigm of Ooyama (1969) and the rotating convection paradigm reviewed by Montgomery and Smith (2014, 2017) and Smith and Montgomery (2016b). The present paper seeks to explore issues surrounding the spin up of the low-level tangential winds within and just above the boundary layer in the geometry of the E12 model, but starting from fundamental principles.

2 The E12 theory in brief

As a preamble, we present here our understanding of the mathematical formulation of the axisymmetric E12 model and the physical constraints embodied in it. In particular, we seek to articulate our understanding of how spin up in the model comes about. The assumed flow configuration in radius-height coordinates (r, z) is sketched in Figure 1. Air is assumed to converge in a shallow frictional boundary layer of constant depth h, acquiring moisture from the surface as it does so. As air parcels ascend out of this layer at inner radii, they are assumed to flow upwards and radially outwards into the upper troposphere, conserving their saturation specific entropy, s^*, and absolute angular momentum, M. These quantities are defined in the usual way with $s^* = c_p ln\theta_e{}^*$ and $M = rv + \frac{1}{2}fr^2$, where c_p is the specific heat of dry air, θ_e^* is the saturation equivalent potential temperature (assuming pseudo-adiabatic ascent in which all condensed water instantly precipitates), v is the tangential velocity component, and f is the Coriolis parameter, assumed constant.

The master prognostic equation is one for the moist entropy, s_b, in the thermodynamic boundary layer, which is assumed to have depth h also. It is assumed further that $s^* = s_b$ at $z = h$. In turn, M and its time derivative are constrained by the assumptions above the boundary layer that: (1) the flow there is in hydrostatic and gradient wind balance and therefore thermal wind balance; (2) the M surfaces are congruent with the surfaces of saturation moist entropy s^*; and (3) a closure assumption in the upper-tropospheric outflow relating the partial derivative of outflow temperature T_o with respect to M, i.e., $\partial T_o/\partial M$, to the derivative of s^* with respect to M, i.e. ds^*/dM. The M and s^* surfaces are assumed to flare outwards with height and not fold over[3] so that the flow remains everywhere symmetrically stable. The closure assumption, in essence a parameterization for $\partial T_o/\partial M$, is based on the premise that "the thermal stratification of the outflow ($\partial T_o/\partial M$, our insertion) is set by small-scale turbulence that limits the Richardson number" to a critical value (E12, p988).

Significantly, the E12 model does not include a classical boundary layer in which both the tangential and radial components of flow satisfy Newton's equations of motion. Rather, the mean radial inflow in the layer influenced by friction, which in physical coordinates would be required to predict the time evolution of s_b, is determined by integrating vertically the tendency equation for M, across the layer, assuming that both horizontal velocity components, and therefore M, are essentially uniform through the depth of the layer. This formulation is represented by Equation (A13) in Peng et al. (2018), which gives the inflow as a function of the temporal and radial derivatives of M together with the surface torque.

In physical coordinates, spin up in the model will occur in the layer of friction if the M surfaces therein move inwards at a sufficient rate that the radial advection of M exceeds the azimuthal torque per unit depth. Since the radial flow above the friction layer is radially outwards, spin up of the flow above the friction layer has to occur by the vertical advection of M from the friction layer. It follows that, in physical coordinates, spin up of the maximum winds in the E12 model must occur in the friction layer.

While the mathematical constraints leading to vortex spin up in the E12 model are reasonably clear, the physical processes involved remain obscure. Despite the fact that the mixing parameterization relating ds^*/dM to $\partial T_o/\partial M$ in the upper-tropospheric outflow layer is a crucial element of the theory, without which the vortex will not spin up, the physics of spin up brought about by this mixing are mysterious, at least to us. It is unclear also how spin up in the E12 model relates to the classical paradigm for spin up articulated by Ooyama (1969) or its extensions to include a nonlinear boundary layer spin up mechanism

[3]It may be significant that, in axisymmetric balance calculations in which the heating rate is specified along M surfaces centered around the radius of maximum tangential wind, which is qualitatively equivalent to assuming the s^* is constant along M surfaces, the M surfaces are found to turn over, e.g., Fig 8 of Smith et al. (2018). Some hours after this overturning occurs, the balance calculations break down.

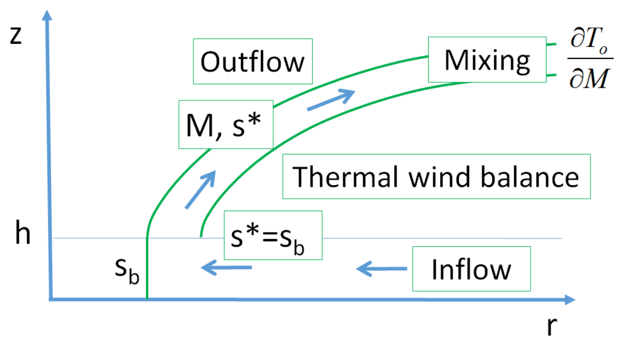

Figure 1. Geometric configuration of the axisymmetric Emanuel (2012) formulation for an intensifying tropical cyclone in cylindrical polar coordinates (r, z). In the schematic, h denotes the depth of the layer influenced by friction, which is assumed constant, s_b is the specific entropy in the boundary layer, s^* is the saturation specific entropy above the boundary layer, and M is the absolute angular momentum. It is assumed that s_b is independent of height. The arrows indicate the secondary circulation with radial inflow in the friction layer and outflow above it. Mixing by shear-stratified turbulence in the outflow layer of the developing tropical cyclone is assumed to determine the thermal stratification of the outflow ($\partial T_o/\partial M$, where T_o is the outflow temperature, our insertion).

articulated by Smith and Vogl (2008) and others (see e.g. Smith and Montgomery (2016a) and refs.).

While spin up in the friction layer in the E12 model might be interpreted as akin to the boundary layer spin up mechanism, the latter mechanism is one that occurs in a classical nonlinear and unbalanced vortex boundary layer. Indeed, the radial inflow velocity in the E12 model is not constrained to satisfy Newton's equation of motion and it is pertinent to enquire how the inclusion of this additional constraint would affect the E12 theory. We explore this question in the next section.

3 The boundary layer equations

The geometric configuration of a boundary-layer formulation relevant to the axisymmetric E12 model in physical space is sketched in Figure 2[4], which defines also the quantities used below. By definition, all resolved eddy fluxes in the radial and vertical directions are zero in an axisymmetric formulation.

In cylindrical polar coordinates, the tendency equation for the tangential wind component, v_+, at $z = h_+$, just above the boundary layer is:

$$\frac{\partial v_+}{\partial t} = -u_+(\zeta_+ + f) - w_{h-}\frac{\partial v_+}{\partial z}, \qquad (1)$$

where u_+ and w_h are the radial and vertical velocity components at h_+, f is the Coriolis parameter, assumed constant, ζ_+ is the vertical component of relative vorticity at h_+, $w_{h-} = \frac{1}{2}(w_h - |w_h|)$, and t is the time. In this equation, it has been assumed that air parcels leaving the boundary layer do not change the tangential momentum at $h+$, consistent with the assumptions that $\partial v_b/\partial z = 0$ and that $v_b = v_+$.

The surface stress (τ_r, τ_λ) may be approximated in terms of a drag coefficient C_D by $C_D \rho_b v_b(u_b, v_b)/h$, where ρ_b is the density in the boundary layer. In this formula, the total velocity which normally appears in the expression for the surface drag has been approximated by v_b following E12, based on the assumption that $u_b^2 << v_b^2$. Moreover, for simplicity, C_D and ρ_b are assumed to be constant. Then, the tendency equation for the depth-averaged tangential velocity, v_b, in the boundary layer, which is assumed to be of uniform depth h, is

$$\frac{\partial v_b}{\partial t} = -u_b(\zeta_b + f) - C_D\frac{v_b^2}{h}, \qquad (2)$$

where u_b is the depth-averaged radial velocity. We have assumed that the tangential velocity is continuous at $z = h$, but we allow the radial velocity to be discontinuous.

The tendency equation for the depth-averaged radial velocity, u_b, in the boundary layer, which E12 does not use, is

$$\frac{\partial u_b}{\partial t} = -u_b\frac{\partial u_b}{\partial r} \underbrace{- \frac{u_+ - u_b}{h} w_{h-}}_{A} \underbrace{+ \frac{v_b^2}{r} + f v_b - \frac{1}{\rho_b}\frac{\partial p}{\partial r}}_{B} - C_D\frac{v_b u_b}{h}. \qquad (3)$$

The terms A and B identified by an underbrace represent the effect of dilution of boundary layer radial momentum by the downward transport of radial momentum from above the boundary layer and the radial acceleration by the agradient force per unit mass, respectively.

Integrating the continuity equation across the boundary layer and using the condition that the vertical velocity at the surface is zero gives

$$w_h = -\frac{h}{r}\frac{\partial r u_b}{\partial r} \qquad (4)$$

Making the assumption of E12 that there is gradient wind balance above the boundary layer, we do not have an independent radial momentum equation for u_+. This quantity must be determined by another constraint as explained below.

4 Deductions from the equations

E12 assumes that M is always continuous at the top of the boundary layer, again implying that $v_b = v_+$, $\zeta_b = \zeta_+$. (recall that, $M = rv + \frac{1}{2}fr^2$, where v is either v_b or v_+.) In this scenario, it follows that the tendencies in Equations

[4]Our interpretation of the E12 model is consistent with that in Peng *et al.* (2018), Figure 10b.

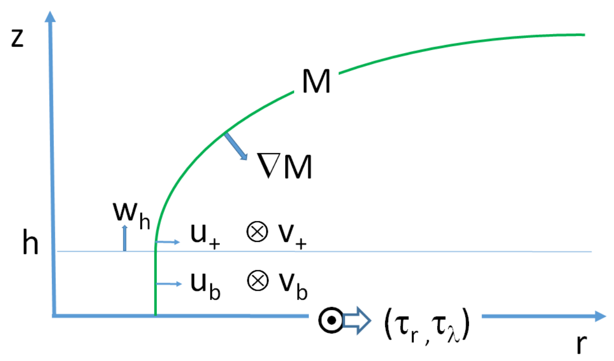

Figure 2. Configuration of the axisymmetric boundary-layer formulation in cylindrical polar coordinates (r, z) using the Emanuel geometry according to Peng et al. (2018). Again, h is the boundary layer depth, assumed constant, (u_b, v_b) are the radial and tangential velocity components in the boundary layer, (u_+, v_+) are the corresponding velocity components just above the boundary layer and w_h is the vertical velocity at the top of the boundary layer, M is the absolute angular momentum, and (τ_r, τ_λ) represent the radial and azimuthal turbulent momentum fluxes at the surface. It is assumed that u_b, v_b and M in the boundary layer are independent of height to leading order. It is assumed further that the tangential velocity component and M are continuous at $z = h$, whereas the radial velocity is generally discontinuous at this level. The downward pointing arrow on the M-contour shows the gradient of M assuming a centrifugally (inertially) stable vortex. In an intensifying tropical cyclone, the radial velocity vectors would be inwards.

(1) and (2) must be equal. The continuity of the tangential wind tendencies then implies that

$$(u_b - u_+)(\zeta_+ + f) = w_{h-}\frac{\partial v_+}{\partial z} - C_D\frac{v_+^2}{h}, \tag{5}$$

which ensures that M is continuous at $z = h$. Alternatively,

$$u_+ = u_b - \frac{1}{(\zeta_+ + f)}\left(w_{h-}\frac{\partial v_+}{\partial z} - C_D\frac{v_+^2}{h}\right). \tag{6}$$

This equation determines u_+ in terms of u_b when v_+ and $\partial v_+/\partial z$ are known.

Consistent with the assumption that $v_b = v_+$ and that gradient wind balance applies at the top of the boundary layer, Equation (3) reduces to

$$\frac{\partial u_b}{\partial t} = -u_b\frac{\partial u_b}{\partial r} - \frac{u_+ - u_b}{h}w_{h-} - C_D\frac{v_+}{h}u_b. \tag{7}$$

Note that, in this equation, both u_+ and w_{h-} are functions of u_b. In particular, u_+ is given by Equation (6) and w_{h-} is determined by Equation (4) when $w_h < 0$. Unlike a realistic vortex boundary layer, there is no agradient force to drive flow inwards, or to decelerate an existing inward flow that has become supergradient.

4.1 Case $w_h = 0$

One solution of this equation is $u_b = 0$ for all time, implying that $w_h = 0$ from Equation (4). Thus if u_b is zero initially, it will remain zero indefinitely, as will w_h. A zero initial secondary circulation is typically assumed in the prototype spin up problem (see e.g. Montgomery and Smith, 2014, p38).

In this case, Equation (6) reduces to

$$u_+ = C_D\frac{v_+^2}{h(\zeta_+ + f)}, \tag{8}$$

whereupon $u_+ > 0$. This outflow above the boundary layer leads to spin down there according to Equation (1), i.e. the M surfaces will be advected radially outwards.

4.2 Case $w_h > 0$

Where there is ascent out of the boundary layer, $w_{h-} = 0$ and u_b is not initially zero everywhere. Then, Equation (6) reduces to

$$u_+ = u_b + C_D\frac{v_+^2}{h(\zeta_+ + f)}, \tag{9}$$

and using Equation (4), Equation (7) gives

$$\frac{\partial u_b}{\partial t} = -u_b\left(\frac{\partial u_b}{\partial r} + C_D\frac{v_+}{h}\right). \tag{10}$$

Now one must solve for u_b as part of a coupled system involving also v_b, v_+ and u_+ using equations (1), (2) and (6). Provided that the expression in parentheses in Equation (10) is positive, then $\partial u_b/\partial t > 0$, i.e. the inflow u_b will be damped. Then, from Equation (9), even if u_+ is negative initially, it will ultimately become positive when the magnitude of u_b becomes smaller than that which would occur if radial inflow were determined from the assumption of quasi-steady torque balance (i.e. a balance between the surface drag torque per unit depth and the radial advection of absolute angular momentum). Thus, if the maximum tangential velocity is located in the region of ascent, which it generally is, and the radial inflow and its radial derivative are not too large in magnitude, the tangential velocity maximum will ultimately decrease, even though there could be a temporary period of spin up. The nature of the solution would be expected to be one of a damped nonlinear inertial oscillation in the inner region of the vortex.

4.3 Case $w_h < 0$

When there is descent into the boundary layer, $w_{h-} = w_h$ and u_b is not initially zero everywhere. The region where $w_h < 0$ occurs principally in the outer region of the vortex. Now there is no obvious simplification of Equation (7). Again, one must solve for u_b as part of a coupled system. Except in the pathological case where u_+ is negative and substantially larger in magnitude than u_b, $\partial u_b/\partial t > 0$ and the inflow will be damped. Again, the solutions would be expected to have the nature of a frictionally damped nonlinear inertial oscillation in the outer region of the vortex.

4.4 A linear approximation for the outer region

In the outer region of the vortex the secondary circulation is relatively weak compared with the primary circulation and it would seem reasonable to linearize Equation (7) to

$$\frac{\partial u_b}{\partial t} = -C_D \frac{v_+}{h} u_b. \tag{11}$$

Thus, as long as v_+ is positive, u_b decreases locally in magnitude exponentially to zero with time. It follows then from Equation (4) that w_h decreases exponentially to zero with time also and the boundary layer flow collapses.

5 Discussion and Conclusions

We have presented here our understanding of the mathematical formulation of Emanuel's 2012 axisymmetric theory for tropical-cyclone intensification and the physical constraints embodied in it. We have shown that, when viewed in physical coordinates, spin up in the model must originate in the layer of friction when the M surfaces therein move inwards at a sufficient rate that the radial advection of M exceeds the azimuthal torque per unit depth. This rate is determined by the parameterization of turbulent mixing in the upper troposphere and is unconstrained by a radial momentum equation in the boundary layer. Since the radial flow above the friction layer is radially outwards, spin up of the flow above the friction layer has to occur by the vertical advection of M from the friction layer, itself. Despite the foregoing interpretations, the physics of spin up in the friction layer brought about by the mixing in the upper troposphere remain to be explained.

One could imagine that the neglect of a radial momentum equation in the boundary layer could be problematic in such an explanation. Echoing a conclusion of the study by Smith and Montgomery (2015), we would argue that "a minimum requirement of any acceptable theory for tropical cyclone intensification is that consideration be given to all dynamic and thermodynamic equations in a consistent manner". For this reason we have explored the consequences of incorporating a full radial momentum equation in the geometry of the Emanuel model.

The boundary layer formulation explored builds on approximations consistent with the Emanuel model, i.e. the boundary layer is treated as a slab of uniform depth in which the radial and tangential velocity components are independent of height to leading order and the tangential velocity and absolute angular momentum are assumed to be continuous at the top of the layer. This formulation is consistent with the interpretation of Emanuel's model given by Peng et al. (2018) as embodied in the cartoon in their Figure 10b.

We argued that, since the tangential velocity at the top of the friction layer is assumed to be in gradient wind balance, the tangential wind in the boundary layer is in gradient wind balance also. This implies that there is no agradient force to accelerate air parcels down the radial pressure gradient. Consequently, for an initial vortex with zero secondary circulation, there is no mechanism for amplifying the tangential winds in the boundary layer or just above it, and the vortex would have to spin down. If, on the other hand, the initial radial inflow and its radial derivative are not too large in magnitude, then the boundary layer dynamics would damp this inflow and would lead to outflow just above it so that the vortex would ultimately spin down.

A weakness of the present boundary layer formulation is the lack of a momentum equation for u_+ and an equation linking u_+ to the distribution of vertical velocity in the interior flow using a mass continuity equation. A resolution of this issue would be a topic for future research.

While there may be a more sophisticated way to incorporate a boundary layer into the Emanuel 2012 model in which one or more of the foregoing assumptions are relaxed, additional problems may be anticipated as articulated by Smith et al. (2008), see sections 5 and 6. In particular, a relaxation of gradient wind balance in the boundary layer implies a relaxation of the assumption that the tangential velocity in the boundary layer is equal to that above. This mismatch makes it difficult to relate the boundary layer flow to that in the vortex above where flow ascends out of the boundary layer, without introducing an additional matching zone. These issues are obstacles to extending Emanuel's theory, linking a full nonlinear boundary layer to the circulation and thermodynamics of the vortex above. They would seem to be obstacles also in providing a physical interpretation of spin up in the friction layer brought about by mixing in the upper troposphere.

As a final remark, the premise that spin up in the Emanuel 2012 model is controlled by mixing in the upper tropospheric outflow layer may a red herring. Based on the summary of the model in section 2, it seems that the prescription of *any* functional relationship between ds^*/dM and $\partial T_o/\partial M$ that leads to the M surfaces in the friction layer moving inwards at a sufficient rate that the radial advection of M exceeds the azimuthal torque per unit depth would yield a model for vortex spin up! This is true, whether or not the functional relationship has any physical meaning. In fact, attempts to test the foregoing premise on the basis of idealized numerical model experiments showed that mixing had no appreciable effect on vortex intensification (Montgomery et al., 2018). Thus, an attempt to find a physical interpretation of spin up in the friction layer brought about by mixing in the upper troposphere may be a fruitless exercise.

Acknowledgements

This paper arose out of stimulating discussions we had with Rich Rotunno of the National Center for Atmospheric Research during our participation in the 33rd American Meteorological Society Conference on Hurricanes and Tropical Meteorology in Ponte Vedra, Florida in 2018.

We are grateful also to Kerry Emanuel and an anonymous reviewer for their comments on an earlier version of the paper. RKS acknowledges financial support for tropical cyclone research from the Office of Naval Research Global under Grant N62909-15-1-N021. MTM acknowledge the support of NSF grant AGS-1313948, ONR grant N0001417WX00336, and the U. S. Naval Postgraduate School. The views expressed herein are those of the authors and do not represent sponsoring agencies or institutions.

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