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LONG-TERM GOALS

This project addressed the following aspects of ocean model predictability:

(i) using the first passage time conception as the theoretical tool for the quantitative analysis of model predictability skill; (ii) calculating a low-order statistics of the prediction skill for the Princeton Oceanographic Model (POM) applied for modeling of shallow-water wind driven circulation in a semi-closed basin; (iii) comparison of numerical and analytical methods, such as the ensemble prediction technique (EPT) , singular vectors (SV) and iterative analytical solution of Pontryagin – Kolmogorov equation (PKE); (iv) introducing a new measure (sensitivity indexes) for the sensitive analysis of numerical oceanographic models; (v) the analysis of contributions of different kinds of uncertainties such as errors in initial data, stochastic variations of wind and normal velocity along an open boundary into the losing of model predictability skill.

OBJECTIVES

The objectives for the research were as follows
- the prediction skill of the POM applied for modeling of wind driven shallow-water circulation in a semi-closed basin;
- the probability density of prediction (PDP), the mean and the most probable predictability times(τ_m and τ_{mp} , respectively), variance of predictability time ($\tau^2{}_m$) ,sensitivity indexes χ_n (n=1,..,N is the number of external or/and input model parameters), singular vectors and amplification factors;
- different approaches for the analysis of the model prediction skill ;

APPROACH

In order to determine the prediction skill, the POM was reduced to a simpler stochastic dynamical system. The system is a set of stochastic ordinary differential equations whose parameters are embedded in probabilistic properties of the full model. We have found the special variate

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transformation to reformulate the problem of the POM stability to uncertainty of normal velocity along an open boundary as the probabilistic stability of a dynamical system with multiplicative stochastic forcing. Then, PDP was calculated for the full model, for dynamical systems of 10th and 100th orders by EPT and only for a 10-dimensional dynamical system from PKE. We can vary ensemble size between 50 and 10000 members. The mean time of predictability and its variance were calculated as the low-order moments of PDP.

Singular vectors and amplification factors were found from the solution of special equations describing the evolution of the predictability ellipsoid into the phase space of 100-dimensional dynamical system. Such an approach allows considerably saving computer resources.

We varied bottom friction coefficient (external parameter), mean values of wind stress and normal velocity along an open boundary (input parameters). Giving distribution functions of such variations as homogeneously distributed within some interval of values we calculated the sensitivity indexes.

WORK COMPLETED

For a stochastic process, the first passage time (τ) is defined as the time when the process, starting from a given point (position), reaches a predetermined level for the first time, and is a random variable. We used this time as a measure of the prediction skill for the POM. For chosen mean wind stress $\tau_w = 10^{-3} m^2 s^{-2}$ and basin geometry (a rectangular basin with the geometrical sizes of $1000km \times 1050km \times 2km$, plane bottom and a single open boundary) we found that for 50-60 days the flow starting from the state of rest develops in the quasi-stationary inferred circulation. We ignored the horizontal diffusion, and only the bottom friction was considered. The details of numerical experiments can be found in Chu et al., 2001.

The non-stationary circulation developed within a two-month period was chosen as the reference solution for the study of the POM stability. We focused only on short and intermediate predictions limited by 2-2.5 months. Three kinds of uncertainties can distort the reference solution: stochastic errors of initial conditions**,** stochastic wind and normal velocity along the open boundary.

We calculated numerically by EPT and analytically from PKE the probability density of prediction that is the probability of forecasting error to be less than some given tolerance (the prediction accuracy) δ ,τ_m ,τ_{mp}, $\tau^2{}_m$ and χ_n (n=1,..,3).

The mean and the most probable times, the variance of the mean time are the local characteristics of the probabilistic model stability in the perturbation geostrophic stream function L_2 norm and are described in Chu and Ivanov, 2001. Three sensitivity indexes are determined through the ratio of appropriate conditional variances of the predictability time to its unconditional variance. The indexes allow separating contributions of one external parameter (χ_1 for the bottom friction coefficient) and two input parameters (χ_2 and χ_3 for the mean wind stress and normal velocity along the open boundary, respectively).

RESULTS

1.We have developed the numerical techniques for calculating PDP, the mean time predictability, its variance and sensitivity indexes for circulation models based on primitive equations.

2. We have developed the set of orthogonal basis functions (normal modes) for semi-closed seas. That allowed constructing dynamical systems corresponded to the full POM. We have found that the 10-dimensional dynamical system approximated the reference solution with the accuracy around 1-2 %. In comparing with ten normal modes three empirical orthogonal functions contents more than 0.1% of total kinetic energy of the reference circulation.

3. We have suggested a new method for calculating Singular vectors and amplification factors for a perfect circulation model. Our method requires considerably less computer resources than the well-known approaches and can be easily generalized on imperfect models.

4. An approach based on iterative analytical solution of PKE was examined. We found that the analytical approach is highly efficient to calculate the predictability time and sensitive indexes for stochastically forcing models but is less accurate than SV when e-folding time is determined for perfect models without stochastic forcing.

5. We have constructed the general variate transformation and reformulated the stability problem of the reference solution relative to perturbations of an open boundary into the analysis of a dynamical system with multiplicative stochastic forcing.

6.We have found that the forecasting error can be strongly depressed on the initial stage of growth. For example, we observed the zero or negative growth of the forecasting error within 30-day period if Gaussian stochastic noise with the correlation radius equaled to 300 km was added into initial conditions and uncertainty of open boundary conditions was small. Reducing the correlation radius, i.e. transiting to white noise, we reanimated effective growth of initial perturbations. The amplification factor was equal to 5 for a one-month reference period and weakly depends on value of initial errors within this period. The prediction skill of the POM is the most sensitive to the systematic errors of normal velocity along an the open boundary. Stochastic perturbations of the normal velocity can often result only in super-slow growth of the forecasting error.

7.The sensitivity indexes $\chi_1 = 0.9$, $\chi_2 = 0.02$, $\chi_3 = 0.05$ and $\chi_1 = 0.85$, $\chi_2 = 0.019$, $\chi_3 = 0.07$ calculated by EPT and from PKE, respectively, demonstrated that the POM was strongly sensitive to choice of the bottom friction coefficient. The critical value of the coefficient was 0.001.Below this value the model blows up.

8. Three typical scenarios of the predictability process have been found.
Scenario 1. After a rapid growth of initial perturbations a non Gaussian probability density function (PDF) for the first passage time had a symmetric form with narrow variance (Figure 1 a,d,e; $\delta = 10^{-1}$). Herein, $\eta = \tau_m / \tau_{mp} \sim 1$. The $\tau_m \{\text{days}\} (\tau^2{}_m \{days^2\})$ calculated by EPT, SV and from PKE were equal to 27.6 (11.7); 26.9 (--) and 25.6 (10.5), respectively. The prediction skill of the model in this case could be considerably improved through the filtration of the most dangerous initial perturbations of initial conditions.

Scenario 2. PDF had a non- symmetric form with a long tail in domain of the long-term prediction (Fig.2 b,e,h ; $\delta = 10^{-3}$.) The $\eta \sim 1.3$, τ_m (τ^2_m) calculated only by EPT and from PKE were 1.32 (0,31); and 1.31 (0.33), respectively. We called such abnormal long-term predictions the outlier events. The outlier prediction time was determined by intrinsic properties of a model attractor and weakly depended on uncertainty of initial conditions. Therefore, improving the prediction skill of model required other approaches differ from the depression of initial errors of initial conditions. Note, that such scenario is often take place for small tolerances.

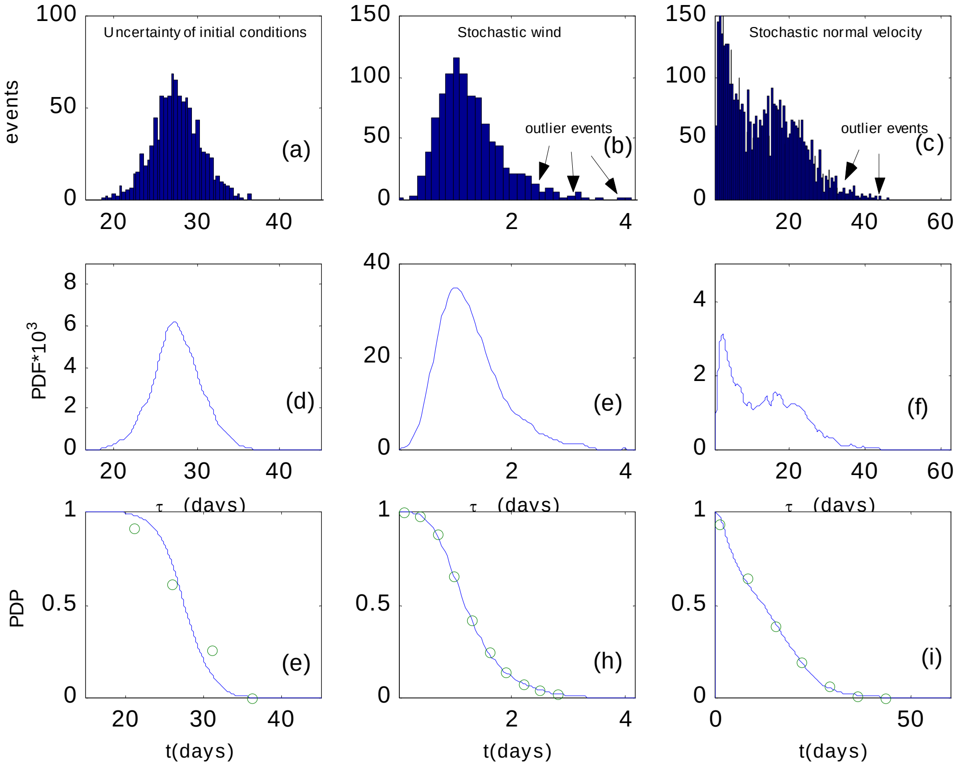

Figure 1

Statistics of the first passage time calculated from PKE (dots) and by EPT (curves)

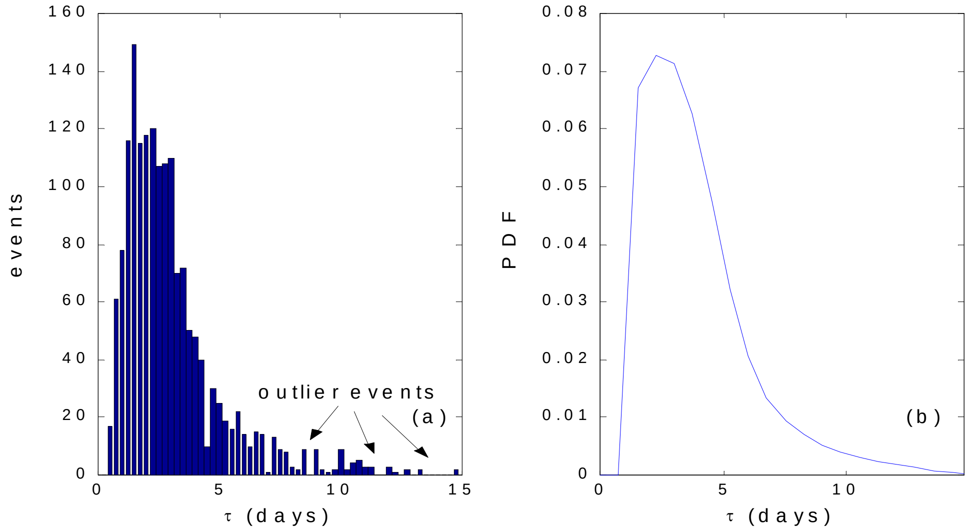

Figure 2
Statistics of the first passage time for the Kantha's high resolution model

Our estimations have shown that outlier events satisfy non-Gaussian statistics and can be described through PDP moments of third and higher orders. Calculations of such moments were not planned in the current proposal and therefore there are not present here.

However, accounting the significance of outlier events for the improving of model prediction skill we checked existing outlier events in the real ocean prediction. To do so we estimated the prediction skill of the well-known high resolution Kantha's model applied to the Gulf of Mexico through the drifter data collected from 50 buoys drogued at 50 m depth. We compared divergence of real buys' and synthetically simulated particle trajectories. Our calculations have clearly revealed outlier events. For example, for 50- km tolerance and $\tau_m \approx 3$ *days* the outlier predictability time achieved 12-15 days (Fig.2 a,b).

Scenario 3. Instead of unique, PDP had several maximuma, $\eta \approx 5.2$ (Fig.1 c, f, i; $\delta = 5.10^{-3}$). The τ_m $(\tau^2{}_m)$ in notations of scenario 2 was 10.6 (84.3) and 12.1 (88.3), respectively. Herein, we could expect both short-term and long-term predictions. Note, that the uncertainty of initial conditions plays a second role again. The tail of PDF can be directed along both short and long-term predictions.

IMPACT/APPLICATIONS

1.An original mathematical technique for the construction of orthogonal basis functions (normal modes) for semi-closed seas has been developed. This technique was successfully applied for nowcast of the daily circulation on Louisiana-Texas shelf reconstructed from the SCULP-1 drifter and LATEX data.

2.Several quantitative measures of the prediction skill have been suggested and examined. All these measure were based on the definition of the first passage time. We have demonstrated their explicit usefulness for the practical using in full oceanographic models.

3.Along with discovering the outlier events (predictions) numerically we confirmed their existence through the comparison of a high-resolution model and drifter data. The knowledge of statistics of outlier events should allow to considerably improve the prediction skill of oceanographic models.

TRANSITIONS

None

RELATED PROJECTS

None

PUBLICATIONS

Chu , P.C., and L. M. Ivanov, 2001: Probabilistic stability of a simple atmospheric model to various amplitude perturbations. J. Atmos. Sci., (revised).

Chu, P.C., L.M.Ivanov, T.P.Korzhova, T.M.Margolina, and O.V.Melnichenko, 2001: Reconstruction of ocean currents from sparse and noisy data in an open domain. Part 1.Theory. J. Geophys. Res., (in revision)

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