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Submitted to the Department of Aeronautics and Astronautics on 1 May 1959 in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

This study considered the feasibility of obtaining a self-adapting pitch rate control system of a hypothetical airframe, utilizing linear compensation of the invariant, i.e., normal operating state, parameters of the aircraft.

The relationship between the flight dynamics and aircraft parameters was established, and each was reduced to an invariant and variant component. A method of representing the system with, and without parameter variation was then developed. Linear compensation of the normal operating state was developed to provide nearly flat, minimum phase shift, frequency response; as the primary performance criterion.

The compensated and uncompensated aircraft was analyzed in widely varying operating conditions. Theoretical frequency response and analog computer transient response simulation, yielded the following results:

The desired frequency response in all flight conditions can almost be obtained, since the compensated system introduced varying attenuation with a maximum of 35%. Six of the seven conditions produced acceptable transient response.

To eliminate the attenuation, a variable gain amplifier, sensitive to the conditions is proposed. Using linear compensation the complete system, including the model, amplifier, and pitch rate system, acceptably reproduced the desirable model output, through most of the various flight conditions. This satisfies the basic requirements of a self-adaptive system.

Thesis Supervisor: Y.T. Li.

Title: Associate Professor of Aeronautics and Astronautics.

May 1, 1959.

Mr. Alvin Sloane
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39,
Massachusetts.

Dear Mr. Sloane:

In accordance with the regulations of the faculty, we hereby submit a thesis entitled, A Study of a Linear Self-Adaptive System, in partial fulfillment of the requirements for the degree of Master of Science.

ACKNOWLEDGEMENT

The authors express their appreciation to all personnel of the Massachusetts Institute of Technology, who assisted in the preparation of this thesis.

Particular thanks is due to Professor Y.T. Li who, as thesis supervisor, guided the project, and initially suggested the concept of invariant parameter compensation; and to Mr. H. Philip Whitaker who acted as thesis technical advisor.

The graduate work for which this thesis is a partial requirement was performed while the authors were assigned to the U.S. Naval Administrative Unit, Massachusetts Institute of Technology.

TABLE OF CONTENTS

OBJECT

The object of this study is to propose and investigate a self-adaptive control system by means of compensation of normal operating state aircraft parameters, using linear techniques.

CHAPTER 1

INTRODUCTION

Modern Control System Requirements

With the advent of high performance aircraft, satellites, rocket aircraft, guided missiles, ICBM's, and eventually, space ships, there has been an accelerated requirement for replacement of the human pilot with automatic flight control systems. This replacement is not necessarily complete, and in many cases is used only to augment the human characteristics in order to add damping, increase accuracies, and in extreme cases, to provide control capability. This class of automatic control system has been further instrumented, and is more popularly known as a self-adapting control, or servo, system.

The human servo system is an excellent example of self-adaptation. However, in the present day supersonic vehicles, reaction time is too slow, and muscular coordination is not sufficient to cope with the various extremes of velocities, temperatures, forces and other modifying

influences; since, modern aircraft have flight capability over extreme ranges of altitude, air speed and Mach number. This capability is mostly attainable due to propulsive thrusts which approach maximum aircraft gross weight. Automatic control systems and stability, augmentation devices have thus been installed in aircraft to provide satisfactory stability and control throughout the various flight regimes. The stability and control characteristics of all aircraft vary as functions of equivalent air speed, Mach number, altitude, aircraft gross weight, center of gravity position and external configuration.[1]

System Representation and Classification.

Self-adaptation may often be represented by an ordinary negative feedback control loop which in itself, is the first of the basic servo concepts. However, the quality of this adaptation may then be used as a measurement of performance capability.

Some basic classifications of the adaptive concept: The first, and most easily understood is that of negative feedback. If there is a high gain in the forward loop, assuming non unity feedback then the closed loop transfer function then becomes approximately that of the negative feedback performance equation. The forward loop has no

effect essentially, on the entire closed loop if the combined gains are much greater than one. Thus, if the forward loop is the controlled member, then the system operation will be completely independent of the controlled member, e.g., an aircraft, which is an ideal situation in self-adaptation. However, actual implementation with mechanical or electrical devices are not as easily found as the transfer equation would have us believe.

There are many ways of re-writing the block diagram representing the above concept in order to overcome the basic difficulties with normal feedback. The method that is introduced by a number of authors[2,3] is that of the conditional feedback loop based on a performance model, that is designed such that the over-all system approximates the characteristics of the idealized model, over wide ranges of operation.

Sperry Gyroscope Company[2] have further described systems such as conditional feedback with a pre-filter, which will be further discussed in the later chapters of this report.

Compensation and Effects.

Automatic control systems are generally multiple closed loop with feedback paths utilizing variously:

displacement, velocity, and acceleration of the aircraft as control quantities. Therefore, the static and dynamic response characteristics have a great effect and cannot be ignored by using only high gain feedback, since saturation and power obtainable from the physical system often falls far short of the theoretical desires. It becomes obvious, then, that some compensation must be made for the aforementioned effects.

This compensation may take many forms as predicated by Whitaker, Yamron, and Kezer;[3] namely, changing system sensitivities as calibrated, programmed functions of indicated airspeed, Mach number and other flight quantities. The adjustments to the system sensitivities are then made such that they are open loop, from the standpoint of the effect of the adjustment on system performance. Fast action adapting control systems are now becoming of greater importance, as stated before, and will become even more so with the advent of space flight. There will be colossal changes in environmental parameters as well as the basic system parameters themselves. Large changes in mass with attendant changes of inertia, aerodynamic heating due to high velocities in the earth's atmosphere, and changes of the density of the medium in which the controlled vehicle is operating, are a few examples of the changes that must be anticipated and corrected for by the vehicles control system. Recently

it was announced that the problem of re-entry of a manned satellite would be undertaken. Let us assume that the satellite would re-enter at an altitude of 100 kilometers, and utilize a bounce-glide technique of re-entry, i.e., decelerate gradually in the upper layers of the sensible atmosphere until a tolerance of temperature is obtained, then bouncing back to the next higher level and then descending again, in gradual sweeps, for re-entry. This requires that a theoretical aircraft, weighing 20 tons with a wing loading of 30 kilograms per square meter, undergoes skin temperature variations of from 800 degrees C, to 400 degrees C, to 1400 degrees C, to 0 degrees C, while undergoing a velocity change from 8.5 kilometers per second to zero during a 2-hour flight.[(4)]

It is seen, then, that it is necessary either to change certain system parameters; or to modify command signals, as the environment changes, in order to maintain satisfactory dynamics. It is further necessary, since flight testing in outer regions of space vehicles is not physically realizable, to be able to design a theoretical system that can be implemented at a later date that is intelligent enough to know when dynamic performance has changed and to utilize this knowledge to adapt itself, so that its final performance will be within a satisfactory range at all times.[(3)]

The present trend to supplement, or even eliminate, a human pilot, has led to severe requirements for control performance, since the capability of many applications requires control through violent maneuvers and yet must still be sensitive to small input command signals.[10] In many cases a control system may be extremely complex; however, when dealing with a specific vehicle such as an aircraft, we are limited by size, weight and structural considerations. The result, then, is often a servo with a limited output rate. If we are able to make this assumption, the controller may be considered as a simple linear system with a limit on the rate of control.[14] But, it is this very limit on the control surface rate that is a major difficulty in a system requirement for stability for both large and small commands. Compensation and adjustment of the system parameters to give the required stable response generally results in undesirable slow response to small signals. There have been many theories on compensation for such undesirable effects, more specifically: to design system parameters as functions of error and error rates. This has led to development of non-linear elements to improve the performance of both linear and non-linear devices.[5]

There have been methods proposed to also accomplish this, in which a control system tends to adjust

itself to a pre-determined operating condition. The system imposes known continuous variations on system inputs to determine system parameters, to maintain output at a maximum.

Series compensation and unity feedback with a controlled component in which the series compensation is varied by an external computer is yet another proposal.[6]

Summary

Thus, adaptive mechanisms may be cataloged into two general types, the first of which is the simple high gain system using linear negative feedback. The second, which is of primary importance in the recent history of self-adaptation, is where the alteration of the controller as a function of system response, is involved in a non-linear system. This type leads ultimately to an opportunity to design and manufacture a controller that is optimum insofar as the continuous determination of the static and dynamic characteristics of the controlled members is concerned.

To correct the difficulties in stability already mentioned, it has been suggested that uniform stability may be obtained by compensating for changes in aircraft stability directly.[12] This is accomplished by adjusting

control parameters as functions of response in order to maintain a uniform response.[8] The basic instabilities generally result from high feedback gain, due to the dynamic effects of the servo actuators. Todays manufacturing techniques, however, have been able to synthesize controllers with better rates of response, and thus higher gains are obtainable, leading to more practical value in the theory of negative feedback.

By definition, optimum control of the aircraft results if the control system provides close control of the transient and steady state responses of the aircraft to command and disturbance inputs, so as to use the capabilities of the aircraft most effectively for fulfilling the specifications of its flight mission. The characteristics involved being response time, damping, dynamic and static errors, and control of interference effects.[2] This very definition allows great flexibility in utilizing the aircrafts capability, without forcing an impractical response from the physical vehicle. There are as many design criteria as there are designers, however, and in selecting such a criteria it must be remembered that the desired performance itself may vary from one phase of a flight to another.

CHAPTER 2

SCOPE

The purpose of this study is to investigate the feasibility of obtaining a self-adaptive system by employing a linear technique, herein defined as invariant parameter compensation. The basic philosophy underlying this technique is the estabilishment of a state in which the airframe parameters, and hence the airframe dynamics are defined to be invariant. Further, having established the airframe in this state, it is then assumed that suitable compensation can be devised to produce an essentially flat response within specified tolerances, of the airframe controlled system over a wide range of frequencies.

The design tolerance criteria is as follows:

a. Amplitude ratio $1 \pm 1\%$ variation

b. Phase shift of less than $\pm 10^{\circ}$

all within an excitation frequency range of 0-50 radians/second.

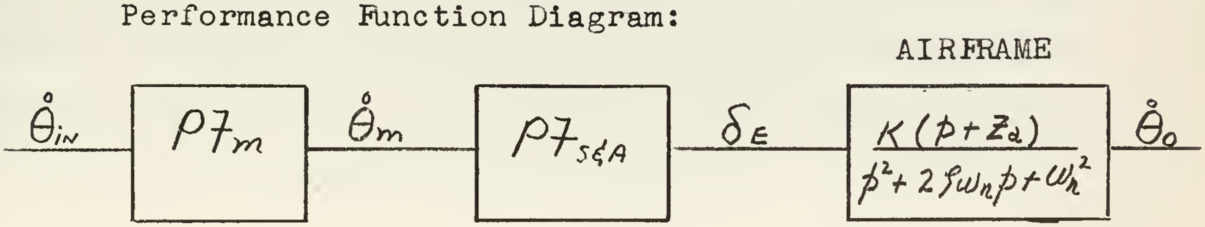
The value of this technique lies in the capability of establishing an effective natural frequency very much

greater than any frequency to which the system will normally be subjected, and an effective damping ratio which is dynamically acceptable. Through this technique, it is believed that the normal range of parameter variations to which the aircraft will be subjected will not change the performance function to any great extent, over the range of interest of input frequencies. It is further desirable that the aircraft perform in a manner as specified by the performance of a reference model system which precedes the controlled loop, and whose output will be the input to the controlled loop. The performance function of the model will be developed in later chapters.

If the aircraft, as compensated, does reproduce as its output, the output of the reference model system for all normal variations of parameters, and for all frequencies within a range of interest; then the aircraft control system will truly be self-adaptive.

The scope of this paper will be limited to the pitch rate performance of a hypothetical aircraft, whose performance function and dynamics are:

Performance Function Diagram:



Where:

PF_m = Reference model system performance function.

$PF_{S\&A}$ = Servo and Actuator performance function.

Za = First order zero; an airframe dynamic.

W_n = Airframe natural frequency; an airframe dynamic.

ζ = Airframe damping ratio, an airframe dynamic.

$\dot{\theta}$ in = Commanded pitch rate.

$\dot{\theta}$ m = Model commanded pitch rate.

δ E = Elevator deflection angle.

$\dot{\theta}_o$ = Airframe pitch rate response.

In order to employ this method of obtaining self-adaptation, it will be necessary to establish the relationship between the above airframe dynamics and their corresponding parameters. Further, since compensation is to be performed on only the invariant parameters, it is necessary that a method be devised wherein the invariant parameters may be handled separately from the variant components.

This development will lead to representation of parameter variations by synthesis of a fictitious system containing both the invariant and variant components. The variant components represented in this fictitious system are the inner loops, which are active only when the aircraft is not in its invariant state.

Although it is anticipated that this technique can be utilized by any aircraft configuration, a hypothetical airframe was employed, since by this method the contributions of the various parameters to the aircraft dynamics and the effect of variations of these parameters on the dynamics could be more adequately demonstrated.

The invariant system parameters are those which exist when the subject aircraft is in the state of altitude, airspeed, and loading specified as the invariant state of operation. The specification of this state appears to be somewhat arbitrary, although it would seem logical to designate the "normal" operating state of the aircraft as its invariant state. For aiframes not possessing such a state, selection of the invariant parameters becomes a matter of additional study and specifications of criteria to establish the "best" set of invariant parameters for such types of vehicles as ballistic missiles, may well prove to be of sufficient magnitude to justify further work in this field.

CHAPTER 3

THE HYPOTHETICAL AIRFRAME

Definition of Parameters

As indicated in Chapter 2, the effectiveness of linear compensation is to be tested on a hypothetical airframe. In this chapter the airframe pitch rate equation, dynamics and parameters are established. In addition, the invariant state will also be defined.

For the purpose of this development, the following table of parameters is applicable:

Dimensional

ρ = air density

ρ_{SL} = air density at sea level

V = airframe airspeed

S = a characteristic surface area of airframe

c = a characteristic chord length of airframe

C_m = moment coefficient

C_L = coefficient of lift

C_{ij} = derivative coefficient $= \frac{dC_i}{dj}$; i = m, L ; $j = \delta_E, \alpha, Q, \dot{\alpha}$

δ_E = control surface deflection

α = airframe angle of attack

θ = airframe pitch angle

Q = airframe pitch rate; i.e. $\frac{d\theta}{dt}$

I_{yy} = longitudinal axis moment of inertia $= k_y^2 m$

k_y = longitudinal axis radius of gyration

m = mass of airframe

Non-dimensional

$$\sigma = \frac{\rho}{\rho_{SL}} \quad ; \quad \mu = \frac{m}{\rho_{SL}\, Sc} \quad ; \quad K_y = \frac{k_y}{c}$$

Aircraft Performance Function

The short period performance function relating elevator angle (δ_E) input and aircraft pitch rate ($\dot{\theta}_o$) output may be represented in terms of dynamics as follows:

$$\frac{\dot{\theta}_o}{\delta_E} = -\frac{K[p + Z_\alpha]}{[p^2 + 2\zeta\omega_n p + \omega_n^2]}$$

(Note: The negative relationship is introduced by sign conventions for δ_E & θ_o (11). It should be apparent that the same sign convention will apply for δ_E & θ_i, thus making a positive relationship for θ_i & θ_o.)

It is assumed that each of the dynamics my be represented as the sum of a variant and an invariant component, as follows:

$$Za = Zao + \Delta Za \;; \quad K = Ko + \Delta K \;; \zeta Wn = (\zeta Wn)_o + \Delta(\zeta Wn)$$

$$W_n^2 = (W_n^2)_o + \Delta(W_n^2)$$

The subscript zero signifies the parameter is invariant. The selection of W_n^2 & ζWn as basic parameters was done to eliminate cross product terms in later development.

<u>Dynamics in terms of airframe /flight path parameters</u>

$$W_n^2 = \frac{-A_1 V^2 \sigma \mu + A_2 \sigma^2 V^2}{\mu^2} \quad \text{where;} \quad A_1 = \frac{C_{m\alpha}}{2c^2 K_y^2} \;; A_2 = \frac{C_{mq} C_{L\alpha}}{8c^2 K_y^2}$$

$$2\zeta Wn = -A_3 \frac{V\sigma}{\mu} \quad \text{where;} \quad A_3 = -\left[\frac{C_{mq} + C_{m\dot{\alpha}} + 2K_y^2 C_{L\alpha}}{4c\, K_y^2}\right]$$

$$Za = \frac{A_4 V\sigma}{A_5 \mu + A_6 \sigma} \quad \text{where;} \quad A_4 = \frac{2}{c}\left[C_{m\alpha} C_{L\delta_E} - C_{m\delta_E} C_{L\alpha}\right]$$

$$A_5 = 4C_{m\delta_E} \;; \quad A_6 = C_{m\dot{\alpha}} C_{L\delta_E}$$

$$K = \frac{A_7 V^2 \sigma}{\mu^2}\left[\mu + A_8 \sigma\right] \quad \text{where;} \quad A_7 = -\frac{C_{m\delta_E}}{2c^2 K_y^2} \;; \quad A_8 = \frac{C_{m\dot{\alpha}} C_{L\delta_E}}{4C_{m\delta_E}}$$

The following parameters are assumed to be constant: S, c, K_y^2, $C_{m\delta_E}$, $C_{m\alpha}$, $C_{L\alpha}$, $C_{m\dot{\alpha}}$, $C_{L\delta_E}$, & C_{mq} . Actually, the derivative coefficients vary somewhat with Mach number. However, the variations of these quantities are not significant when compared to variations of other parameters. Consequently, the results obtained will not be too great in

error as a result of this simplifying assumption.

Just as the airframe dynamics are considered to be composed of a variant and an invariant component, so too, may the airframe/flight path parameters be considered. The following then define the dynamic components in terms of the corresponding parameter components:

(a) $\omega_n^2 = (\omega_n^2)_0 + \Delta\, \omega_n^2$

$$(\omega_n^2)_0 = -\frac{A_1 V_0^2 \sigma_0 \mu_0 + A_2 \sigma_0^2 V_0^2}{\mu_0^2}$$

$$\Delta\, \omega_n^2 = \frac{1}{(\mu_0 + \Delta\mu)^2} \Big\{ -A_1 \left[(V_0 + \Delta V)^2 (\sigma_0 + \Delta\sigma)(\mu_0 + \Delta\mu) - V_0^2 \sigma_0 \mu_0 \right]$$
$$+ A_2 \left[(\sigma_0 + \Delta\sigma)^2 (V_0 + \Delta V)^2 - (\sigma_0 V_0)^2 \right] - (\omega_n)_0^2 (2 \Delta\mu \mu_0 + \Delta\mu^2) \Big\}$$

(b) $2\zeta\omega_n = 2(\zeta\omega_n)_0 + 2\Delta(\zeta\omega_n)$

$$2(\zeta\omega_n)_0 = -A_3 \frac{V_0 \sigma_0}{\mu_0}$$

$$2\Delta(\zeta\omega_n) = \frac{1}{(\mu_0 + \Delta\mu)} \Big\{ -A_3 \left[(V_0 + \Delta V)(\sigma_0 + \Delta\sigma) - V_0 \sigma_0 \right] - 2(\zeta\omega_n)_0 \Delta\mu \Big\}$$

(c) $Z_a = Z_{a0} + \Delta Z_a$

$$Z_{a0} = A_4 \frac{V_0 \sigma_0}{A_5 \mu_0 + A_6 \sigma_0}$$

$$\Delta Z_a = \frac{1}{A_5(\mu_0 + \Delta\mu) + A_6(\sigma_0 + \Delta\sigma)} \Big\{ A_4 \left[(V_0 + \Delta V)(\sigma_0 + \Delta\sigma) - V_0 \sigma_0 \right]$$
$$- Z_{a0} \left[A_5 \Delta\mu + A_6 \Delta\sigma \right] \Big\}$$

(d)

$$K = K_0 + \Delta K$$

$$K_0 = \frac{A_7 V_0^2 \sigma_0 \mu_0 + A_7 A_8 V_0^2 \sigma_0^2}{\mu_0^2}$$

$$\Delta K = \frac{1}{(\mu_0 + \Delta\mu)^2} \Big\{ A_7 \left[(V_0 + \Delta V)^2 (\sigma_0 + \Delta\sigma)(\mu_0 + \Delta\mu) - V_0^2 \sigma_0 \mu_0 \right]$$
$$+ A_7 A_8 \left[(V_0 + \Delta V)^2 (\sigma_0 + \Delta\sigma)^2 - (V_0 \sigma_0)^2 \right]$$
$$- K_0 (2 \Delta\mu \mu_0 + \Delta\mu^2) \Big\}$$

Specification of the Invariant State

The hypothetical airframe is assumed to be at its invariant state, i.e. normal operating state, at an altitude of 35,000 ft., with an airspeed of 1000 ft./sec. and a mass of 497 slugs. This condition produces the following values, defined to be the invariant values, of the parameters:

$\rho_0 = .835 \times 10^{-3}$ slug/ft^3 $\quad \sigma_0 = .35$

$V_o = 1000$ ft/sec. $\quad \mu_0 = 79.1$ (see below)

$M_o = 497$ slugs

In addition, the parameters previously defined as constants have been assigned the following values:

Parameters		Coefficients	
$k_y^2 =$	81 ft^2	$A_1 =$	-0.00275
$K_y^2 =$	1.046	$A_2 =$	0.0485
$S =$	300 ft^2	$A_3 =$	-0.475
$c =$	8.8 ft	$A_4 =$	-0.486
$C_{L\alpha} =$	-4.5	$A_5 =$	-2.0
$C_{m\alpha} =$	-.45	$A_6 =$	-0.6
$C_{m\dot{\alpha}} =$	-3.0	$A_7 =$	0.00306
$C_{mq} =$	-7.0	$A_8 =$	-0.30
$C_{L\delta_E} =$	-0.2		
$C_{m\delta_E} =$	-0.5		

Utilizing the foregoing parameters, the invariant dynamics are determined to be:

$$(W_n^2)_o = 13.1$$

$$2(\zeta W_n)_o = 2.1$$

$$(Z_a)_o = 1.07$$

$$(K)_o = 13.55$$

Once the dynamics have been defined, the problem is reduced to one of compensation; to obtain acceptable aircraft performance.

CHAPTER 4

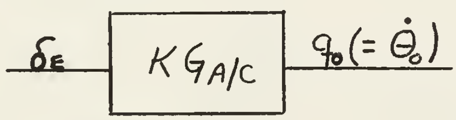
COMPENSATION AND EFFECT OF PARAMETER VARIATIONS

Method

As previously indicated, this study is concerned with compensating the air frame for one state of operation and then testing the effectiveness of this system when the vehicle is in states other than that in which it is compensated. It was found that handling of variational effects could be greatly facilitated by employing the analog representation of variant and invariant parameters shown below.

(a) Functional Diagram and Exact Equation of $\frac{\dot{\theta}_o}{\delta_E}$:

Diagram of elevator to aircraft



Define $KG_{A/C}$ such that:

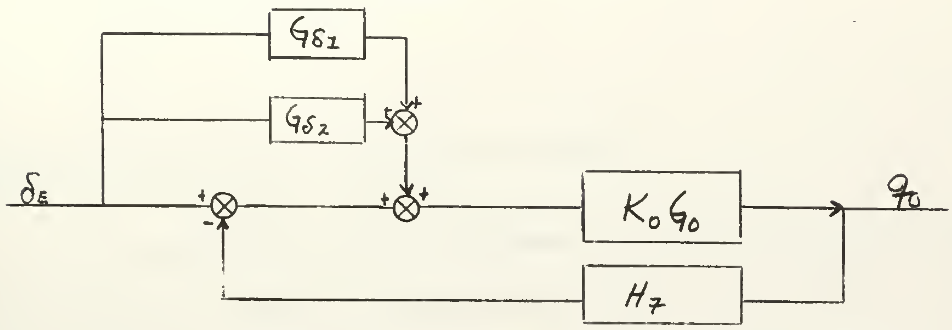
$$(p^2 + 2\zeta W_n\, p + W_n^2)\quad q_o = K\,(p + Za)\delta_E$$

(1) $$\left[p^2 + 2([\zeta\omega_n]_o + \Delta\zeta\omega_n)p + (W_n^2)_o + \Delta(W_n^2)\right]\, q_o =$$

$$(Ko + \Delta K)\;(p + Zao + \Delta Za)\,\delta_E$$

Where the subscript o refers to the invariant parameters.

(b) Analog Representation of Equation (1)



$$(2)\quad q_o = KoGo\,(1 + G_{\delta_1} + G_{\delta_2})\delta_E - (KoGoH_F)\,q_o$$

where $KoGo = \dfrac{Ko\,(p + Zao)}{p^2 + 2(\zeta W_n)_o p + (W_n^2)_o}$

select: G_{δ_1} , G_{δ_2} and H_F such that substitution into (2) yields (1)

let: $G_{\delta_1} = \dfrac{\Delta Za}{(p + Zao)}$; $G_{\delta_2} = \dfrac{\Delta K}{Ko}\left(\dfrac{p + Zao + \Delta Za}{(p + Zao)}\right)$;

$$H_F = \frac{2\Delta(\zeta Wn)p + \Delta(W_n^2)}{Ko\,(p + Zao)}$$

then: $(p^2 + 2(\zeta\omega_{n_0})p + W_n^2)_o \quad q_o = Ko(p+Zao)$

$$\left\{1 + \frac{\Delta Za}{(p + Zao)} + \frac{\Delta K\,(p + Zao + \Delta Za)}{Ko\,(p + Zao)}\right\}\delta_E$$

$$- \left\{2\Delta(\zeta Wn)\,p + \Delta(W_n^2)\right\}\, q_o$$

or: $\left\{p^2 + 2[(\zeta Wn)o + \Delta(\zeta Wn)]\;p + (W_n^2)o + \Delta(W_n^2)\right\}\,q_o =$

$(Ko + \Delta K)(p + Zao + \Delta Za)\,\delta_E$

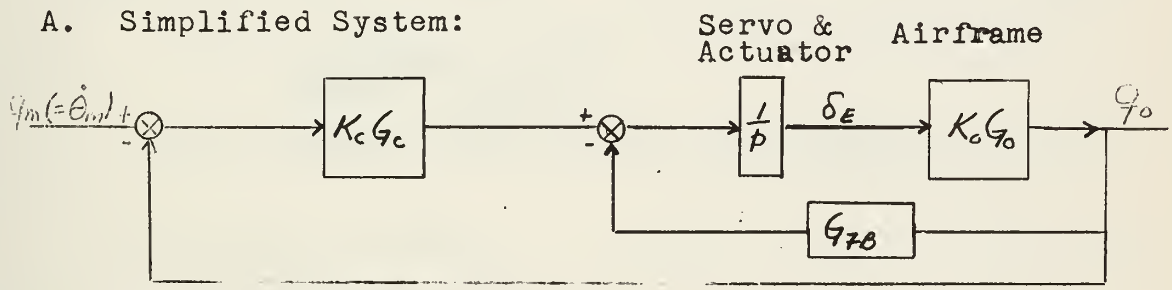
which is identically equal to equation (1)

Hence, to compensate the invariant system, the system can be treated as though the inner loops are opened.

Compensation of the Invariant System

To compensate the airframe in its invariant state, a simplified system was employed. The servomotor and elevator actuator dynamics are reduced and the dynamics of the rate gyro in the feedback loop were ignored. There is no loss of effectiveness caused by making these assumptions. Since one of the objects is to compensate entirely for the servo and actuator dynamics, and the dynamics of rate gyros currently employed in practice are such that they are of small consequence when compared to the dynamics of the other elements in the system.

A. Simplified System:



Open Loop Performance Equation

It is desired to increase the damping ratio to a value of approximately .7 since this damping ratio produces desirable dynamic performance. It is further desired to increase W_n^2 to a value much greater than $(W_n^2)o$, since such an increase will nullify, to a large extent, the effect

of parameter variations which occur when the airframe is not in its normal operating state. Furthermore, a high effective natural frequency of the second order system ensures a "flat" response of this component over a wide frequency band.

$$q_0 = \frac{K_0 G_0}{p}\left(G_c q_m - G_{7B} q_0\right)$$

$$\left\{p^2 + 2(\zeta\omega_n)_0 p + (\omega_n^2)_0\right\} q_0 = K_0 \frac{(p+Z_{a0})}{p}\left\{G_c q_m - G_{7B} q_0\right\}$$

let: $G_{7B} = p\,\frac{(p+Z_b)\,K_1}{(p+Z_{a0})}$; or the equivalent thereof.

then: $\left\{p^2 + \left[2(\zeta\omega_n)_0 + K_0K_1\right]p + (\omega_n^2)_0 + K_0K_1Z_b\right\} q_0 = \frac{K_0}{p}(p+Z_{a0})(K_cG_c)q_m$

or:

$$\left\{p^2 + 2\zeta'\omega_n' p + \omega_n'^2\right\} q_0 = \frac{K_0}{p}(p+Z_{a0})(K_cG_c)q_m$$

where: $\omega_n'^2 = (\omega_n^2)_0 + K_0K_1Z_b$; $2\zeta'\omega_n' = 2(\zeta\omega_n)_0 + K_0K_1$

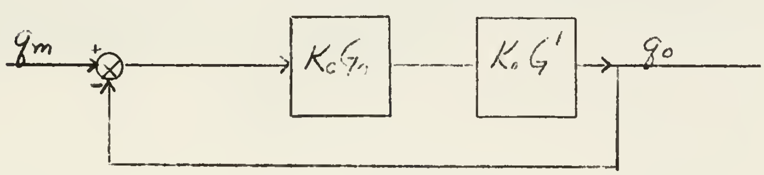
$$\zeta' = \frac{2(\zeta\omega_n)_0 + K_0K_1}{2\sqrt{(\omega_n^2)_0 + K_0K_1Z_b}}$$

Choose values of K_1 & Z_b such that $\omega_n'^2 \gg (\omega_n^2)_0$ and $\zeta' \doteq .7$

B. Determination of KcGc:

let: $K_cG_c = K_c p\,\frac{(p+Z_c)}{(p+Z_{a0})}$; or the equivalent thereof.

Then the system is effectively reduced to:



where:

$$G_0 = (p+Z_c)$$

$$G' = \frac{1}{p^2 + 2\zeta'\omega_n' p + \omega_n'^2}$$

A frequency response analysis was made on the resultant system employing a damping ratio of .7, an effective natural frequency of 20 rad/sec, a loop gain of 25 and a value of Zc = 8 rad/sec. The open and closed loop frequency responses are shown in the chart below:

From the frequency plots:

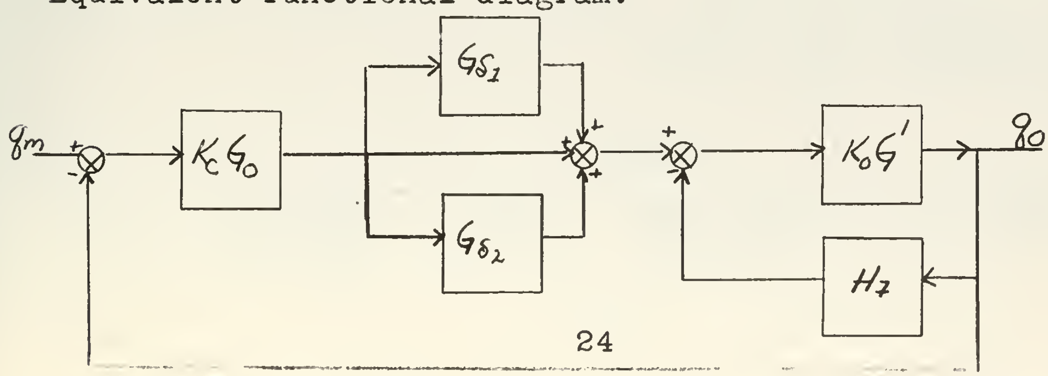
W	Open Loop		Closed Loop	
	AR	ϕ	M	N
.2	1	2	.965	0
.5	1	3	.965	0
1.26	1	6	.965	0
3.10	1	10	.965	0
8.0	1.35	12	.966	0
13.0	1.55	4	.966	0
20.0	1.6	-20	.976	0
50.0	.8	-64	.976	-2.5

where: W rad/sec., ϕ = phase angle $\frac{q_o}{Kq_m}$; $M = \left|\frac{q_o}{q_m}\right|$; N = phase angle $\frac{q_o}{q_m}$

$AR = \left|\frac{q_o}{Kq_m}\right|$

Effect of aircraft/flight path parameter variations on compensated system

Equivalent functional diagram:



As indicated on the first page of this Chapter, G_{δ_1}, G_{δ_2} & H_F are fictitious functions used to represent the effects of parameter variations. The existence of such variations is equivalent to closing the loops containing these functions. With these loops "closed" the following analysis of the system is made.

Performance Function:

Unity feedback loop open;

$$\frac{q_o}{q_m} = \frac{K_o K_c G_o G'(1 + G_{\delta_1} + G_{\delta_2})}{1 + K_o G' H_F}$$

The frequency response of the first bracketed term is the open loop frequency response of the compensated invariant system. The frequency response of the second term is determined by substituting the functions for G_{δ_1} & G_{δ_2} previously derived, and the resultant function becomes: $K_1 \frac{(\tau_{n_1} p + 1)}{(\tau_{d_1} p + 1)}$

where: $K_1 = \frac{(K_o + \Delta K)}{K_o}\frac{(Z_{ao} + \Delta Z_a)}{Z_{ao}}$

$$\tau_{n_1} = (Z_{ao} + \Delta Z_a)^{-1} \; ; \; \tau_{d_1} = (Z_{ao})^{-1}$$

The frequency response of the third term is similarly determined and the resultant function of $(1 + K_o G' H_F)^{-1}$ becomes:

$$\frac{1}{1 + \dfrac{K_2(\tau_{n_2} p + 1)}{\left(\dfrac{p^2}{\omega_n^2} + \dfrac{2\zeta'}{\omega_n} p + 1\right)(\tau_d p + 1)}}$$

where:

$$K_2 = \frac{\Delta \omega_n^2}{Z_{ao}\,\omega_n^2} \; ; \; \tau_{n_2} = \frac{2\Delta(\zeta \omega_n)}{\Delta \omega_n^2} \; ; \; \tau_{d_2} = (Z_{ao})^{-1}$$

Thus, to produce the open loop frequency response of the controlled system, it is necessary to determine:

$$\frac{q_o}{q_m}(jw) = AR_T \angle \phi_T$$

where:

$$AR_T = \log^{-1}\left\{ \log AR_{(KoKcG_DG')} + \log AR_{(1+G_{\delta_1}+G_{\delta_2})} - \log AR_{(1+KoG'H_F)} \right\}$$

$$\phi_T = \phi_{(KoKcG_DG')} + \phi_{(1+G_{\delta_1}+G_{\delta_2})} - \phi_{(1+KoG'H_F)}$$

Using this method of variations representation; frequency analysis plotting and transient response, by analog simulation, were made for the compensated airframe in seven different flight conditions.

The flight conditions tested were chosen as a result of the desire to impose the most severe tests of the effectiveness of this technique of compensation. Limits of altitude, airspeed, and loading were determined from current literature, and from operational Naval aviators, in an effort to maintain the conditions within practical limits.

Definition of Flight Conditions

Condition A represents the aircraft operating in its invariant (normal) state. Condition B represents the aircraft operating at high altitude, at high speed, and with payload expended. Condition C shows the aircraft making an approach for landing, with payload expended. Condition

D simulates the aircraft at a "medium" altitude and at reduced speed, while fully loaded. Condition E represents the aircraft at high altitude, reduced speed, and payload expended. The F condition shows the vehicle at the same altitude and speed conditions as E, but fully loaded. Finally, condition G simulates the aircraft in takeoff condition, fully loaded.

The resultant transient and frequency responses of the aircraft in the seven test states, both in a compensated and uncompensated control state, for comparison purposes, are included as Appendices A and C.

CHAPTER 5

MODEL DESIGN, VARIABLE GAIN AMPLIFIER AND OVERALL SYSTEM PERFORMANCE

Model Design

As indicated in Chapter 2, the system design objective is to cause the system output response to approximate as closely as possible the response of the model to various inputs. The model is the physical embodiment of the design specifications of a system, and a change in the model is exactly equivalent to a change in system specifications. The first consideration, then, is to decide upon the system specifications, which must be compatible with performance capabilities of the aircraft. The model can then be designed independently of the design of the control system.(3)

Since the theoretical air frame was chosen with no specific mission in mind, the model reference system for this study was arbitrarily selected to be a second order system characterized by an undamped natural frequency of 3.5 rad/sec and a damping ratio of .7.

Variable Gain Amplifier

It is shown in Appendix A that the compensated controlled system response has little phase shift from the controlled system input, for the different conditions tested over a wide range of frequencies. However, the system does attenuate the input to varying degrees for the seven test condition.

In order that the requirement, that the system output response duplicates as closely as possible the model response, be fulfilled, it is required that an amplifier be inserted between the model and the controlled system. However, since the system attenuates its input by varying amounts, depending upon the flight condition, it is necessary that the amplifier be a variable gain device sensitive to the flight condition. The performance criterion of this device is such that:

$$\left|\frac{q_o}{q_m}\right| \doteq 1$$

Employing the following development; an acceptable performance, within the above criterion, of the variable gain amplifier is obtained:

Define: $g_o = \frac{V_o \sigma_o}{\mu_o}$; $g_x = \frac{V_x \sigma_x}{\mu_x}$; K_{var} = gain of variable amplifier

Where o subscript refers to the invariant state of the parameter. The x subscript refers to the general state

of the parameter, and V, σ and μ are as defined in Chapter 3.

$$\text{Required: } K_{var} \left[\frac{25\ g_x/g_o}{1+25\ g_x/g_o}\right] = 1$$

$$\text{or: } K_{var} = \left[\frac{1+25\ g_x/g_o}{25\ g_x/g_o}\right]$$

As indicated above, the use of a variable gain amplifier having this performance enables the controlled system output response to duplicate the model output response.

Overall System Performance

With the reference system model and variable gain device established, a frequency response plot and an analog computer transient response were made for the overall system. The results of these tests are included herein as Appendix D.

It was found that for all frequencies of interest the overall system, i.e., the model, variable gain amplifier and closed loop pitch rate system; can be represented by the following functional diagram:

Where K_{att} represents the controlled system attenuation of the input to the controlled system and K_{var}

$K_{att} \doteq 1$

This representation was found to be valid regardless of the condition in which the aircraft was operating.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Using only the linear technique as described heretofore, the pitch rate control system can be compensated to the extent that the aircraft response follows the reference system model output with a phase shift of no more than $\pm 10^{\circ}$ for all practicable input rates. This method of compensation is not completely self-adaptive, since the compensated system attenuates its input (the reference system model output) amplitude in varying degrees up to 25%. Unless this attenuation is acceptable, a variable gain amplifier should be employed to eliminate this feature of the compensated system. Since the variable gain amplifier is programmed so that it is sensitive to the flight condition of the aircraft, it is an overall nonlinear device from state to state. But, with the aircraft in a given state, the amplifier performs as a

linear element. Utilizing the combination of linear compensation and a programmed variable gain amplifier, a pitch rate control system having essentially a "flat" frequency response, i.e., $\pm 10^\circ$ phase shift and an amplitude ratio of $1 \pm 1\%$, is feasible.

As indicated in the scope, the basic objective of this method of compensation is the attainment of a flat response. In other words, the system was designed with frequency response characteristics as the principal consideration. This system when so considered became self adaptive, to a great degree of accuracy.

Of secondary consideration in this design, but of great significance in actual practice, are the transient characteristics of the compensated system. Ideally, the transients of the compensated system would be of small significance relative to the reference system model transients, or at worst, no greater than the model transients. Under this ideal arrangement, the response time of the overall system used in this paper is the response time of the second order model, i.e., .85 seconds to come within 5% of correspondence. The analog computer tests of the overall system, Appendix D, shows that this system does approach the ideal transient performance. However, the computer results are misleading as a result of

mechanization of the problem, in that the computer analysis was made for the general case and conditions. The general case was then extrapolated by changing of dial settings only, which did not allow the required flexibility for proper solution to flight condition F.

The step response equations representing the air frame dynamics in its various operating states as tabulated in Appendix C, show that with the aircraft in flight conditions A, B, and C, the transients are of no consequence relative to those of the model. Conditions D, E, and G yield transients, which are of the same order of magnitude as those of the model. With the aircraft in condition F the response time of the control system is noticeably greater than that of the reference model. Additional study of performance equations revealed that the minimum value of the quantity $\frac{V\sigma}{\mu}$ required to produce transients which are ideal, within the previous definition, is 2.3. All of the flight conditions tested, except F, have values of this quantity which are greater than this minimum.

Exclusive of the above limitation, the method of invariant (normal state) parameter compensation is effective in obtaining self adaptation in the pitch rate control system. It is reasonable to presume that this same technique will be valid and useful in obtaining the desirable level

of self-adaptability, in any control system.

Recommendations

It is recommended that:

1. The system as developed in this study be mechanized and tested further. In the mechanization of this system, it is envisioned that a rate gyro, such as the MIT 10^4 rate gyro, with a static sensitivity of 1 mv/mr/sec be employed. Such a device ensures the attainment of gain requirements without saturation problems. To attain certain of the performance functions employed in the development in Chapter 4, it may be necessary to utilize high gain devices fed back upon themselves through devices having transfer functions which are the reciprocals of those desired.

2. As an intermediate testing and analysis procedure, it is evident that another type of analog simulation of the linear adaptive system should be made, using basic mechanisms and associated electronics. The characteristics of these devices should closely approximate those specified in recommendation #1.

Further, some of the amplification factors and controls may be simulated by simple hydraulic valves and connections.

This "breadboard" type of study would satisfy a

basic design goal; that of using available components that are already capable of design and/or manufacture. These devices, once proven, can be extrapolated and modified as designs for the specific devices to be used in the control system desired.

3. Additional study be made on this technique of compensation with the objective of obtaining a compromise system which yields an acceptable frequency response and an acceptable transient performance, i.e., near ideal, for all operating states of the aircraft.

APPENDIX A

Frequency Response of the Pitch Rate System.

1. Definition of the Basic Flight Conditions.

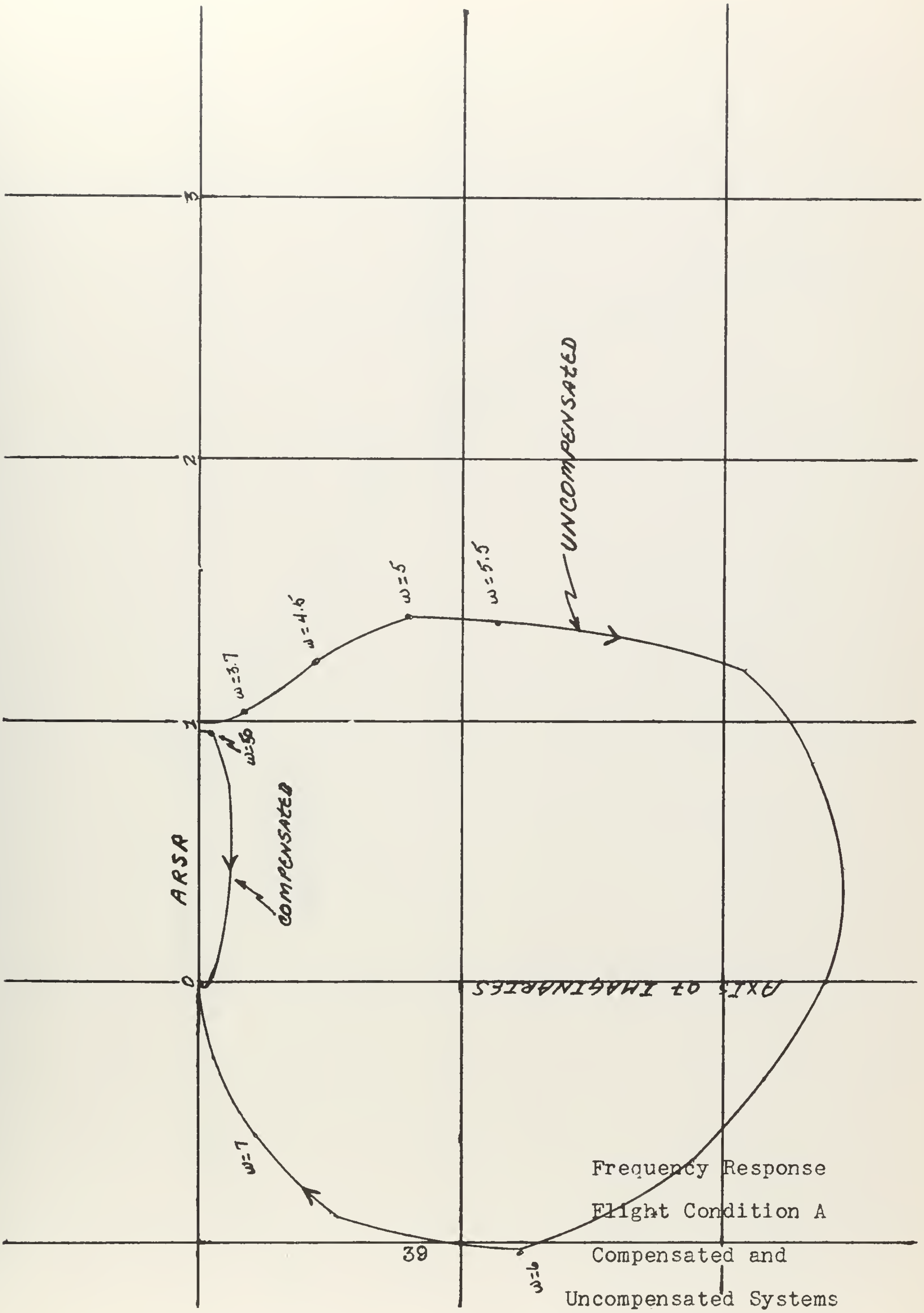
2. Polar Diagrams of the Compensated System.

3. Polar Diagrams of the Uncompensated System, Unity Feedback only.

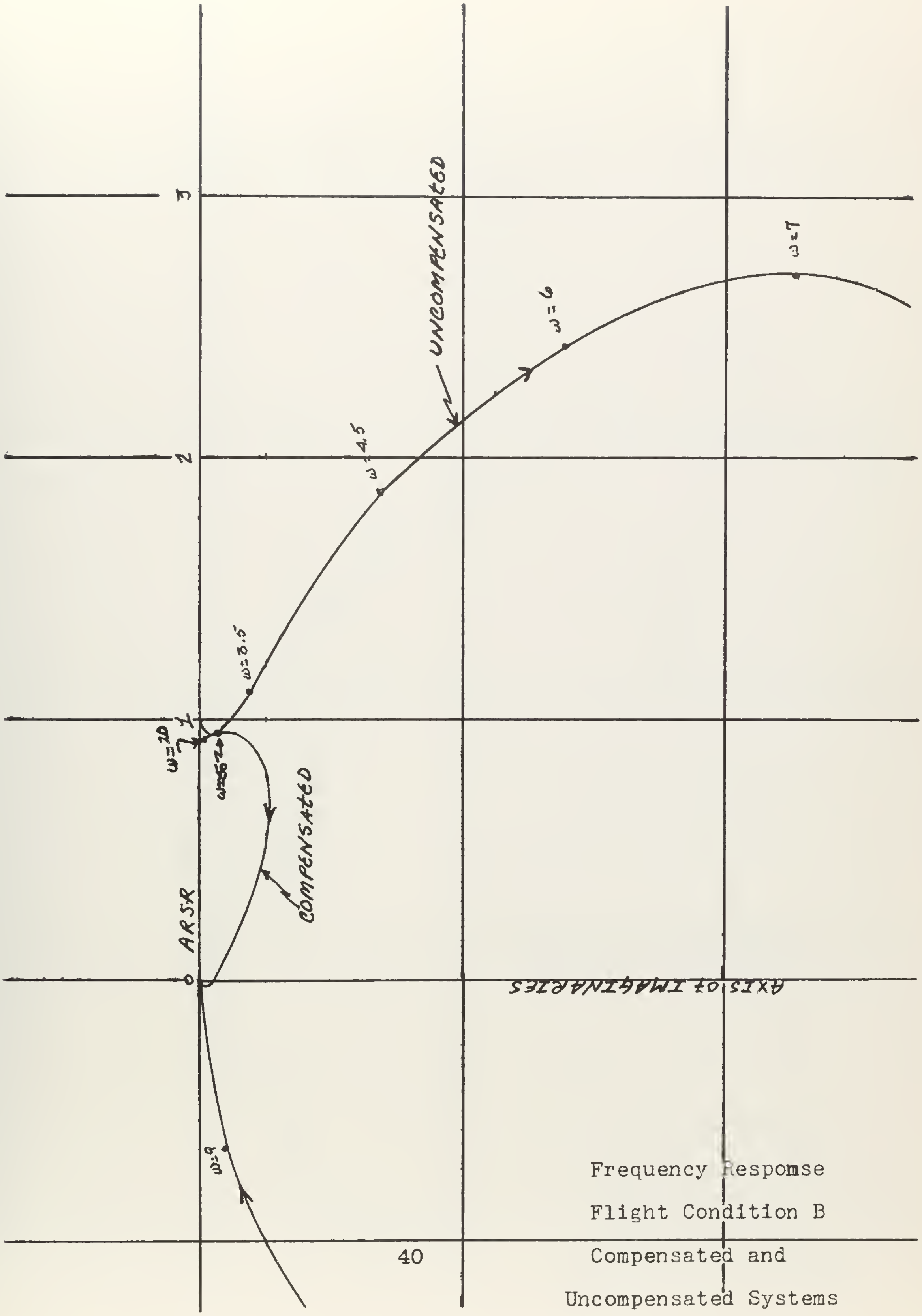
1. The Basic Flight Conditions Defined:

Condition	Altitude (ft)	Airspeed (fps)	Mass(slugs)
A	35,000	1000	497
B	70,000	1000	248.5
C	Sea Level	500	248.5
D	35,000	500	497
E	70,000	750	248.5
F	70,000	750	497
G	Sea Level	250	497

2. $$(ARSR)_{A-G} = \left| \frac{q_{out}}{q_{model}} \right|$$



Frequency Response
Flight Condition A
Compensated and
Uncompensated Systems

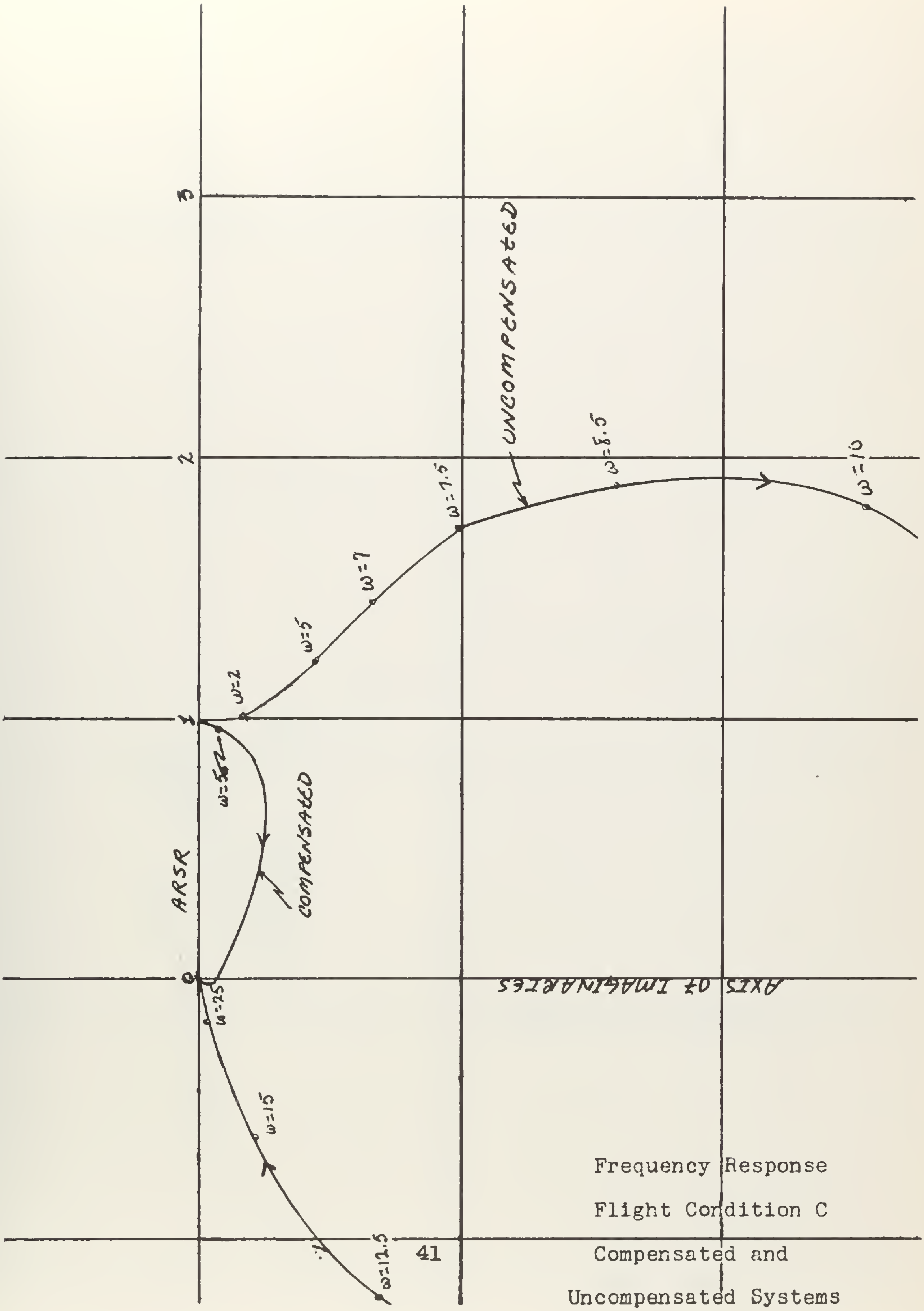


Frequency Response

Flight Condition B

Compensated and

Uncompensated Systems

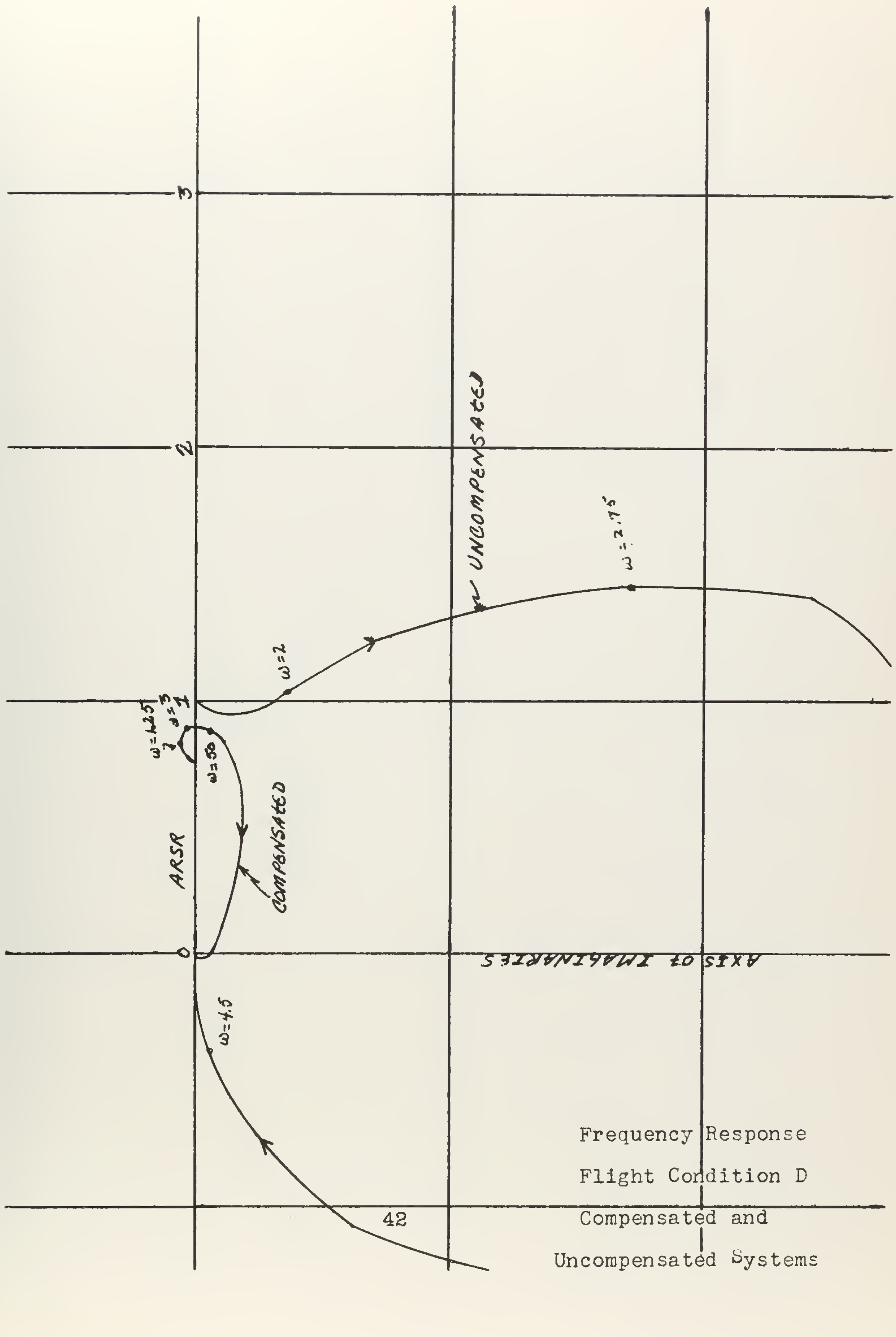


Frequency Response

Flight Condition C

Compensated and

Uncompensated Systems

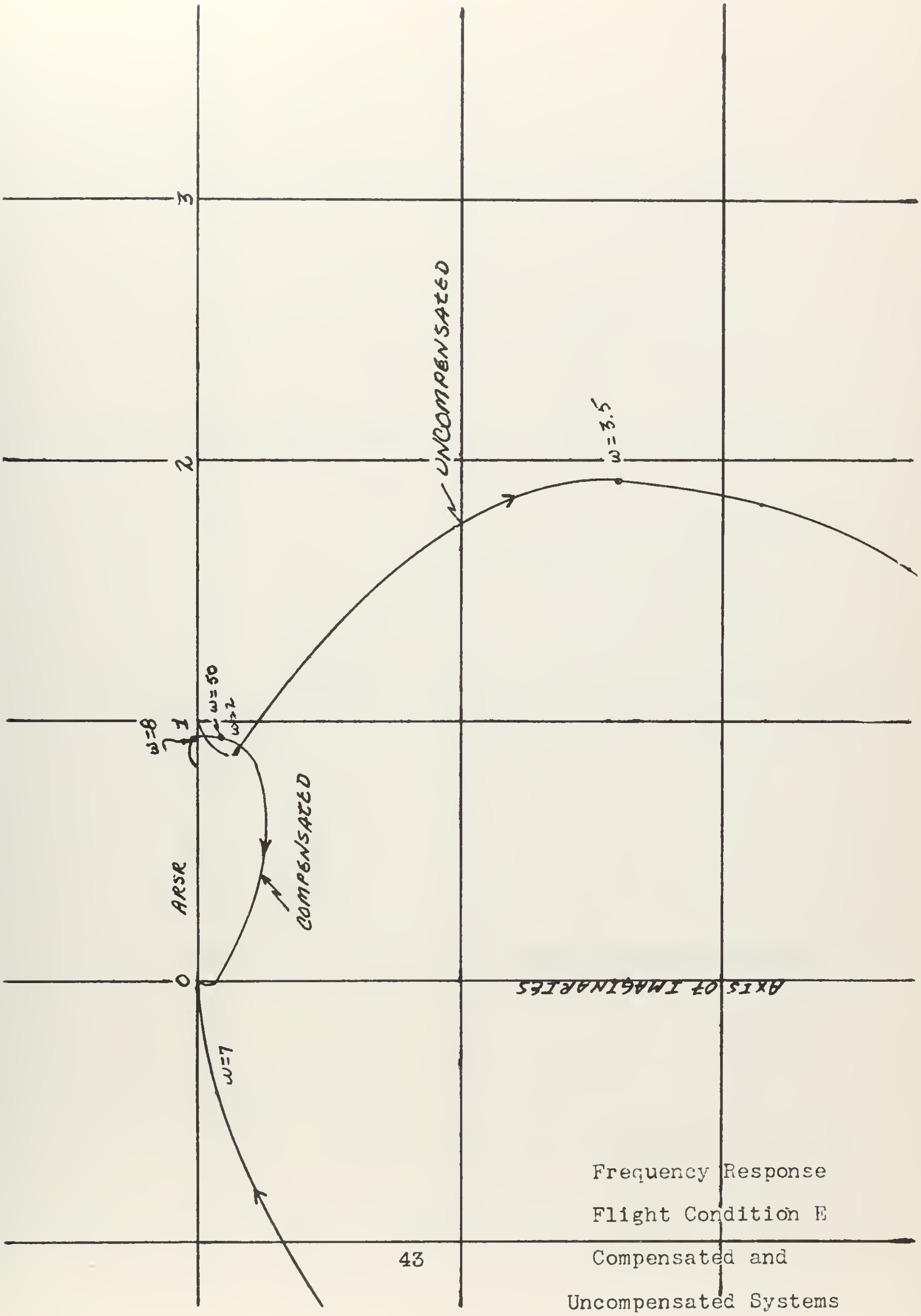


Frequency Response

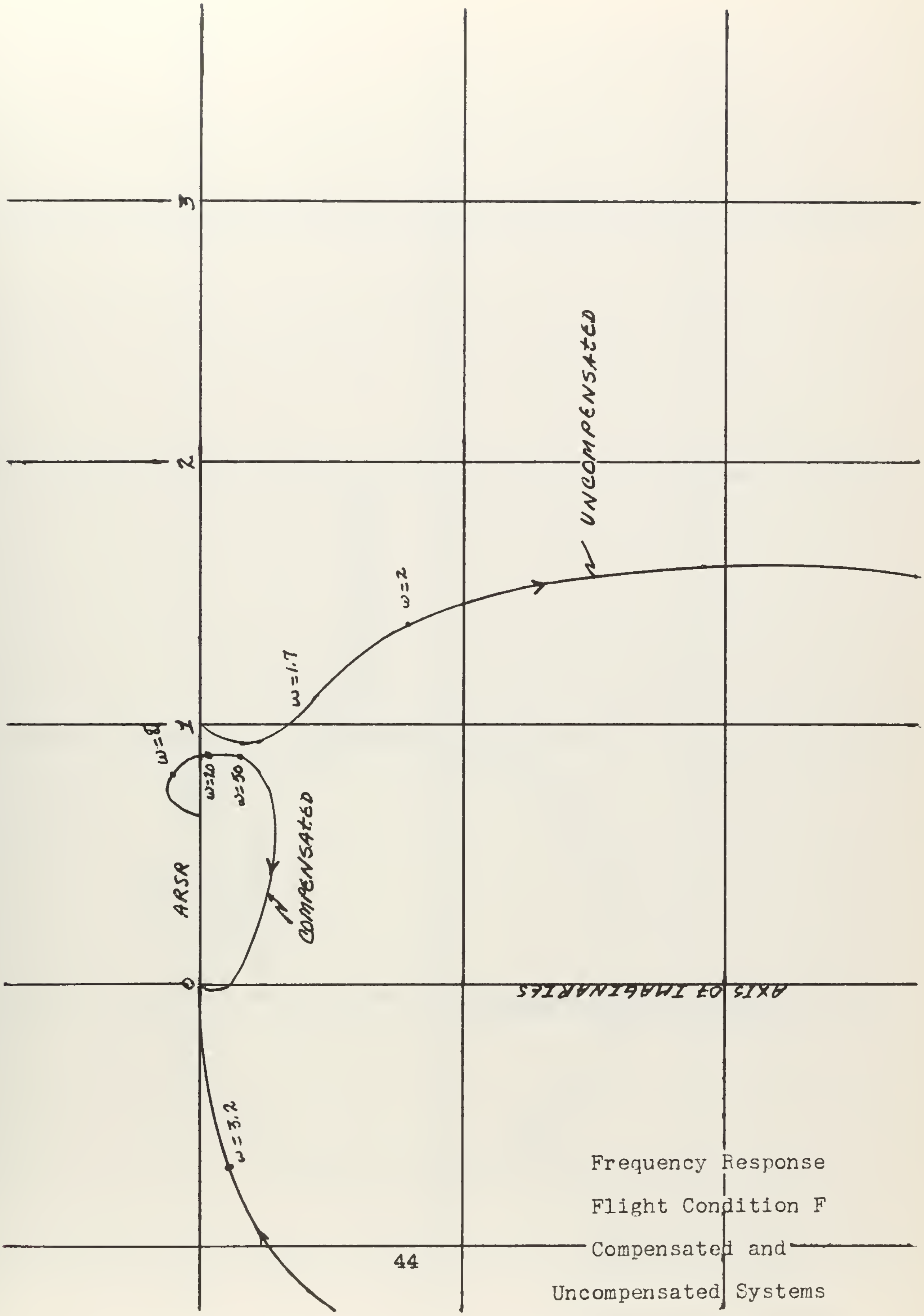
Flight Condition D

Compensated and

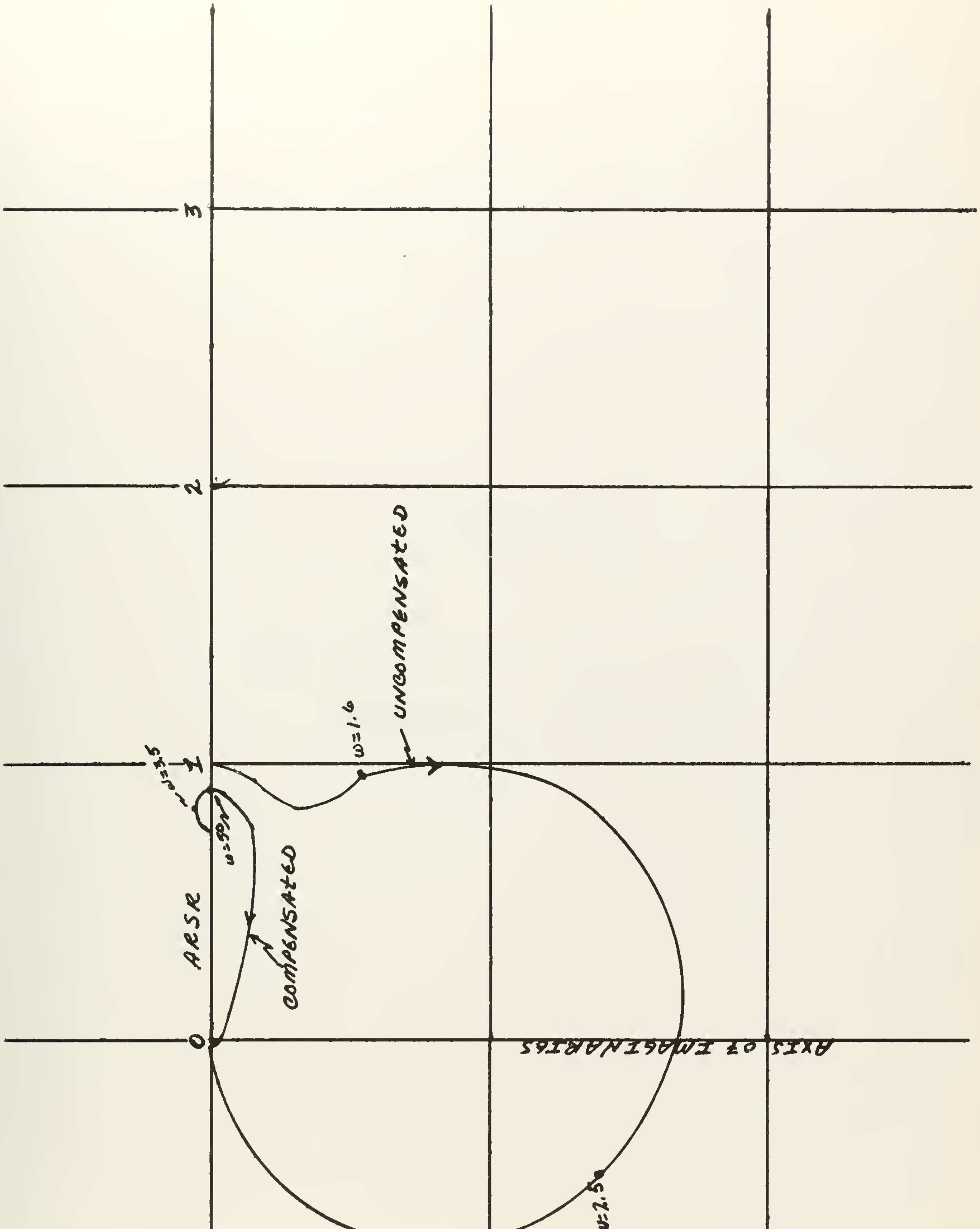
Uncompensated Systems



Frequency Response
Flight Condition E
Compensated and
Uncompensated Systems



Frequency Response
Flight Condition F
Compensated and
Uncompensated Systems



Frequency Response
Flight Condition G
Compensated and
Uncompensated Systems

APPENDIX B-1

The GAP/R Analog Computer; General Information & Techniques.

General Information

The G.A. Philbrick Researches Incorporated, Electronic, high speed analog computer was used to verify the transient response characteristics of the uncompensated aircraft pitch rate system with unity feedback, the compensated system, and the idealized model transient response.

It was decided to test the pitch rate system, uncompensated and with unity feedback only, since this is an obvious first approximation to self adaptability, and then compare this with the response of the final compensated unity feedback system.

The Computer used is one of the earlier models of the GAP/R unit. The idea involved is that of a combination of "black boxes" or modules, that perform specific functions within themselves, such as multiplication, addition, integration, differentiation, squaring, and simulation of nonlinear functions. Further, these capabilities may be extended to include complete first and second order servo systems. The modules were designed to operate in "fast" time, so that results may be seen on any standard oscilloscope trace, from which permanent photographic records can be made. In the modules, herein after referred to as boxes, that are concerned with time functions, there are

characteristic time constants (T_O) settings of 4 and .4 milliseconds. Modifications were made at MIT that allowed "real" time operations, by changing the value of the condensers in the box that is concerned, so that 4ms, .4ms and 1 second, are available for T_O settings.

The boxes actually used in the computer simulation are described more fully in the latter section of this appendix.

The nominal voltage range for all variables is 100 volts, that is, minus 50 to plus 50. The manufacturer specifies that electronic variations affect the sensitivities by less than 1%. Each box is made up of one or more DC minimum drift amplifiers that have been modified by chopper stabilization. The drift problem was negligible, forafter initial alignment, checks on the less complicated units showed very little drift when noted daily. However, the more complicated systems, such as the first order lead/lag networks, and the second order generating system, which involves eight amplifiers in the units, required several checks in the course of solution.

As will be seen in the attendant circuit diagrams, the operational amplifier potentiometers, which regulate

the magnitude of multiplication factors, are of the proportional type, i.e., one portion in the feedback loop of the DC amplifier, and the other is in the feed forward loop. These are normally set so that clockwise rotation of the potentiometer dial puts more resistance in the feedback, while reducing that in the forward loop, thus, causing multiplication. In the K4DY second order however, the dial A_2 is wound oppositely, therefore division is performed for clockwise dial rotation. This is necessitated by the form of the standard second order differential equation.

Electronic clamping of the integrating boxes back to zero potential is necessary, if the solution to the problem is not reached prior to the characteristic time of the integration units that are involved in an open loop solution. If these units are connected in closed loop, clamping is generally not required; however, due to drifting DC levels, it is best to clamp all integrating boxes, as a matter of course.

Calibration of the dial settings on the multiplication and DY boxes was difficult, since normal bridge resistance measurements would not give satisfactory results. Rather, known voltages were fed into the boxes

being calibrated, and the dial set to give the desired value of output voltage. This is not too time consuming, since other calibration methods on components require as much care and time, for accurate results.

The feature of high speed solution and simultaneous viewing is the greatest advantage of this type of computer. It is readily useful in simulating any control system, due to the many functions that can be generated by a series of interconnected black boxes, and a quick solution, or quick evidence of an improper solution, becomes apparent almost immediately. Since so many solutions, to physical systems, are in the form of the standard second order differential equation, then the K4DY box inherently is ideal for a quick approximate solution.

Method

In setting up the pitch rate control system on the computer, there were some modifications required:

In that the time constant of the unit lead box is set from zero to one, and it is desired that all T_o be set at .4ms (for ease in scaling) then the uncompensated open loop system equation of the form:

$$q_o = \frac{K(pT+1)q_m}{p\ (\text{2nd order equation})}$$

is rewritten as:

$$q_o = \frac{K(T + \frac{1}{p})q_m}{(\text{2nd order equation})}$$

where $\frac{1}{p}$ can be simulated by an integration box.

The same type of problem arises from the compensated closed loop system form:

$$q_o \quad \frac{K(pT_1 + 1)\ (pT_2 + 1)q_m}{(\text{2nd order equation})} \quad \text{where } T_1 > 1 \text{ and } T_2 < 1$$

rewrite as:

$$q_o = \frac{K(p + \frac{1}{T_1})\ (pT_2 + 1)T_1 q_m}{(\text{2nd order equation})}$$

and KT_1 becomes a new K', and p can be approximated by a derivative box. This derivative box, since a perfect derivative is almost impossible to generate, causes a time lag error of 1.5 seconds (problem time) in the parallel network in which it is operating. This leads to an undesirable time lag in the computer simulation. Realizing, however, that this particular shortcoming exists; permits its removal in the final analysis. This unwanted lag is not apparent in the unit lead box.

The compensated system, has the open loop form:

$$q_o = \frac{K'(pT+1)\ (p+Z)q_m}{(\text{2nd order equation})}$$

Where $\frac{1}{T}$ approaches one of the roots of the 2nd order equation, and Z approaches the other root. There is high gain in the forward loop, also. Then, to a close approximation, the equation may be written in the final closed loop form as:

$$q_o = \frac{K'\ (pT_1+1)q_m}{(pT_2+1)}$$; K' includes K and the T's that are determined, as are T_1 and T_2, according to the problem involved.

Thus, the equation can be mechanized in the closed loop by use of a multiplication box in series with a unit lag and a unit lead box, with no feedback connection required.

Scaling

Magnitude scaling was carried out, assuming that the peak signal voltage available from the square and sine wave generator was 13.5 volts and noting that 50v was the limiting factor of the DC amplifiers, and that the maximum input signal to the pitch rate system was from .1 to .3 radians/second. For scaling purposes, the quantities E and X are introduced to the problem as intermediate quantities.

E, the error, is the difference between q_m and q_o. X is the input to the unity gain second order system.

For simplified presentation, the output gain of the system was changed to show the final results, in the photographs, in the proper relative aspect. That is, the output is scaled relative to the input. Thus, if the input is 10 squares, and the output is 8 squares in steady state, then the final value is 80% of the input, and no extra magnitude scaling is required of the reader.

Time scaling for this problem was set up to be a special case, in that all units involving time were set for T_o equal to .4ms. Thus, 1 second of "real time" on the computer; in computer (problem) time is 2500 seconds.

A dual beam oscilloscope was used for final presentation, in order that both input and output may be viewed simultaneously.

Input Signals

Response to a step function, to simulate violent maneuvers; and a ramp function, to simulate more gradual maneuvers, was used. The frequency of the square wave step function was 30 cps. In problem time, this is 42 seconds/step. The slope of the negative input ramp is

.12 units of input/computer second. Each small square on the grid of the photographs represents 1.67 seconds of problem time.

Frequency Response

The scope of this paper does not include experimental frequency analysis. However, in preliminary study of the problem it was found that the more complicated boxes have their own frequency characteristics. It was also noted that to make proper response for lead, or differentiating systems, the following must be done. Connect a system to simulate a lead lag transfer function made up of integrators, potentiometers and adders. Make the denominator term of the network far enough out on the real axis, so as to essentially leave a pure lead term in the final result, that gives the best frequency response. The unit lag boxes exhibited better frequency characteristics. The second order box was accurate for the frequencies tested, and should adapt well to measurement of simulated sinusoidal gust type responses.

Circuit Representation

A more detailed study of the unit boxes, their functions and capabilities, along with a shorthand method of representation, is presented in later sections for the

following reasons:

1. Since most present-day manufacturers of analog computers have all but abandoned the modular technique; the internal circuitry is shown for comparison with present simplified "plug in" R and C component methods.

2. To demonstrate, in block diagram form, the computer final circuit connections; to simulate the pitch rate system, model, and associated compensation devices.

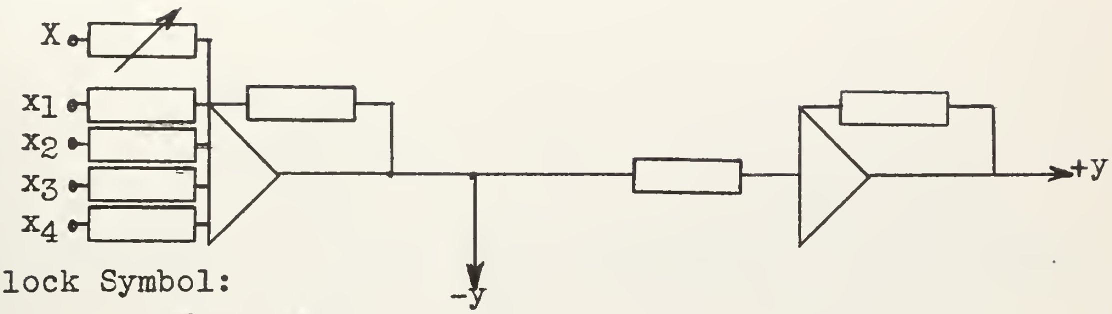
APPENDIX B-2

Circuit and Block Symbols of the

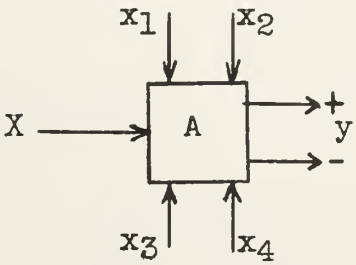
GAP/R Components.

K3-A Adding Component

Equation: $y = x_1 + x_2 + \ldots X$



Block Symbol:



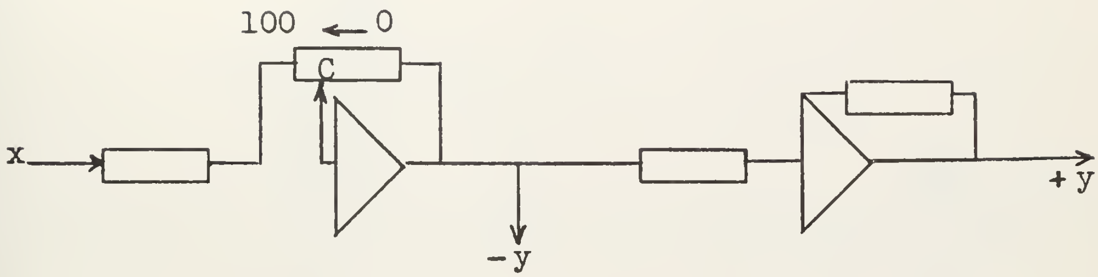
Note: for simplicity, the JIC and NEMA symbol for control resistance is utilized:

DESCRIPTION: The "Adder" computes the instantaneous sum of four or fewer input signals. By interconnecting Adders in tandem, any number of signals may be combined in sums and differences. With N such Components, 3N + 1 signals may be added. Each Adder features a calibrated dial whereby a steady voltage may be added in, amounting at the extremes to plus or minus 10 volts, or 20% of the maximum signal excursion. This is convenient for manual addition of a constant term in an equation.

Signals receive neither gain nor attenuation through the Adding Component.

K3-C Coefficient Component

Equation: $y = \pm Cx$



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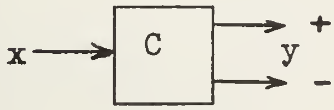
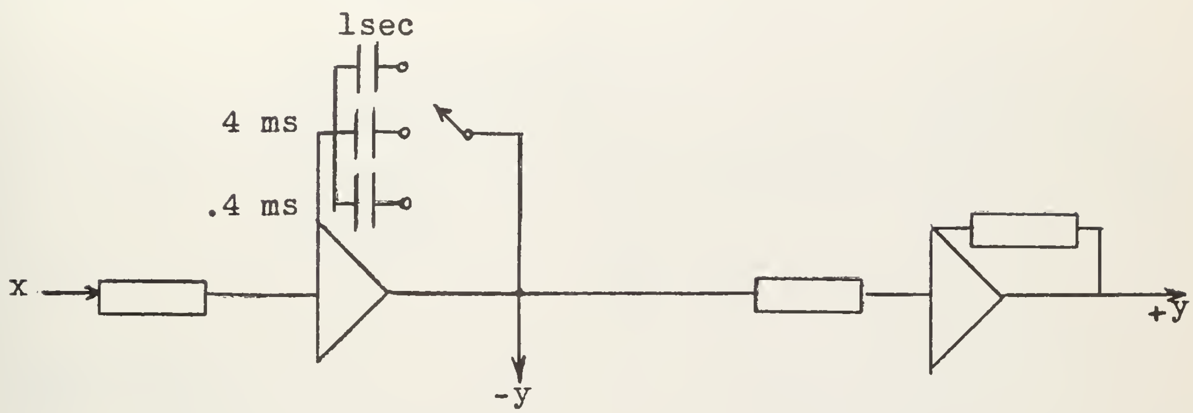


DESCRIPTION: An input signal to this Component is automatically multiplied by a numerical factor, then carried directly and inversely to the two outputs. The factor may be set on a specially calibrated nonlinear dial, extending from 0 to 100. Technically, gains from a precise zero to infinity are within range, but, the best linear range is 0.1 to 10.

K3-J Integrating Component

Equation: $\pm y = \frac{1}{T_o}\int x\,dt = \frac{x}{T_o p}$



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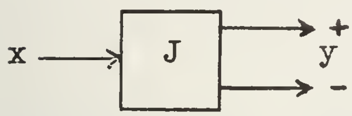
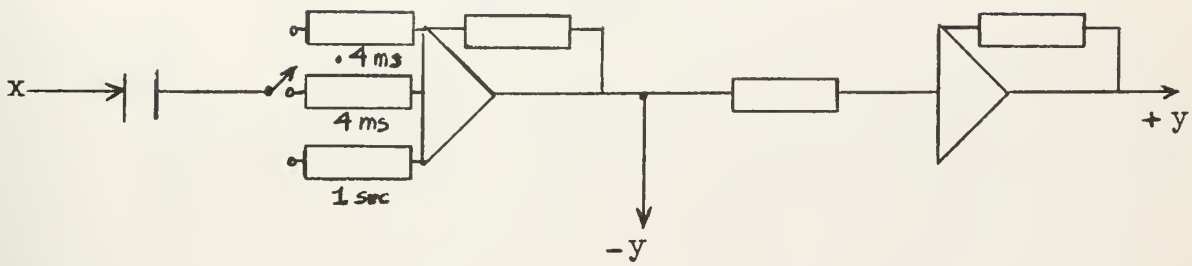


DESCRIPTION: This Component, the "Integrator", computes an integral with respect to time of the voltage supplied to the lower input. The time-factor T_o is 0.4 millisecond. For normal computing, in closed loops, the right-hand switch position,is chosen for maximum sensitivity; the left-hand switch position introduces a stabilizing network and provides for open-cycle operating when desired, as for the purpose of display alone.

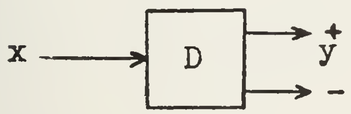
The integrated outputs are artificially and automatically returned to zero by means for a "clamping" signal supplied to the upper input.

K3-D Differentiating Component

Equation: $\pm y = T_o \frac{dx}{dt} = T_o px$



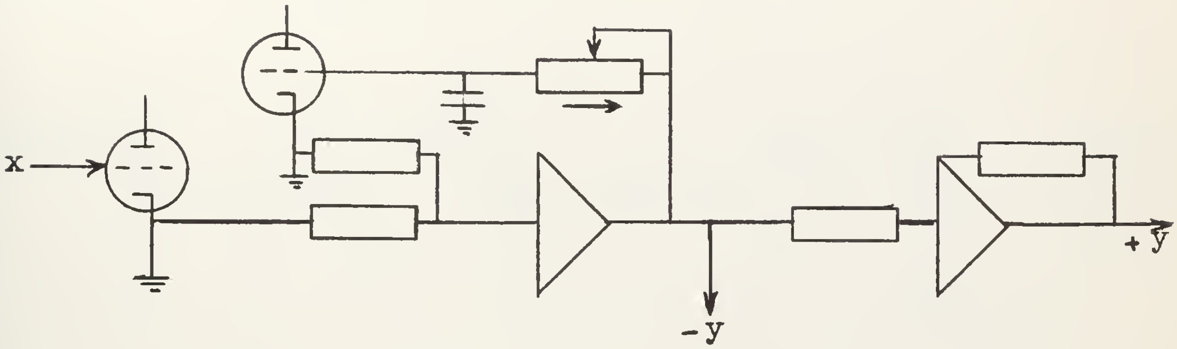
Block Symbol:



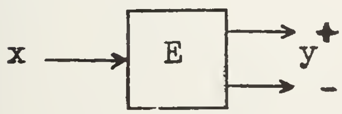
DESCRIPTION: Operationally, the "differentiator" performs inversely to the Integrator. It computes time derivative of the input signal. The time-factor T_o is 0.4 milli-second. With the switch at the left-hand setting, as in this case, a lag is introduced having a time constant of about 50 microseconds; about 1/10 the fixed time-factor of the Component. This setting is an approximate derivative, which did affect accurate representation showing as spurious lag.

K3-E Augmenting Differentiator

Equation: $\pm y = x + T\frac{dx}{dt} = (1 + Tp)x$



Block Symbol:

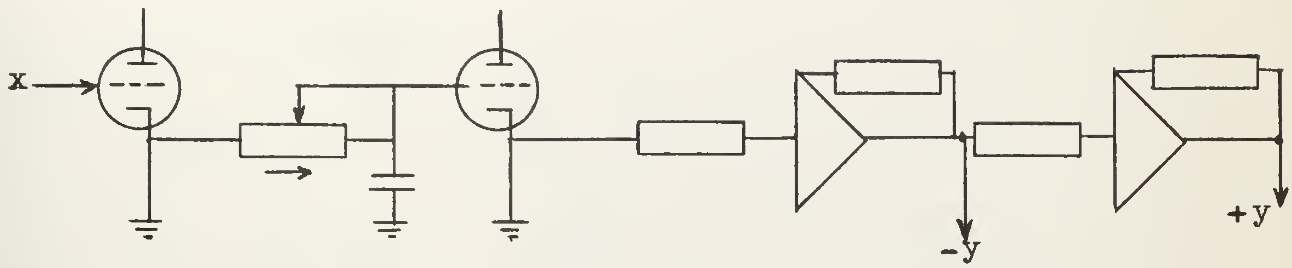


DESCRIPTION: This Component is related in a simple way to the K3-D Differentiator, since it passes the input signal and adds to it the first time derivative of that signal. The time-factor T, the derivative sensitivity, is continuously adjustable from zero up to 0.4 milli-second on a linear percentage scale. The operator embodied in inverse to that of the K3-L Unit-lag Component.

For certain purposes this Component may replace a combination of K3-A, K3-D, and K3-C. It represents the introduction of a derivative response.

K3-L Unit-Lag Component

Equation: $y + T\frac{dy}{dt} = (1 + Tp)y = \pm x$



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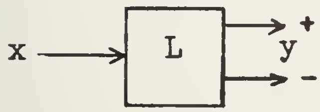
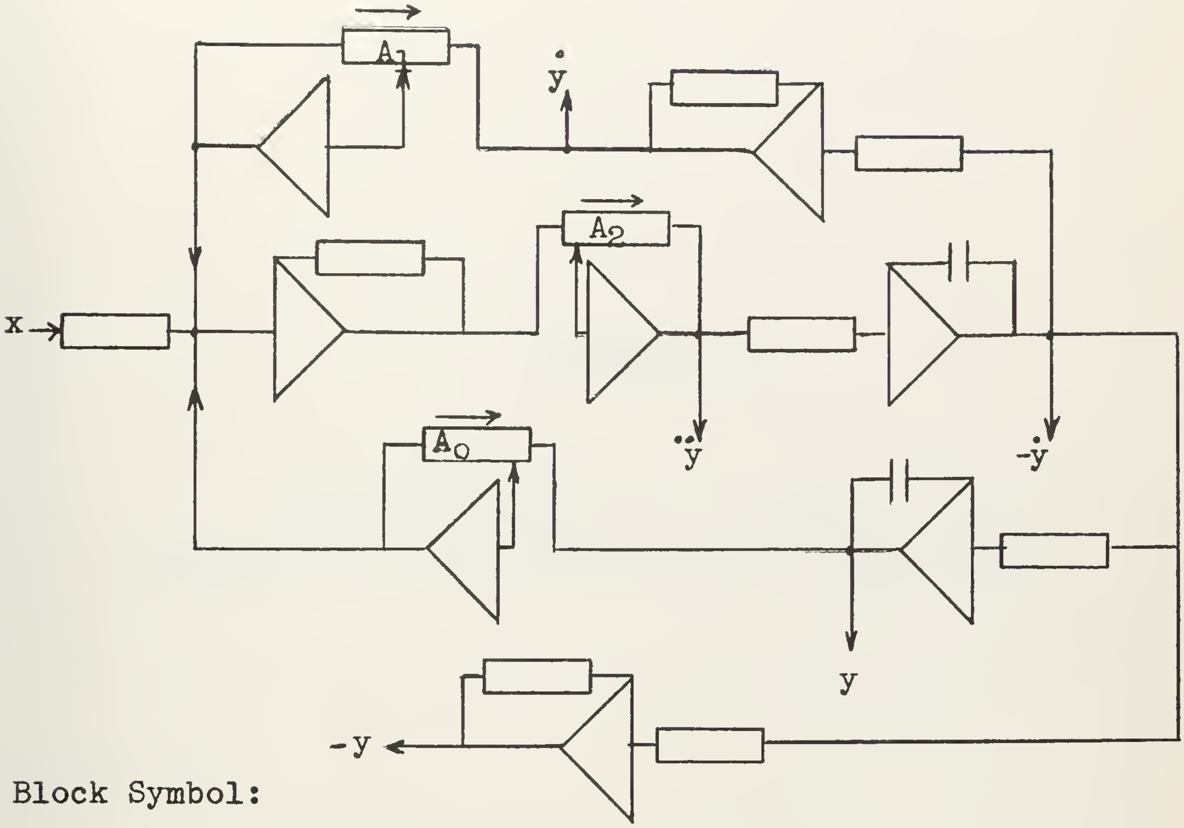


DESCRIPTION: The "Unit-lag" performs a first-order lagging operation. By means of its 0-100 dial, the time constant T may be adjusted from zero up to a maximum of 0.4 millisecond. The final, or long-term, sensitivity is unity. This type of operation or dynamic characteristic may be obtained by a simple loop involving a K3-A, a K3-J, and a K3-C, which loop also affords several other characteristics. However, the K3-L Component provides a compact and convenient version of this frequently recurring operator.

K4DY Dynamic Component

Equation: $A_2 T_o^2 \frac{d^2y}{dt^2} + A_1 T_o \frac{dy}{dt} + A_o y = x(t)$



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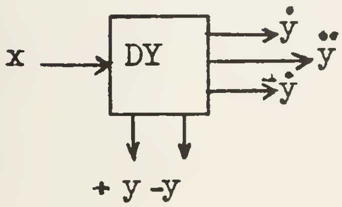
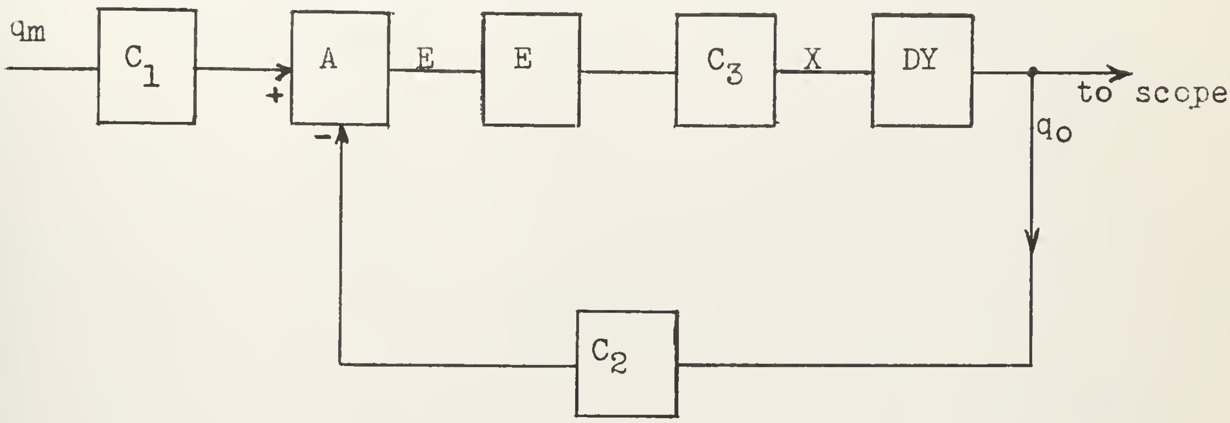


DESCRIPTION: This box combines the functions of eight assorted A,C, and J boxes into one simplified unit. The standard second order equation satisfied by this unit, is an intrinsic part of most actual physical systems. T_o is equal to .4 milliseconds, and A_1, A_2, A_o are directly calibrated on 0-100 dials of the Coefficient type. Dial A_2 is wound inversely, as previously described.

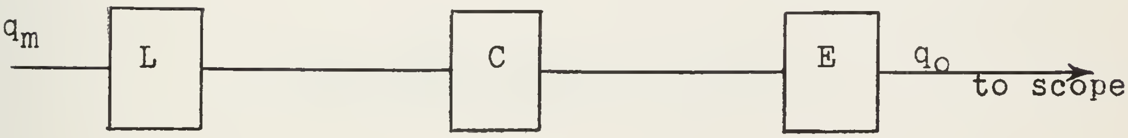
APPENDIX B-3

GAP/R Block Connection Diagrams and Dial Settings, for the Pitch Rate System Simulation.

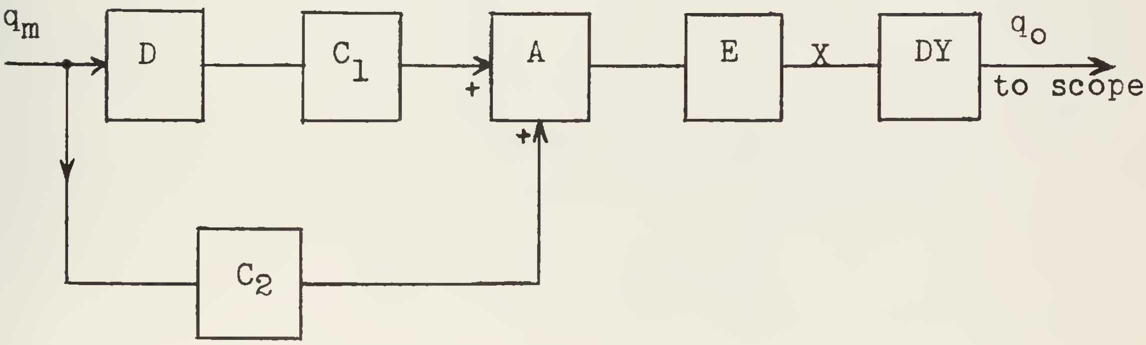
FLIGHT CONDITION A



FLIGHT CONDITION B,C

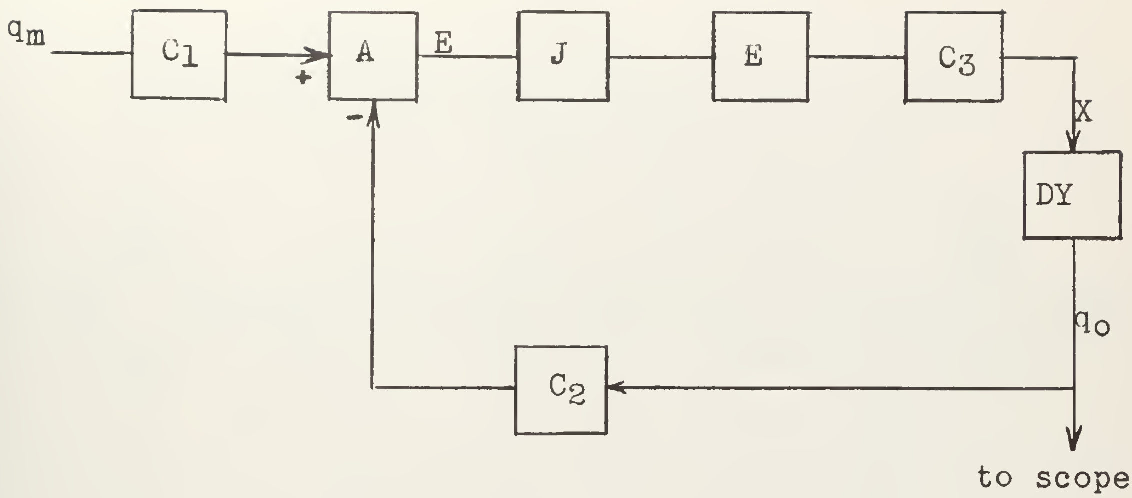


FLIGHT CONDITION D,E,F,G

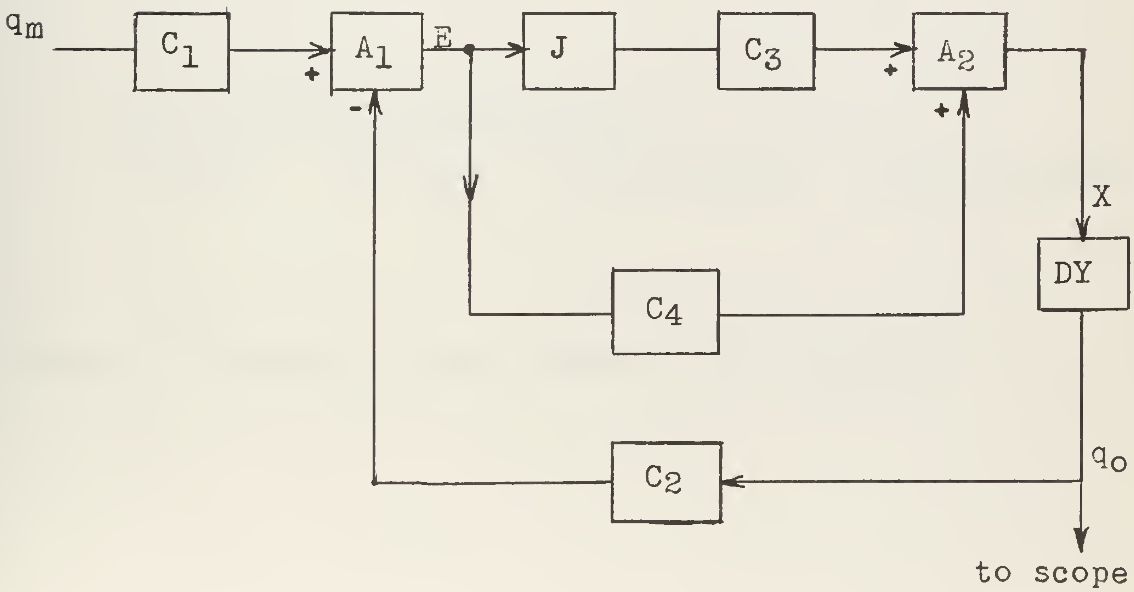


Computer Block Diagrams for the Compensated Pitch Rate System, During the Various Flight Conditions.

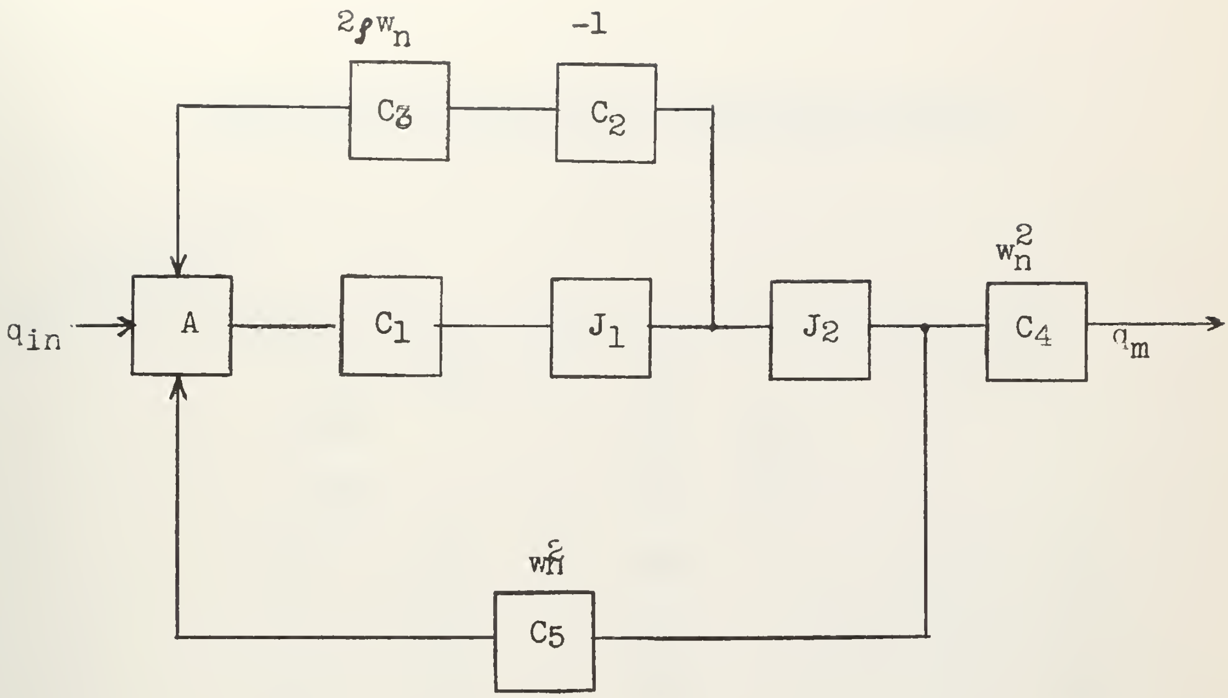
FLIGHT CONDITION A,C



FLIGHT CONDITION B,D,E,F,G



Computer Block Diagrams for Uncompensated Unity Feedback during the Various Flight Conditions,Pitch Rate System.



$w_n = 3.5$ radians/second

$\zeta = .7$

Dial Settings for the Model

$C_1, C_2 = 1$ $\quad C_3 = 4.9$ $\quad C_4, C_5 = 12.25$ $\quad J_1, J_2 = .4$msec.

Computer Diagram and Dial Settings for the Model

Unit Dial Settings for Compensated System.

Flight Condition \ Unit	C_1	C_2	C_3	E	L	A_2	A_1	A_o
A	.185	.500	1	.125		.1	2.8	4
B	.936			.126	.123			
C	.999			.125	.125			
D	8.14	4.37		.124		5	4.56	1.03
E	8.43	4.82		.125		5	4.43	1.03
F	8.02	2.32		.125		5	4.71	1.44
G	7.98	6.15		.125		5	4.73	1.43

a. K4DY and K3D set for T_o = .0004 sec.

b. A_2, A_1 and A_o are the dial settings of the K4DY 2nd order equation module.

Unit Dial Settings for the Uncompensated System. (with unity feedback)

Flight Condition \ Unit	C_1	C_2	C_3	C_4	E	A_2	A_1	A_o
A	.925	.925	.69		.935	2.5	5.25	1.21
B	1.11	1.00	.073	.094		.500	.300	.092
C	.925	.925	.167		.325	2.5	15.1	2.32
D	1.11	1.00	.018	.034		.500	.214	.033
E	1.11	1.00	.031	.054		.500	.230	.05
F	1.11	1.00	.01	.025		.500	.115	.025
G	1.11	1.00	.02	.025		.500	.30	.026

a. K4DY and K3J are set for T_o = .0004 sec.

b. A_2, A_1, and A_o are the dial settings of the K4DY 2nd order equation module.

APPENDIX C

Transient Reponse of the Pitch Rate System.

1. General information and definitions.
2. Step input, simulating violent maneuver.
3. Ramp input, simulating gradual maneuver.

GENERAL INFORMATION AND DEFINITIONS

1. Theoretical step response of the compensated pitch rate system, determined by inverse transform methods.

Flight Condition	Step Response
A	$q_o = .965 + .0222\, e^{-8.2t}$
B	$q_o = .945 + .0245\, e^{-8.155t}$
C	$q_o = .990 - .00497\, e^{-8.04t}$
D	$q_o = .768 + .092\, e^{-.605t} + .0545 e^{-8.5t}$
E	$q_o = .84 + .0357\, e^{8.3t} + .073\, e^{-.62t}$
F	$q_o = .6 + .268 e^{-.425t} + .0552 e^{-8.5t}$
G	$q_o = .761 + .0587 e^{-.83t} + .067 e^{-8.64t}$

2. The uncompensated, unity feedback, system reduces to a closed loop type 1 servo system. The final value theorem predicates that the final amplitude ratio of output to input is equal to 1.

3. Each small block on the photographs is equal to 1.67 seconds of problem time.

4. For the ramp input function; the output ramp is extrapolated, and the "dynamic error" is estimated.

Dynamic Error (DE) is defined as the difference between the output response level at t, and the forcing function point for the same instant:

or in equation form:

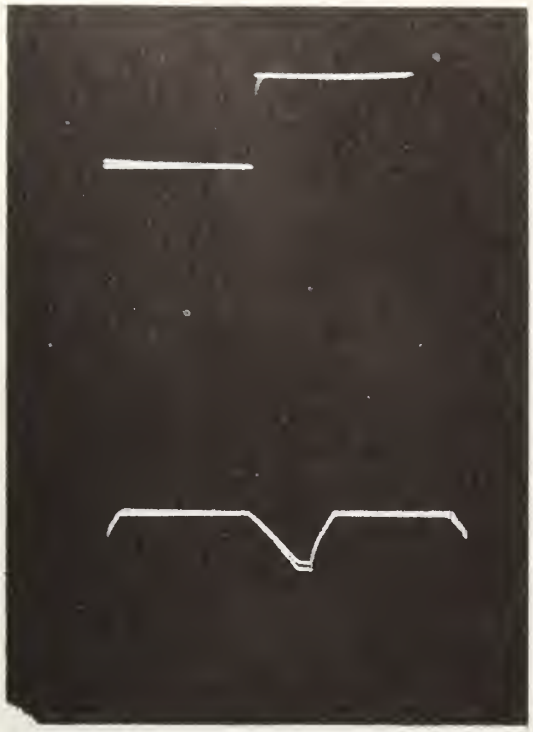
$$(DE)_{t_1} = q_{(out)t_1} - q_{(out)\,qs\,t_1}$$

where qs = quasi-static, the estimate of the output level rate, for quasi-static conditions.

(DE) is then nondimensionalized as $^{DE}/_{INPUT}$

5. For the step input function, the "response time" is determined.

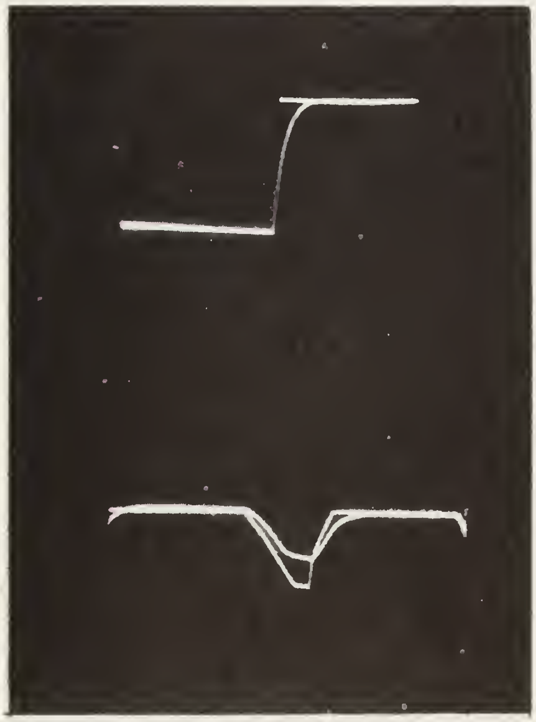
Response time (RT) is defined to be the time required for the output response to change from an initial arbitrary initial dynamic error to within five percent of the forced output response, or to within 95% of its steady state value.



(A) Compensated

RT .835 seconds

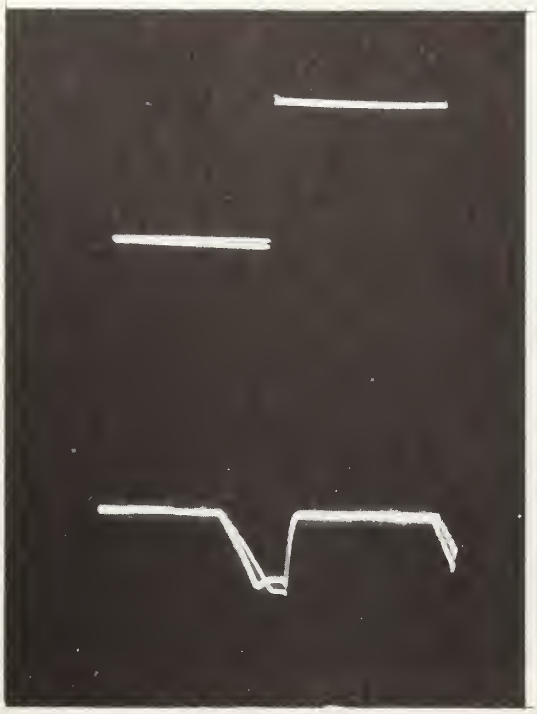
DE/Input .05



(A) Uncompensated

RT 5.85 seconds

DE/Input .1



(B) Compensated

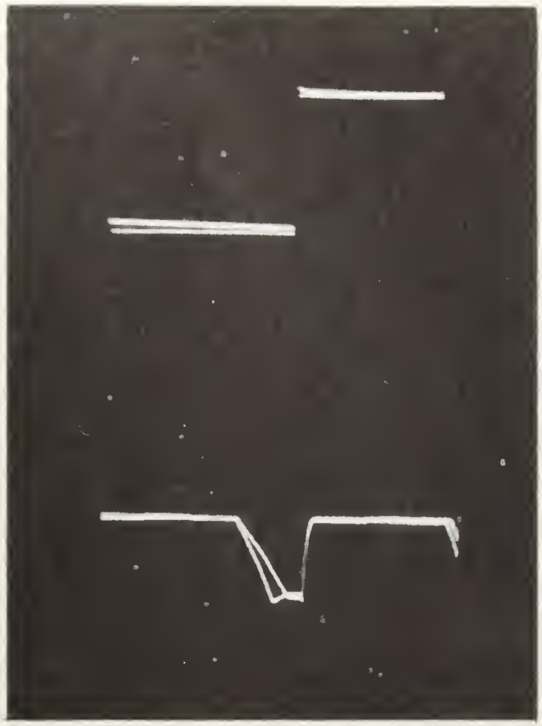
RT .167 seconds

DE/Input .1

(B) Uncompensated

RT 10.85 seconds

DE/Input .561



(C) Compensated

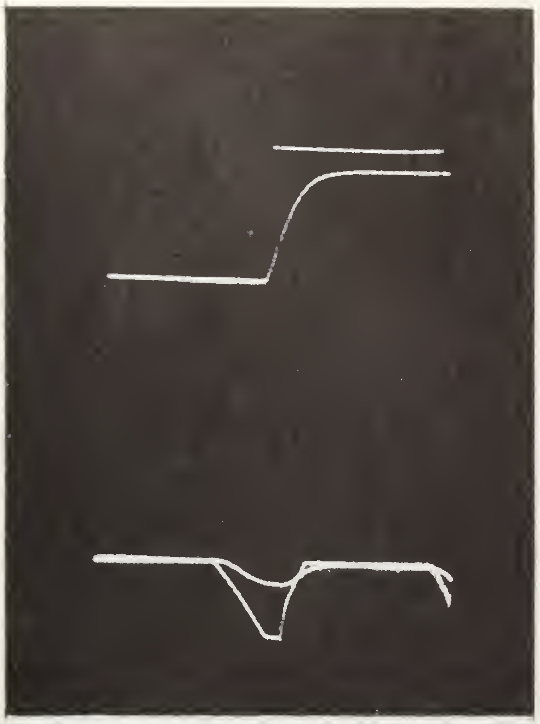
RT .334 seconds

DE/Input .1

(C) Uncompensated

RT 10.85 seconds

DE/Input .561



(D) Compensated

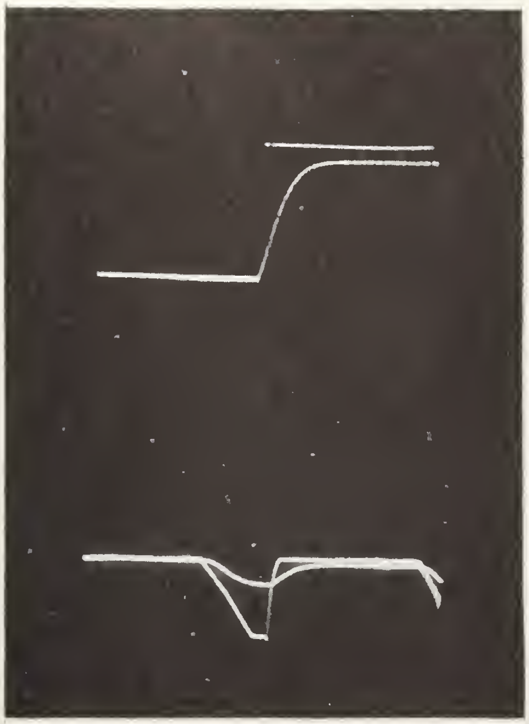
RT 5.0 seconds

DE/Input .474

(D) Uncompensated

RT 13.4 seconds

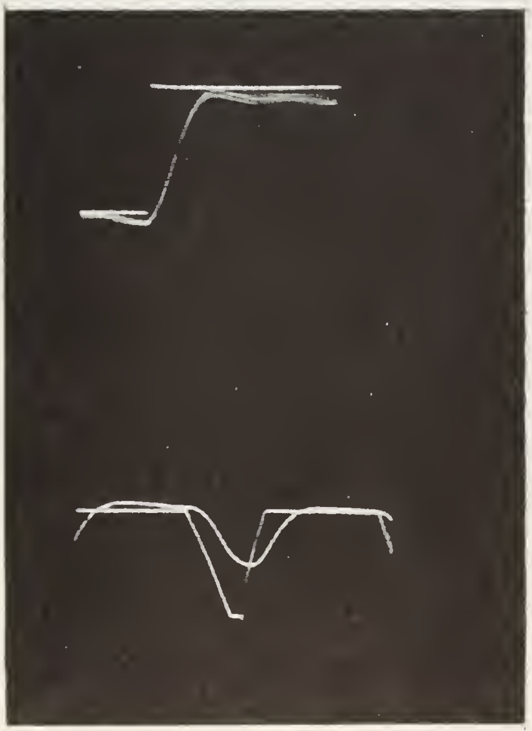
DE/Input .5



(E) Compensated

RT 7.51 seconds

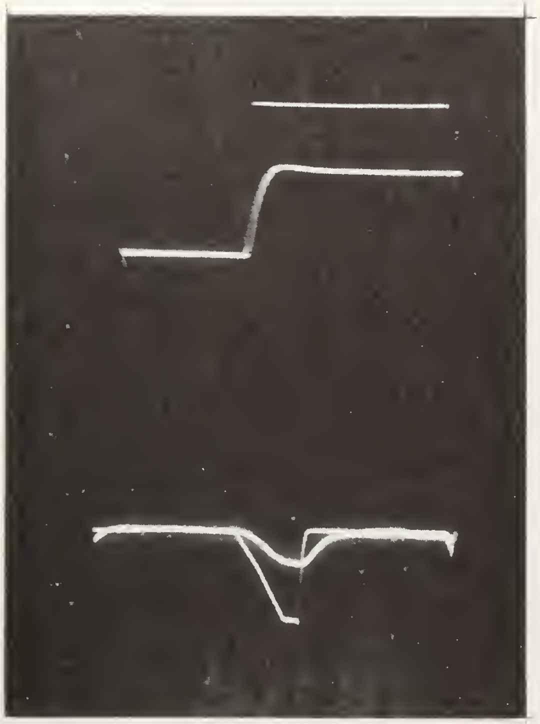
DE/Input .5



(E) Uncompensated

RT 10.85 seconds

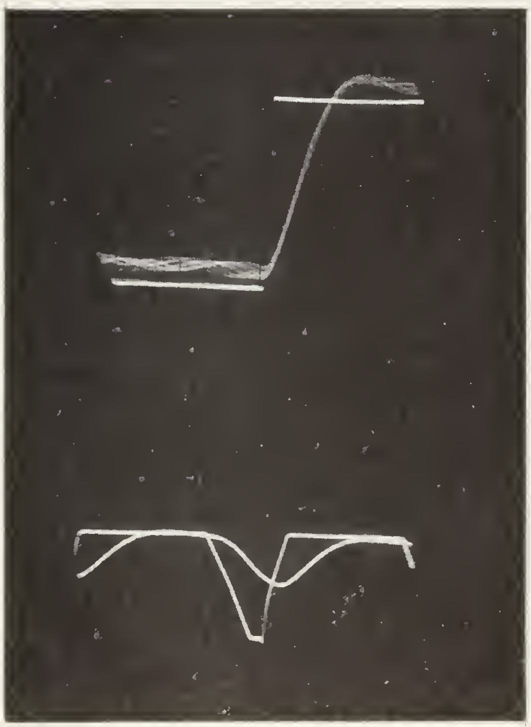
DE/Input .308



(F) Compensated

RT 4.58 seconds

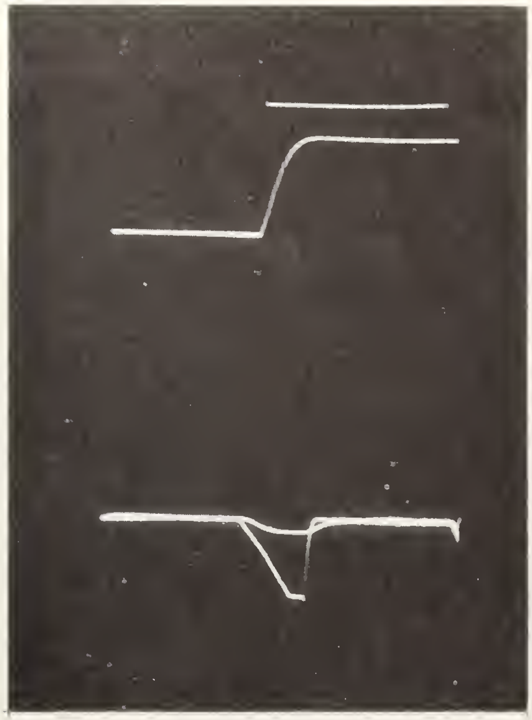
DE/Input .465



(F) Uncompensated

RT 15.08 seconds

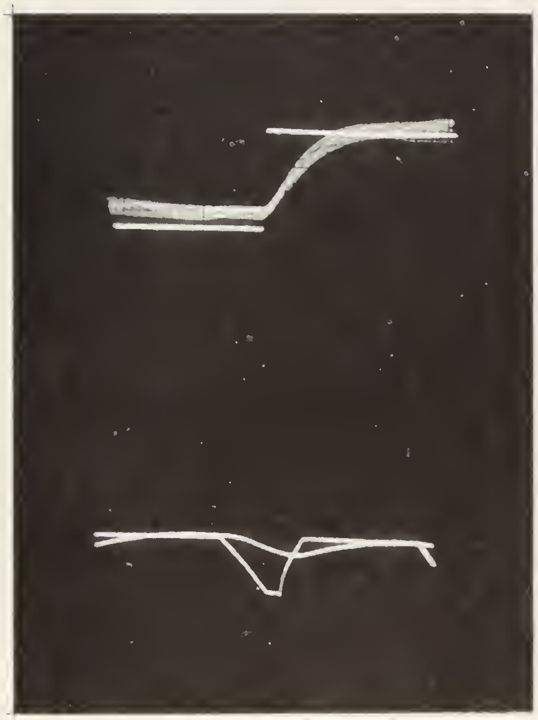
DE/Input .615



(G) Compensated

RT 6.51 seconds

DE/Input .25



(G) Uncompensated

RT 17.5 seconds

DE/Input .80

APPENDIX D

Transient and Frequency Response of the Combined Model and Pitch Rate System.

The performance function of the model in closed loop is:

$$q_m = \frac{W_n^2 \, q \, in}{p^2 + 2 \zeta W_n p + W_n^2}$$

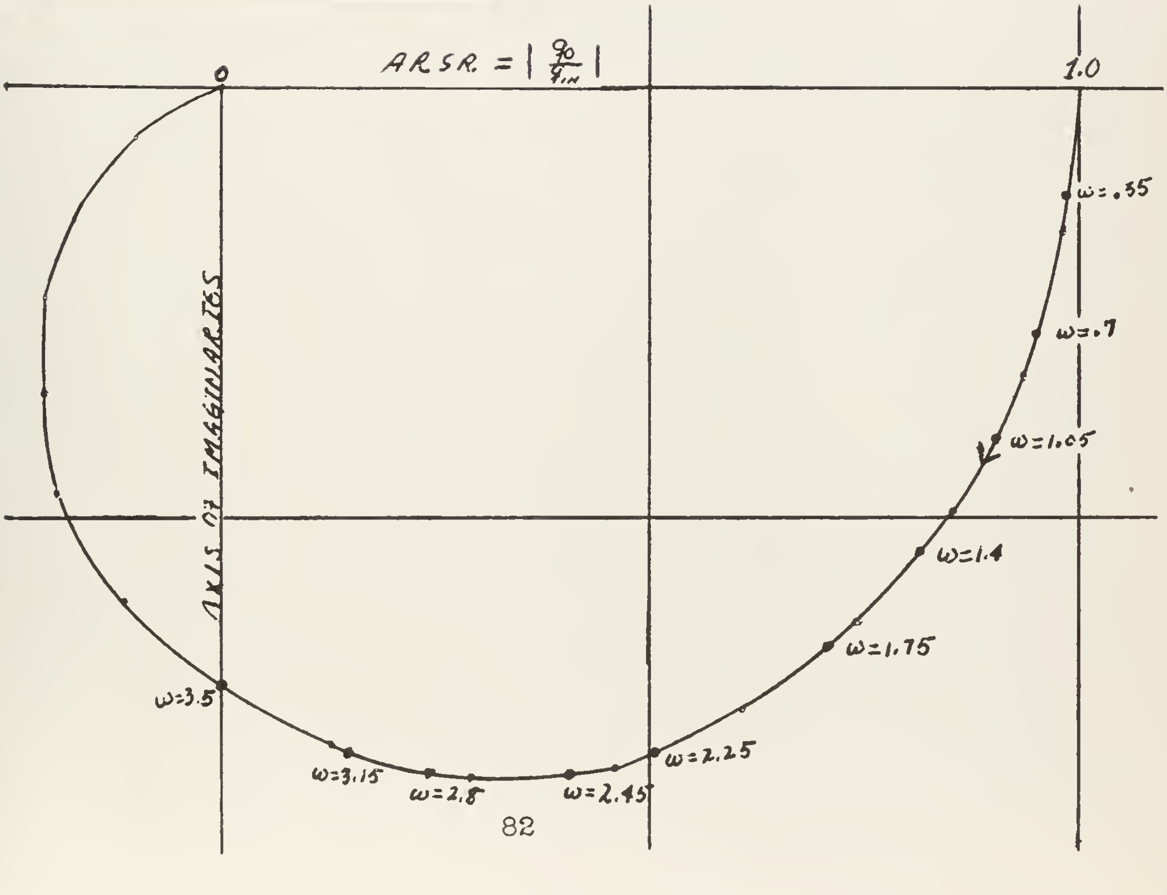
where: $W_n = 3.5$ radians/second

$\zeta = .7$

Combined System

RT 1.1 seconds

DE/Input 0



APPENDIX E

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