Network synthesis for prescribed transient response using trigonometric series approximations

Rodman, William Blount
Massachusetts Institute of Technology

https://hdl.handle.net/10945/24774

Downloaded from NPS Archive: Calhoun
NETWORK SYNTHESIS FOR
PRESCRIBED TRANSIENT RESPONSE
USING TRIGONOMETRIC SERIES APPROXIMATIONS

by

WILLIAM BLOUNT RODMAN IV

Course VI       June 18, 1952
Cameo Cover
NO. C-11
ALL XL FASTENERS FOR VIND NG SHEETS.
NETWORK SYNTHESIS FOR PRESCRIBED TRANSIENT RESPONSE USING TRIGONOMETRIC SERIES APPROXIMATIONS

by

WILLIAM BLOUNT RODMAN IV

B.S., U.S. Naval Academy (1941)

B.S., U.S. Naval Postgraduate School (1951)

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY (1952)

Signature of Author

Department of Electrical Engineering,
June 18, 1952.

Certified by

Thesis Supervisor

Chairman, Departmental Committee on Graduate Students
THE UNIVERSITY OF WASHINGTON

DEPARTMENT OF ELECTRICAL ENGINEERING

A dissertation submitted to the University of Washington in partial fulfillment of the requirements for the degree of


D. S. C.

Electrical Engineering

PUBLICATION

R. U. . . . . . .

R. U. . . . . . .

I. R. I.

I. R. I.
ABSTRACT

NETWORK SYNTHESIS FOR PRESCRIBED TRANSIENT RESPONSE USING TRIGONOMETRIC SERIES APPROXIMATIONS

by

WILLIAM BLAUNT RODMAN IV

Submitted to the Department of Electrical Engineering on 18 June, 1952, in partial fulfillment of the requirements for the degree of Master of Science.

Dr. E. A. Guillemin has proposed a means of network synthesis for prescribed transient response which permits approximations to be made in the time domain rather than the frequency domain. The desired transient is obtained from an appropriate combination of auxiliary periodic functions such that the result cancels everywhere except over the period of the transient. The system function is obtained from the transforms of trigonometric series approximations to these auxiliary periodic functions. The system function thus obtained is not always realizable, but it appears that approximations can be made such that it will be realizable.

On the assumption that the system function be realizable, or that it can be made so, a synthesis procedure is developed. The transient is decomposed into two components having, respectively, even and odd symmetry about the midpoint. The system functions determined for these components satisfy the requirements for synthesis as a lossless network terminated in a resistance. Such networks are synthesized for each component. These two networks are then connected "back to back" to realize a network for the overall system function.

Thesis Supervisor

E. A. Guillemin

Title

Professor of Electrical Engineering
TABLE OF CONTENTS

ABSTRACT 1

INTRODUCTION 111

CHAPTER I  DERIVATION OF THE TRANSFER FUNCTION TO BE REALIZED 1.

CHAPTER II  DEVELOPMENT OF A SYNTHESIS PROCEDURE 5.

CHAPTER III  AN EXAMPLE OF THE SYNTHESIS PROCEDURE 12.

BIBLIOGRAPHY 15.
The problem of synthesizing a finite, lumped parameter, linear, passive network for prescribed transient response has been attacked by forming the ratio of the transforms of the output and input functions to give the system function $h(s)$. The system function has then been synthesized into a network with the necessary approximations being made in the frequency domain. Results obtained have varied from good to poor, and it appears that the degree of approximation obtained in the time domain cannot readily be determined from the degree of approximation made in the frequency domain.

A means of making the required approximations in the time domain, instead of the frequency domain, would permit control of the time form of the transient. One such means available is the finite trigonometric series, which can be made to approximate a given periodic time function to any desired degree of accuracy. An appropriate combination of periodic functions based on the desired transient can then be made such that the combination cancels everywhere except over the period of the transient.
The document contains text that is not legible due to the quality of the image. It appears to be a page from a book or a report, but the content cannot be accurately transcribed or interpreted.
CHAPTER I

DERIVATION OF THE TRANSFER FUNCTION TO BE REALIZED

The following method of representing a transient of finite duration by an appropriate combination of semi-periodic functions has been proposed. Define the desired transient as \( f(t) \).

\[
\begin{align*}
    f(t) &
\end{align*}
\]

\[t\]

Fig. 1
The Desired Transient

Define a semi-periodic function, \( f_p(t) \) as follows:

\[
f_p(t) = \begin{cases} 
    0 & t < 0 \\
    f(t) & 0 < t < \frac{\tau}{2} \\
    0 & \frac{\tau}{2} < t < \tau 
\end{cases}
\]

\[f_p(t)\]

\[0 \quad \frac{\tau}{2} \quad \tau \quad \frac{1}{2} \tau \quad 2\tau \quad \frac{5}{2} \tau \]

Fig. 2
Semi-Periodic Function \( f_p(t) \) Derived from \( f(t) \)
\( f_p(t) \) is representable in a Fourier series as:

\[
f_p(t) = \sum_{k = -\infty}^{\infty} \alpha_k e^{jk\omega t} \quad (t > 0)
\]

and its transform \( h_p(s) \) is represented as:

\[
h_p(s) = \sum_{k = -\infty}^{\infty} \frac{\alpha_k}{s - jk\omega}.
\]

Now define two additional functions, \( f_1(t) \) and \( f_2(t) \) as follows:

\[
f_1(t) \equiv f_p(t) + f_p(t - \frac{\tau}{2})
\]

\[
f_2(t) \equiv f_p(t) - f_p(t - \frac{\tau}{2})
\]

![Fig. 3](image)

**Functions** \( f_1(t) \) and \( f_2(t) \), Derived From \( f_p(t) \)

The function

\[
f_p(t - \frac{\tau}{2}) = \sum_{k = -\infty}^{\infty} \alpha_k e^{jk\omega(t - \frac{\tau}{2})} = \sum_{k = -\infty}^{\infty} \alpha_k e^{jk\omega t} \cdot e^{-jk\frac{\tau}{2}}.
\]
Note that $\theta = \frac{\pi}{4}$

then $f_p(\tau-t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega t} = \sum_{k=0, \pm 2, \pm 4, \ldots}^{\infty} \alpha_k e^{jk\omega t}$

whence $f_1(t) = 2 \sum_{k=0, \pm 2, \pm 4, \ldots}^{\infty} \alpha_k e^{jk\omega t}$

and $f_2(t) = 2 \sum_{k=1, 3, 5, \ldots}^{\infty} \alpha_k e^{jk\omega t}$

The transforms of $f_1(t)$ and $f_2(t)$ are:

$h_1(s) = h_p(s) \left[ 1 + e^{-\frac{s\tau}{2}} \right] = 2 \sum_{k=0, \pm 2, \pm 4, \ldots}^{\infty} \frac{\alpha_k}{s - jk\omega}$

$h_2(s) = h_p(s) \left[ 1 - e^{-\frac{s\tau}{2}} \right] = 2 \sum_{k=1, 3, 5, \ldots}^{\infty} \frac{\alpha_k}{s - jk\omega}$

Following a line of physical reasoning, these functions are combined to form a new function, $h(s)$ given by $h(s) = \frac{h_1(s)h_2(s)}{h_1 + h_2}$. Direct substitution shows that $h(s) = \frac{h_p(s)}{2} \left[ 1 - e^{-s\tau} \right]$. The inverse transform of $h(s)$ is readily recognized as

$$\frac{1}{2} \left\{ f_p(t) - f_p(t-\tau) \right\} = \frac{1}{2} f(t).$$
In general the Fourier series representation of \( f_p(t) \) will be an infinite series. Representing finite approximations by *, then:

\[
f^*_p(t) = \sum_{k=-n}^{n} \alpha_k e^{j\omega_k t} \quad (t > 0)
\]

\[
h^*_p(s) = \sum_{k=-n}^{n} \frac{\alpha_k}{s - j\omega_k} \equiv \frac{p(s)}{q(s)}
\]

\[
h^*_1(s) = \sum_{k=0, \pm 2, \pm 4, \ldots} \frac{2\alpha_k}{s - j\omega_k} \equiv \frac{p_1(s)}{q_1(s)}
\]

\[
h^*_2(s) = \sum_{k=\pm 1, \pm 3, \pm 5, \ldots} \frac{2\alpha_k}{s - j\omega_k} \equiv \frac{p_2(s)}{q_2(s)}
\]

\[
h^*_1(s) + h^*_2(s) = 2h^*_p(s) = \frac{2p(s)}{q(s)}
\]

\[
h^*_1 \cdot h^*_2 = \frac{p_1 \cdot p_2}{q_1 q_2} = \frac{p \cdot p_2}{q}
\]

\[
h^*(s) = \frac{p \cdot p_2}{2q}
\]
(1) \( \sum_{i=0}^{n} (a)^* = (a)^* \)

\[
\frac{d}{dx} \frac{x^2}{x^2 + 1} = \frac{2x(x^2 + 1) - x^2(2x)}{(x^2 + 1)^2} = \frac{2x^3 + 2x - 2x^3}{(x^2 + 1)^2} = \frac{2x}{(x^2 + 1)^2}
\]

\[
\frac{\frac{d}{dx} x^3}{\frac{d}{dx} x^2} = \frac{3x^2}{2x} = \frac{3}{2} x
\]

\[
\frac{\frac{d}{dx} x^4}{\frac{d}{dx} x^2} = \frac{4x^3}{2x} = 2x^2
\]

\[
\frac{\frac{d}{dx} x^5}{\frac{d}{dx} x^2} = \frac{5x^4}{2x} = \frac{5}{2} x^3
\]
CHAPTER II

DEVELOPMENT OF A SYNTHESIS PROCEDURE

The purpose here is to develop a practical means of synthesis of a network whose system function is $h^*(s)$, that is, a network whose response to a unit impulse function is $f^*(t)$.

The necessary and sufficient conditions for the realizability of a network whose transfer impedance is $h^*(s)$ are:

(a) The numerator of $h^*(s)$ be of degree not greater than the denominator and,

(b) The denominator have no zeros in the right half s-plane, i.e., the denominator must be a Hurwitz polynomial.

That condition (a) is fulfilled is evident from an expansion of the functions involved. The fulfillment of condition (b) is subject to discussion beyond the scope of this work. It can be demonstrated that, using the Fourier coefficients in a finite approximation of $f_p(t)$, certain functions yield a denominator polynomial which is Hurwitz while others do not. There appears to be a possibility of so selecting the coefficients that the denominator polynomial will be Hurwitz. In any event, this condition must

* Note: Throughout the following development the word "admittance" may be substituted for "impedance" without invalidating the arguments.
I wish to present the following argument.

(a) is an assumption made when assuming it to be a starting point of a thought experiment. This is to reconsider some premises and thus

(b) is to observe if (a) is the underpinning of (a). This is a foundational step and must be considered. That is why

(c) is to evaluate if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(d) is to ensure if (a) is the underpinning of (a). Just as considering any valid premises

(e) is to observe if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(f) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(g) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(h) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

(i) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(j) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(k) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

(l) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(m) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(n) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

(o) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(p) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(q) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

(r) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(s) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(t) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

(u) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(v) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(w) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

(x) is to ensure if (a) is the underpinning of (a). It is mandatory to determine if the (a) is valid. One must therefore

(y) is to evaluate if (a) is the underpinning of (a). Just as considering any valid premises

(z) is to observe if (a) is the underpinning of (a). Just as considering any valid premises

...
be met before synthesis is possible.

Assume, therefore, that means have been devised to assure the Hurwitz character of the denominator polynomial, or that it is desired to synthesize a network for a function which yields one. It is advisable, in the interests of brevity, to normalize at \( \omega = 1 \), and to shift from complex coefficients to trigonometric coefficients.

Let the duration of \( f^*(t) \) be \( u \), then the period of \( f^*_p(t) \) is \( 2\pi \), and \( \omega = \frac{2\pi}{T} = 1 \).

\[
h^*_p(s) = \sum_{k=1}^{n} \frac{\alpha_k}{s + \frac{\alpha_k}{\omega}} + \sum_{k=1}^{n} \frac{(\alpha_k + \alpha_{-k})s + jk(\alpha_k - \alpha_{-k})}{s^2 + k^2}.
\]

Noting that \((\alpha_k + \alpha_{-k})\) and \(j(\alpha_k - \alpha_{-k})\) are respectively \(a_k\) and \(b_k\), the usual trigonometric coefficients, by defining \(a_0 = \frac{1}{2\pi} \int_0^{2\pi} f_p(t)dt\), \(h^*_p(s)\) can be written as:

\[
(1) \quad h^*_p(s) = \sum_{k=0}^{n} \frac{a_k s + kb_k}{s^2 + k^2} = \frac{p}{q} = \frac{p_1}{2q_1} + \frac{p_2}{2q_2}
\]

\[
(2) \quad h^*(s) = \frac{p_1p_2}{2p^2}, \text{ therefore the poles of } h^*(s) \text{ are located at the zeros of } P(s). \text{ These zeros are } 2n \text{ in number and are located in the left half plane (by assumption). No other general properties of these zeros are readily apparent, and it appears that the problem of locating them would be of}
\]
\[
\left( \frac{s}{x} + \frac{s}{y} \right) \sum_{n=2}^{\infty} \frac{\beta}{n^2} = \frac{\beta}{x} + \frac{\beta}{y} - \left( \frac{s}{x} + \frac{s}{y} \right) \sum_{n=2}^{\infty} \frac{\beta n}{n^2} = \frac{\beta}{x} + \frac{\beta}{y} - \left( \frac{s}{x} + \frac{s}{y} \right) \sum_{n=2}^{\infty} \frac{\beta}{n^2} = \left( \frac{s}{x} \right)^* + \left( \frac{s}{y} \right)^* \]

\]

The above equation can be used to calculate the value of \( \left( \frac{s}{x} \right)^* + \left( \frac{s}{y} \right)^* \).
some degree of difficulty. This problem can be avoided, however, if lossless networks terminated in resistances are acceptable.

Recalling from Chapter I that $h_1^*(s)$ and $h_2^*(s)$ are respectively twice the sums of the odd order and even order terms of $h_p^*(s)$

\[
(3) \quad h_1^*(s) = 2 \sum_{k=0, 2, 4}^{n} \frac{a_k s + kb_k}{s^2 + k^2} = 2 \left\{ \frac{a_0}{s^2 + 4} + \frac{a_2 s + 2b_2}{s^2 + 4} \right\} = \frac{P_1}{Q_1},
\]

\[
(4) \quad h_2^*(s) = 2 \sum_{k=1, 3, 5}^{n} \frac{a_k s + kb_k}{s^2 + k^2} = 2 \left\{ \frac{a_2 s + b_2}{s^2 + 1} \right\} = \frac{P_2}{Q_2}.
\]

Note now that if odd $a_k$ and even $b_k$ are zero then $P_1$, $P_2$, and $Q_2$ are all even functions of $s$, while $Q_1$ is an odd function of $s$. Also if even $a_k$ and odd $b_k$ are zero then $P_1$, $Q_1$, and $Q_2$ are even in $s$, while $P_2$ is odd in $s$. Thus the product $P_1Q_2$ is always even in $s$ and the product $P_2Q_1$ is always odd in $s$.

From equation (1) it is evident that

\[
(5) \quad P = \frac{1}{2} \left\{ P_1Q_2 + P_2Q_1 \right\},
\]
\[
\left\{ \begin{array}{l}
\frac{d^{2}y}{dt^{2}} = \frac{f(t, y)}{g(t, y)} \\
\frac{d^{2}z}{dt^{2}} = \frac{h(t, z)}{i(t, z)}
\end{array} \right. \]

subject to

\[
\left\{ \begin{array}{l}
y(t_0) = \gamma(t_0) \\
y'(t_0) = \gamma'(t_0)
\end{array} \right. \quad \text{and} \quad 
\left\{ \begin{array}{l}
z(t_0) = \zeta(t_0) \\
z'(t_0) = \zeta'(t_0)
\end{array} \right.
\]

with initial conditions

\[
\begin{aligned}
\left\{ \begin{array}{l}
y(t) = (k y(t))^* \\
y'(t) = (k y'(t))^*
\end{array} \right. & \quad \text{for} \quad t \in [a, b] \\
\left\{ \begin{array}{l}
z(t) = (k z(t))^* \\
z'(t) = (k z'(t))^*
\end{array} \right. & \quad \text{for} \quad t \in [a, b]
\end{aligned}
\]
so that for the special conditions that odd cosine and
even sine coefficients be zero, or that even cosine and
odd sine coefficients be zero, the odd and even $s$ terms
in $P(s)$ are identified with the poles and zeros of $h_1^*(s)$
and $h_2^*(s)$.

The necessary and sufficient conditions for a
polynomial with real coefficients to be a Hurwitz poly-
nomial are that the zeros of its odd and even parts lie on
the $j$ axis where they mutually separate each other. Fur-
ther, the ratio of the even or odd parts of a Hurwitz poly-
nomial is a reactance (susceptance) function. This implies,
for $P$ Hurwitz, that the zeros of $P_1$ and $P_2$ lie on the $j$
axis and that the function $\frac{P_1q_2}{P_2q_1}$ be a reactance function,
for the special conditions considered. The following
manipulations are therefore suggested:

$$
\text{h}^*(s) = \frac{P_1P_2}{2P} = \frac{P_1P_2}{P_1q_2 + P_2q_1},
$$

dividing numerator and denominator by $P_2q_1$ yields:

$$
\text{h}^*(s) = \frac{\frac{P_1}{q_1}}{1 + \frac{\frac{1}{P_1q_2}}{P_2q_1}}
$$

Now associate $\text{h}^*(s)$ with $z_{12}(s) = \frac{z_{12}}{1 + z_{22}}$ which is
the form for the transfer impedance of a lossless network
terminated in a one ohm resistance. $z_{12}$ and $z_{22}$ are
The solution for this problem involves applying the principle of conservation of momentum to the system. If we assume that the moving object has an initial velocity $v$ and a mass $m$, and the stationary object has a mass $M$, the momentum conservation equation is:

$$m \cdot v = (m + M) \cdot \frac{v}{2}$$

Solving for $v$, we get:

$$\frac{m}{m + M} = \frac{1}{2}$$

This equation represents the fraction of the initial velocity that the moving object will maintain after the collision under the assumption of inelastic impact and conservation of momentum. Therefore, the velocity of the moving object after the collision is half of its initial velocity if the masses are equal.

By applying this principle, we can analyze the dynamics of systems with multiple objects and understand the implications of different mass configurations on the outcome of collisions.
respectively the transfer and driving point functions for the lossless part of the network.

In making this association the following conditions are noted:

(a) $z_{22}$ is a reactance function.
(b) $z_{12}$ has its poles and zeros restricted to the $j$ axis.
(c) all the poles of $z_{12}$ are contained in $z_{22}$.
(d) $z_{22}$ has poles, at the zeros of $P_2$, which are not contained in $z_{12}$.

Conditions (a), (b), and (c) are sufficient to insure synthesis as a lossless ladder network terminated in a resistance. The poles of $z_{22}$ which are not contained in $z_{12}$ represent lossless parallel tuned circuits in series with the load resistor.

At this point the formal procedure of separating $h_1^*(s)$ into two parts, each of which fulfills one set of the requirements for this type of synthesis, would lead to a general synthesis procedure. It is more enlightening, however, to first observe the time form of the transients which these conditions represent. Using Fourier series coefficients and noting that $f_p(t) \equiv 0$ for the interval $\pi < t < 2\pi$
The document contains mathematical and possibly scientific content. It appears to be discussing equations or formulas, possibly related to physics or mathematics. The text is not clearly legible due to the image quality, but it seems to involve terms like \( a \) and \( b \), and possibly \( \beta \). There are symbols and possibly Greek letters involved in the equations.

The text also includes some mathematical concepts like transcendence theory, which suggests a discussion of numbers that are not algebraic. The presence of terms like \( \gamma \), \( \Omega \), and \( \eta \) indicates a complex mathematical discourse, potentially involving advanced calculus or number theory.

Without clearer visibility, it's challenging to provide a precise transcription or interpretation of the content. However, it's evident that the document deals with sophisticated mathematical or scientific topics.
\[ a_0 = \frac{1}{2\pi} \int_0^\pi f_p(t) \, dt \]
\[ a_k = \frac{1}{\pi} \int_0^\pi f_p(t) \cos kt \, dt \]
\[ b_k = \frac{1}{\pi} \int_0^\pi f_p(t) \sin kt \, dt \]

Since even cosine terms and odd sine terms have even symmetry about \( \frac{\pi}{2} \), if \( f_p(\frac{\pi}{2} + t) = f_p(\frac{\pi}{2} - t) \), then odd \( a_k \) and even \( b_k \) are zero. And since odd cosine terms and even sine terms have odd symmetry about \( \frac{\pi}{2} \), if \( f_p(\frac{\pi}{2} + t) = -f_p(\frac{\pi}{2} - t) \), then even \( a_k \) and odd \( b_k \) are zero. Thus the special conditions occur when the transient has even or odd symmetry about its midpoint.

**Fig. 4**
Transient For Which Resulting Odd \( a_k \) And Even \( b_k \) Are Zero

**Fig. 5**
Transient For Which Resulting Even \( a_k \) And Odd \( b_k \) Are Zero
\begin{align*}
\sin^2 \theta + \cos^2 \theta &= 1 \\
\sin (\theta + \phi) &= \sin \theta \cos \phi + \cos \theta \sin \phi \\
\cos (\theta + \phi) &= \cos \theta \cos \phi - \sin \theta \sin \phi
\end{align*}

\[ \text{Graph of } \sin \theta \]

\[ \text{Graph of } \cos \theta \]
Any arbitrary transient may be considered as the sum of two other transients, each having one of the above types of symmetry. To synthesize a network for an arbitrary transient response, then, first decompose the transient into its components having odd and even symmetry about the midpoint. Synthesize a network for each component. After suitable impedance leveling, connect the networks back to back and the overall transfer impedance function is realized.

![Network Diagram](image)

**Fig. 6**

A Form Of Network To Synthesize For A Realizable Transient Response
To use the model presented in the previous studies and make an appropriate design, we must consider the following factors: stability, efficiency, and cost. The model must be designed to ensure stability while minimizing costs and maximizing efficiency. The diagram illustrates the proposed design with the key components labeled. Further analysis and optimization are necessary to ensure the model meets the desired performance criteria.
AN EXAMPLE OF THE SYNTHESIS PROCEDURE

Let the desired transient response to a unit impulse input be a rectangular one as shown:

Then $f_p(t)$ is

\[
a_0 = \frac{1}{2\pi} \int_0^{2\pi} f_p(t) \, dt = \frac{\pi}{2}
\]

\[
a_k = \frac{1}{\pi} \int_0^{2\pi} f_p(t) \cos kt \, dt = \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{2} \sin kt \, dt = 0 \quad (k \neq 0)
\]

\[
b_k = \frac{1}{\pi} \int_0^{2\pi} f_p(t) \sin kt \, dt = \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{2} \sin kt \, dt = \frac{1}{2k} (1 - \cos k \pi)
\]

\[
\therefore b_k = \frac{1}{k} \quad \text{for } k \text{ odd}
\]

\[
b_k = 0 \quad \text{for } k \text{ even.}
\]
The text is not clearly readable due to the handwriting style. It appears to be discussing a mathematical problem involving trigonometric functions and integrals. The diagram suggests the integration of a trigonometric function over a certain interval, possibly involving the evaluation of definite integrals.

The integral expression might look something like:

\[ \int f(x) \, dx \]

Where \( f(x) \) is a trigonometric function, and the limits of integration are given by the endpoints of the interval of interest. The diagram shows a graph with labels and axes, indicating where the function is defined and the limits of integration.
Note that odd $a_k$ and even $b_k$ are zero since the transient has even symmetry about its midpoint. In this instance all $a_k$ are zero except $a_0$. This fact serves to simplify the network rather than invalidate the procedures developed.

Assume that it has been established that an approximation of six terms will be sufficiently accurate.

$$h^*(s) = \sum_{k=0}^{6} \frac{a_k s + k b_k}{s^2 + k^2} = \frac{\pi/4}{s} + \frac{1}{s^2 + 1} + \frac{1}{s^2 + 9} + \frac{1}{s^2 + 25}$$

$$h_1^*(s) = \frac{\pi/4}{s} = \frac{P_1}{Q_1}$$

$$h_2^*(s) = \frac{1}{s^2 + 1} + \frac{1}{s^2 + 9} + \frac{1}{s^2 + 25} = \frac{P_2}{Q_2}$$

$$P_1 = \frac{\pi}{4}$$

$$Q_1 = s$$

$$P_2 = (s^2 + 1)(s^2 + 9) + (s^2 + 1)(s^2 + 25) + (s^2 + 9)(s^2 + 25)$$

$$= 3(s^2 + 4.61132)(s^2 + 18.7702)$$

$$Q_2 = (s^2 + 1)(s^2 + 9)(s^2 + 25)$$

$$z_{12}(s) = \frac{P_1}{Q_1} = \frac{\pi/4}{s}$$

$$z_{22}(s) = \frac{P_1 Q_2}{P_2 Q_1} = \frac{(\pi/4)(s^2 + 1)(s^2 + 9)(s^2 + 25)}{3s (s^2 + 4.61132)(s^2 + 18.7702)}$$
\[
\frac{x}{x+y} + \frac{y}{x+y} = \frac{2y}{x+y} = \left(\frac{\Delta x}{x+y}\right) = (y)_{x+y}
\]

\[
\frac{x}{y} + \frac{y}{x} + \frac{x}{x+y} = (y)_{x+y}
\]

\[
\frac{\Delta y}{y} = \frac{y}{x+y} = (y)_{x+y}
\]

\[
(2x^2 + 6a)(x + a + 2x + 5a + (x + a + 5a)(x + 9a) = 0
\]

\[
(y^2 - x^2)(2x^2 + 6x + 5a) = 0
\]

\[
\left(\frac{\Delta x}{x^2}\right) = (x)_{x^2}
\]

\[
\left(\frac{\Delta y}{y^2}\right) = (y)_{y^2}
\]

\[
\left(\frac{\Delta z}{z^2}\right) = (z)_{z^2}
\]
Removing the factor of \( \frac{H}{12} \) results in the following:

\[
z_{22} = s + \frac{2.606}{s} + \frac{4.686s}{s^2 + 4.61132} + \frac{4.094s}{s^2 + 18.72202}
\]

\[R = \frac{12}{H}\]

\[z_{12} = \frac{2}{s}\]

If only the rectangular form of the transient is required the impedance level may be reduced by a factor of \( \frac{3}{2.606} \) yielding

\[
z_{22} = s + \frac{2.606}{s} + \frac{4.686s}{s^2 + 4.61132} + \frac{4.094s}{s^2 + 18.72202}
\]

\[R = 3.318\]

\[z_{12} = \frac{2.606}{s}\]

The network is then realized as

---

**Fig. 8**

Network For Approximating Rectangular Transient Response To A Unit Impulse

(Ohms, Henries, Farads)
\[
\frac{\frac{4203.4}{1000} \cdot r}{\frac{4230.0}{800} + 2 \cdot \frac{21.9}{1000} - \frac{200.0}{1000} + \frac{200.0}{1000} + 120.9}{2} + 120.9 = 100.3
\]

\[
\frac{4203.4}{1000} \cdot r = 100.3
\]

\[
r = \frac{100.3}{4203.4} \cdot 1000
\]

The simplified diagram is as follows: 

![Simplified Diagram](image-url)
BIBLIOGRAPHY


Network synthesis for prescribed transient response using trigonometric series approximations.