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A COMPARISON OF ALL-DAY TIME STUDY VERSUS WORK SAMPLING FOR MEASURING WORK IN A NAVY SUPPLY DEPARTMENT

A Thesis

Submitted to the Faculty

of

Purdue University

by

Benedict Joseph Scott

In Partial Fulfillment of the

Requirements for the Degree

of

Master of Science

in Industrial Engineering

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To Captain R. F. Scott, USN, Commanding Officer, Naval Ordnance Plant, Indianapolis, Indiana; to Commander H. R. Johnson, USN, Supply Officer; and to the employees of the Material Division of the Supply Department of the Naval Ordnance Plant whose sincere and extremely helpful cooperation greatly aided this investigation, the author wishes to express his deepest gratitude.

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ABSTRACT

ABSTRACT

Scott, Benedict Joseph. MS(IE), Purdue University, June 1955. "A Comparison of All-day Time Study Versus Work Sampling For Measuring Work in a Navy Supply Department." Major Professor: H. H. Young.

The basic objective of this study was to compare the merits of All-day Time Study vs. Work Sampling as methods of measuring work for variable cycle operations and for indirect labor activity. Comparison was made on the basis of experience and training of analysts necessary for each type of study, the possible psychological effects on the worker caused by the presence of the observer, the preparation time necessary to organize each type of study, the time required to make the observations, and the accuracy of each method.

Necessarily, all of the factors of comparison were subjective except for "time required" and "accuracy" of each method. Data collected in the laboratory was treated by the statistical analysis of variance technique, using procedures developed in Appendix A. This technique gives estimates of error of each method, and, thereby, gives an objective basis for comparison of the accuracy of, and the time required for, each method.

Eight operators in the Supply Department, U. S. Naval Ordnance Plant, Indianapolis, Indiana, were observed for four days under a typical work sampling plan, and four of these eight operators were observed for two different days each using all-day time study. The primary item of data collected was the time spent on the different functions of the job. This collected data was grouped and studied by the analysis of variance technique to isolate and estimate the error in the average time spent on each function.

Both of the studied methods of work measurement require about the same amount of training time to assure that the observer has sufficient know-how to make a reliable work measurement study. Three weeks of formalized training would be considered as the minimum to provide competent data takers under the guidance of an experienced industrial engineer or time study analyst. However, the observer requires more knowledge of the work being measured to obtain accurate data by work sampling than is necessary for the all-day study. This is primarily due to the fact that the continuity of actions observed in the all-day time study aids the analyst to recognize the functions, while in the work sampling study the analyst must decide on a function from an instantaneous observation.

The continued presence of the observer in an all-day study has a larger influence on the actions of the operator than does the occasional appearance of the observer in work

sampling. In this study, due to the harmonious relationship between the observer and the operator, the effect was beneficial. This is the subjective opinion of the writer, but was substantiated by the statistical data. The results for the daily variability of each worker (σ^2_{wd}) were less for the all-day study, indicating that the operators endeavored to cooperate with the writer by showing him their version of a typical day each time they were observed. This gave a smaller error to the measured mean, but indicates an acute awareness of the presence of the analyst. Had relations been less cordial the results could have been less satisfactory.

Work sampling will require more time on the part of the analyst to set-up a study. This time will be spent drawing random numbers and coding the parameters in order to obtain a truly random sample of the total available population of observation times. In addition, to obtain the desired accuracy or to measure the accuracy obtained, the data for either work sampling or all-day study should be treated by an analysis of variance. A little practice at the analysis of variance technique will soon enable an analyst to obtain a high degree of skill and accuracy.

The overall statistical results indicate that the work sampling plan used gave just as accurate a measurement as the all-day study, but required only one half of the observation time (and cost). Individual compon-

ents of error showed no difference between the two methods for detecting the variance between men and days, but, as stated above, the all-day study gave a better estimate of the daily variability of each man. However, this component is subject to the cooperative attitude of the operator and could be strongly influenced by an antagonistic worker. Also, this advantage was more than compensated for, in this study, by the increased efficiency of the work sampling study.

INTRODUCTION

This study was conducted in an endeavor to compare all-day time study with the work sampling, or ratio-delay, technique as methods of work measurement for variable cycle operations and for indirect labor activity. The basic objective was to decide which is the more desirable method of estimating the average work distribution throughout an accounting period (day or week), when the actual distribution varies from day to day and is only constant over a longer time element such as a week, month, or quarter. Comparison was made on the basis of experience and training of analysts necessary for each type of study, the possible psychological effects on the worker, the preparation time necessary to set-up each study, the time required to make the observations and the accuracy of each method. Obviously, observation time and the accuracy of each method are closely related and must be considered as variables to a single factor.

The estimate of error of each method can be calculated from observed variations among men, days, and the daily variation of each man. This experiment was developed to estimate these three components of variance. Certain restrictions were placed on the results because of experimen-

tal limitations imposed by practical situations. Other factors for comparison are somewhat subjective, and these results will therefore be an expression of the writer's opinion of the difficulties associated with each method of study, at least under the particular conditions of this analysis.

Experience and training needed to prepare time study analysts varies according to the complexity of the industrial situation being studied and the intended use of the results of the work measurement program. In order to obtain a knowledge of the actual mechanics of taking all-day time studies, making adjustments and allowances, and presenting the data, a three week period of instruction and practice will normally suffice (1). This presumes, of course, that the trainee will be under the close guidance of an experienced time study analyst when he starts to apply his newly learned procedures. The course of training given at the Rock Island Arsenal by the Army (2) consists of three weeks of concentrated instruction and practice and appears to satisfy the needs of the users. Some

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(1)Rock Island Arsenal, "Ordnance Management Engineering Training Program-Work Measurement Course Summary Session Outline", Rock Island, Ill., Rock Island Arsenal, undated.

(2)Ibid

time is devoted in this course to the use of standard data, and, in addition, one day is devoted to work sampling. There is some question about the value of only one day spent to cover the phases of work sampling that differ from all-day time study.

In a survey of the previous work done in this branch of the work measurement field, the writer noted a few prominent names. L. C. H. Tippett (3) is credited with introducing statistical methods to work measurement in 1935. The name "Ratio-Delay", commonly associated with this statistical approach, probably came from Mr. R. L. Morrow (4); however, it has been used in statistical quality control for some time. C. L. Brisley (5) receives credit for coining the name "work sampling" which is gaining favor because of its broader connotation. In this paper, work sampling will be used to indicate this broader application; and in subscripts, "R" for Random observations, will be employed for brevity. Some outstanding work in the field has been carried out by

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(3)Tippett, L. C. H. "Statistical Methods in Textile Research. Uses of the Binomial and Poisson Distribution. A Snap Reading Method of Making Time Studies of Machine and Operatives in Factory Surveys." Manchester, England, Journal of Textile Institute Transactions, vol. 26, Feb. 1935, 51-70.

(4)Morrow, R. L. "Time Study and Motion Economy." New York, Ronald Press Co., 1946, Chap. 16.

(5)Brisley, C. L. "How You Can Put Work Sampling to Work." Factory Management and Maintenance, July 1952, 84-89.

Abruzzi (6), who has included a thorough treatment of procedures for control chart application in his text "Work Measurement" (7). Increased interest in the use of work sampling is evidenced by the fact that most of the literature on the topic has been published since 1946.

In contrast, all-day time study has been used for measuring indirect labor and office work since the days of the Gilbreths (8). Dr. Fredrick W. Taylor (9) suggested the use of time studies for variable operations, but he spent most of his efforts on direct production applications. The work by Mitchell (10) in 1927 and Bills (11) in 1928, under the sponsorship of the American Management Association, seems to have been the first concentrated effort to develop standard procedures for use in the clerical field. Today, most scientifically managed

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(6)Abruzzi, A. "Delay Allowances by Statistical Methods." Columbia Engineering Quarterly, May 1948, 6-8, 23.

(7)Abruzzi, A. "Work Measurement - New Principles and Procedures." New York, Columbia University Press, 1952.

(8)Gilbreth, F. B. "Motion Study." New York, D. Van Nostrand Co., 1911, 88.

(9)Merrick, D. V. "Time Studies on a Basis for Rate Setting." New York, The Engineering Magazine Co., 1919.

(10)Mitchell, J. "Measuring Office Output." New York, American Management Association, Office Executive Series No. 29, 1927.

(11)Bills, M. A., et. al. "Measuring Office Output." New York, American Management Association, Office Executive Series No. 32, 1928.

enterprises use some form of work measurement, both for establishing incentives and for estimating budgets or labor requirements.

PROCEDURES

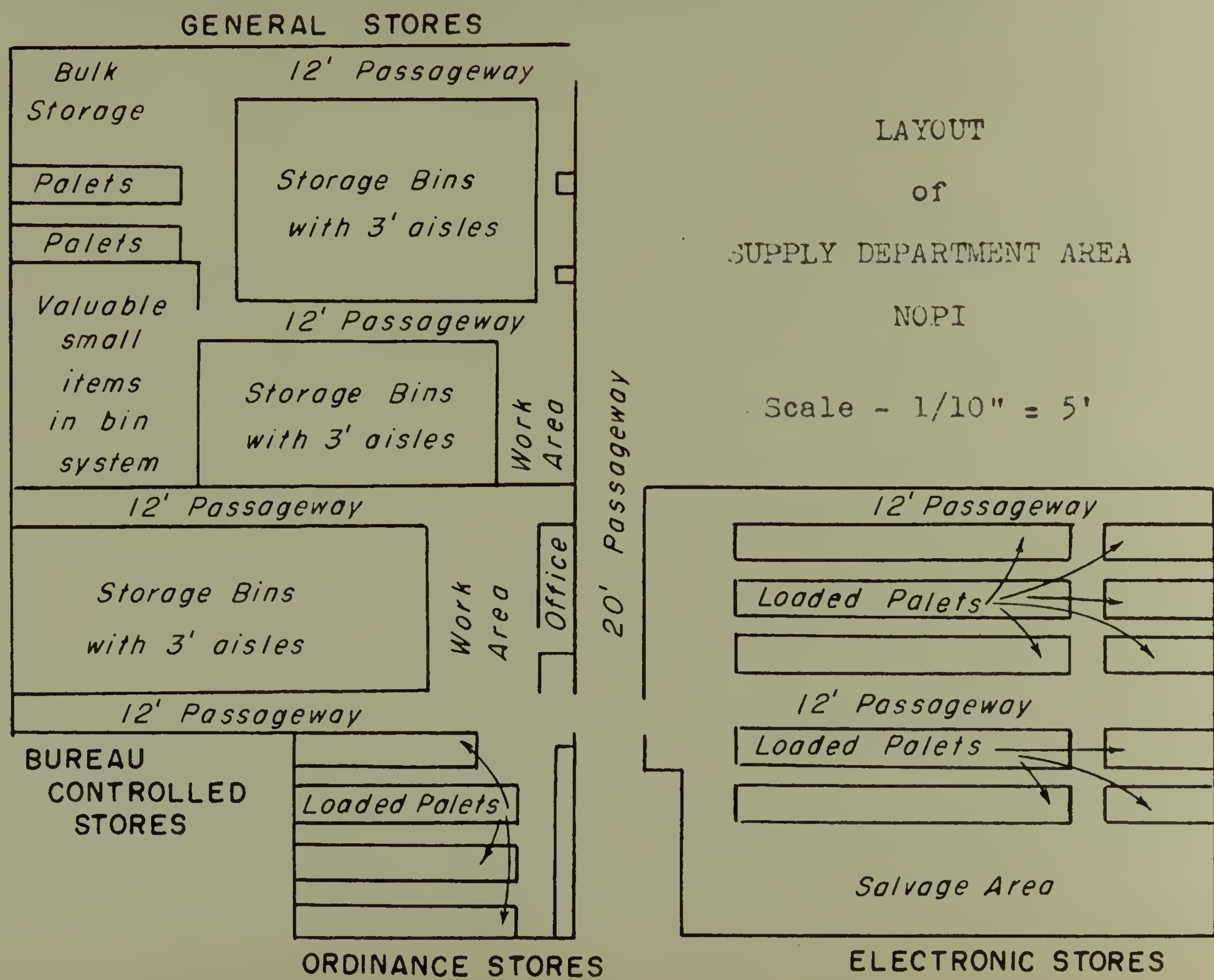
The procedures used in the collection of data will be outlined in detail. It was necessary because of practical considerations to depart slightly from the desired experimental procedures, and these departures have all been either justified or noted. The first and most important consideration is that the data were collected under actual operating conditions, and, as in all industrial situations, the idealistic experiment gave way to practical necessity, requiring adjustment of some procedures. Contrary to the implication that this might detract from the value of the study, it is the writer's belief that this will give a better basis for relying on any data collected in this manner as being realistic and indicative of what might normally be expected.

The Supply Department, U. S. Naval Ordnance Plant, Indianapolis, Indiana, hereinafter abbreviated as NOPI, was used as a typical non-cyclic work area for the collection of this data. The Supply Department is made up of five divisions: Administration and Planning, Inventory, Control, Material, and Fiscal. Of these the Material division and the Control division each had a fairly large

number of employees performing relatively homogeneous functions within the division. The Material division was selected because of the lesser time needed for the writer to learn to recognize the functions performed.

The sections within the Material division selected for the study were General Stores, Ordnance Stores, Electronic Stores, and Bureau Controlled Stores. This selection was based on the layout of the plant. The four sections were sufficiently close together that a work sampling study of eight operators could be conducted with a minimum of lapsed time between observations. In practice, one minute was taken as the period of observation, and only three observations were missed during the entire period of the study. Fig. 1 is a sketch of the plant layout of these four Material sections.

The statistical design of the experiment called for an all-day time study on four operators for two days each (a total of eight days), and a work sampling study on a minimum of eight operators for four other days. Only one operator was observed at a time in the all-day study. The four operators observed under the all-day study, Operators 1 through 4, were also observed in the work sampling study. The other four operators in the work sampling study were selected from the same sections and perform approximately the same type of work. Thus, Operator 1, from Bureau Controlled Stores, was studied under both plans; and



LAYOUT
of
SUPPLY DEPARTMENT AREA
NOPI
Scale - 1/10" = 5'
GENERAL STORES
Bulk Storage
Palets
Palets
Valuable small items in bin system
12' Passageway
Storage Bins with 3' aisles
12' Passageway
Storage Bins with 3' aisles
Work Area
12' Passageway
Storage Bins with 3' aisles
Work Area
Office
12' Passageway
BUREAU CONTROLLED STORES
Loaded Palets
ORDINANCE STORES
20' Passageway
12' Passageway
Loaded Palets
12' Passageway
Loaded Palets
Salvage Area
ELECTRONIC STORES

FIGURE 1.

Operator 5, also from Bureau Stores, and performing the same type of work, was included only in the work sampling study. The same pattern is true for Operators 2 and 6, 3 and 7, and 4 and 8.

The four sections within the Material division had forty-six full time employees. Since the study was to be conducted over several months, in order to fit into the academic schedule of the writer, employees to be observed were picked for their low record of absenteeism. Elimination of supervisors and employees with high absentee rates, and the necessity to have at least two employees in each section who performed fairly similar functions, somewhat narrowed the selections. Since the purpose of the study was to compare two methods of work measurement and not to endeavor to accurately estimate the work done within the division, the fact that the quality of the employees selected was above average is not important. It was desirable, however, that all of the operators to be studied be selected in the same way. Since four of the eight operators were included under both plans, it was only necessary to make a random selection of two employees from each section.

Because the writer was only casually familiar with the type of work performed in the Supply Department at NCPI, the data for the all-day time study was taken first. This was advantageous in that the continuity of elements

that added up to a discrete function could be observed, and the function easily recognized. At the same time the writer was, in effect, subjected to eight days of training in the recognition of the functions by recognizing a single element, a series of elements, or a form used; training that was mandatory before a function could be discerned in a single minute's observation as required by work sampling. Nevertheless, it was necessary to spend two days in the plant, preliminary to the taking of any data, in order for the writer to become acquainted with the employees and to learn to recognize the functions performed. The word "function" as used in this paper is intended to denote a series of elements that compose a task, but includes, in addition, the paper work and housekeeping chores associated with but not normally considered a part of the task.

This period served also to enable the work to be broken up into discrete functions for the recording of time data. Seven functions were discerned to be discrete and independent and to be common to all sections: issuing, receiving, screening records, x-tra, delay, and unavoidable delay. Definitions of these terms are included in Appendix B. Delay and unavoidable delay were later combined because two operators used their pockets in which to store work-order forms. As a result, the only method of discerning whether the operator was in delay because of lack of work or because of just loafing was to ask.

This was tried; however, some evidence of untruthfulness was noted, and, since these were the only functions not considered unquestionably discrete, it was considered advisable to combine the functions rather than to risk a biased error.

Procedures for the all-day time study were simple. Each of the four operators was studied on a Tuesday and on a Thursday. No particular pattern was maintained; the writer made a trip to the plant each week on whichever of the two days happened to be convenient. The operator to be studied that day was selected by drawing a slip of paper from a hat. As the study progressed the choices were forced by the need for a Tuesday or a Thursday observation on one or another of the operators. The time element was taken as one minute, and whenever an operator changed functions the last minute was included in the function in process at the beginning of the minute.

The procedures for work sampling were the best possible under the circumstances, and appear to meet all of the requirements for a proper statistical analysis (12). The time population was taken as four days numbered in sequence; each day consisted of 480 minutes (0730 to 1600) with the thirty minutes for lunch not included. An element

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(12)McAllister, G. E., "Random Ratio-Delay", University of California Industrial Logistics Research Project, Research Report No. 12, Los Angeles, April 10, 1953.

of the time population was taken as one minute. Each operator was numbered (1 to 8). These parameters were codified into five digits, and, by use of a five digit table of random numbers, a sample was drawn (13). Sample size was taken to give the largest possible coverage of the four days - a total of 345 observations. This gave an average of an observation every six minutes which was sufficiently frequent due to the large area to be covered. The use of an unequal number of observations per operator unbalances the design of the statistical model. It would be better to use an equal number of observations, but, since the purpose of this experiment is to estimate the components of variance, the lack of balance causes only minor inconvenience.

At this period of the study a change in the academic schedule of the writer required a shift from Tuesdays and Thursdays to Tuesdays and Wednesdays. A survey of the average time per day spent on each function showed no significant variation in the functions performed due to the day of the week so the change was considered permissible. As later brought out in the results, a significance test for means showed that at the 5% level there was no significant difference in the sample means for work sampling and for all-day time study. They may then be

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(13) Ibid

presumed to have come from the same population. Again, no pattern was maintained in the visits to the plant. The writer conducted the study on whichever Tuesday or Wednesday of the week was convenient, until the need for a particular day forced a choice.

RESULTS

The tabulated statistical results are presented on the next three pages. The compilation of data collected is presented in Appendix D. Formulae and Sample Calculation are presented in Appendix C. Appendix A has been used as the authority for the statistical analysis.

All statistical calculations were made in percentages. The results for the all-day study are in per-cent of total time, and the work sampling results are in percent of total observations. This presents the means and the standard deviations in a highly desirable form, but, unfortunately, the variances are in square percent, a less meaningful term. It is hoped that the advantage of the former will offset the disadvantage of the latter.

TABLE I

STATISTICAL RESULTS

RECEIPT

Work Sampling	All-day Time Study
$\bar{X}_R = 24.7$	$\bar{X}_C = 22.9$
$\bar{X}'_R = 22.4$	
$\sigma_w^2 \doteq 230.10$	$\sigma_w^2 \doteq 219.35$
$\sigma_w \doteq 15.2$	$\sigma_w \doteq 14.8$
$\sigma_d^2 \doteq 35.49$	$\sigma_d^2 + \sigma_{wd}^2 \doteq 67.99$
$\sigma_d \doteq 6.0$	
$\sigma_{wd}^2 \doteq 298.56$	
$\sigma_{wd} \doteq 17.3$	$\sqrt{\sigma_d^2 + \sigma_{wd}^2} \doteq 8.3$
$\sigma_{\bar{X}_R} \doteq 8.0$	$\sigma_{\bar{X}_C} \doteq 9.0$

$$\left| \frac{\bar{X}_R - \bar{X}_C}{\sqrt{\sigma_{\bar{X}_R}^2 - \sigma_{\bar{X}_C}^2}} \right| = 0.15 < 1.96$$

ISSUE

Work Sampling	All-day Time Study
$\bar{X}_R = 27.1$	$\bar{X}_C = 38.2$
$\bar{X}'_R = 27.9$	
$\sigma_w^2 \doteq 397.12$	$\sigma_w^2 \doteq 643.60$
$\sigma_w \doteq 19.9$	$\sigma_w \doteq 25.4$
$\sigma_d^2 \doteq 0$	$\sigma_d^2 + \sigma_{wd}^2 \doteq 131.94$
$\sigma_{wd}^2 \doteq 246.37$	
$\sigma_{wd} \doteq 15.7$	$\sqrt{\sigma_d^2 + \sigma_{wd}^2} \doteq 11.5$
$\sigma_{\bar{X}_R} \doteq 7.9$	$\sigma_{\bar{X}_C} \doteq 13.3$

$$\left| \frac{\bar{X}_R - \bar{X}_C}{\sqrt{\sigma_{\bar{X}_R}^2 - \sigma_{\bar{X}_C}^2}} \right| = 0.81 < 1.96$$

TABLE 2

STATISTICAL RESULTS

DELAY

Work Sampling		All-day Time Study	
$\bar{X}_R$	$\doteq 32.8$	$\bar{X}_C$	$= 29.5$
X'_R	$\doteq 40.0$		
σ_w^2	$\doteq 130.32$	σ_w^2	$\doteq 239.65$
σ_w	$\doteq 11.4$	σ_w	$\doteq 15.5$
σ_d^2	$\doteq 0$	$\sigma_d^2 + \sigma_{wd}^2$	$\doteq 188.75$
σ_{wd}^2	$\doteq 160.92$		
σ_{wd}	$\doteq 12.7$	$\sqrt{\sigma_d^2 + \sigma_{wd}^2}$	$\doteq 13.7$
$\sigma_{\bar{X}_R}$	$\doteq 5.2$	$\sigma_{\bar{X}_C}$	$\doteq 9.1$

$$\left| \frac{\bar{X}_R - \bar{X}_C}{\sqrt{\sigma_{\bar{X}_R}^2 - \sigma_{\bar{X}_C}^2}} \right| = 0.32 < 1.96$$

RECORDS

Work Sampling		All-day Time Study	
$\bar{X}_R$	$= 10.7$	$\bar{X}_C$	$\doteq 5.0$
$\bar{X}'_R$	$= 4.2$		
σ_w^2	$\doteq 140.51$	σ_w^2	$\doteq 19.40$
σ_w	$\doteq 11.9$	σ_w	$\doteq 4.4$
σ_d^2	$\doteq 0$	$\sigma_d^2 + \sigma_{wd}^2$	$\doteq 37.26$
σ_{wd}^2	$\doteq 171.17$		
σ_{wd}	$\doteq 13.1$	$\sqrt{\sigma_d^2 + \sigma_{wd}^2}$	$\doteq 6.1$
$\sigma_{\bar{X}_R}$	$\doteq 5.0$	$\sigma_{\bar{X}_C}$	$\doteq 3.1$

$$\left| \frac{\bar{X}_R - \bar{X}_C}{\sqrt{\sigma_{\bar{X}_R}^2 - \sigma_{\bar{X}_C}^2}} \right| = 0.97 < 1.96$$

TABLE 3

STATISTICAL RESULTS

SCREENING

Work Sampling	All-day Time Study
$\bar{X}_R = 0.6$	$\bar{X}_C = 1.0$
$\bar{X}'_R = 0.6$	
$\sigma_w^2 \doteq 0$	$\sigma_w^2 \doteq 1.28$
$\sigma_d^2 \doteq 0$	$\sigma_w \doteq 1.1$
$\sigma_{wd}^2 \doteq 0$	$\sigma_d^2 + \sigma_{wd}^2 \doteq 0.09$
	$\sqrt{\sigma_d^2 + \sigma_{wd}^2} \doteq 0.3$

X-TRA

Work Sampling	All-day Time Study
$\bar{X}_R \doteq 10.3$	$\bar{X}_C = 9.2$
$\bar{X}'_R = 10.5$	$\sigma_w^2 \doteq 271.18$
$\sigma_w^2 \doteq 0$	$\sigma_w \doteq 16.5$
$\sigma_d^2 \doteq 0$	$\sigma_d^2 + \sigma_{wd}^2 \doteq 15.10$
$\sigma_{wd}^2 \doteq 0$	$\sqrt{\sigma_d^2 + \sigma_{wd}^2} \doteq 3.9$

DISCUSSION OF RESULTS AND CONCLUSIONS

Statistical results are subjected to two types of error: sampling errors consisting of random and systematic errors, and process errors consisting of systematic errors (among operators, machines, and time periods) and random errors. Due to the relatively small sample size used in this study the random sampling errors could be fairly large.

All functions were tested to see if the sample means could be considered as coming from the same population, i.e. if $\bar{X}_R$ AND $\bar{X}_C$ can be presumed to be estimates of the same mean. The sample means, $\bar{X}_R$ and $\bar{X}_C$ for all of the functions were not significantly different at the 5% level and may be presumed to be equal - and estimates of the same population mean, $\bar{\bar{X}}$. Thus the work sampling study was as accurate as the all-day study in the measurement of the work performed, although only one-half as much time was spent on the collection of data.

The results on "screening" and "x-tra" functions for work sampling indicate that the number of observations taken was probably too small to detect any variance in these functions. Since the estimates do not have the χ^2 distribution (there can be no negative values), the algebraically negative solutions for the components of variance in these cases indicate very small or no variance. Therefore, more observations would have been needed to

obtain a more accurate measurement.

The estimates of components of error (σ^2_w, σ^2_d, σ^2_{wd}) can not be checked to establish their accuracy. Because of the necessarily small samples, these components of error are not well estimated, and the large differences in the variances, as shown in the tables of data, may be due in part to chance variation. A look at the trends of these values may help to substantiate some assumptions.

The values of the estimate of variance between workers (σ^2_w) shows no significant bias. Neither method of observation seems to have caused significant changes in the difference between workers.

In all of the functions of work sampling, the variance due to days was zero, except for receipt where the value is small. It was not possible to separate the component of variance due to days (σ^2_d) from that due to the variability of each man on different days (σ^2_{wd}) in the all-day study because no two operators were observed on a common day. If it could be presumed that in the all-day study the variance due to days is approximately the same as for work sampling, small or zero, then the term ($\sigma^2_d + \sigma^2_{wd}$) could largely be due to σ^2_{wd}. This assumption should be valid since the values for σ^2_d should not depend on the method of observation. On this assumption, the lesser values for variance obtained in the all-day study seem to indicate

that the operators tried to present a more typical or routine day's performance and thus cut down on the day to day variability.

If this reaction were typical in studies conducted for actual measurement of work, then the all-day study would show a lesser component of variance. However, this advantage for all-day study is more than compensated for in this study by the increased efficiency of work sampling.

In the computation of total variation, $\sigma_{\bar{X}_R}^2$ and $\sigma_{\bar{X}_C}^2$, the component of variance due to workers (σ_w^2) is the term which has the largest effect on the results (Equations 4 and 5, Appendix A). This, as pointed out in Appendix A, makes the more desirable work measurement plan the one that requires observations of a large number of workers a minimum number of times each. In the results of the data collected at NOPI, the standard deviations, $\sigma_{\bar{X}_R}$ and $\sigma_{\bar{X}_C}$, for all but one of the functions are smaller for work sampling. This tends to give strength to the assumption that the work sampling plan, with only one-half as much time required, gave as good an estimate of the function means as the all-day study.

The experience and training of the analyst for either all-day time study or work sampling can be broken into two parts. First and most obvious is the actual mechanics of making the study, including rating, adjustments, and the

presentation of data. The second part is the indoctrination of the analyst in the philosophy of Motion and Time Study, Labor Relations, and Industrial Psychology in order to obtain the best atmosphere in which to make the study and apply the results. This second part will not be discussed in this paper on the assumption that any interested parties will have established policies in the field of labor relations and industrial management.

As previously mentioned, a three week course, such as the one used at Rock Island Arsenal, seems to be sufficient to train the analyst to perform the mechanics of all-day study. In addition to this training, the inexperienced analyst may require as much as a full working day (sometimes more) of preliminary observation of each job with which he is not thoroughly familiar in order to break down the job into the functions of interest. As noted in the first part of this paper, observing the continuity of the work pattern in the all-day study aids the analyst in adding up the elements for a total function. This is true, of course, in an indirect labor or non repetitive situation where the measurement is of total functions and not of the elements which make up the functions. This job study period can also be utilized to establish a harmonious relationship with the employee to be observed.

For work sampling, current literature implies that little or no training is required. Most of these articles

presume, however, the changing of a trained time study analyst to a work sampling approach. If this is not the case, some formal training probably is desirable. A broad knowledge of statistics is not required, and, since only elementary mathematics are involved, the same mental capacities needed for all-day time study will be sufficient. A period of training covering some statistical theory, including the recognition of discrete and independent functions, the definition of a binomial distribution, the use of random digits, and the computation of standard deviation; the procedures for timing; the presentation of data; and a heavy concentration of practice problems can easily be covered in three weeks. Concentration on rating would be a point of emphasis since rating is more difficult in sampling techniques. Some background and related information would be included to advise the trainee that the technique presented had definite limitations and that these should be recognized. This training should establish a degree of proficiency in work sampling equal to that of the three weeks of all-day time study training at Rock Island Arsenal. In either case, the trained analyst would need to work under the close supervision and guidance of an experienced time study man or member of the industrial engineering department.

In addition to the factors noted above, work sampling requires the recognition of a function from only one minute

of observation. For a complicated job or one necessitating a detailed breakdown, even the most experienced analyst may have to spend some time studying the functions before he can recognize them with only a brief observation. If a harmonious atmosphere exists, the operator being studied can be trusted to give a reliable clarification when the analyst is in doubt. The training of an analyst who has had some experience in the department being studied would simplify matters some. The writer rejected a study of a clerk-typist section at NOPI because of his inability to recognize functions from single elements. To illustrate; in a telephone call is the stock clerk checking on a receipt, on an issue, or only making a social call? This difficulty varies with the degree of refinement desired. The writer's experience, based on very little contact with industrial operations, was to require him to study the functions from one-half day to one day to be capable of a (1) work, (2) delay, (3) unavoidable delay breakdown and at least a week of study in an office requiring an (1) incoming paperwork, (2) outgoing paperwork, (3) housekeeping (or other), (4) delay, (5) unavoidable delay breakdown.

Having presumed in the analysis of the cost of training that the employees to be trained had only the required mental capacity and little or no industrial experience, it is unrealistic to try to analyze the question of preparation

time to organize a study unless the analyst has had some experience or is under the close guidance of a highly competent time study analyst. The industry wide assumption that either all-day time study or work sampling will give reliable data on which to base budgets or manhour requirements is based on the assumption that the current estimated values foretell future requirements. This is only true if the process is stable and the average work level is constant over the projected time period and equal to the estimated level (14). The validity of the assumptions made in the planning stage will have strong influence on the accuracy of the estimated data, and the corresponding value of this data for forecasting future requirements. An experienced time study analyst or industrial engineer should review or approve any plans or procedures for the collection of data.

The preliminary steps for planning a study would be identical for either all-day time study or work sampling. Once the process to be studied were known to be in control and the mean was presumed to be reasonably stable, the objective of the study and the areas to be measured could be outlined. A preliminary study would be necessary to define the functions of interest, verify that they were discrete and independent, and establish some measure of acceptance

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(14)Davidson, H. O., "Activity Sampling and Analysis - Present State of Theory and Practice", New York, ASME, Paper No. 53-F-24, Aug. 1953.

among the employees. Some standardization of procedures might be necessary in order to obtain discrete functions. For example, had Operators 3 and 4, in this research study, maintained their work-order forms in a box or specific place on the workbench instead of their shirt pockets no difficulty would have been met in maintaining "delay" and "unavoidable-delay" as discrete elements.

Once the preliminary steps are accomplished, work sampling will require additional work besides that necessary to present the data. The operators, days, and minutes of the population to be sampled must be codified, and a selection of random numbers made. The listing of the selected observation times in chronological order on a work sheet, with space for the data to be collected, is essential for planning the sequence of the study (15). Detailed procedures will not be outlined because of the abundance of current literature on this phase of the topic. From the bibliography listed at the end of this paper the writings of McAllister, Rowe, and Young are recommended.

The industry wide practice of using the binomial formula

$$\sigma = \sqrt{\frac{\mu(1-\mu)}{n}}$$

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(15)McAllister, op. cit.

to decide the number of sample observations required, in order to estimate work or delay elements with maximum assurance, (2 σ limits are typical) is correct only for measuring the work of one man or crew working at a stable process which has a constant mean, μ . However, if more than one worker or crew is involved, then the terms in Equation 5 (Appendix A) for σ^2_w, σ^2_d, σ^2_{wd} must be considered. Presumably in the past the use of only the last term of Equation 5, the binomial formula, has been based on the assumption that σ^2_w, σ^2_d, σ^2_{wd}, were small or equal to zero. This is not true where more than one worker is studied, but, conversely, the term for the binomial distribution is small compared to the components of error caused by the workers and by the varibility of each worker on different days.

Two criteria may be used for determining the extent of the sampling study. If available time and/or money for the work measurement study are fixed, the plan requiring the observation of the maximum number of workers the maximum number of days allowable will be selected. On the other hand, if a certain accuracy is required, the plan requiring the observation of the maximum number of workers as many days as are required to reduce $\sigma^2_{\bar{X}_R}$ to the desired figure will be used. For the latter plan an analysis of variance can be made at the end of each day's study, after the first 2 days, to see if further observation is required.

The presence of the analyst in the work area has some influence on the action of the worker and may cause the work activity to differ from that normally performed. It seems likely that a time study man standing a few feet from the worker all day long may have a considerable influence on work performances while the casual and some times unnoticed presence of the work sampling observer would have a lesser influence. The degree of the worker's reaction is not limited by his knowledge of the presence of the analyst, but is also influenced by his knowledge of and approval, disapproval, or disinterest in the study. The accepted belief that less bias is introduced in work sampling is based on valid knowledge of human reaction, and the findings of this writer can only substantiate this belief.

Excellent worker cooperation and interest in this experiment was obtained by explaining that the observer was a student only interested in conducting an experiment on two methods of measurement. The workers were told, and accepted from the start, that no effect on pay scales or hours would result from the study.

Throughout the all-day time studies, the operators were most cooperative in making sure that the writer knew what they were doing and even why they did it in some particular manner. Despite encouragement to "just ignore" the writer, none of the operators hesitated to

stop and explain every time some interest was shown in a procedure. One operator even offered to save up some interesting jobs until the next visit of the writer in order that he might observe how they differed from the routine. In other words, the bias was there, and no amount of explanation could reduce it. Had the operators been critical or antagonistic to the observer's presence the bias would have been shown in other forms.

In the work sampling study the writer endeavored with a fair degree of success to appear in the operators presence just at the beginning of the time element desired. The operators were thus seldom aware of the writer until after he had observed the function being performed. There was no question of popping around corners or from the aisles, but simply a knowledge of distances involved and the habit of walking at a steady pace so that arrival within the sight of the work area was usually within a few seconds of the desired time. Results were excellent; few of the operators even noticed the presence of the writer unless he needed to ask which function they were performing.

It may be concluded, then, that the technique of work sampling, or ratio-delay, seems to offer reliable estimates of the functions of interest in indirect or non repetitive work situations within a shorter period of observation, and at correspondingly less cost, than the all-day time study method. Results of this experiment further

indicate that the opportunity to observe more operators on similar work, within the observation period, by the work sampling method is primarily responsible for shortening the observation time necessary to obtain statistically accurate results.

The procedures recommended in Appendix A for the design of a work measurement plan to obtain the smallest variance and thus the best estimate of the mean are statistically sound and should be of value to industry. Practical limitations deny the full theoretical value of this solution to the users since there are seldom areas in industry where the number of men, who perform the same functions, exceed fifty or sixty. The potentials of this method of deciding on a desirable work measurement plan can be realized if a reliable estimate of variance can be obtained. In small or restricted areas of use, the observations to obtain a reliable estimate of variance can be almost as large as those needed to conduct the work measurement program.

A practical use of this procedure may be obtained by combining a work measurement study with an analysis of variance. The first two day's observations could be used to make an analysis of variance in order to estimate the error. Using each subsequent day's collected data to include in an extended analysis of variance the process may be repeated until the desired level of confidence is obtained. If it is desired to have the means ($\bar{X}$) accurate

to $\pm$ 5% (δ = .05) at least 95% of the time, then

$$P\left\{\left|\frac{\bar{X}-\mu}{\sigma}\right| \leq \frac{.05}{\sigma}\right\} \geq .95 \quad (16)$$

$$\left|\frac{\bar{X}-\mu}{\sigma}\right| = t \geq .95$$

$$t = 1.96$$

$$P\left\{1.96 \leq \frac{.05}{\sigma}\right\} \geq .95$$

$$\sigma = \frac{.05}{1.96} = 2.55\,\%$$

$$\sigma^2 = 6.5\,\%$$

Thus, once the total variance is reduced to 6.5%, the true means will be within limits of $\pm$ 5% from the estimated means 95% of the time. Rougher approximations of these values are usually acceptable in certain uses, but if accuracy is important, Appendix A gives methods of determining the accuracy obtained or of establishing the study to obtain the desired accuracy.

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(16)Burr, I. W., "Engineering Statistics and Quality Control", McGraw-Hill Book Co., New York, 1953, 69.

Training of analysts for work sampling or all-day time studies will require approximately the same amount of time, presuming the initial capabilities of the trainee are the same. The end product of a typical three week course should be competent data takers who have sufficient background to avoid obvious errors. In either case, the trainee would need to work under the close guidance of an experienced leader. Use of standard forms and procedures will increase the speed and accuracy of any time study analyst.

Preparation time to organize a study is shorter for all-day time study. This is primarily due to the steps required to assure sound statistical procedures and reliable results in the work sampling study. Any use of time study data for forecasting of budgets, workloads, or manpower requirements is based on certain assumptions, and, therefore, all plans for time study should be based on a knowledge of the use of the results and the validity of the assumptions made. If at any place in time study the supervision of a skilled and experienced analyst is required, this must be the time and place to utilize his knowledge. Although a knowledge of statistics is not needed for work sampling, in the planning stage some statistical background is essential. There is a plentiful supply of literature written on the use of statistics in work measurement, and most of it is well within the mental capabilities of the average engineer. Some recommendations have already been made

on what seems to the writer to be the better literature in this area. Others are included at the end of this paper and are marked with an asterisk.

Work sampling is commonly assumed to produce less bias than all-day time studies. In this study, the operators were less aware of the presence of the observer in the work sampling study, and, presumably, that which was observed was more accurate. However, due to the endeavor of the operators to present a "typical" day when observed in the all-day study, the daily variability of each operator was less and the all-day study was presented in a more favorable light. This advantage is also a weakness since the values received were subject to the cooperation of the employees.

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APPENDIX A

STATISTICAL THEORY for this EXPERIMENT

prepared by

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THEORY

Consider observing a worker at several instants on a given day and recording what he is doing at each instant. We might put down the complete day (480 minutes) as a line and darken the parts in which he actually was engaged in the task of interest (A). The proportion of darkened parts is the number of minutes at task A divided by 480, $Y_{ij}/480 = X_{ij}$, for worker i on day j. If we pick a point at random on the line, the chance that we land in a dark part is $Y_{ij}/480$, corresponding to the probability that we observe him at task A at a random instant during the day. Picking n points independently at random, the number of times we observe him has the binomial distribution $B(n, X_{ij})$, and, as is well known, the proportion of task A observations to n, is an estimate of X_{ij}. If p_{ij} is this sample proportion, p_{ij} will estimate X_{ij}. Let

(1) $$\delta_{ij} = p_{ij} - X_{ij}$$

be the random error of estimation of X_{ij}. It is well known that δ_{ij} has mean 0, and variance

(2) $$\frac{1}{n}X_{ij}(1-X_{ij})$$

This is for a given man, i, and day, j.

Now suppose we observe a worker all day long. Assuming this does not change the bias, we will obtain X_{ij} without error. Our goal in any case is to estimate the average proportion of time per day spent at task A by all the work-

ers on all days. To this end we will observe several workers chosen at random for several days chosen at random. The resulting X_{ij}'s will be random also. We can conceive of these as being a sum of various components

a) μ — the overall average we are seeking a non random quantity,

b) w_i — the bias due to worker i's average daily proportion of time at task A . We can think of the random choice of a worker as being the choice of one of these numbers from a large population of all possible numbers. The population average of these is zero since we have taken out μ . The population variance we will call σ_w^2. This is a measure of the error introduced by sampling workers.

c) d_j — The bias due to the amount of task A to be done in day j . Random choice of days, as of workers, means we choose d's from a population of mean 0 and variance σ_d^2,

d) $(wd)_{ij}$ — the actual difference between the work done by worker i on day j and the average $\mu + w_i + d_j$. The mean of the population of these is 0 , and the variance is σ_{wd}^2. This is sometimes called the experimental error.

Writing X_{ij} in terms of these, we have

$$X_{ij} = \mu + w_i + d_j + (wd)_{ij}$$

We are now in a position to calculate the variance of an all-day time study average. Suppose we choose a men at random and observe each all day for b days, taking k men on each day. Then there are ab X_{ij}'s and ab/k different days in the experiment. We sum the proportion of time each day on job A for all the men and all the days and divide by the number of items in the sum;

$$\bar{x}_C = \frac{1}{ab} \sum \sum x_{ij}$$

$$= \frac{1}{ab} \sum \sum \left[\mu + w_i + d_j + (wd)_{ij}\right]$$

from (3)

All the μ's are the same so the average μ is μ itself. Of the w_i's, there are a different ones each repeated b times so the result is $\bar{w}$, an average of a items. Similarly there are ab/k different d_j's so that the average is $\bar{d}$, an average of ab/k items. Finally the average of the $(wd)_{ij}$ is $(\overline{wd})$, an average of ab different items. Thus

$$\bar{x}_C = \mu + \bar{w} + \bar{d} + (\overline{wd})$$

Since each of the averages has mean zero, $\bar{x}_C$ is an unbiased estimate of μ. The variance of this estimate is the sum of the variances of the component averages. Since the variance of an average is the variance of one item divided by the number of items,

$$(4) \qquad \sigma_C^2 = \frac{1}{a}\sigma_w^2 + \frac{k}{ab}\sigma_d^2 + \frac{1}{ab}\sigma_{wd}^2$$

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In random sampling we choose a men, b days and observe each of k mean at n random times each day. We then take the observed proportions p_{ij} for each man-day, and average them as we did for the X_{ij} in the all-day study.

$$\bar{X}_R = \frac{1}{ab}\sum\sum p_{ij}$$

$$= \frac{1}{ab}\sum\sum(X_{ij} + \delta_{ij}) \qquad \text{from (1)}$$

$$= \frac{1}{ab}\sum\sum\left[\mu + w_i + d_j + (wd)_{ij} + \delta_{ij}\right] \qquad \text{from (3)}$$

$$= \mu + \bar{w} + \bar{d} + (\overline{\overline{wd}}) + \frac{1}{ab}\sum\sum\delta_{ij}$$

Thus we have the same random errors as in all-day study plus the amount $\frac{1}{ab}\sum\sum\delta_{ij}$. Since this has mean zero, $\bar{X}_R$ is also an unbiased estimate of μ. However the variance of $\bar{X}_R$ will be equal to that of $\bar{X}_C$ (for the same a, b, k) plus the variance of $\frac{1}{ab}\sum\sum\delta_{ij}$.

Each δ_{ij} is different and is the result of random processes, the choice of a man and day, and the choice of the n sampling times during the days. We can compute its variance using conditional expectations:

$$\text{var } \delta_{ij} = E(\delta_{ij}^2) = E_x\left[E(\delta_{ij}^2/X_{ij})\right]$$

$$= E_x\left[\frac{1}{n}X_{ij}\ (1 - X_{ij})\right] \qquad \text{from (2)}$$

$$= \frac{1}{n}\ (\mu - E\,X_{ij}^2)$$

$$= \frac{1}{n}\ (\mu - \sigma_x^2 - \mu^2)$$

$$= \frac{\mu(1-\mu)}{n} - \frac{\sigma_x^2}{n}$$

$$= \frac{\mu(1-\mu)}{n} - \frac{\sigma_w^2 + \sigma_d^2 + \sigma_{wd}^2}{n}$$

Since this does not depend on i or j, we get for the average

$$\operatorname{var}\left(\frac{1}{ab}\sum\sum \delta_{ij}\right) = \frac{\mu(1-\mu)}{nab} - \frac{\sigma_w^2 + \sigma_d^2 + \sigma_{wd}^2}{nab}$$

Combining this with (4), we get

$$(5) \quad \sigma_R^2 = \frac{1}{ab}\sigma_w^2 + \frac{k}{ab}\sigma_d^2 + \frac{1}{ab}\sigma_{wd}^2 - \frac{\sigma_w^2 + \sigma_d^2 + \sigma_{wd}^2}{nab} + \frac{\mu(1-\mu)}{nab}$$

The possible advantage of the random method other than reduction of bias lies in reducing the cost of observation. Whereas one observer can handle one or two $(k = 1,2)$ men a day on an all-day basis, he will be able to handle a larger number on a random basis. Thus both a and b can be made larger while the number of days of observation (ab/k) remains the same. This will usually reduce all the components of variance of the mean except the one due to days, and will put in an additional component which depends on the mean we would like to know. Depending on the values of σ_w^2, σ_d^2, σ_{wd}^2, and μ the random variance may be less than the all-day variance for two plans which cost the same.

If an expression can be obtained for the cost of sampling under each scheme, the variance of the estimate can be minimized for a fixed cost. That is a, b, k, and n can be chosen so as to make the error smallest. Then the smallest possible all-day error variance can

be compared with the minimum random error variance for the same cost, and conclusions drawn as to which plan is best.

Under the following very simple cost arrangement the results are not realistic but indicate the general idea. Assuming that the cost is the daily wage of the observer and that he can take care of two men under all-day time study, the minimum variance is obtained when he observes as many days as possible, two different men each day. (The cost is the number of days spent sampling). This makes the number of men a, as large as possible, as well as the number of man days. For random sampling suppose the number of men one observer can handle is inversely proportional to the number of times per day each man is observed. Then the result is that he should observe as many men as possible each as few times per day as possible. With proper interpretation, these results can indicate the things to aim at when choosing a sampling plan. However, if only a few sampling plans are feasible, the variances of each should be calculated and compared. For this a knowledge of σ_w^2, σ_d^2, σ_{wd}^2, and μ are necessary.

L. COTE

APPENDIX B

SYMBOLS AND DEFINITIONS

SYMBOLS AND DEFINITIONS

Symbol	Definition
a	number of men observed in work sampling.
a'	number of men observed in all-day time study.
b	number of days each man was observed in work sampling.
b'	number of days each man was observed in all-day time study.
C	subscript to denote all-day time study (Continuous).
i	subscript denoting rows (men) in statistical matrix.
j	subscript denoting columns (day) in statistical matrix.
k	number of men observed each day. Equal to a in this study but not necessarily so.
n	number of times per day each man is observed on work sampling.
n_{ij}	from statistical matrix, number of times man, i, is observed on day, j. Used in this sequel for the case where each man is observed a different number of times.
N	total number of observations (n x a x b).
p	proportion.
R	subscript to denote work sampling (Random).
$\bar{X}$	sample mean or average.
δ	difference.
μ	true mean or average.
σ	standard deviation
Σ	sum of or summation
σ^2	variance

issue	function consisting of: from information on standard forms for material requests clerk checks cardex to verify location, procures material, verifies identification, signs invoice, and places in designated area for pick up.
receipt	function consisting of: checking incoming stock against invoice, inspection report or returned material form; counting or weighing material; inspection of material for condition (some material is returned from the field for overhaul): verifying the location from cardex or making up new card; entering number and stock number: placing in bin or on pallet in proper location.
delay	function consisting of: all non-working time (not including lunch time), including personal time and time lost to lack of work or interruption by foreman, supervisor, or other workers.
screening	function consisting of: time spent in assisting production personnel in verifying the identification or availability of desired stock prior to the preparation of a material request. Used primarily in highly technical material where a stock number in the catalogue is not a precise description or the interchangeability of material is doubtful. Also used when records are not accurate.
records	function consisting of: maintenance of IBM cards and Function sheets for cost accounting, logging of serial numbers on items having numbers, changing stock number or custody designator on reclassified material, and miscellaneous paperwork.
x-tra	function consisting of: rewarehousing to improve storage, issuing patterns and molds to shop personnel, surveying overaged or unrepairable material, cyclic preservation, supervising or directing an assistant, issuing and receiving shop laundry, miscellaneous work including cleanup of area.

APPENDIX C

FORMULAE AND SAMPLE CALCULATIONS

FORMULAE AND SAMPLE CALCULATIONS

ISSUE:

Work Sampling

Row Sum of Squares (W) $= \frac{1}{ab}\left[a \sum_{i=1}^{a} X_{i.}^2 - X_{..}^2\right]$

$= \frac{1}{32}\left[8\ (149{,}224.33) - 752{,}278.68\right]$

$= 13{,}797.38$

where $X_{i.} = X_{i1} + X_{i2} + X_{i3} + X_{i4}$

and $X_{..} = \sum_{i=1}^{a} X_{i.} = \sum_{j=1}^{b} X_{.j}$

and $X_{.j} = X_{1j} + X_{2j} + \cdots\cdots + X_{8j}$

Column Sum of Squares (D) $= \frac{1}{ab}\left[b \sum_{j=1}^{b} X_{.j}^2 - X_{..}^2\right]$

$= \frac{1}{32}\left[4\ (192{,}489.28) - 752{,}278.68\right]$

$= 552.45$

Total Sum of Squares $= \frac{1}{ab}\left[ab \sum_{i=1}^{8} \sum_{j=1}^{4} X_{ij}^2 - X_{..}^2\right]$

$= \frac{1}{32}\left[32(45{,}892.29) - 752{,}278.68\right]$

$= 22{,}383.58$

Interaction (I) = Total - M - D = 22,383.58 - 13,797.38 - 552.45

= 8033.75

$A = \frac{1}{ab} \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{1}{n_{ij}} = \frac{1}{32}(3.27136) = 0.10223$

$$\sigma_w^2 \doteq \frac{1}{b}\left[\frac{W}{a-1} - \frac{I}{(a-1)(b-1)}\right]$$

$$\doteq \frac{1}{4}\left[\frac{13,797.38}{7} - \frac{8,033.75}{21}\right]$$

$$\doteq 397.12$$

$$\sigma_d^2 \doteq \frac{1}{a}\left[\frac{D}{b-1} - \frac{I}{(a-1)(b-1)}\right]$$

$$\doteq \frac{1}{8}\left[\frac{552.45}{3} - \frac{8033.75}{21}\right]$$

$= 0$ (negative number has no meaning)

$$\sigma_{wd}^2 \doteq \frac{1}{1-A}\left[\frac{I}{(a-1)(b-1)} - A\left\{\bar{x}(1-\bar{x}) - \sigma_w^2 - \sigma_d^2\right\}\right]$$

$$\doteq 1.11387\left[382.56 - 0.10223\left\{27.10(72.90) - 397.12 - 0\right\}\right]$$

$$\doteq 246.37$$

$$\text{est } \sigma_{\bar{x}_C}^2 \doteq \frac{1}{a}\sigma_w^2 + \frac{k}{ab}\sigma_d^2 + \frac{1}{ab}\sigma_{wd}^2 + \frac{A}{ab}\left[\mu(1-\mu) - \sigma_w^2 - \sigma_d^2 - \sigma_{wd}^2\right]$$

$$\doteq \frac{1}{8}(397.12) + 0 + \frac{1}{32}(242.36) + \frac{0.10223}{32}\left[27.10(72.90) - 397.12 - 0 - 246.37\right]$$

$$\doteq 61.60$$

$$\sigma_{\bar{x}_R} = \sqrt{\sigma_{\bar{x}_R}^2} = 7.85$$

$$\bar{x}_R = \frac{x_{..}}{ab} = \frac{867.34}{32} = 27.10$$

$$\bar{x}'_R = \frac{\sum_{i=1}^{4} x_{i.}}{4b} = \frac{447.08}{16} = 27.94$$

<u>All-day Time Study</u>

B Sum of Squares $= \sum_{i=1}^{a'} \sum_{j=1}^{b'} X_{ij}^2 - \frac{1}{a'b'} X_{..}^2$

$= 16446.83 - \frac{1}{8}(93,299.70)$

$= 4,784.37$

C Sum of Squares $= \sum_{i=1}^{a'} \sum_{j=1}^{b'} X_{ij}^2 - \frac{1}{b'} \sum_{i=1}^{a'} X_{i.}^2$

$= 16,446.83 - \frac{1}{2}(31,838.18)$

$= 527.74$

$$\sigma_w^2 \doteq \frac{1}{b'(a'-1)} \left[B - (a'b' - 1) \frac{C}{a'(b'-1)} \right]$$

$$\doteq \frac{1}{6} \left[4,784.37 - 7/4(527.74) \right]$$

$$\doteq 643.60$$

$$\sigma_d^2 + \sigma_{wd}^2 \doteq \frac{C}{a'(b'-1)} \doteq \frac{527.74}{4} \doteq 131.94$$

$$\text{est } \sigma_{\bar{X}_C}^2 \doteq \frac{1}{a'} \sigma_w^2 + \frac{1}{a'b'} (\sigma_d^2 + \sigma_{wd}^2)$$

$$\doteq \frac{1}{4}(643.60) + \frac{1}{8}(131.94)$$

$$\doteq 177.39$$

$$\sigma_{\bar{X}_C} = 13.32$$

$$\bar{X}_C = \frac{X_{..}}{a'b'} = \frac{305.45}{8} = 38.18$$

If $\bar{X}_R \doteq \bar{X}_C$, that is if they are estimates of the same mean

$$\left| \frac{\bar{X}_R - \bar{X}_C}{\sqrt{\sigma_{\bar{X}_R}^2 - \sigma_{\bar{X}_C}^2}} \right| \leq 1.96$$

will indicate that at the 5% level there is no significant difference and the two estimates may be presumed to be of the same mean, $\bar{\bar{X}}$.

$$\left|\frac{27.10 - 38.18}{13.61}\right| = \frac{11.08}{13.61} = .814 < 1.96$$

APPENDIX D

TABLES of DATA

TABLE 4

Work Sampling

Record of Observations

Day	1	2	3	4
Men				
1	7	10	14	13
2	6	7	7	8
3	11	10	19	10
4	9	16	6	7
5	14	9	15	13
6	13	11	10	7
7	13	6	11	14
8	10	12	19	15

$$\sum N_{ij} = 345$$

TABLE 5

Work Sampling

Receipt in % of Observations

Day / Man	1	2	3	4	Totals
1	28.57	30.00	14.29	76.92	149.78
2*	x	x	x	x	
3	45.45	10.00	33.33	10.00	98.78
4	0	6.25	0	14.29	20.54
5	7.14	55.56	60.00	92.31	215.01
6*	x	x	x	x	
7	0	33.33	18.18	14.29	65.80
8	10.00	0	5.26	26.67	41.93
Totals	91.16	135.14	131.06	234.48	591.84

$\Sigma X_{i.}^2 = 84930.49$ $\qquad X_{..}^2 = 350274.59$

$\Sigma X_{.j}^2 = 98730.56$ $\qquad \bar{X}_R = 24.66$

$\Sigma X_{ij}^2 = 29200.58$ $\qquad \bar{X}'_R = 22.43$

	Sum of Squares	d.f.	Mean Squares	σ^2
Men (W)	6637.84	5	1327.57	230.10
Days (D)	1860.32	3	620.11	35.49
Interactions (I)	6107.65	15	407.18	298.56
Totals	14605.81	23		

est $\sigma_{\bar{X}}^2 = 64.81$

*Function not normally performed by observed workers

TABLE 6

Work Sampling

Issue in % of Observations

Day / Man	1	2	3	4	Totals
1	14.29	20.00	28.57	0	62.86
2	50.00	100.00	57.14	62.50	269.64
3	0	40.00	25.00	10.00	75.00
4	0	6.25	33.33	0	39.58
5	57.14	11.11	13.33	0	81.58
6	76.92	36.37	30.00	85.71	229.00
7	7.69	33.33	18.18	7.14	66.34
8	20.00	16.67	0	6.67	43.34
Totals	226.04	263.73	205.55	172.02	867.34

$\sum X_{i.}^2 = 149,224.33$ $\qquad X_{..}^2 = 752,278.68$

$\sum X_{.j}^2 = 192,489.28$ $\qquad \bar{\bar{X}}_R = 27.10$

$\sum X_{ij}^2 = 45,892.29$ $\qquad \bar{\bar{X}}_R' = 27.94$

	Sum of Squares	d.f.	Mean Squares	σ^2
Men (W)	13797.38	7	1971.05	397.12
Days (D)	552.45	3	184.15	0
Interactions (I)	8033.75	21	382.56	246.37
Totals	22383.58	31		

est $\sigma_{\bar{X}}^2 = 61.60$

TABLE 7

Work Sampling

Delay in % of Observations

Day / Man	1	2	3	4	Totals
1	57.14	30.00	28.57	23.08	138.79
2	0	0	42.86	25.00	67.86
3	54.44	40.00	41.67	80.00	216.22
4	55.56	68.75	50.00	42.86	217.17
5	35.72	11.11	26.67	7.69	81.19
6	0	27.27	40.00	14.29	81.56
7	23.08	0	9.09	71.43	103.60
8	40.00	16.67	47.37	39.99	144.03
Totals	266.05	193.80	286.23	304.34	1050.42

$\sum x_{i.}^2 = 162,502.99$ $\qquad x_{..}^2 = 1,103,382.18$

$\sum x_{.j}^2 = 282,891.49$ $\qquad \bar{x}_R = 32.83$

$\sum x_{ij}^2 = 48,994.64$ $\qquad \bar{\bar{x}}'_R = 40.00$

	Sum of Squares	d.f.	Mean Squares	σ^2
Men (W)	6145.06	7	877.87	130.32
Days (D)	880.74	3	293.58	0
Interactions (I)	7488.14	21	356.58	160.92
Totals	14513.94	31		

est $\sigma_{\bar{x}}^2 = 27.43$

TABLE 8

Work Sampling

Screening in % of Observations

Day / Man	1	2	3	4	Totals
1	0	10.00	0	0	10.00
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	7.69	0	0	0	7.69
7	0	0	0	0	0
8	0	0	0	0	0
Totals	7.69	10.00	0	0	17.69

$\sum x_{i.}^2 = 159.14$ $\quad x_{..}^2 = 312.95$

$\sum x_{.j}^2 = 159.14$ $\quad \bar{\bar{x}}_R = 0.55$

$\sum x_{ij}^2 = 159.14$ $\quad \bar{\bar{x}}'_R = 0.63$

	Sum of Squares	d.f.	Mean Squares	σ^2
Men (M)	30.00	7	4.29	0
Days (D)	10.11	3	3.37	0
Interactions (I)	109.25	21	5.20	0
Totals	149.36	31		

TABLE 9

Work Sampling

Records in % of Observations

Day	1	2	3	4	Totals
Man					
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	33.33	18.75	0	14.28	66.36
5	0	0	0	0	0
6	7.69	9.09	30.00	0	46.78
7	69.23	0	0	0	69.23
8	30.00	66.66	36.84	26.67	160.17
Totals	140.25	94.50	66.84	40.95	342.54

$\Sigma x_{i.}^2 = 37,039.24$ $\qquad x_{..}^2 = 117,333.65$

$\Sigma x_{.j}^2 = 37,744.80$ $\qquad \bar{\bar{X}}_R = 10.70$

$\Sigma x_{ij}^2 = 14,912.96$ $\qquad \bar{X}'_R = 4.15$

	Sum of Squares	d.f.	Mean Squares	σ^2
Men (W)	5593.13	7	799.02	140.51
Days (D)	676.42	3	225.47	0
Interactions (I)	4976.73	21	236.99	171.17
Totals	11246.28	31		

est $\sigma_{\bar{X}}^2 = 24.97$

TABLE 10

Work Sampling

X-tra in % of Observations

Day / Man	1	2	3	4	Totals
1	0	10.00	28.57	0	38.57
2	50.00	0	0	12.50	62.50
3	0	10.00	0	0	10.00
4	11.11	0	16.67	28.57	56.35
5	0	22.22	0	0	22.22
6	7.70	27.27	0	0	34.97
7	0	33.34	54.55	7.14	95.03
8	0	0	10.53	0	10.53
Totals	68.81	102.83	110.32	48.21	330.17

$\sum x_{i.}^2 = 19527.43$ $\qquad x_{..}^2 = 109{,}012.23$

$\sum x_{.j}^2 = 29803.53$ $\qquad \bar{\bar{X}}_R = 10.32$

$\sum x_{ij}^2 = 10435.85$ $\qquad \bar{\bar{X}}'_R = 10.46$

	Sum of Squares	d.f.	Mean Squares	σ^2
Men (W)	1475.23	7	210.75	0
Days (D)	318.81	3	106.27	0
Interactions (I)	5235.18	21	249.29	0
Totals	7029.22	31		

TABLE 11

All-day Time Study

Receipt in % of Time Observed

Day	1	2	Totals
Man			
1	32.93	34.58	67.51
2*	x	x	x
3	21.25	39.17	60.42
4	0	9.17	9.17
Totals	54.18	82.92	137.10

$\Sigma X_{i.}^2 = 8292.27$ $\quad X_{..}^2 = 18796.41$

$\Sigma X_{ij}^2 = 4350.10$ $\quad \bar{X}_c = 22.85$

	Sum of Squares	σ^2
Men	203.97	219.35
Days + Interactions	1217.37	67.99

*Function not normally performed by operator

TABLE 12

All-day Time Study

Issue in % of Time Observed

Day / Man	1	2	Totals
1	9.16	5.63	14.79
2	69.59	68.57	138.16
3	61.25	36.25	97.50
4	17.29	37.71	55.00
Totals	157.29	148.16	305.45

$\Sigma X_{i.}^2 = 31838.18$ $\qquad X_{..}^2 = 93299.70$

$\Sigma X_{ij}^2 = 16446.83$ $\qquad \bar{X}_C = 38.18$

	Sum of Squares	σ^2
Men	527.74	643.60
Days + Interactions	4784.37	131.94

TABLE 13

All-day Time Study

Delay in % of Time Observed

Day / Man	1	2	Totals
1	12.29	17.50	29.79
2	28.75	29.79	58.54
3	15.83	21.04	36.87
4	74.38	36.24	110.62
Totals	131.25	104.57	235.82

$\Sigma X_{i.}^2$ = 17910.56 $\quad X_{..}^2$ = 55611.07

ΣX_{ij} = 9710.29 $\quad \bar{X}_C$ = 29.48

	Sum of Squares	σ^2
Men	755.01	239.65
Days + Interactions	2758.91	188.75

TABLE 14

All-day Time Study

Screening in % of Time Observed

Day	1	2	Totals
Man			
1	1.87	2.71	4.58
2	1.66	1.46	3.12
3	0	0	0
4	0	0	0
Totals	3.53	4.17	7.70

$\sum X_{i.}^2 = 30.71$ $X_{..}^2 = 52.29$

$\sum X_{ij}^2 = 15.73$ $\bar{X}_c = 0.96$

	Sum of Squares	σ^2
Men	0.37	1.28
Days + Interactions	8.32	0.09

TABLE 15

All-day Time Study

Records in % of Time Observed

Day	1	2	Totals
Man			
1	15.00	0	15.00
2	0	0	0
3	0	0	0
4	8.33	16.88	25.21
Totals	23.33	16.88	40.21

$\sum X_{i.}^2 = 860.54$ $\quad X_{..}^2 = 1616.84$

$\sum X_{ij} = 579.32$ $\quad \bar{X}_C = 5.03$

	Sum of Squares	σ^2
Men	149.05	19.40
Days + Interactions	377.22	37.26

TABLE 16

All-day Time Study

X-tra in % of Time Observed

Day	1	2	Totals
Man			
1	28.75	39.58	68.33
2	0	0	0
3	1.67	3.54	5.21
4	0	0	0
Totals	30.42	43.12	73.54

$\sum X_{i.}^2 = 4696.13$ $\qquad X_{..}^2 = 5408.13$

$\sum X_{ij}^2 = 2408.46$ $\qquad \bar{X}_C = 9.19$

	Sum of Squares	σ^2
Men	60.39	271.18
Days + Interactions	1732.44	15.10

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