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Optimizing a military supply chain in the presence of random, non-stationary demands

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THESIS

OPTIMIZING A MILITARY SUPPLY CHAIN IN THE PRESENCE OF RANDOM, NON-STATIONARY DEMANDS

by

Yew Soon, Ng

December 2003

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Demand for supplies, such as ammunition, during a military operation is a scenario-dependent random variable that may be subject to high variance. The challenge is to design an efficient military logistics supply chain that satisfies uncertain, non-stationary demands, while taking into account the volatility and singularity of military operations. This research focuses on the development of a modeling framework that determines the optimal deployment of transportation assets and supplies at the operational level, with possible interdiction by enemy forces. We term this model, Optimal Military Logistics Supply Chain (OPTiMiLSC). This is a two-level, multiple time period scenario-based stochastic model. OPTiMiLSC uses a combination of optimization, scenario-based simulation and statistical analysis. We use a "scenario tree" method to generate the demand scenarios. The results show a positive correlation between the number of demand scenarios and the probability that a random demand scenario is satisfied. We compare OPTiMiLSC with two deterministic optimization approaches. The first approach is where demands are fixed at the 90^{th} percentile, which tends to over-supply when compared to OPTiMiLSC. The mean value approach, on the other hand, tends to under-supply. OPTiMiLSC enables military planners to establish a robust logistic plan that responds more adequately to an intra-theater operation.

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OPTIMIZING A MILITARY SUPPLY CHAIN IN THE PRESENCE OF RANDOM, NON-STATIONARY DEMANDS

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Demand for supplies, such as ammunition, during a military operation is a scenario-dependent random variable that may be subject to high variance. The challenge is to design an efficient military logistics supply chain that satisfies uncertain, non-stationary demands, while taking into account the volatility and singularity of military operations. This research focuses on the development of a modeling framework that determines the optimal deployment of transportation assets and supplies at the operational level, with possible interdiction by enemy forces. We term this model, Optimal Military Logistics Supply Chain (OPTiMiLSC). This is a two-level, multiple time period scenario-based stochastic model. OPTiMiLSC uses a combination of optimization, scenario-based simulation and statistical analysis. We use a "scenario tree" method to generate the demand scenarios. The results show a positive correlation between the number of demand scenarios and the probability that a random demand scenario is satisfied. We compare OPTiMiLSC with two deterministic optimization approaches. The first approach is where demands are fixed at the 90^{th} percentile, which tends to over-supply when compared to OPTiMiLSC. The mean value approach, on the other hand, tends to under-supply. OPTiMiLSC enables military planners to establish a robust logistic plan that responds more adequately to an intra-theater operation.

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EXECUTIVE SUMMARY

Demand for supplies, such as ammunition, during a military operation is a scenario-dependent random variable that may be subject to high variance. Other uncertainties in military operations are associated with the malevolent nature of the environment – attrition, interdiction and friction. An effective military logistic supply plan should reflect explicitly these uncertainties, and in particular, the stochastic nature of a military logistics network.

In this research, we develop a stochastic, two-level, multiple-period logistics model that considers supplies, means of transportation and possible interdiction of supply lines by the enemy. The model uses a relatively simple optimization problem within a scenario-based simulation setting, and obtains pertinent statistics for analysis. We use a scenario tree method to generate the reference set of demand scenarios, with each branch giving k scenarios. The reference set is then used to obtain an optimal deployment that satisfies all the demand scenarios in the set. This model, called the Optimal Military Logistics Supply Chain model (OPTiMiLSC), helps to determine the optimal deployment of transportation assets and supplies at the two levels.

We make the following assumptions. First, we assume that demands are normally distributed and are dependent on the operational plans. Next, we assume that the transitions in demand between successive time periods are Markovian, that is, that they depend on the last time period only. We also assume that there are no capacity constraints. Last, we assume that transportation times are fixed, but supply routes may be subject to interdiction.

We seek a least-cost deployment that satisfies the demand of the reference set. Once this deployment is obtained, we randomly draw additional scenarios and, utilizing Bernoulli experiments, we estimate the probability that this deployment is adequate for any scenario. We seek an optimal deployment for which this probability satisfies some required operational threshold. The optimization procedure comprises two steps. First, we determine the initial deployment of supply and the required periodic supply that meets

all demand at the points of demand. The results are then fed as inputs into the second stage of the optimization, where transportation needs are determined.

The OPTiMiLSC is implemented using a reference data set for the demand and the cost parameters. As expected, the results show a positive correlation between the size of the reference set of demand scenarios and the probability that an arbitrary demand scenario is satisfied. This probability is called the "responsiveness probability."

The results obtained are analyzed statistically to determine whether the size of the reference set is sufficient to achieve a certain responsiveness probability (say, 0.90). The null hypothesis, H_0, is that the responsiveness probability is 0.90 or less. We use a sample size of 60 randomly generated independent demand scenarios. We find that for values of *k,* the number of draws at each branch of the scenario tree, greater or equal than 11, we consistently reject the null hypothesis H_0 when this is tested at the 10% significance level. We infer that to obtain, for our data set, a deployment plan with at least 90% probability of success, *k* has to be at least 11.

We also compare the OPTiMiLSC with two deterministic approaches: one which fixes demands at the 90th percentile and another which fixes them at the mean values. Using the 90th percentile deterministic approach gives approximately the same responsiveness probability (0.98) as the OPTiMiLSC model, but it requires more supply and trucks to obtain this result. The deterministic approach using mean values gives only a 0.10 responsiveness probability. We infer that the use of deterministic optimization approaches produces results that are misleading and unreliable.

The OPTiMiLSC, being a stochastic model, enables military planners to establish a robust logistic plan that responds closely to the intra-theater situation while hedging against future demand scenarios. It combines optimization, simulation and statistical analysis in a novel, simple and easily applicable way. The model as presented here considers only a few selected parameters, so it does not cover all the possible requirements of battle. But it can provide a foundation for a more general planning framework for operational logistics. Subsequent studies can extend for multiple commodities, to allow for multiple interdictions, or to factor in different supply methods, means of transportation, and capacity constraints for logistic nodes and edges.

I. INTRODUCTION

Military operational logistics constitutes one of the most important and essential components of warfare (Kress 2002). In a military operation, deployed combat units consume supplies. And the demand for supplies, such as ammunition, is scenario-dependent and subject to high variance. Any military logistic supply plan should directly address the stochastic nature of the military logistics network. While mean values may be appropriate estimates for commercial supply chains (e.g. manufacturing operations), these measures may not be appropriate in the military context, which is typically transient and singular (Kress 2002). An approach that aims only at meeting the average demand for supplies could lead the military operation to fail at a moment of critical need.

The challenge is how to design, deploy and employ a military supply chain which satisfies uncertain non-stationary demands in the most efficient manner, taking into account the risks, high stakes and singularity of military operations.

In this research we embed a relatively simple optimization scheme within a scenario-based simulation setting to obtain pertinent statistics for analysis. We develop a stochastic, two-level, multiple-period logistics model that considers supplies, means of transportation and possible interdiction of supply lines by the enemy. We assume demands to be normally distributed and dependent on the operational plan. We also assume that transitions in demand scenarios between successive time periods are Markovian.

A. BACKGROUND

Military logistics, which comprises movement, supply and maintenance of forces during military operations, is one of the most important and essential components of warfare. It consists of the following functions (Mason 2003):

- Supply – This refers to the processes of acquisition, management, storage and issuance of material.
- Transportation – Movement of units, personnel, equipment, and supplies.

- Maintenance – Actions that are taken to keep weapons and other equipment in usable condition.
- General Engineering – Construction, repair, and operation of facilities for logistic operations.
- Health Services – Evacuation, treatment, hospitalization, medical supplies and other medical services to the battle troops.
- Other Services – Troop support functions like aerial delivery, laundry, clothing, meals and graves registration.

Three Levels of Logistics

There are three levels of logistics that correspond to the three levels of war – strategic, operational and tactical, as shown in Figure 1 (Kress 2002).

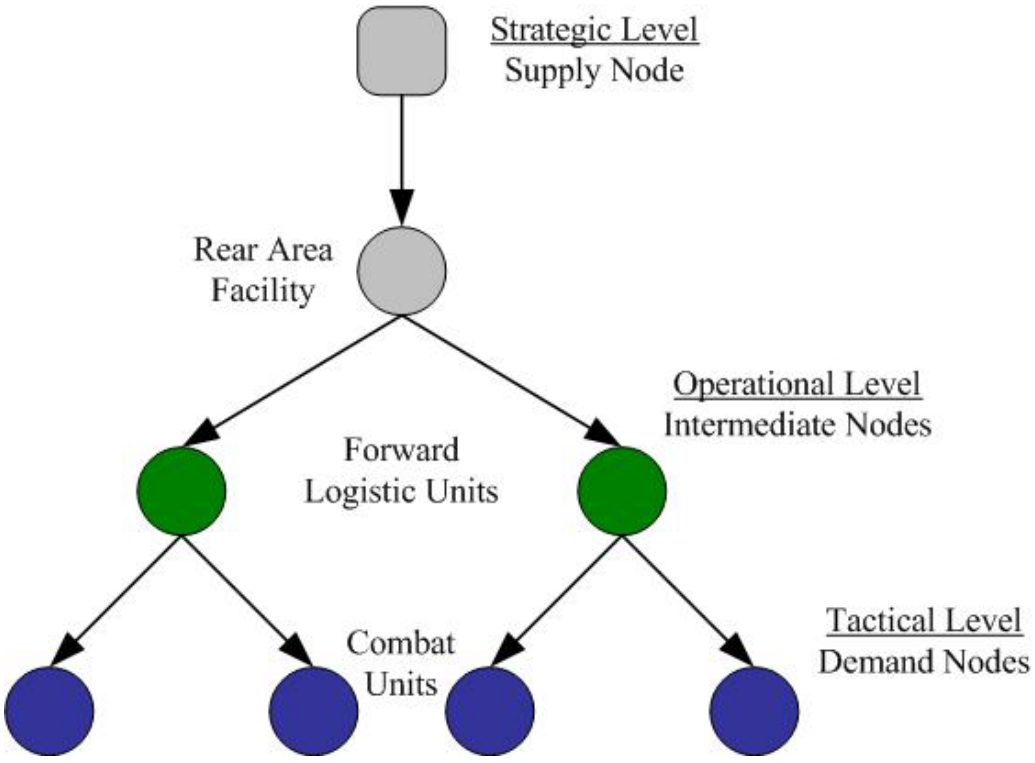

Figure 1. Logistic Network From Kress (2002)

Strategic logistics concerns the build-up and maintenance of the national military-related infrastructure. This infrastructure includes technology, industry, inventory, storage and transportation. At this level, major defense-related decisions have long-lasting impact on national security as well as the economy. Some of these decisions include investments in defense-related research and development, procurement plans and replenishment policies. Economic constraints drive the logistics capabilities and the interplay of these two factors determines operational capabilities. (Kress 2002)

Operational logistics (OpLog) is a collection of means, resources, organizations and processes whose common goal is to sustain campaigns and large-scale military operations. Campaign leaders use this collection, an output of strategic logistics, as input for tactical logistics. The purpose of OpLog is to sustain battles across time and space; it focuses primarily on theater-level activities and operational moves, not combat units. (Kress 2002)

Tactical logistics, which lies at the other end of the logistic spectrum, affects the battle in progress. Tactical logistics involves basic and practical activities that facilitate the "production" of military gains. These activities are generally technical, prescriptive, normative and easily quantified. Examples of theses activities include the replenishment of ammunition, maintenance and repair of equipment, supply of personal needs items (e.g. rations), supply of medical aid and support in the event of evacuation needs. (Kress 2002)

Strategic logistical decisions are made during peacetime. They concern national supply levels for the force, doctrine and operational plans. Operational decisions, on the other hand, are taken with respect to a certain operational scenario. Different operational scenarios require different logistics infrastructure and it is imperative that the supply chain, resource allocation and deployment be optimized (Kress 2002). The current research focuses on the operational level.

B. THE STRUCTURE AND FEATURES OF OPERATIONAL-LEVEL SUPPLY CHAINS

MLSC is a conceptual cyclic sequence of logistics related processes and events (Kress 2002). The objective of the MLSC is to sustain the military operation. The MLSC encapsulates both the demand and supply functions. At the demand side, the tactical units convey their requirements, to the operational or strategic logistic sources. At the supply side, supplies flow through the logistics network which links the source or intermediate nodes to the tactical destinations as shown in Figure 1.

During a military operation, the deployed combat unit consumes a variety of supplies that range from basic amenities like food and clothing to weaponry and ammunition. The wide range of supplies also means varying levels of demands for the supplies. The demand for basic amenities like food and clothing is relatively stable, as it is dependent on the number of troops deployed, which generally remains constant. The demand for supplies like ammunition, on the other hand, is highly scenario-dependent and as a result, has a larger variance. Another dimension that contributes to the high variance of the demand is the high-risk nature of the combat environment where supply lines may be interdicted. In such situations demand may not be satisfied and thus will accumulate. Therefore demand in any military operation is random, non-stationary and affected by possible (random) attrition.

C. PROBLEM DEFINITION

This research looks at the design and the deployment of a MLSC in the theater of operations. This is a least-cost deployment model that satisfies, in the probabilistic sense, uncertain and non-stationary demands. Cost minimization, in this context, refers not only to quantifiable costs but also to the operational burden created by the logistic tail.

The situation we look at is a single depot (e.g, a theater level supply unit), which provides logistic support to several battalions through the MLSC. Because the demand for supplies, such as ammunition, is scenario-dependent and subject to high variance, we employ a two-level multiple time period logistics model which represents the time-dependent dynamics of the logistics flow. We consider two types of flows: the supplies

and the transportation trucks. We also consider the possibility of interdiction to the supply line by the enemy.

D. OBJECTIVE AND SIGNIFICANCE

The model developed in this research provides a structure for determining the optimal deployment of transportation assets and supplies at the operational levels of a hierarchy. At the same time, it takes into account the imbedded uncertainties of any given military operation. We refer to this model as The Optimal Military Logistic Supply Chain (OPTiMiLSC) from this point onward. The results of this research may help military planners to establish an optimal, robust plan for logistics deployment in a theater of operations.

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II. LITERATURE REVIEW

The focus of this research is to design, deploy and employ a MLSC, which satisfies uncertain non-stationary stochastic demands in the most efficient way. This chapter provides the literature review of some of the relevant subjects that are useful to this thesis.

A. INVENTORY MODELS FOR STOCHASTIC NON-STATIONARY DEMAND

There are inherent difficulties when dealing with uncertain and non-stationary demands. Hadley and Whitin (1963) provide one of the earliest references to a stochastic non-stationary demand problem and its solution. They formulate a solution to a finite inventory problem for which there is a known obsolescence date and whose demand function follows a Poisson distribution.

Karlin (1960) analyzed a dynamic (non-stationary and stochastic) system in which the demand distribution varies in each time period, an extension of the classical Arrow-Harris-Marshak dynamic inventory model. This model's main characteristic is its ability to show in a quantitative manner how the optimal inventory level varies over time, which is a function of the various demand densities. The Arrow-Harris-Marshak model has shown that as demand densities decrease stochastically in consecutive periods, the optimal inventory level also decreases. On the other hand, when the demand densities increase, the optimal inventory level may or may not increase. And if the optimal inventory level decreases while the demand densities increase in successive periods, the optimal inventory level decreases further in the following period.

Graves and Willems (2002) propose an adaptive base-stock inventory policy for a non-stationary, single item when there is a deterministic lead time problem. This demand model produces an integrated moving average based upon the assumed inventory policy. Rather than pursuing optimality, their heuristic, determines a required safety stock level, and they refer to their policy as a critical fractal policy. The periodic demand is

estimated using an exponential smoothing procedure. With the emphasis on current period demand and assumptions made on the demand model, the model predicts the distribution of the next demand.

Song and Zipkin (1992) discuss a general modeling framework operating in a fluctuating demand environment. They also define the characteristics of optimal policies. They show that optimal policies are dependant on the unit cost, holding cost and penalty cost. They have suggested two different algorithms to solve a linear cost problem. Song and Zipkin also demonstrated how inventories should be managed in the face of possible obsolescence. One critical assumption they make is that the current Markov chain is always known exactly and it describes the current demand process. The optimal policy is then calculated using two heuristics. The first heuristic, a blind policy, forecasts demand over a lead time, assuming no change in the demand over the lead time. The second heuristic, a myopic policy, manages demand over a lead time and accounts for changes in demand over the lead time but not beyond.

B. SCENARIO-BASED STOCHASTIC PROGRAMMING APPROACH

The modeling framework we adopt for our research is a scenario-based, multi-stage linear programming optimization model. Stochastic models differ from deterministic models in that the selection of decision variables comes with imperfect knowledge of the future, as there are a number of possible futures. As such, an important requirement of these models is non-anticipativity. This means that at each stage, decisions have to be made without the luxury of knowing the exact value of the random variables in future stages. We find applications of stochastic optimization models in numerous commercial operations studies.

We find alternative approaches to scenario-based stochastic programming in Kelman, et al. (1990), *Stochastic Dynamic Programming* and Hooper, et al. (1991), *Stochastic Optimal Control*. The advantage of scenario-based stochastic programming over these alternatives lays in its flexibility in the decision process and scenario definitions. The disadvantage is the complexity and size of the programming models, which then require special solution algorithms. A common algorithm for two-stage and

multi-stage stochastic programming is based on the L-shaped method (Benders 1962) which is applied to demand management. This method is useful because it allows for scenario analysis within a large-scale problem. In addition, the technique if used in a nested manner allows for multi-stage problems to be decomposed by both scenario and decision period.

Alternative solution techniques for multi-stage stochastic programs include decomposition via augmented Lagrangian methods and direct solution by interior point methods (Carpenter et al. 1991; Lustig et al. 1994). In these techniques, the scenarios are generated prior to the solution procedure. Alternative approaches, as found in Dantzig and Glynn (1990) and Higle and Sen (1991) sampling-based cutting plane methods and Gaivoronski (1988) stochastic quasi-gradient algorithms, are based on "internal" sampling where new scenarios are generated in each iteration of the algorithm. The texts of Infanger (1994), Higle and Sen (1996), and Birge and Louveaux (1997) provide more discussion on sampling-based methods and other modern developments in stochastic programming.

C. CHANCE-CONSTRAINED PROGRAMMING

Chance constrained programming is a type of mathematical programming which incorporates stochastic elements into the constraint functions (Birge 1997). Charnes and Cooper (1959) introduced chance constraint programming as a means of handling randomness or uncertainty in data in an optimization setting.

Chance-constrained programming specifies a confidence level α for each constraint such that it can be violated only $(1-\alpha)$ percent of the time. The operator assigns to α a value he deems to be an appropriate safety margin.

In the chance-constrained approach, the focus is on the system's ability to meet feasibility in an uncertain environment, in other words, system reliability. This reliability is expressed as a minimum requirement on the probability, that satisfying the constraints. A chance constraint can be converted into a deterministic form in which a solution can be

found using standard linear programming formulation. If the random variables follow a multivariate normal distribution, there are solution methods readily available. (Charnes and Cooper 1959)

The objective function in chance constrained programming may contain expected values. However, in our application the objective function is deterministic. This method has been used in such other stochastic programming problems as supply chain operation, portfolio selection and reservoir management (Hooper 1991). The approach to model the random variables in a mathematical program is one of several approaches we have explored that is relevant and useful to the thesis topic.

III. THE OPTIMIZATION MODELS

A. OPTiMiLSC STRUCTURE

A military logistics system has a hierarchical structure, where logistics facilities at the higher echelon feed supplies to subordinate units (lower echelons) through the MLSC. Figure 2 shows a basic two-level network model that depicts this hierarchy.

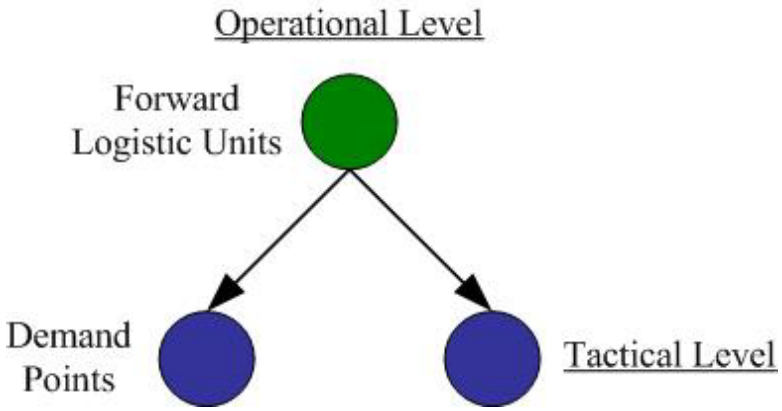

Figure 2. Basic Two-Level Logistic Network

The single node at the top represents the logistics source, which supplies the logistic flow. The destination nodes are combat units (battalions). They constitute the demand nodes of the MLSC. From now on, we refer to these demand nodes as Demand Points (DPs).

We consider a two-level logistics system with multiple time periods that supports an operation that lasts through several phases. To model the time-phased flow of logistics supply during a military operation, we expand the graph of the basic logistic network in Figure 2 to create a dynamic multiple-periods network model. The nodes of OPTiMiLSC are created from the basic logistic network by duplicating its nodes for each time period of the planning horizon. The edges of the OPTiMiLSC reflect the inter-nodal directions of the flow in the basic logistic network. Figure 3 shows a two-level OPTiMiLSC in three periods.

The node at the first level represents the supplier (depot) node, while each node at the second level corresponds to a DP. The time periods in this case are $t = 1,..,3$.

OPTiMiLSC consists of two types of edges: horizontal edges and diagonal edges. A horizontal edge (labeled by H in Figure 3) represents flow that stays in a certain node from one time period to the next. A diagonal edge (Labeled by D in Figure 3) represents a flow from the supplier to a DP across at least one time period. In this model, we assume that each diagonal edge connects nodes in two consecutive time periods.

With reference to Figure 3, U denotes the supply to be deployed at the depot and X_n denotes the supply to be deployed at DP n at the beginning of the military operation. $Y_{n,s,t}$ denotes the supply sent from the depot to DP n for scenario s at the beginning of period t. $d_{n,s,t}$ denotes the demand by DP n for scenario s at period t.

In general, the size and intensity of the operation determine the number of nodes, levels (two in our case) and edges in each period, while the length of the operation and the time resolution determine the number of time periods.

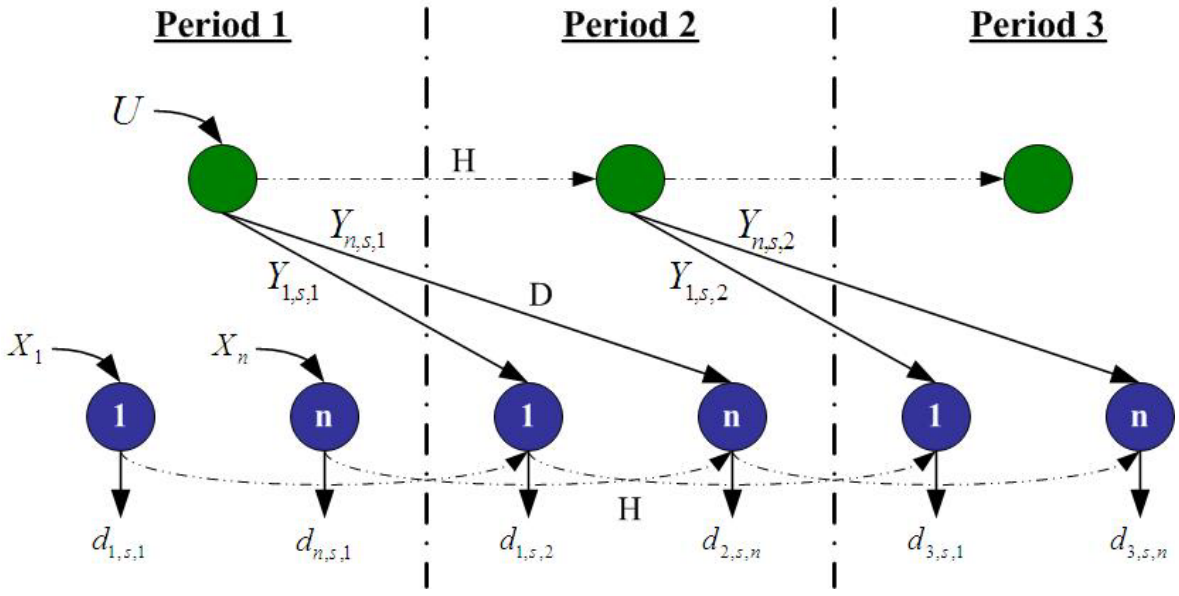

Figure 3. Two-Level Three Periods OPTiMiLSC After Kress (2002)

B. OPTiMiLSC CHARACTERISTICS

1. The Flow

The flow in the logistics network comprises two types:

- Supply (e.g, ammunition)
- Transportation vehicles that carry the supply (e.g, Trucks).

From now on we use the terms supply and trucks to denote these two types of flow, respectively. The two types of flow are not independent. The flow of supply is contingent on the availability of the trucks which carry it.

2. Time Periods

A military operation is typically divided into time periods (days, hours) requiring different levels of supply. The OPTiMiLSC we develop here encompasses this characteristic, reflecting the time-dependent dynamics of the logistics flow. The base-line period ($t = 0$) corresponds to the time period prior to the beginning of the military operation. It represents the logistic situation of the forces at the staging area. Time periods $1,...,T$ correspond to the active combat phase while ($t = T + 1$) corresponds to the end state of the MLSC after the operation is over. The outflow from the end-state nodes represents operational requirements to insure the force retains logistic readiness at the end of the operation.

The time resolution is a key parameter in OPTiMiLSC. It is determined by the operational and logistic considerations that reflect typical time parameters of various processes such as movement, transportation and unloading. The time resolution is usually determined according to the frequency of the logistics "pulses," or the tempo of the logistics support chain. A typical length of a time period at the operational level is 24 hours (Kress 2002, p. 224).

3. Means of Transportation

We consider only ground transportation for the OPTiMiLSC. We consider two types of trucks – regular and armored. When the transportation route has been assessed to be relatively safe, that is, it has a low probability of being interdicted by enemy forces,

regular trucks are used. On the other hand, if the transportation route is assessed to be of high risk, the armored trucks are used. We assume that the armored trucks have only half the carrying capacity of the regular trucks.

Neither the regular nor the armored trucks are assigned to specific battalion units and, unlike the flow of supply, they can move from lower echelon to higher echelon. Specifically, unloaded trucks travel back to the depot after completing the transportation mission.

There are two transportation deployment methods – "pull" and "push." A "pull" method implies a relatively larger deployment of vehicles at the receiving end, the DPs, while a "push" method implies a larger deployment at the supplying end, the depot (Kress 2002, p. 219). For OPTiMiLSC, we are only concerned with the "push" method. Figure 4 shows the truck routing network between the two levels.

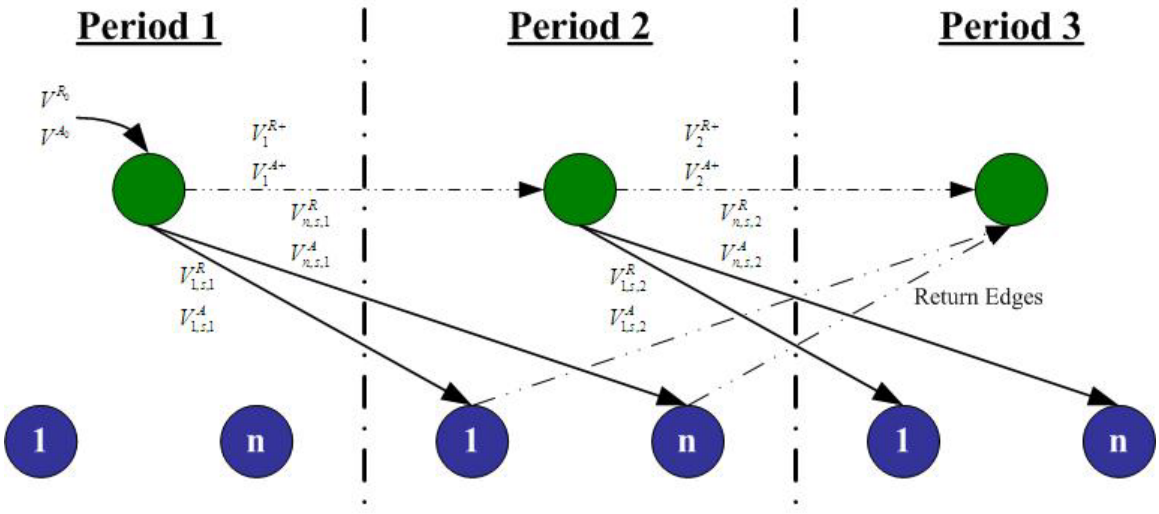

Figure 4. Trucks Routing Network After Kress (2002)

In Figure 4, V^R denotes regular trucks and V^A the armored trucks. V^{R_0} and V^{A_0} denote the initial deployment for the regular and the armored trucks, respectively, at the depot level. $V^R_{n,s,t}$ and $V^A_{n,s,t}$ denote the number of regular trucks and armored trucks required to deliver the supply from the depot to DP *n* during period *t* for scenario *s*. V^{R+}_t

and V_t^{A+} denote the excess flows of regular and armored trucks that stay in the depot from time period t to the next period $t + 1$.

C. OPTiMiLSC ASSUMPTIONS

To simplify the model, we make the following assumptions:

- Transportation routes have no capacity constraints.
- The duration of a transportation mission between the depot and a DP is one time period.
- During each time period, one randomly selected route between the depot and a DP is interdicted. The interdiction factor ($\delta_{n,s,t}$) is equal to "1," if the edge to DP n for scenario s at period t is interdicted; if it is not, this factor equals "0." We assume that the selection of the interdicted edge is uniformly distributed among the DPs.
- The demands in a given period t are normally distributed random variables $N\left(\mu_n^t, \sigma_n^t\right)$ where μ_n^t and σ_n^t are the mean and standard deviation of the demand at DP n at time t. The parameters of the distribution are dependent on the battle intensity at each time period t.
- No transshipment of flow is allowed among DPs. This assumption reflects common practice at the tactical logistics level.

D. OPTiMiLSC SCENARIO TREE GENERATION

The demands at a certain time period are associated with a scenario. Each scenario induces a vector of demand values – one for each DP. Therefore demands in our model are discrete random variables.

Given the scenario history up to a particular time, the uncertainty in the next period is characterized by several possible demand scenarios. To obtain discrete outcomes for the demand we use scenario tree generation (Nalan, Rustem, and Settergren 2001). In multistage models, at each time period new scenarios branch from the old, creating a scenario tree. Using this scenario generating procedure we can preserve the

historical spatial correlation of demand. We denote the number of scenario branches that emerge from any given scenario at time *t*-1 by *k* for all *t*, as shown in Figure 4. Therefore, the total number of scenarios at a period *t* is k^t .

A scenario is defined as a possible realization of the stochastic variables $d_{n,s,t}$. Hence, the set of scenarios $s \in \{1,.....,S\}$ is in a one-to-one correspondence with the set of leaves of the scenario tree. We associate scenario $s \in \{1,.....,S\}$ with the ***s**th* leaf of the scenario tree at period *T*.

The root node in the scenario tree represents the demand realization of "present period" and the nodes further down represent conditional realizations of demand values. The arcs linking the nodes represent various realizations of the uncertain variables. Ideally, a generated set of scenarios would represent the whole universe of possible outcomes of the random variable. To approximate this ideal, scenarios include both optimistic and pessimistic projections.

A scenario in the model is considered to be a sequence of periodic demand in which present demand distribution at period *t* is dependent on the actual realization of the demand at period (*t*-1) (Nalan, Rustem, and Settergren 2001).

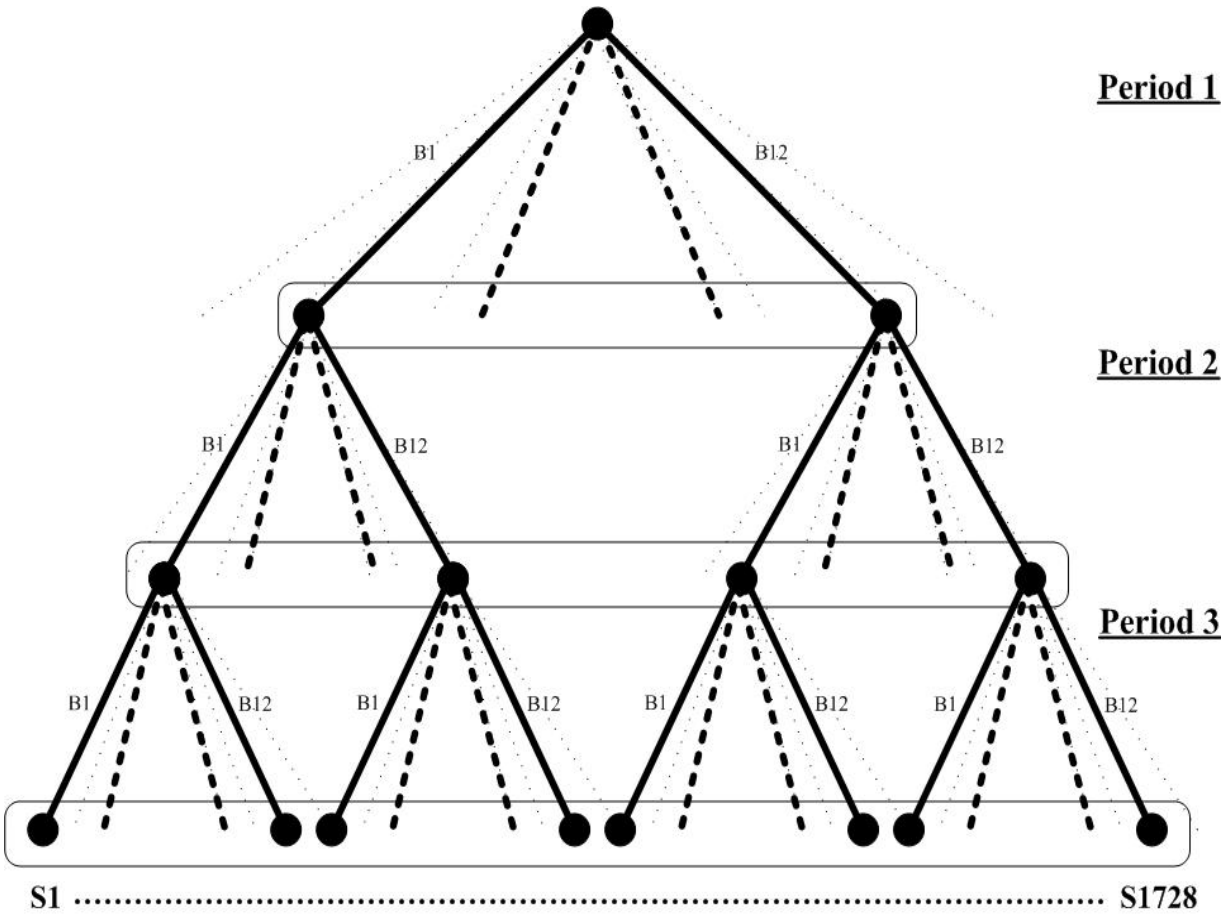

Figure 5. Demand Scenarios Tree Generation (For k = 12 and T = 3)

1. Simulation and Probabilistic Approach

To generate the event tree of demand, we use simulation and a probabilistic approach. The generated demands are then fed as input for the solving of first stage optimization. The basic data structure is the scenario tree node, which contains a cluster of various generated demands scenarios. The nodes of the tree correspond to the random variable, $d_{n,s,t}$, which correspond to the possible realizations of demand at DP n during period t (For $t \geq 1$). These sets of possible realizations at period t are, in general, dependent on the preceding observations of $d_{n,s,t-1}$. The transitions in demands between successive time periods are as discussed below.

2. Demand Categories

Three categories of demand levels are being considered and they are "High," "Nominal" and "Low." A realized demand is considered to be "Low" if it is lower than the mean value by 10%. It is considered to be "Nominal" if it is within a deviation of 10% from the mean. It is considered to be "High" if it is higher than the mean value by 10%.

Formally, at period t-1,

- $d_{n,s,t-1} < 0.9\mu_n^{t-1}$, Implies Low (L) demand.
- $0.9\mu_n^{t-1} \le d_{n,s,t-1} \le 1.1\mu_n^{t-1}$, Implies Nominal (N) demand.
- $d_{n,s,t-1} > 1.1\mu_n^{t-1}$, Implies High (H) demand.

3. Transition Probabilities

The nodes in the scenario tree are associated with the sequential transition process, so that each node at period t corresponds to a demand scenario, which is dependent on the demand scenario in the previous period. That mean the transitions in demands between successive time periods are Markovian. We use the following transition matrix:

$$\begin{array}{c} \\ L \\ N \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & N & H \end{array} \\ \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.05 & 0.9 & 0.05 \\ 0.3 & 0.6 & 0.1 \end{bmatrix} \end{array}$$

The above transition matrix reads as follows:

$P_{LL} = 0.1$ - Probability that the demand in period t is "Low," given that the demand at previous period (t-1) is "Low."

$P_{LN} = 0.6$ - Probability that the demand in period t is "Nominal," given that the demand at previous period (t-1) is "Low."

$P_{LH} = 0.3$ - Probability that the demand in period t is "High," given that the demand at previous period (t-1) is "Low."

4. Scenario Generation Algorithm

To obtain the demand realizations, we use an algorithm outlined in the following steps:

Step 1: (Initialization) Create a root node, with k scenarios. Initialize all the scenarios with the desired starting point ("Present" demand). That is, draw for each DP n, the demand value, $d_{n,s,1}$, from $F_1 \sim N\left(\mu_n^I, \sigma_n^I\right)$ and obtain a realization $d_{n,1,1}$. Repeat the drawing k times.

Step 2: (Transition) For a given demand realization at each one of the k newly branched nodes in the scenario tree $d_{n,s,t-1}$, check if the demand is "Low," "Nominal" or "High." At each of these nodes of the scenario tree, conduct a random number draw to obtain the transition probability for the next stage of the scenario-generation (simulation) process.

Step 3: (Simulation) At each node of scenario tree, based on the transition probabilities matrix generated in Step 2, draw the next demand value from $F_t \sim N\left(\mu_n^t, \sigma_n^t\right)_{\text{L or N or H}}$ and obtain a demand realization $d_{n,s,t}$. Repeat the drawing k times.

Step 4: (Termination) Repeat Step 2 and 3. Terminate the algorithm when we have reached the specified time period T.

E. OPTiMiLSC OPTIMIZATION MODELS

OPTiMiLSC consists of two stages. First, we solve the logistic supply problem to determine the supply to be deployed at the depot and at the DPs before the start of the operation. In this stage we also determine the required periodic supply to the depot and from the depot to each of the DPs such that the demands (based on all the scenarios generated) are satisfied. The results are then fed as input into the second stage of the optimization, in which we determine the optimal initial deployment of the transportation trucks, both regular and armored.

We formulate the models as follows:

Sets and Indices

T	Set of time periods, $t \in \{1,...T\}$
N	Set of battalion nodes in the network, $n \in N$
S	Set of scenarios, $s \in S$

1. First Stage Optimization

a. Data

$d_{n,s,t}$ Realized demand by DP *n* for scenario *s* at period *t.*

C_t^U Cost of unit deployment of supply at the depot at the beginning of period *t.*

C_n^X Cost of unit deployment at DP *n* at the beginning of the operation.

b. Variables

U_t The supply to be delivered at depot at beginning of period *t.*

$Y_{n,s,t}$ The supply from the depot to DP *n* for scenario *s* at the beginning of period *t.*

X_n The supply to be deployed at DP n before the operation.

c. Objective Function

$$\text{Min} \sum_t \left(C_t^U * U_t \right) + \sum_n \left(C_n^X * X_n \right) \quad (1)$$

d. Constraints

$$X_n \geq d_{n,s,1} \qquad \forall \text{ n}, s, t = 1 \quad (2)$$

$$X_n + \sum_{t'=1}^{t-1} Y_{n,s,t'} \geq \sum_{t'=1}^{t} d_{n,s,t} \qquad \forall \text{ n}, s, t > 1 \quad (3)$$

$$\sum_{t'=1}^{t-1} U_{t'} \geq \sum_{t'=1}^{t-1} \sum_n Y_{n,s,t'} \qquad \forall \text{ s}, t \quad (4)$$

e. Description of the Formulation

The objective function (1) expresses the cost of unit deployment at DP n at the beginning of the operation and the cost of unit deployment of supply at the depot at the beginning of period t. With $C_{t-1}^U < C_t^U$ for $2 \leq t \leq T$, supply is not sent before it is required. Constraint (2) requires that the supply to node n at the beginning of the operation must be greater or equal to the realized demand at period 1 for each scenario s. Constraint (3) ensures that for each period t, the total accumulated supply received by DP n at beginning of period t is not less than the total accumulated demand up to period t, for each scenario s. This constraint ensures that the demand for each DP is satisfied and any excess supply would be brought forward and consumed in subsequent periods. Constraint (4) ensures that the total accumulated supply to the depot level at beginning of period t is not less than the total accumulated supply to all the DPs for each scenario s.

2. Second Stage Optimization

a. Data

$\overline{Y}_{n,s,t}$ The supply from depot to DP *n* for scenario *s* at the beginning of period *t*, which equals the output from the first stage of optimization.

$\delta_{n,s,t}$ As noted above, the interdiction factor ($\delta_{n,s,t}$) is equal to "1," if the route from the depot to DP *n* for scenario *s* at period *t* is interdicted and "0," otherwise. We assume that the selection of the possible interdiction route is uniformly distributed among the DPs, [1, *N*]. If the randomly drawn number is equal to *n,* then $\delta_{n,s,t} = 1$. We also assume that at each time period, there is only one random possible interdiction to the routes from the sources node to any of the destination nodes ($\sum_{n} \delta_{n,s,t} = 1$).

C^R Cost of a regular truck.

C^A Cost of an armored truck.

RCap Capacity of a regular truck.

ACap Capacity of an armored truck.

b. Variables

V^{R_0} Initial total number of regular trucks to be deployed at the depot before the operation.

$V^{R+}_{s,t}$ Total number of excess regular trucks for scenario *s* at the end of period *t.*

$V^{R}_{n,s,t}$ Number of regular trucks required for DP *n* for scenario *s* at period *t*.

V^{A_0} Initial total number of armored trucks to be deployed at the depot before the operation.

$V^{A+}_{s,t}$ Total number of excess armored trucks for scenario *s* at the end of period *t.*

$V^{A}_{n,s,t}$ Number of armored trucks required for DP *n* for scenario *s* at period *t.*

c. Objective Function

$$Min\ C^{\mathrm{R}} * V^{R_0} + C^{A} * V^{A_0} \tag{5}$$

d. Constraints

$$V^{R_0} - \sum_n V^{R}_{n,s,t} - V^{R+}_{s,t} = 0 \qquad \forall s,t = 1 \tag{6}$$

$$V^{R+}_{s,t-1} - \sum_n V^{R}_{n,s,t} - V^{R+}_{s,t} = 0 \qquad \forall s,t = 2 \tag{7}$$

$$V^{R+}_{s,t-1} + \sum_n V^{R}_{n,s,t-2} - \sum_n V^{R}_{n,s,t} - V^{R+}_{s,t} = 0 \qquad \forall s,t = 2,....,T-1 \tag{8}$$

$$V^{R_0} - \sum_n V^{R}_{n,s,T-2} - \sum_n V^{R}_{n,s,T-1} - V^{R+}_{s,T-1} = 0 \qquad \forall s,t = T \tag{9}$$

$$V^{A_0} - \sum_n V^{A}_{n,s,t} - V^{A+}_{s,t} = 0 \qquad \forall s,t = 1 \tag{10}$$

$$V^{A+}_{s,t-1} - \sum_n V^{A}_{n,s,t} - V^{A+}_{s,t} = 0 \qquad \forall s,t = 2 \tag{11}$$

$$V^{A+}_{s,t-1} + \sum_n V^{A}_{n,s,t-1} - \sum_n V^{A}_{n,s,t} - V^{A+}_{s,t} = 0 \qquad \forall s,t > 2 \tag{12}$$

$$V^{A_0} - \sum_n V^{A}_{n,s,T-2} - \sum_n V^{A}_{n,s,T-1} - V^{A+}_{s,T-1} = 0 \qquad \forall s,t = T \tag{13}$$

$$\left(1-\delta_{n,s,t}\right) * V^{R}_{n,s,t} * RCap + \delta_{n,s,t} * V^{A}_{n,s,t} * ACap \geq \overline{Y}_{n,s,t} \qquad \forall n,s,t \tag{14}$$

e. Description of the Formulation

The objective function (5) for the second-stage optimization is the minimization of the total number of trucks, both the regular and the armored types, to be deployed at the depot level before the operation. Constraints (6) to (8) ensure that for each scenario *s*, the total inflow and outflow of normal trucks at all nodes and periods are balanced. Constraint (13) ensures that the total number of trucks, both regular and armored, is greater or equal to the number required to bring supplies from the depot to each battalion unit *n*.

IV. DESIGN OF EXPERIMENT AND RESULTS

A. OVERVIEW OF OPTiMiLSC ANALYSIS

The analysis procedure comprises three stages:

- **Sampling**: We generate a sample set of demand scenarios ($d_{n,s,t}$) with respect to the number of draws (k) at each branch of the scenario tree as input to the optimization models.

- **Optimization**: We solve the optimization models and obtain a deployment for the supply and the trucks that meets the demands at the depot and DPs.

- **Test of Robustness**: We generate a sample set of m number of independent demand scenarios. We check for feasibility and obtain an estimate for the probability that a randomly selected demand scenario is satisfied. Next, we conduct a statistical analysis to determine the number of draws (k) at each branch of the scenario tree sufficient to achieve the estimated responsiveness probability.

1. Generating the Sample Set for Optimization

a. Demand Distribution

The demand for each combat unit at each period in a certain demand scenario is a normally distributed random variable with mean of μ_n^t and standard deviation of σ_n^t. Each random variable corresponds to the battle intensity (Low, Normal or High) at each period. The demand distribution can be estimated based on an operational plan, past experience and statistical data. In this research, we consider five DPs and three time periods. The following table presents the distributions data:

	Period 1	Period 2	Period 3
DP_n	$N\left(\mu_n^I, \sigma_n^I\right)$	$N\left(\bar{\mu}_n^{II}, \bar{\sigma}_n^{II}\right)$	$N\left(\bar{\mu}_n^{III}, \bar{\sigma}_n^{III}\right)$
DP_1	*N(100, 10)*	*N(90, 10)*	*N(80, 10)*
DP_2	*N(90, 10)*	*N(85, 10)*	*N(75, 10)*
DP_3	*N(100, 10)*	*N(90, 10)*	*N(80, 10)*
DP_4	*N(90, 10)*	*N(85, 10)*	*N(75, 10)*
DP_5	*N(110, 10)*	*N(100, 10)*	*N(95, 10)*

Table 1. Demand Data for Combat Units at each Period

b. Cost Function

The operational plans of the combat units drive the cost parameters. In this research, we assume the "supply" deployment cost at the depot level is relatively lower than that at the DPs ($C_t^U < C_n^X$). Specifically for this research, we assume the following:

$$C_t^U = 100 \; for \; \mathrm{t} = 1, 2, 3$$
$$C_n^X = 120 \; for \; \mathrm{n} = 1, ..., n.$$

Because we only have three time periods and travel times will not permit us using a truck more than once, C_t^U can be the same for all *t*. The deployment cost for an armored truck is twice the cost of a regular truck ($C^R < C^A$). Specifically for this research, we assume the following:

$$C^R = 100$$
$$C^A = 200$$

c. Generating the Demand Scenarios

We use the scenario tree as shown and discussed in Chapter III to generate the demand scenarios. One critical factor in the scenario generation process is the number of draws (*k*) at each branch of the scenario tree. The number of draws

determines the total number of demand scenarios in the reference set of the optimization problem.

2. Optimization

At the optimization stage, we utilize a two-step optimization approach. Reiterating the model formulation outlined in Chapter III, we first determine the initial supply deployment and the required periodic supply that meets all demands at the DPs, based on all the scenarios generated. The results are then fed as inputs into the second optimization stage. The outcomes generated in the second stage provide the optimal deployment for the supply trucks, regular and armored, prior to the operation.

The OPTiMiLSC is coded in General Algebraic Modeling System (GAMS) and solved using the Optimization Solutions Library (OSL) solver (Brooke, Kendrick, Meeraus, Raman 1998). The size of an OPTiMiLSC instance depends on k, the number of draws at each branch of the scenario tree. First stage optimization problem (Supply) consists of about 150 constraints and 250 variables when $k = 2$ and 60,000 constraints and 80,000 variables when $k = 15$. Second stage optimization problem (Trucks) consists of about 200 constraints and 300 variables when $k = 2$ and 80,000 constraints and 100,000 variables when $k = 15$. The run-time for both problems takes as little as one minute, for the smaller scale problem, to approximately 45 minutes for the larger scale problem. Of the total run-time taken, the second stage optimization problem constitutes 60% of the time.

3. Test of Robustness

In order to evaluate the robustness of the planned logistic deployment one needs to define appropriate measures of effectiveness (MOE). These measures must be quantifiable, so that assessments can be objectively made and tracked. The MOE used in our analysis is the probability that a randomly selected demand scenario is satisfied. We call this measure the "responsiveness probability."

The following table outlines the steps of this process:

(a)	Fix the optimal deployment plan obtained from the reference set of demand scenarios.
(b)	Generate (simulate) a sample of *m* independent demand scenarios. For each scenario compute the respective deployment requirements for the depot and the DPs.
(c)	For each sampled scenario, check if it is "feasible" or "infeasible." A scenario is said to be "logistically feasible," or in short, "feasible," if the demand of at least 80% of the DPs (in our example, four DPs out of five) and the initial deployment for both the regular and armored trucks at the depot are satisfied. Otherwise, a demand scenario is "infeasible."
(d)	Compute the estimate $\hat{P}$ for the responsiveness probability. We estimate the responsiveness probability by computing the fraction of the feasible ones among the *m* number of independent demand scenarios. $$\hat{P} = \frac{Number\ of\ "feasible"\ scenarios}{Number\ of\ "feasible"\ \&\ "infeasible"\ scenarios}$$

Table 2. Steps for Robustness Check

4. Statistical Analysis

The next step for the experiment is the statistical analysis. The statistical test is based on the Binomial probability distribution. It describes the probability of obtaining a given number of successes or "feasible" outcomes in a fixed number of independent trials. The sample consists of the outcomes of *m* independent trials. Each outcome is either "feasible" with probability *p*, or "infeasible," with probability $q = 1 - p$. We use the exact binomial test to establish the test statistic. For a 10% test level, using a sample size $m = 60$ and probability of "feasible" set as $p = 0.90$, the critical value (t) is 57. We reject the null hypothesis if the number of observed "feasible" outcomes (T) is greater

than t = 57. When we reject the null hypothesis for a particular number of randomly generated draws (k) at each branch of the scenario tree, we can conclude that the optimal deployment corresponding to the draws k is able to satisfy the requirement of having the responsiveness probability greater than 0.90.

Two statistical issues relevant to OPTiMiLSC are discussed. They are (a) sample size and (b) types of hypothesis testing.

a. Sample Size

The sample size is crucial to any experimental design as it influences whether or not a false null hypothesis will be rejected. A type II error is more likely to be committed if the sample size is too small. When choosing a sample size for any experimental design, we need to consider the significance level (α), the Type II error rate (β) we wish to achieve and the value (p') we wish to detect. For the purpose of our analysis, the Type II error rate is set to $\beta = 20\%$. The power of the test is referred to as $1-\beta = 80\%$. In other words, the study has enough power to detect the smallest worthwhile effects 80% of the time.

Table 3 shows the required sample size correspond to the value to detect (p'), such that $\beta(p') = 20\%$ for the significance level $\alpha = 0.05$ and $\alpha = 0.10$, as provided by the S-Plus statistical package (Crawley 2002). The lower the p' value, the more stringent the conditions will be for the test. For example, assume we want to conduct a hypothesis test in which

$$H_0 : p \le 0.90$$
$$H_1 : p > 0.90.$$

For $\alpha = 0.10$ and with $p' = 0.96$, we would require a sample size of $m = 60$. For $\alpha = 0.05$ and with $p' = 0.92$, we would require a sample size of $m = 1200$. This illustrates that the required sample size increases as the conditions for the test becomes more stringent. There is a trade-off between the precision and the sample size we can

generate for our experiment. For this research, we use the sample size of m = 60, when $p' = 0.96$ and $\alpha = 0.10$.

10% Type I Error		5% Type I Error	
Value to Detect (p')	**Sample Size (m)**	**Value to Detect (p')**	**Sample Size (m)**
0.92	950	0.92	1200
0.93	350	0.93	500
0.94	180	0.94	250
0.95	100	0.95	150
0.96	60	0.96	100
0.97	40	0.97	60
0.98	20	0.98	30

Table 3. Sample Size for $\beta = 20\%$

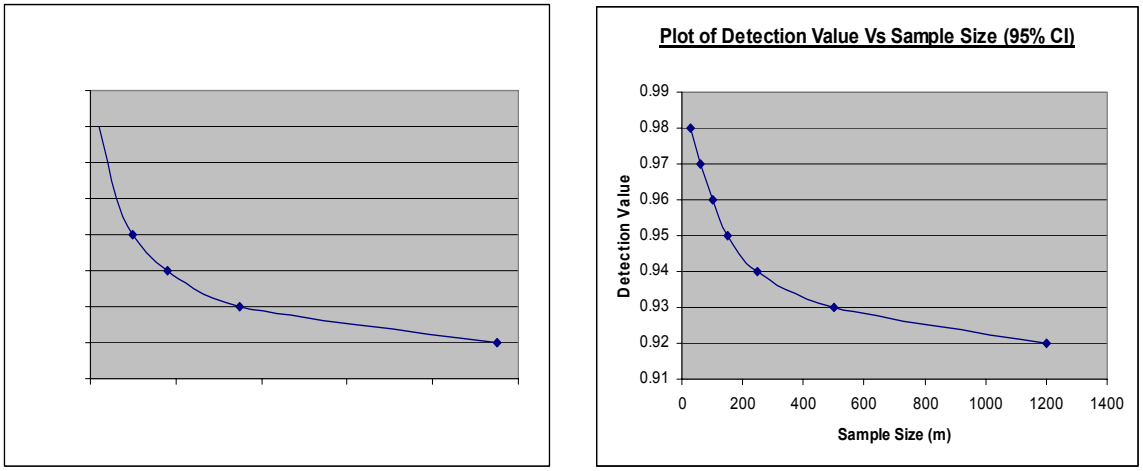

Figure 6. Plots of Detection Value Versus Sample Size for $\beta = 20\%$

b. Hypothesis Tests (Upper-Tailed or Lower-Tailed)

The alternative hypotheses represent the condition we are investigating. For this research, we want to know whether the optimal deployment we have obtained, based on a number of draws (k) at each branch of the scenario tree, is sufficient to achieve a responsiveness probability greater than 0.90.

In the statistical test, we control the probability of a Type I error by specifying its value (the significance level). There are two possible errors: concluding that a deployment plan corresponding to the number of draws (k) is sufficient when it is not, and concluding that the optimal deployment corresponding to the number of draws (k) is not sufficient when it is. Suppose we believe that the risk of underestimating the optimal deployment corresponding to a number of draws (k) is higher than the potential loss of not having the model. Then the error we wish to avoid is the erroneous conclusion that the optimal deployment based on a particular number of draws is sufficient. We define this as Type I error. As the result, the burden of proof is placed on the system to deliver sufficient statistical evidence that the responsiveness probability is greater than 0.90.

For this research, since we are developing a new model to be used in military logistic planning which is mission-critical, we would want the risk of an incorrect conclusion to be small enough that we can feel confident we are not observing a freak random event; rather, we are seeing strong evidence against the null hypothesis. It is important that we have sufficient statistical evidence to infer that the optimal deployment corresponding to the number of draws (k) is sufficient to achieve a responsiveness probability greater than 0.90. Thus the upper-tailed test is more appropriate for our analysis. The null and alternative hypotheses are formulated as follows:

$$H_0 : p \leq 0.90$$
$$H_1 : p > 0.90$$

B. IMPLEMENTATION AND RESULTS

The situation we consider here is a single depot (e.g, a theater level supply unit), which provides logistic support to several DPs. We simulate, and then optimize, a two-level, three-time period logistics system, which is subject to possible interdiction.

1. Optimal Deployment

Solving the optimization problem generates the optimal initial deployment for two types of resources: supply in the depot and in the DPs, and trucks at the depot. The optimization is performed with respect to a set of k^t randomly generated scenarios, where t is the number of time periods in the planning horizon. Recall that k is the expansion rate of the scenario generation process (the number of draws at each branch of the scenario tree). In our numerical analysis k ranges between 2 and 15. The following table presents an optimal deployment for a sample of scenarios when $k = 12$.

First Stage Optimization	
Optimal Deployment of Supply at the Depot (U^{t*})	
U^{0*}	916
Optimal Deployment of Supply at the DPs (X_n^*)	
X_1^* (DP_1)	118
X_2^* (DP_2)	107
X_3^* (DP_3)	120
X_4^* (DP_5)	103
X_5^* (DP_5)	126
Second Stage Optimization	
Optimal Deployment of Trucks at the Depot (V^{0*})	
V^{R_0*}	74
V^{A_0*}	42

Table 4. Optimal Deployment of Supply and Trucks (For $k = 12$)

The optimal deployment plan in Table 4 shows that in the theater of operations the depot requires an initial supply of 916 units. The five DPs require an initial supply ranging from 103 units to 126 units. The total numbers of trucks to be deployed at the depot are 74 regular trucks and 42 armored trucks.

2. Robustness Test

In order to evaluate the robustness of the optimal deployment we generate sixty independent new demand scenarios and for each scenario we compute the respective deployment requirement for the depot and the DPs. We use d_m^U to denote the demand at depot for a random selection *m* of demand scenarios. We use $d_m^{DP_n}$ to denote the demand at DP_n for a random selection *m* of demand scenarios. Then d_m^{VR} and d_m^{VA} denote the required number of regular and armored trucks, respectively, for a random selection *m* of demand scenarios.

Each newly drawn demand scenario *m* is checked for feasibility with respect to the optimal deployment obtained for each *k* ranging between two and fifteen. Table 5, column (*OpD*), shows the sample results of the robustness test for the optimal deployment obtained when *k* = 12. Columns (*d*1) to (*d*60) are the required demands with respect to each of the demand scenarios *m*. If the optimal deployment obtained can satisfy the demand, (e.g, $U^{0*} \geq d_m^U$), it is measured as "1." Otherwise, it is considered equal to"0."

A randomly drawn scenario is said to be "feasible" if

- The demands for the depot are met ($U^{0*} \geq d_m^U$),
- The demand for 80% of the DPs (four out of five DPs) are met ($X_n^* \geq d_m^{DP_n}$) and
- The demands for both regular and armored trucks are met ($V^{R_0*} \geq d_m^{VR}$ and $V^{A_0*} \geq d_m^{VA}$).

Otherwise the scenario is said to be "infeasible." Table 5 presents the results of the robustness test with respect to the example shown in Table 4. Out of 60 randomly generated scenarios 59 were feasible, which gives a responsiveness probability of 0.98.

Optimal Deployment		Sixty Independent Demand Scenarios												
k = 12			$m1$		$m2$			$m58$		$m59$		$m60$		
	(OpD)		($d1$)		($d2$)			($d58$)		($d59$)		($d60$)		
U^{0*}	**916**	d_m^U	828	1	845	1	…	881	1	804	1	804	1	
X_1^*	**118**	$d_m^{DP_1}$	95	1	107	1	…	100	1	112	1	119	0	
X_2^*	**107**	$d_m^{DP_2}$	100	1	88	1	…	86	1	104	1	117	0	
X_3^*	**120**	$d_m^{DP_3}$	115	1	93	1	…	103	1	101	1	111	1	
X_4^*	**103**	$d_m^{DP_4}$	78	1	85	1	…	96	1	86	1	90	1	
X_5^*	**126**	$d_m^{DP_5}$	121	1	99	1	…	119	1	92	1	108	1	
V^{R_0*}	**74**	d_m^{VR}	67	1	69	1	…	73	1	63	1	64	1	
V^{A_0*}	**42**	d_m^{VA}	32	1	32	1	…	30	1	35	1	34	1	
			Feasible		**Feasible**			**Feasible**		**Feasible**		**Infeasible**		

Table 5. Sample Results for the Robustness Test (For k =12)

3. Statistical Analysis

In this research, we want to know whether there is sufficient statistical evidence to infer that the number of draws at each branch of the scenario tree (k) is sufficient to achieve a responsiveness probability greater than 0.90. At each robustness test for each number of k from two to fifteen, there is a hypothesis test. Table 6 presents the results of the hypothesis tests. There is a positive correlation between the number of observed "feasible" scenarios and the number of draws, as observed in Figure 8.

Number of Draws (k)	Number of Scenarios (k^T)	Number of Feasible Scenarios	Responsiveness Probability ($\hat{P}$)	Hypothesis Test
2	8	23	0.38	Not Reject H_0
3	27	24	0.40	Not Reject H_0
4	64	27	0.45	Not Reject H_0
5	125	30	0.50	Not Reject H_0
6	216	50	0.83	Not Reject H_0
7	343	53	0.88	Not Reject H_0
8	512	54	0.90	Not Reject H_0
9	729	58	0.97	Reject H_0
10	1000	57	0.95	Not Reject H_0
11	1331	58	0.97	Reject H_0
12	1728	59	0.98	Reject H_0
13	2197	58	0.97	Reject H_0
14	2744	59	0.98	Reject H_0
15	3375	59	0.98	Reject H_0

Table 6. Results of Hypothesis Test

We note a positive relationship between the number of feasible scenarios and the number of draws (k) at each branch of the scenario tree as shown in Figure 7. The optimal deployment plan is more robust with a wider coverage of scenarios. For example, when k = 2, eight scenarios are generated whereas when k = 15, total of 3,375 scenarios are generated. We can expect that the greater the number of branches for each scenario tree, the more robust the solution.

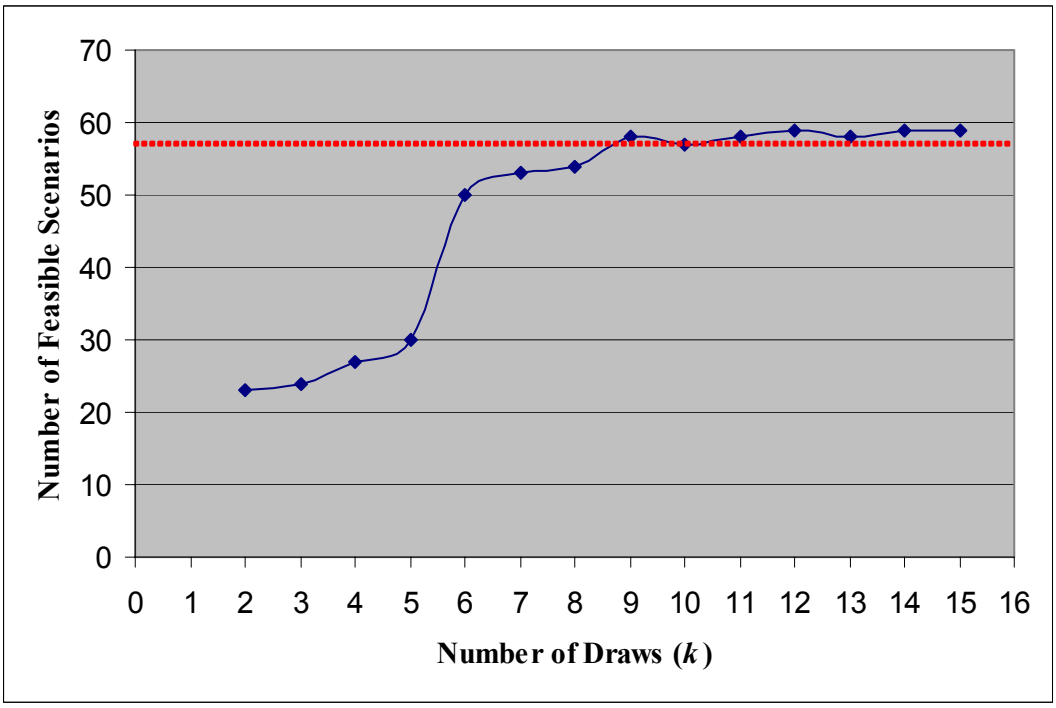

Figure 7. Plot of Number of Feasible Scenarios Versus Number of Draws (k)

The critical test statistic for this experiment is $t = 57$ using a significance level of 10%. This means that we reject the null hypothesis, H_0, for values of t greater than 57. This translates to a required responsiveness probability of 0.90 or greater. We find that for values of k greater or equal than 11, we consistently reject H_0. This implies that to obtain a deployment plan with at least a 90% responsiveness probability, we need, in our case, to determine the optimal deployment on a model in which k is at least 11.

C. EXPLORATORY ANALYSIS

1. Comparison with Deterministic Approaches

The deterministic optimization approach assumes that future demands are known with certainty. We gain interesting insights when we compare the OPTiMiLSC to two deterministic approaches.

In the first approach we use the mean values of the demands to determine the optimal deployment plan. In the second approach we use the 90th percentile of the demand distribution. Table 7 compares the optimal deployment plans using the three approaches, where the OPTiMiLSC refers to the results of the k = 12 case presented in Table 2.

	OPTiMiLSC (k=12)	**Using Mean Value**	**Using 90th Percentile**
Optimal Deployment of Supply at Depot Level (U^{t*})			
U^{0*}	916	855	1094
Optimal Deployment of Supply at DPs (X_n^*)			
X_1^* (DP_1)	118	100	128
X_2^* (DP_2)	107	90	115
X_3^* (DP_3)	120	100	128
X_4^* (DP_4)	103	90	115
X_5^* (DP_5)	126	110	141
Total Supply	1490	1345	1721
Optimal Deployment of Trucks at Depot Level (V^{0*})			
V^{R_0*}	74	69	88
V^{A_0*}	42	34	44

Table 7. Optimal Deployment for the OPTiMiLSC and the Two Deterministic Approaches

Table 8 gives a summary of the results of responsiveness probability and the percentage difference in supply and trucks among the three approaches. We see that both the OPTiMiLSC and the deterministic approach using the 90th percentiles give the same responsiveness probability (0.98). The deterministic approach using mean values gives a 0.10 probability of success, which is not acceptable for any military operation.

	OPTiMiLSC	Using Mean Value	Using 90th Percentile
Probability of Success	0.98	0.10	0.98
Percentage Difference			
- Supply	-	-9.73%	15.50%
- Regular Truck	-	-6.76%	18.92%
- Armored Truck	-	-19.05	4.76%

Table 8. Comparison of Results for OPTiMiLSC and Two Deterministic Approaches

In terms of the requirements for supply and trucks, we observe that the deterministic approach using the 90th percentile requires more as compared to the OPTiMiLSC.

We infer that the use of deterministic optimization approaches produces results that are misleading and unreliable. The use of the 90th percentile approach seems to yield an excess of supply in most cases while the mean value approach provides inadequate responsiveness.

2. Change in Battle Intensity

This research considers two cases of demand variation such that the demand values are 20% above and below the mean demand value obtained. This is part of the sensitivity analysis, which reflects an expected change in the battle intensity from the normal operational plan. Table 9 shows the results of the sensitivity analysis with respect to the three levels of battle intensity – Low (decrease demand by 20%), Normal, and High (increase demand by 20%).

	Demand Low		Demand Normal		Demand High	
Number of Draws (K)	**Number of Feasible Observations**	**Hypothesis Test**	**Number of Feasible Observations**	**Hypothesis Test**	**Number of Feasible Observations**	**Hypothesis Test**
2	23	Not Reject H_0	23	Not Reject H_0	22	Not Reject H_0
3	31	Not Reject H_0	24	Not Reject H_0	32	Not Reject H_0
4	27	Not Reject H_0	27	Not Reject H_0	32	Not Reject H_0
5	33	Not Reject H_0	30	Not Reject H_0	33	Not Reject H_0
6	51	Not Reject H_0	50	Not Reject H_0	50	Not Reject H_0
7	51	Not Reject H_0	53	Not Reject H_0	52	Not Reject H_0
8	54	Not Reject H_0	54	Not Reject H_0	54	Not Reject H_0
9	57	Not Reject H_0	58	Reject H_0	56	Not Reject H_0
10	53	Not Reject H_0	57	Not Reject H_0	55	Not Reject H_0
11	57	Not Reject H_0	58	Reject H_0	57	Not Reject H_0
12	59	Reject H_0	59	Reject H_0	56	Not Reject H_0
13	58	Reject H_0	58	Reject H_0	56	Not Reject H_0
14	59	Reject H_0	59	Reject H_0	58	Reject H_0
15	58	Reject H_0	59	Reject H_0	59	Reject H_0

Table 9. Results of Hypothesis Test for Three Different Level of Demands (Normal, Low and High)

Figure 8 shows the relationship between the number of feasible scenarios and the number of draws for the three levels of battle intensity. From the plot, we see that all three levels of battle intensity give similar upward trends. We infer that the probability of success is not sensitive to variations in the mean value of demand – that is, the OPTiMiLSC is not sensitive to overall changes in demand profiles.

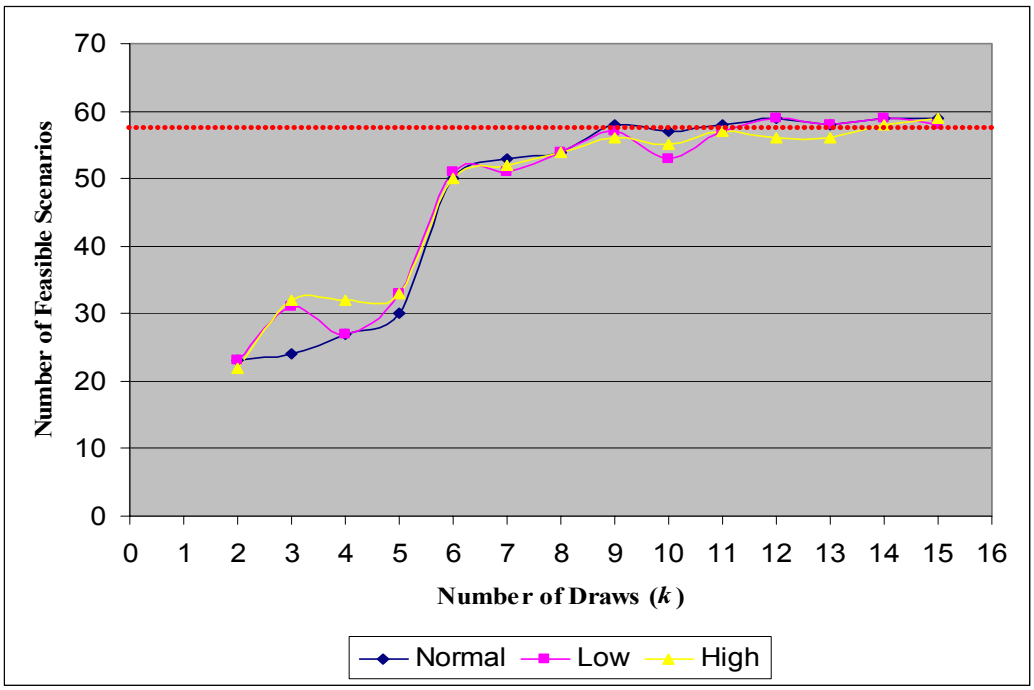

Figure 8. Plot of Number of Feasible Scenarios Versus Number of Draws (k) for Three Different Level of Demands (Normal, Low and High)

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This research presents a general model for determining a two-level optimal deployment of transportation assets and supplies in a theater of operation. The proposed model, called the optimal military logistics supply chain model (OPTiMiLSC), takes into consideration the imbedded uncertainties in military operations. This research explores the control of a military supply system in a dynamic setting, where demand is random, non-stationary and scenario dependent.

OPTiMiLSC is a two-stage scenario-based multi-period optimization problem. It can help military logistics planners determine how much supply and transport to deploy at each logistic node or DP before the start of the operation. This type of model can provide insights different from a purely deterministic optimization model. By explicitly considering a number of demand scenarios, the stochastic model can determine a deployment plan that responds to demand requirements with a certain and prespecified confidence level (probability). Thus the modeling framework of OPTiMiLSC may enable military planners to establish a robust plan for a logistics force structure at the theater of operations.

This model may be used as a first step or a building block in a more general planning framework for operational logistics. The methodology is new and novel in the sense that it combines optimization, simulation and statistical analysis in a simple and easily applicable way. We embed a relatively simple optimization scheme within a scenario-based simulation setting and obtain pertinent statistics for analysis. The demand scenarios are generated based on operational plans and their transitions between successive time periods are assumed to be Markovian. The model also reflects the malevolent environment in which military supply chains operate. Possible threats to supply lines are explicitly represented by an "interdiction" variable.

B. USE OF MODEL RESULTS

OPTiMiLSC is a stochastic planning and simulation model. It helps to provide an estimate of the risks involved in the MLSC. It also determines the appropriate level of supply to enable the military units to hedge against future demand scenarios. The most attractive feature of the OPTiMiLSC developed in this research is the flexibility introduced in the operational logistic planning procedure.

We have to bear in mind that the OPTiMiLSC is not a prescriptive model. It serves more as a guide to translating demand profiles and operational parameters to logistic design requirements. Care must be taken not to overstate the benefits of the stochastic optimization model, since it considers only a moderate number of scenarios, and these are constructed from a rather small data set. We have also made some assumptions in order to simplify the OPTiMiLSC. We assume that there are no capacity constraints at either the logistic nodes (Depot and DPs) or on the transportation routes. We also assume that the logistic nodes are well protected and are robust in protecting against any possible enemy attack. Finally, we assume that there is a guaranteed transportation time between the depot and a DP.

Therefore the contribution of such a modeling approach can be evaluated only with time, after it is accepted by commanders and logistics planners and proves capable of identifying worthy alternatives that might otherwise have been overlooked. In the typical real world, the OPTiMiLSC probably comprises many times more variables than those outlined in this research, together with a greater number of constraints. To obtain realistic results, one should incorporate into the model multiple commodity demands, more logistic levels, different means of transportation and capacity constraints for logistic nodes and edges.

C. RECOMMENDATIONS FOR FUTURE RESEARCH

The focus of this research was to develop a modeling framework for analyzing concepts used in military logistic operation, and an initial model was developed. This initial model may be a first step in what should become a continuing study of military operational logistics. More detailed analysis and model formulation will assist in the

ongoing development of new MLSC concepts. The following are just a few directions in which this work may be taken in future research:

1. Extensions and Modifications

The basic OPTiMiLSC that has been presented so far may be extended and modified in several directions.

a. Multiple Commodities

The single supply (ammunition) in the current model can be extended to several types of supply (including, for example, rations and spare-parts) that share a common fleet of trucks. The basic structure of the OPTiMiLSC will not be changed but the size of the problem may be increased considerably.

b. Multiple Levels

The current model represents a logistics system that consists of only two levels. The model can be expanded to represent more logistic levels, including the strategic level.

c. Multiple Interdictions

We consider a single possible flow interdiction in each period. Given the current interdiction modeling, it is easy to expand the model to take into account multiple random flow interdictions at each period.

d. Different Supply Methods

The current model considers only the "push" method for the employment of the trucks. This means that the trucks belong to the depot, which allocates them among the transportation missions. Another (more realistic) transport deployment would combine "push" and "pull" methods. In this case X_n can be viewed as trucks that are initially loaded with the supply that is needed by the DPs in the first few periods and then, after being unloaded, are used to "pull" supply from the depot to the DPs.

e. Different Means of Transportations

The current model assumes only two types of trucks are used and it does not take into account the possibility of variable delivery times. Other modes of transportation such as air transportation may also be incorporated into the model in order to add more realism to it.

f. Capacity Constraints for Logistic Nodes and Edges

The current model assumes that there are no capacity constraints, neither at the logistic nodes (the depot and the DPs) nor at the edges. Capacity constraints for both logistic nodes and edges should be considered in the model in order to add more realism to it. However, unlike the capacity of a logistic node, which is a static attribute that is measured in terms of storage area or volume, edge capacity is a dynamic attribute that represents the rate at which the logistic flow can move through that edge. The capacity of an edge depends on the type, width and topography of the corresponding LOC, and on the number, capacity and speed of the means of transportation that are assigned to that edge. The capacity of an edge is measured by the maximum possible throughput of flow on that edge.

2. Using Actual Field Data

The thrust of the research is modeling and not analysis. Therefore the data set is invented, not drawn from actual field data. A real world application will require an effort to generate realistic scenarios and estimate the probability distributions of a more complex set of demand vectors.

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