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Full Viscous Modeling in Generalized Coordinates of Heat Conducting Flows in Rotating Systems

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A computational and analytical study has been carried out that assesses viscous formulations used currently in the state-of-the-art and widely used incompressible viscous flow solution methodologies in generalized curvilinear coordinates. These methodologies use a powerful and simple approximate factorization delta scheme. The conventional thin (viscous) layer form used for steady-state flows is analyzed and is then improved to extend its applicability, in general, to the solution of unsteady incompressible flows. Then, this improved thin-layer formulation is further extended to include more terms of the same order that are ignored in the thin-layer approximation because of the limitation imposed by the use of a conservation form of the viscous flux vector. This is achieved by recasting the viscous vector in nonconservation form. The resulting formulation represents the full-viscous (FV) formulation (except for higher-order cross-derivative terms). Solutions with the full-viscous, improved, and conventional thin-layer formulations are shown to be different, even for laminar heat conducting flow. Also, the steady state full-viscous solutions with and without the cross-derivative terms are compared and are found to be practically the same. This delineates the need, in general, for the FV formulation for turbulent flows, where turbulent viscosity variation in space is large. The methodology developed here is then used to compute flow in a rotating square duct with fast and slow rotation. The limiting solutions obtained are compared with the analytic limiting solutions and previously obtained reduced numerical solutions. Agreement is shown to be very good.

Introduction

THE thin-layer (TL) form of viscous terms in the Navier–Stokes equations in body-conforming grids neglects the viscous diffusion that is not normal to the surface along which there is a flow, and it is an acceptable approximation for flows at a high Reynolds number and without the effects of major flow separation from the surface. The TL form must be invoked so as to incorporate the viscous terms on the implicit side [left-hand side (LHS)] of the approximate factorization (AF) delta scheme[1] in conservation form.

In some incompressible TL methodologies, incomplete TL terms on the LHS (implicit side) have been considered (terms involving cross-metric derivatives are neglected) while seeking a steady-state (SS) solution because the implicit side approaches zero during convergence. This SS solution is achieved as an asymptotic solution in physical time,[2] or as an asymptotic solution in artificial time at each physical time step[3] to obtain a time-accurate solution. In the former case, local linearization is done in physical time; in the latter, local linearization is carried out in artificial time and not in physical time.

The SS formulation is correct in such solution methodologies,[2,3] provided complete TL terms are considered on the right-hand side (RHS) of the scheme. It is not correct, in general, when seeking a solution to the unsteady problem by other methodologies such as those in which local linearization is carried out in physical time.[4,5] In addition, the SS formulation can slow down the convergence rate of the solution to steady-state because of imbalance in the diffusion terms on the LHS and RHS of the AF delta scheme.

Therefore, an improved TL formulation has been developed,[6] which can be used in methodologies[4,5] where local linearization is carried out in physical time and not in artificial time. The results with the improved TL formulation are presented here.

For applications, where the TL approximation breaks down, the TL restriction as entailed by use of the conservation form of the viscous flux vector is removed here by considering the viscous terms in nonconservation form, since the conservation form of the viscous fluxes is not necessary as is that of inviscid fluxes for proper resolution of discontinuities such as shear layers, shock waves, etc. This results in a full-viscous (FV) formulation where all of the lowest-order terms are considered, many of which are neglected in the TL formulation. The cross-derivative terms are found to be of no consequence in the present study. This is observed by comparing the SS results with and without the cross-derivative terms on the right-hand (explicit) side.

The improved TL formulation[6] as well as the FV formulation developed here are incorporated in a 5×5 temperature-dependent incompressible flow solver developed earlier,[7] which is based on the widely used AF scheme[1,8] and the artificial compressibility approach.[9]

In internal flows such as that of coolant flow in rotating turbine blade passages, viscous effects are of primary interest. Such a flow is highly three dimensional in nature because of the complex internal geometry (confined serpentine passages with turbulators, pin fins, etc.) and low Reynolds effects (owing to the presence of a high-temperature environment).

Because of the presence of high temperature, numerical study of these flows can be adequately done with an incompressible flow prediction methodology. Therefore, the present formulation will be discussed mostly in the context of incompressible flows. The results on the extension of the present formulation to compressible flows will be reported in a later study.

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The main goal of the present study is to provide a consistent, accurate, and reliable unsteady computational methodology[10] to predict internal flows, especially internal coolant flows in turbine blades of new-generation jet aircraft engines.

Governing Equations

An incompressible temperature-dependent laminar flow (ignoring buoyancy effects) in a rotating frame of reference is given by the following system of equations[11]:

$$\partial_t u_i^* + u_j^* u_{i,j}^* = P_{ij,j}/\rho + \partial_i(|\boldsymbol{\Omega} \times \boldsymbol{r}|^2/2) + 2\varepsilon_{ijk} u_j^* \Omega_k$$

where

$$P_{ij} = -p^* \delta_{ij} + 2\mu e_{ij}, \qquad e_{ij} = (u_{i,j}^* + u_{j,i}^*)/2$$

Ω is constant angular velocity, δ_{ij} is the Kronecker delta, and P_{ij} represents the stress tensor, and the second and third terms on the RHS represent the centrifugal and Coriolis forces associated with the rotating frame of reference. All of the other terms and symbols have their usual meaning.

The continuity equation via the concept of artificial compressibility[9] is

$$\partial_t p^* + \beta u_{i,i}^* = 0$$

where β is the artificial compressibility parameter.

The temperature equation (assuming perfect gas and ignoring viscous dissipation) is

$$\rho C_v \partial_t T^* + \rho C_v u_j^* T_{,j}^* = (k T_{,j}^*)_{,j}$$

After normalization, these equations take the following form:

$$\partial_t p + \beta u_{i,i} = 0$$

$$\partial_t u_i + (u_i u_j)_{,j} = -p_{,i} + [\mu(u_{i,j} + u_{j,i})]_{,j}/Re$$
$$+ \partial_i[|\boldsymbol{\Omega} \times \boldsymbol{r}|^2/2]/(Ro)^2 + 2\varepsilon_{ijk} u_j \Omega_k/Ro$$

$$\partial_t T + u_j T_{,j} = (k T_{,j})_{,j}/(RePr)$$

where $Ro = U_0/\Omega L$ is the Rossby number, $Re = \rho_0 U_0 L/\mu_0$ is the Reynolds number, and $Pr = C_{v_0}\mu_0/k_0$ is the Prandtl number.

Normalization reference velocity, length, density, viscosity, thermal conductivity, specific heat at constant volume are U_0, L, ρ_0, μ_0, k_0, and C_{v_0}, respectively, and the temperature is normalized as

$$T = (T^* - T_0)/(T_w - T_0)$$

where T_w is the wall temperature.

In the present study, heating caused by viscous dissipation and the buoyancy effects are neglected, although these terms are considered in the flow solver.[7] Only laminar flow will be considered in the present study. This corresponds to a benign case for comparing the SS, improved TL, and FV formulations. This is to avoid any uncertainty in the predictions that may arise using a phenomenological turbulence model. However, even in the temperature-dependent laminar case, the predictions will be shown to be different with the three formulations.

Introducing a compact form,[7] the system of the normalized governing equations can be expressed in conservation form as

$$\partial_t \boldsymbol{U} + \boldsymbol{E}_{i,i} = \boldsymbol{E}_{vi,i} + \boldsymbol{S}$$

where the solution vector $\boldsymbol{U}$ is given by

$$\boldsymbol{U} = \begin{bmatrix} p \\ u \\ v \\ w \\ T \end{bmatrix}$$

the inviscid flux vector $\boldsymbol{E}_i$, by

$$\boldsymbol{E}_i = \begin{bmatrix} \beta u_i \\ u u_i + \delta_{i1} p \\ v u_i + \delta_{i2} p \\ w u_i + \delta_{i3} p \\ u_i T \end{bmatrix}$$

and the viscous flux vector $\boldsymbol{E}_{vi}$, by

$$\boldsymbol{E}_{vi} = \begin{bmatrix} 0 \\ \mu(\partial_x u_i + u_{,i}) \\ \mu(\partial_y u_i + v_{,i}) \\ \mu(\partial_z u_i + w_{,i}) \\ k T_{,i}/Pr \end{bmatrix}$$

and the source terms $\boldsymbol{S}$ by

$$\boldsymbol{S} = \boldsymbol{S}_{\text{cor}} + \boldsymbol{S}_{\text{cen}}$$

where $\boldsymbol{S}_{\text{cor}}$ and $\boldsymbol{S}_{\text{cen}}$ represent Coriolis and centrifugal terms, respectively.

The implicit treatment of source terms within the AF scheme is similar to that described in Ref. 12. Here, only the viscous fluxes will be considered.

Analysis

Consider a viscous flux vector in any given generalized curvilinear coordinate direction i

$$\boldsymbol{F}_i \equiv \boldsymbol{E}_{vi} = \boldsymbol{F}(\alpha_{jk}^i, u^k, u_{,i}^k) \tag{1}$$

where α_{jk} are the coefficients containing metric terms and diffusion coefficients. The representation given previously is also true for any flux vector describing transport of appropriately defined physical variables.

Incompressible Formulation

For incompressible flow without viscous dissipation, we can write the viscous flux vector directly as

$$\boldsymbol{F}_i = \alpha_{jk}^i f(u_{,i}^k) = A_i f_i \tag{2}$$

This is possible because without viscous dissipation, the viscous flux vector is quasilinear for incompressible flow, i.e., A_i can be functions of u^k and perhaps $u_{,i}^k$ for turbulent flows, depending upon the level of complexity of turbulence modeling. But, for compressible flow and incompressible flow with viscous dissipation, the viscous vector being nonlinear, we cannot write it in the form given in Eq. (2); we need to locally linearize the flux vector in Eq. (1). First, we can write Eq. (2) as

$$\Delta \boldsymbol{F}_i = \Delta(A_i f_i)$$

where the operator $\Delta\phi \equiv \phi^{n+1} - \phi^n$; n is the time index.

Further, locally linearizing in time, we have

$$\Delta \boldsymbol{F}_i \approx \left[\frac{\partial(A_i f_i)}{\partial A_i}\right]\left[\frac{\partial A_i}{\partial \boldsymbol{U}}\right] \cdot \Delta \boldsymbol{U} + \left[\frac{\partial(A_i f_i)}{\partial \boldsymbol{U}_{,i}}\right] \Delta \boldsymbol{U}_{,i} \tag{3}$$

where $\boldsymbol{U}$ denotes the vector u^k.

For the present analysis, let A_i be independent of $\boldsymbol{U}$. For laminar and those turbulent flows assumed to satisfy local mechanical equilibrium conditions, one can make an acceptable approximation that A_i are weak functions of $\boldsymbol{U}$. But, for high temperature and especially complex turbulent flows, this approximation often breaks down. Therefore, the dependence of A_i on temperature (for thermal diffusivity and laminar viscosity) through, for example, the Sutherland formula and the dependence of A_i on the transport variables k and ε (for turbulent viscosity and turbulent diffusivities), through, for example, the Boussinesq approximation and mixing length approximation, respectively, can be incorporated into Eq. (3).

The full form of Eq. (3) for viscous modeling is considered in Ref. 7. For the analysis of the full form of Eq. (3) in the case of transport equations, see Ref. 12.

Then

$$\Delta \boldsymbol{F}_i \approx A_i \left[\frac{\partial f_i}{\partial \boldsymbol{U}_{,i}}\right] \cdot \Delta \boldsymbol{U}_{,i}$$

or

$$\Delta \boldsymbol{F}_i \approx A_i \left\{ \left[\left(\frac{\partial f_i}{\partial \boldsymbol{U}_{,i}} \right) \cdot \Delta \boldsymbol{U} \right]_{,i} - \left[\frac{\partial f_i}{\partial \boldsymbol{U}_{,i}} \right]_{,i} \cdot \Delta \boldsymbol{U} \right\}$$

or

$$\Delta \boldsymbol{F}_i = A_i \left\{ \left[\left(\frac{\partial f_i}{\partial \boldsymbol{U}_{,i}} \right) \cdot \Delta \boldsymbol{U} \right]_{,i} - \left[\frac{\partial f_i}{\partial \boldsymbol{U}_{,i}} \right]_{,i} \cdot \Delta \boldsymbol{U} \right\} + \mathcal{O}[(\Delta t)^2] \qquad (4)$$

The previous representation of $\boldsymbol{F}_i$ is suitable for incorporation into the AF delta algorithm in any generalized coordinate direction i for incompressible flow without viscous dissipation. In Eq. (4), coefficients A_i are independent of $\boldsymbol{U}$, but are functions of metrics and diffusion coefficients. The second term on the RHS of Eq. (4) drops out, since for incompressible flow, A_i exists such that

$$f(\boldsymbol{U}_{,i}) = \boldsymbol{U}_{,i}$$

Therefore, Eq. (4) becomes

$$\Delta \boldsymbol{F}_i = A_i(I\Delta \boldsymbol{U})_{,i} + \mathcal{O}[(\Delta t)^2]$$

where I is an identity matrix, or

$$\Delta \boldsymbol{F}_i \approx A_i I \Delta \boldsymbol{U}_{,i} = A_i \Delta f_i \qquad (5)$$

Equation (5) can be derived directly from Eq. (2) when A_i is independent of $\boldsymbol{U}$. However, in general, when A_i is a function of $\boldsymbol{U}$ and possibly $\boldsymbol{U}_{,i}$, local linearization of $\boldsymbol{F}_i$ becomes necessary even for incompressible flow[7,12] and, therefore, we must follow steps such as leading from Eq. (3) to Eq. (4).

The TL form of matrix A_i for incompressible flows as used in the SS formulation[13] is not complete because, although the viscous terms are represented completely by the full viscous vector in Ref. 13, the thin-layer approximation is not applied correctly, so that A_i for the implicit side of the scheme is defined incorrectly. The neglected terms are of the form

$$\alpha^i_{jk}(u^k)_{,i} \qquad j \neq k$$

and are of the same order of magnitude as the included terms for flows where viscosity variation in space is not negligible, such as turbulent flows and strongly temperature-dependent flows, etc., as discussed in Ref. 6, even after incorporating the divergence-free condition for the velocity field. It is only for isothermal laminar flow that these off-diagonal terms vanish along with an additional component of the diagonal for a solenoidal velocity field so that Eq. (8) in Ref. 2 holds. Details pertaining to this discussion are given in Ref. 6.

Compressible Formulation

The solution of thin-layer compressible flow equations in generalized curvilinear coordinates using the AF scheme[1] has received a great deal of attention since the TL approach in generalized coordinates was introduced in Ref. 14.

However, in an attempt to cast the compressible viscous vector in a conservation form, the linearization of the TL viscous vector in Ref. 14 has been presented ambiguously. The definition of matrix M in Eq. (21), as it is written in Ref. 14, is not clear as it relates to Eq. (20), since M is written and referred to as a coefficient matrix therein and, additionally, also as a Jacobian matrix in other subsequent related publications.[15]

Also, it may be noted here that the relation

$$\Delta^i \boldsymbol{F}_v = [V]^i \Delta^i \boldsymbol{U} + \mathcal{O}[(\Delta x)^2]$$

[Eq. (8–83)] on page 445 of Ref. 15, is not complete if $[V]$ is defined to be a Jacobian matrix $[\partial \boldsymbol{F}_v / \partial \boldsymbol{U}]$, as is done in Ref. 15; the matrix $[V]$ is really an operator matrix that is eventually correctly defined on page 447 of Ref. 15, but whose derivation still remains cloudy.

To avoid the ambiguity in this linearization process, a straightforward approach is taken that will also be helpful in deriving the FV formulation.

From the thin-layer form of Eq. (1), we can express each subelement of vector $\boldsymbol{F}_i$ as

$$e_i \equiv E(\boldsymbol{F}_i) = a\phi_{,i} \qquad (6)$$

where $\phi \equiv \phi(\boldsymbol{U})$.

Then, locally linearizing ϕ in time, we have

$$\Delta\phi \approx \left[\frac{\partial \phi}{\partial \boldsymbol{U}}\right] \cdot \Delta \boldsymbol{U} \qquad (7)$$

Also

$$\Delta(a\phi_{,i}) = a(\Delta\phi)_{,i}$$

assuming a is a weak function of time, or

$$\Delta e_i \approx a \left[\left(\frac{\partial \phi}{\partial \boldsymbol{U}} \right) \cdot \Delta \boldsymbol{U} \right]_{,i} \qquad (8)$$

or

$$\Delta e_i \approx a \left[\left(\frac{\partial \phi}{\partial \boldsymbol{U}} \right)_{,i} \cdot \Delta \boldsymbol{U} \right] + a \left[\left(\frac{\partial \phi}{\partial \boldsymbol{U}} \right) \cdot \Delta \boldsymbol{U}_{,i} \right] \qquad (9)$$

Eq. (8) is the conservative form of the linearization relation for the TL viscous flux vector $\boldsymbol{F}_i$ element-by-element.

It would appear that the first term on the RHS of Eq. (9) corresponds to the coefficient or Jacobian matrix M in Ref. 14. However, that is not correct, as seen in the following text.

It will be shown that Eq. (9) is the nonconservative form of the linearization relation for the $\boldsymbol{F}_i$ element-by-element, and is, therefore, useful in developing the FV formulation.

To arrive at the nonconservative form corresponding to $\boldsymbol{F}_i$, a straightforward approach based on the Taylor series expansion is adopted to derive the TL form of the viscous Jacobian matrix (matrices) in a clear manner. Since from various studies in the past, it has been shown that the viscous terms can be cast in the nonconservative form without any appreciable change in the solution from that arising out of the conservative form of the viscous vector, either the conservative or nonconservative form can be employed for the TL approximation.

This approach is also useful for the FV (as opposed to the TL) formulation for compressible flows, results that will be reported in a later study. This is important for flows where the TL approximation breaks down, such as those with low Reynolds number and complex geometry effects.

Also, this alternate formulation is not limited by the assumption of constant diffusion coefficients, as is the case with the linearization process shown in Ref. 14. For the treatment of variable diffusion coefficients, see Ref. 12.

Following the linearization process, we can write

$$\Delta F_i = \left(\frac{\partial F_i}{\partial U}\right)\Delta U + \left(\frac{\partial F_i}{\partial U_\eta}\right)\cdot\Delta U_\eta + \mathcal{O}[(\Delta t)^2]$$

or

$$\Delta F_i = (G - H_\eta)\Delta U + \partial_\eta(H\Delta U) + \mathcal{O}[(\Delta t)^2]$$

where the Jacobian matrices $G = \partial F_i/\partial U$ and $H = \partial F_i/\partial U_\eta$, the matrix $H_\eta = \partial H/\partial\eta$ and η is a coordinate direction i.

The Jacobian matrices H and $(G - H_\eta)$ are given in Ref. 6. It can be seen on inspection that the nonconservative form derived earlier is equivalent to the one given by Eq. (9), element-by-element.

Note that the matrix G (see Ref. 6) looks like the same as matrix M in Ref. 14, but here G is a Jacobian (coefficient) matrix as resulting from the Taylor series expansion, whereas in Ref. 15, M is defined to be an operator matrix [also see Eq. (8)].

Results

Various test cases were considered to assess the differences in predictions with the SS, the improved TL, and the FV formulations as discussed earlier (also see Ref. 10).

The first case was that of a flow in a duct with a square cross section with uniform cooling at a laminar Reynolds number $Re = 1.0 \times 10^2$. Because of uniform cooling, the temperature gradient along the length of the duct wall was held constant. Both the heat transfer and the skin friction distribution along the cross-section boundary were computed. The predictions are compared in Figs. 1 and 2, respectively, with the fully developed exact solution.[16] An excellent comparison in both cases is demonstrated.

A simulation each with the improved TL formulation [Eq. (5)], the SS formulation, and the FV formulation was carried out. The laminar viscosity was calculated using the Sutherland formula. No appreciable difference was seen among the three predictions. Since there was no crossflow or any axial separation in this case, it was expected that the improved TL and the FV formulations would yield about the same result in the square duct case. Since the generalized coordinate grid lines were parallel to the corresponding base coordinate (Cartesian) grid lines, the SS and the improved TL formulations would, in fact, yield identical results since the cross metric derivative terms vanish in this case.[6]

Another simulation, that of a laminar flow in a curved square duct at $Re = 7.9 \times 10^2$, based on the bulk inflow velocity, was carried out with the duct walls at a uniform cooling, except the outer curved wall that was kept insulated downstream of the point where the curved wall begins. For this case, some differences among the predictions corresponding to three viscous formulations as mentioned earlier were noted as shown in Fig. 3a. The same comparison is shown magnified in Fig. 3b. That a temperature-dependent laminar flow (where the temperature dependence of laminar viscosity is known to be mild) can produce such a difference in the predictions shows that for a turbulent flow where the wall gradients are very large and where turbulent viscosity varies between one and two orders of magnitude spatially, the type of viscous formulation used will have an appreciable effect on the accuracy of the solution. This underscores the need for the FV form of viscous terms in simulating such flows. As seen in Fig. 3, the qualitative trend corresponding to

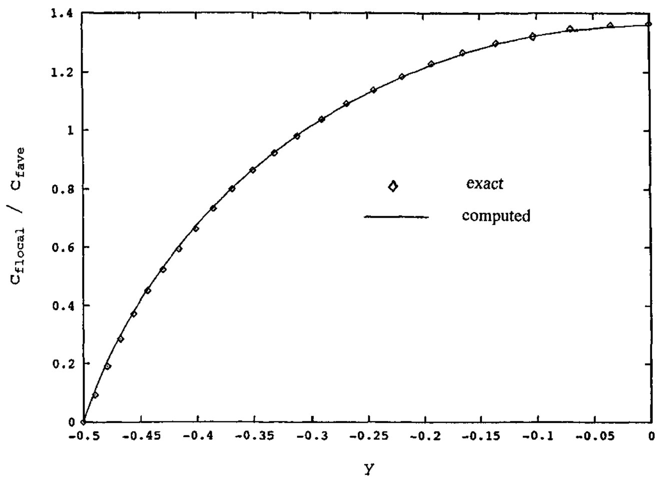

Fig. 2 Skin friction distribution in a square duct.

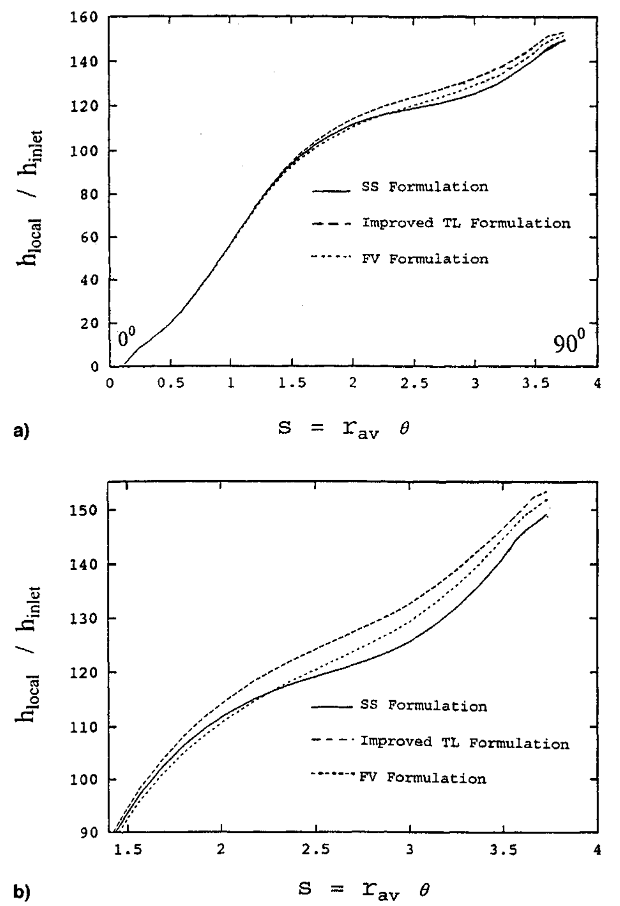

Fig. 3 Heat transfer along the side wall next to the outer curved wall.

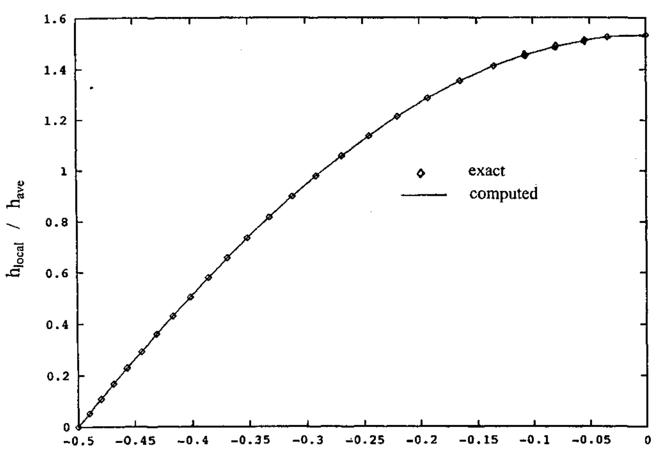

Fig. 1 Heat transfer distribution in a square duct.

the predictions with the improved TL form and the FV form is similar. But, the third prediction, that with the SS formulation, tends to be qualitatively different, as expected, since the grid lines are not parallel to the base (Cartesian) coordinate system.

Finally, a rotating square duct case was considered. The flow was kept isothermal. The centrifugal force term was lumped with the pressure gradient term. The purpose of this test case was to test the code under fast and slow rotation when inertia is weak, since then a comparison can be made readily with the limiting analytical solutions.

A 31×31 crossflow grid was used with an axis system defined in Fig. 4. The axis about which the rotation takes place and the sense of rotation is shown. The crossflow grid was chosen fine enough to resolve the Ekman layers in the fast rotation case.

In the limit of small Rossby number, $Ro = 0.01$, two cases were calculated. The first was at Ekman number $Ek = 0.1$. This is a slow rotation case. The second was at $Ek = 0.001$ and this is a fast rotation case. Axial flow velocity profiles are shown in Figs. 5a and 5b, respectively. The slow rotation case, where essentially pressure gradient balances the viscous force, shows a nearly rectilinear velocity profile (corresponding to a fully developed case) and the fast rotation case, where essentially pressure gradient balances the Coriolis force, shows quite a different profile. This profile looks very similar to the one obtained in Fig. 3c, from Ref. 17, where the numerical solution was obtained corresponding to the reduced limiting problem.

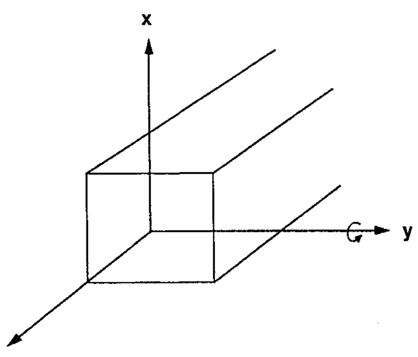

Fig. 4 Axis system for a rotating square channel.

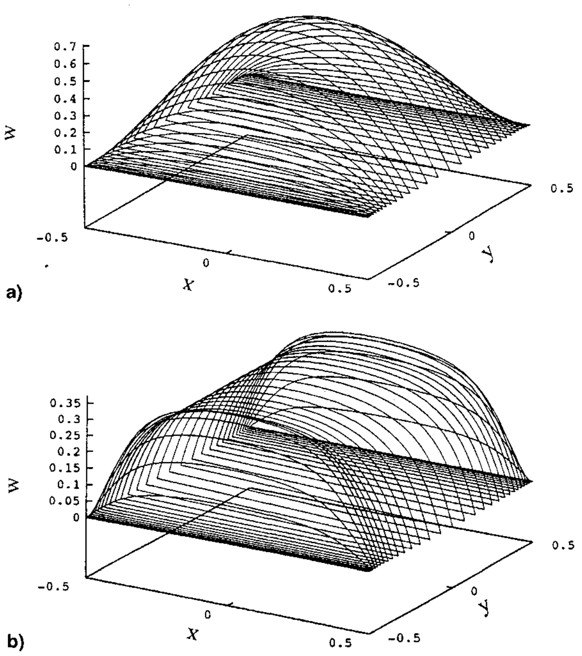

Fig. 5 Rotating channel axial velocity profile over a cross section: a) slow rotation ($Ek = 0.1$) and b) fast rotation ($Ek = 0.001$).

The crossflow velocity vectors corresponding to the two cases are shown in Figs. 6 and 7. In the former, a double-vortex secondary flow is seen in the limit of creeping rectilinear axial flow. In the latter, a strong crossflow pattern is observed that is unlike any secondary flow. Since the magnitude of the crossflow velocity is of the same order as that of the primary flow in this case, the crossflow does not classify as a secondary flow. It is seen that the Ekman layers close to the walls perpendicular to the axis of rotation carry a high-velocity fluid. For a detailed analysis of this flow, see Ref. 17.

It is pointed out here though that in the interior, the crossflow velocity component u attains a magnitude of 0.5 according to the analytical solution, and in the case of present computations, this value was found to be 0.49. Similarly, crossflow velocity component v was numerically found to be negligible in the interior in agreement with the exact solution, $v = 0$.

In Fig. 8, axial velocity along the centerplane parallel to the rotation axis is plotted for both slow and fast rotation cases, i.e., for $Ek = 0.1$ and 0.001, respectively. The predictions are compared with the limiting analytical solution. As seen, the

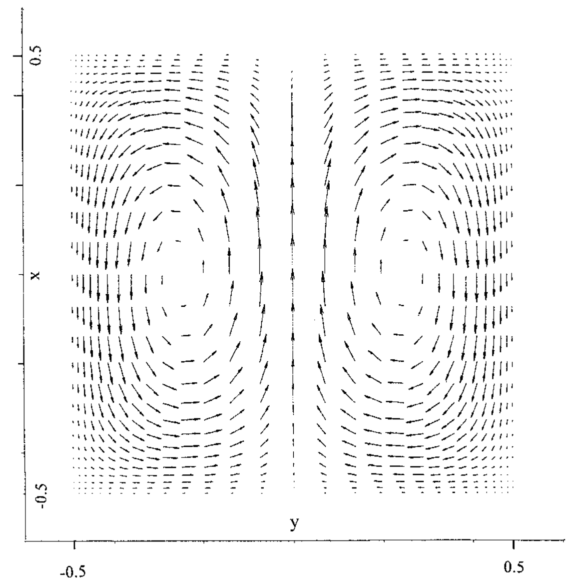

Fig. 6 Rotating channel crossflow velocity vectors: slow rotation ($Ek = 0.1$), limit of rectilinear flow.

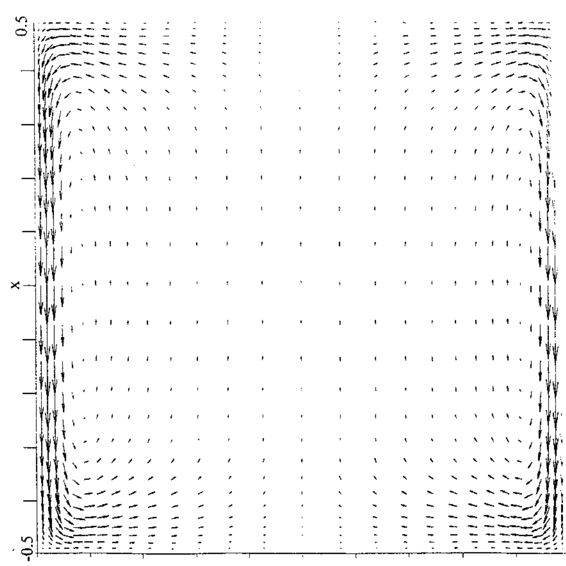

Fig. 7 Rotating channel crossflow velocity vectors: fast rotation ($Ek = 0.001$), Ekman limit.

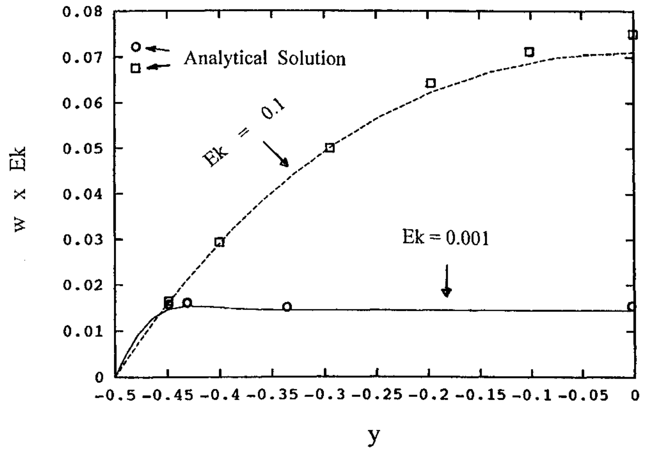

Fig. 8 Axial velocity profile along the centerplane parallel to the axis of rotation.

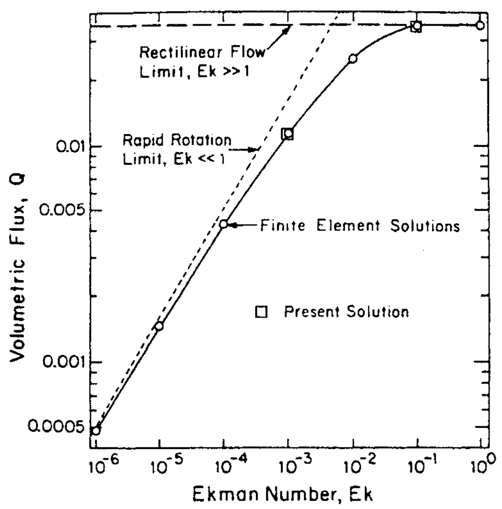

Fig. 9 Volumetric flux through a rotating square duct (*Ro* = 0.01).

agreement turns out to be very good. For $Ek = 0.1$, the solution tends to the rectilinear flow limit as shown.

Finally, Fig. 9 (taken directly from Ref. 17, Fig. 5) shows the volumetric flux through the rotating channel. Although Fig. 5 in Ref. 17 corresponds to $Ro = 0$, two numerical data points from the present simulation shown inserted in the figure correspond to $Ro = 0.01$ and not $Ro = 0$. Owing to a remarkable agreement between the present computed solution and the computed solution of Ref. 17 at $Ro = 0$, it is concluded that $Ro = 0.01$ is small enough for the limiting solutions to hold when either $Ek = 0.1$ or 0.001.

Concluding Remarks

A FV formulation developed here as well as the improved TL formulation developed earlier[6] for incompressible flows have been used with the widely used Beam and Warming AF delta scheme. In the TL formulation, some diffusive terms of the same order as those included in the FV formulation are neglected on the implicit side because of the thin-layer approximation. This can result in errors in the time-accurate predictions of skin friction and heat transfer, depending on the nature of the flow.

The FV formulation has been developed by casting the viscous terms on the implicit side of the scheme in the nonconservation form. Measurable differences in the predictions of heat transfer for a temperature-dependent laminar flow have been observed corresponding to the FV formulation, the conventional TL formulation, i.e., SS formulation, and the improved TL formulation. These differences are clearly a result of different viscous modeling approaches adopted in the SS, the improved TL, and the FV formulations. Consequently, the predictions will progressively improve from the SS to the improved TL to the FV formulations.

Therefore, for unsteady temperature-dependent and/or turbulent flows, in general, it is expected that the predictions with the FV formulation will be more accurate over those corresponding to the TL formulations, especially in rotating systems.

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