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ERNEST STAVELEY

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Lieutenant Commander, United States Navy

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Lieutenant Commander, United States Navy

Submitted in partial fulfillment
of the requirements
for the completion of course

United States Naval Postgraduate School
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AN APPLICATION OF STATISTICAL DECISION THEORY

TO DECISION MAKING IN FOREIGN POLICY

This work is accepted as fulfilling

the thesis requirements for completion of course

by

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from the

United States Naval Postgraduate School

ABSTRACT

Certain aspects of statistical decision theory are applied to assessing objectively the uncertainty involved in estimating which of his courses of action (capabilities) an antagonist is pursuing in an international conflict of interest situation. Observations are made by a nation on its antagonist's conduct and interpreted as being associated with its courses of action. Bayes formula is then used to change a conjectured a priori probability function over the antagonist's courses of action in order to estimate the course of action being pursued by the antagonist. A set of decision rules for selecting an appropriate course of action to use against an antagonist's estimated course of action is developed, based on preference orderings and preference quantification.

The conclusions of this thesis are:

(1) in spite of uncertainty concerning the course of action being pursued by an antagonist, it is possible to select an appropriate course of action to be pursued when using decision rules based on preference orderings and preference quantification .

(2) that an international conflict situation may be quantitatively assessed.

This thesis was written at the U. S. Naval Postgraduate School, Monterey, California, during the period January - May 1960. I am indebted to Professors Thomas E. Oberbeck and Franklin F. Sheehan for their continued patience, encouragement and most capable guidance while acting as faculty advisors. I wish to thank Mrs. Norma Stevens for her meticulous preparation of this manuscript.

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CHAPTER I

INTRODUCTION

This thesis concerns itself with the application of certain aspects of statistical decision theory to decision making on policy implementation at a national level. The specific application made is in the domain of foreign policy as it concerns two nations with a conflict of interest situation existing between them.

Since World War II, the Free World under the leadership of the United States and the Communist Block under the leadership of the Soviet Union have been engaged in a struggle to determine whether the nations of the world will have the right to choose their own way of life and their own form of government or whether the leaders of the Soviet Union will dictate the conditions under which they will live. As yet the outcome of this struggle has not yet been decided but the indications are that we are losing this struggle, for since the close of World War II the Soviet Union has extended its sphere of influence over the major portion of the Eurasian land mass and its peoples and has begun to probe for handholds in South America and Africa.

In the many small engagements that have characterized this struggle so far the United States has had to make many decisions. In Korea for example we were faced with the decisions as to how far north to allow our forces to proceed and whether or not to permit our aircraft the freedom of hot pursuit north of the Yalu River into Manchuria. Both of the resulting decisions made at the time were made in the face of uncertainty as to what Communist China and the U.S.S.R. would do. Hungary is another example where we were faced with the prospect of having to make a

decision in which we were uncertain as to what our antagonist would do. The result in this case was that effectively we did nothing. In Cuba today the United States again is faced with a decision problem in which there is uncertainty as to what Cuba intends to do.

It is interesting to speculate what course of action the United States would have pursued had it known what the intentions of its antagonist were in each of the above situations or, more realistically, had it utilized a systematic and quantitative process for objectively assessing the uncertainty in its estimate as to the intentions of our antagonist. The United States, armed with information as to what course of action our antagonist was pursuing, might have been able to select a course of action, the results of which would have been more advantageous to this nation and the Free World. The word "might" is used because of the fact that assuming such a course of action existed, we might have been unwilling to pursue it since it violated our traditional standards. For example, a course of action which dictates starting a major war against Communist China would violate our tradition of not initiating aggressive action against another nation.

For the foreseeable future these conflict of interest situations will continue to arise as part of the overall "protracted conflict" characterized by Strausz-Hupé, Kintner, Dougherty and Cottrell in their book entitled Protracted Conflict. [6].[1] The United States in this atmosphere will be repeatedly confronted with the enigma of making a

[1]Numbers in brackets refer to references in the bibliography.

decision in the face of uncertainty.

Morton A. Kaplan, Department of Political Science, University of Chicago, in his book System and Process in International Politics [1] discusses and formulates six distinct International Systems based on the alignment of nations and relations within this alignment which characterize them. In the latter portion of his book he discusses game theory and its relation to the "solution" of conflict of interest situations in international politics, pointing out its limitations and drawbacks.

Abraham Wald in his text entitled "Statistical Decision Functions" [4] points out that the statistical decision problem can be interpreted as a two-person game. The author in this thesis chooses to view a conflict of interest situation as a statistical decision problem.

A conflict of interest situation involving the United States and the Soviet Union can be considered as a two person game in which the United States is a contestant playing against the Soviet Union. Both the United States and the Soviet Union have courses of action, designated pure strategies in the language of game theory, which they can utilize in the given situation; these courses, which also may be characterized as capabilities of action, are actually a listing by the United States of all the possible courses of action which it considers the Soviet Union can pursue which are relevant to the particular situation being considered, as well as a listing of its own capabilities. The context of "course of action" is that the actions of a nation indicate that it will utilize the enumerated course of action to gain its objective. For example, troop and combat aircraft buildups in the

[illegible]

vicinity of Berlin by the Soviet Union might be interpreted by the United States that the U.S.S.R. is pursuing a "hot war" course of action in the Berlin crisis, while the U.S.S.R.'s permitting United States aircraft to use routes outside the prescribed air corridors while in flight to Berlin might be interpreted by the United States to mean that the U.S.S.R. is pursuing a "cold war" course of action in the Berlin crisis.

On the other hand, the United States might construe the sending of its armed forces into Lebanon as its "hot war" course of action while the banning of trade with Communist China could be construed as its "cold war" course of action.

Inasmuch as the U.S.S.R. is not going to inform us on the courses of action which it is following, the advisors in the State Department have the problem of estimating which course of action the U.S.S.R. is pursuing in order to provide the President with a basis for making a decision on a course of action by the United States to counter that of the U.S.S.R. The course of action selected will be chosen relative to a decision criterion from the list that the United States has prepared of its capabilities, but the choice of this course of action will depend in part on which estimated course of action the Soviet Union is pursuing.

In the theory of this thesis, the advisors, in order to make their estimates, assign probabilities to the courses of action of the U.S.S.R.. These probabilities represent the probability with which the United States conjectures (estimates) that the U.S.S.R. will use its courses of action or mixture of courses of action. A mixture of courses of action is

interpreted as employment by the U.S.S.R. of all its courses of action or some subset of its courses of action where the frequency with which the individual courses of action are to be utilized has been carefully predetermined by the U.S.S.R. and the order of their employment is determined on a probabilistic basis. The advisors then observe the conduct of the U.S.S.R. and interpret specific acts as one of a set of possible courses of action of the Soviet Union. The observation of the conduct of the Soviet Union may then be used to change the assigned probabilities on the Soviet Union's courses of action through the use of Bayes theorem which provides the quantative basis for changing these probabilities. The a priori probabilities required for the use of Bayes theorem are the probabilities assigned by the United States to the courses of action of the U.S.S.R. in an initial estimate of the situation.

Bayes theorem and statistical decision theory will help the United States to select the course of action it should pursue. Bayes theorem improves the assessment of the uncertainty as to which of the conjectured courses of action the U.S.S.R. is pursuing by changing the a priori probabilities of the courses of action. The changed probabilities are then known as a posteriori probabilities and these represent the United States' estimate of the probabilities that the U.S.S.R. is utilizing these particular courses of action. Statistical decision theory may be employed by representatives of the United States to choose objectively an appropriate course of action.

The onus of "judgment of values" is placed upon the decision maker who utilizes the model, which will be constructed in the next chapter,

for formulating his decision. "Judgment of values" is particularly important in determining the preference pattern for the possible outcomes which result from the United States pursuing its courses of action and the U.S.S.R. pursuing its courses of action. The decision maker must be particularly careful that the resulting preference pattern reflects the "true" ordering of preferences of the United States for the outcomes and is not colored by his personal beliefs.

The solution of the decision maker's problem rests in the final analysis on the selection of the "appropriate course of action" to counter the antagonists estimated course of action. The words "appropriate course of action" are synonymous with the selection of a course of action which is "optimal according to the criterion established for making a decision." This criterion might be for example the maximization of own national interests, these interests may be construed as "utilities" for the possible outcomes which result from the possible combinations of courses of action. It may be the minimization of the "risks" involved in the courses of action or any criterion which has meaning and "value" to the decision maker. In any event the decision maker must establish the criterion and make his selection of the "appropriate" course of action on this basis.

The relationship between the Operations Analysis curriculum and the problem analyzed lies in a definition of the purpose of Operations Analysis which is "to provide a quantitative basis for executive decision." [1]

[1] Paraphrase of a definition of the purpose of Operations Analysis. Robert F. Rinehart, Journal of Operations Research Society of America, 2, 1954.

The author proposes that in this context, this thesis falls within the province of Operations Analysis. More generally, the author believes that every person who calls himself a citizen of a country has an obligation to attempt to apply the discipline in which he has been trained to problems faced by his country.

CHAPTER II

GENERAL FORMULATION

Conjecture a situation involving a conflict of interests between two nations, Nation I and Nation II. Nation I will be the nation making a decision in the face of uncertainty. Nation I has a set of two possible courses of action (pure strategies) that it can pursue and its list of the possible courses of action that it estimates II can pursue consists of two courses of action. These courses of action are denoted by:

$$(a_1, a_2) \text{ for I}$$

$$(b_1, b_2) \text{ for II}$$

Let the courses of action allowed have the following connotation:

a_1 is interpretable as conducive to a non-shooting war or cold war

a_2 is interpretable as conducive to a shooting war or hot war

b_1 is interpretable as conducive to a non-shooting war or cold war

b_2 is interpretable as conducive to a shooting war or hot war

The possible situations which can occur may be described as:

$$(a_1, b_1), \quad (a_1, b_2), \quad (a_2, b_1), \quad (a_2, b_2) \text{ or}$$

$$(C, C), \quad (C, H), \quad (H, C), \quad (H, H)$$

Where C represents cold war and H represents hot war.

Suppose that the order of preference for these situations by Nation I is represented as:

$$(C, C) > (H, C) > (H, H) > (C, H)$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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with the meaning that the situation to the left of the symbol $>$ is preferred (by I) to the situations to the right.

Subject to the general requirements of utility theory [2] : (1) that preference shall be transitive, i.e., if $A > B$ and $B > C$ then $A > C$; (2) any gamble can be decomposed into its basic alternatives according to the rules of probability calculus; and, (3) if there exists a gamble in which A is preferred to B and B to C, then there shall exist a gamble involving A and C which is indifferent to B; a quantification of these preferences by I can be indicated as follows:

$$(C, C) = +1$$

$$(H, H) = -1$$

$$(C, H) = a$$

$$(H, C) = b$$

with the meaning that the "value" of +1 attached to (C, C) is a standard of preference for a desirable situation, the "value" -1 attached to (H, H) is a standard of preference for an undesirable situation. In the example being considered a has a value less than -1 and b has a value less than +1 and greater than -1. If all possible preference patterns are considered, a and b are arbitrary real numbers, "values", assigned to the situations (C, H) and (H, C) respectively relative to the standards +1 and -1.

For example, we might consider $a = -2$ and $b = +1/2$ giving:

$$(C, C) > (H, C) > (H, H) > (C, H) \text{ since}$$

$$+1 > +1/2 > -1 > -2$$

or if $b = +5$ and $a = -5$

$$(H, C) > (C, C) > (H, H) > (C, H) \text{ since}$$

$$+5 > +1 > -1 > -5$$

A matrix representation of the conflict situation with associated preference values assigned by I would appear as illustrated in Fig. 1 where a and b are any real numbers

I's courses of action \ II's courses of action	b_1	b_2
a_1	+1	a
a_2	b	-1

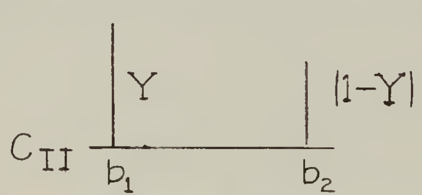
Conflict Situation Matrix and "Values" assigned by I

Figure 1

All the numerical figures in the matrix are assigned by Nation I and as such are meaningful to it. They represent a value to I which it associates with the possible outcomes of the various combinations of courses of action available to I and II. These numerical values are intended to reflect the "utility" to I of each of the possible situations, in the sense described by R. Duncan Luce and Howard Raiffa in Games and Decisions [2]. Nation I must choose an action based on its estimate of which course of action or mixtures of courses of action II is pursuing. I's choice will be that course of action or mixture of courses of action which gives it the greatest expected return. Assuming that I has no information or at best limited information on the course of action II is pursuing, a reasonable procedure for it to follow would be to observe II's actions and from these observations estimate what course of action II is pursuing. The observations represent the stochastic process of the statistical decision problem [4].

An observation by I of an action by II is represented as the

occurrence of one or the other of the pair b_1, b_2. We shall identify this pair as $C_{II} = (b_1, b_2)$. The pair b_1 and b_2 represent the stochastic variables of the statistical decision process [4]. If II employs a mixture of its courses of action, I may identify this mixture as represented in Fig. 2 where Y denotes the relative frequency (or odds) with which II is using b_1, and (1 - Y) the relative frequency with which II is using b_2. Inasmuch as I does not know the value of Y, but knows that Y lies in the interval (0, 1) we regard Y as a stochastic variable. Y represents the true distribution, F, associated with one of the independent variables in the stochastic process. The values of Y constitute the parameter space, Ω, of the statistical decision problem [4]. We shall consider two cases, namely Y is discrete and Y is continuous.



Odds assigned by I to C_{II}

Figure 2

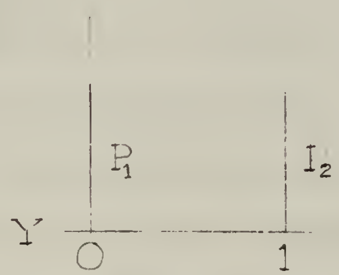
For the case where Y is discrete, only certain values in the interval (0, 1) are allowed; these we shall designate by the notation $(y_n | n = 1, 2, ---, N)$ where N is the number of discrete values of Y in the interval (0, 1). When Y is discrete with N values, we have a probability function $f(y_n)$ such that $Pr\ (Y = y_n) = f(y_n) = P_n$, $P_n \geqq 0$, $\sum_{n=1}^{N} P_n = 1$. Nation I's initial a priori estimate is $f(y_n)$. After I observes II's courses of action one or more times with the results, say, b_1 occurring i times and b_2 occurring j times, the resulting a posteriori distribution which will be designated $f_{ij}(y_n)$, is given by Bayes formula to be:

$$(2.1) \qquad f_{ij}(y_n) = \frac{f(y_n)\, y_n^{\,i}\,(1-y_n)^j}{\sum_{n=1}^{N} f(y_n)\, y_n^{\,i}\,(1-y_n)^j}$$

Assuming stochastic independence of successive observations, the a posteriori probability after any set of observations becomes I's a priori estimate for the next observation. Nation I's initial a priori estimate for the discrete case may be designated $f_{0,0}(y_n) = f(y_n)$. In the discrete case we shall consider several possibilities, namely:

Case (1).

$N = 2$; Y can assume only two values say, $y_1 = 0$ and $y_2 = 1$ with probabilities P_1 and P_2, respectively, as represented in Fig. 3, where the height of the vertical bars represents I's initial a priori estimate.



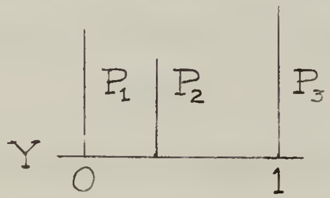
$N = 2$ showing probabilities that $Y = y_n$

Figure 3

A numerical example of this case is worked out in Chapter III using an initial a priori estimate $P_1 = f(y_1) = 3/4$ and $P_2 = f(y_2) = 1/4$ The general results are that I requires only one observation to change its a priori estimate to an a posteriori estimate in which the probability one attaches to one or the other of II's courses of action. This is so regardless of the conjectured a priori estimate.

Case (2).

$N = 3$; Y can assume only the values, say, $y_1 = 0$, y_2 such that $0 < y_2 < 1$ and $y_3 = 1$ with probabilities P_1, P_2, P_3, respectively, as represented in Fig. 4 where the height of the vertical bars represents I's initial a priori estimate.



$N = 3$ showing probabilities that $Y = y_n$

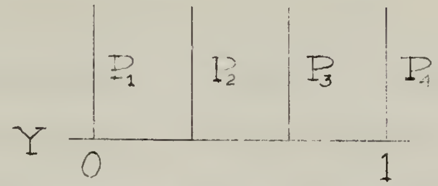
Figure 4

A numerical example of this case is worked out in Chapter III using an initial a priori probability $f(y_1) = 1/3$, $f(y_2) = 1/6$ and $f(y_3) = 1/2$. The general results are that the a priori estimate is changed after one observation to an a posteriori probability in which non-zero probabilities are assigned to either y_1 and y_2 or to y_2 and y_3. As long as the opposite action is not observed in subsequent observations, the probability remains distributed over y_1 and y_2 or over y_2 and y_3 with the probability on y_1 or y_3 increasing with each observation. However, as soon as an action indicating the opposite course of action is observed, the a posteriori probability one attaches to y_2. In this situation all I knows is that II is pursuing a mixture of courses of action, i.e. $0 < y_2 < 1$, but the exact value of y_2 he does not know.

Case (3).

$N = 4$; Y can assume only four values, say, $y_1 = 0$, y_2 and y_3 such that $0 < y_2 < y_3 < 1$ and $y_4 = 1$ with probabilities P_1, P_2,

P_3, P_4, respectively, as represented in Fig. 5, where the height of the vertical bars represents I's initial a priori estimate.



N = 4, showing probabilities that $Y = y_n$

Figure 5

A numerical example of this case is worked out in Chapter III using an initial a priori probability $f(y_1) = 1/4$, $f(y_2) = 1/4$, $f(y_3) = 1/4$, and $f(y_4) = 1/4$. The general results are that the a priori estimate is changed after one observation to an a posteriori probability in which all the probability is distributed over either y_1, y_2 and y_3 or over y_2, y_3 and y_4. As long as the opposite action is not observed in subsequent observations, the probability remains distributed over either y_1, y_2 and y_3 or over y_2, y_3 and y_4 with the probability on y_1 or y_4 increasing with each observation. However, as soon as an action indicating the opposite course of action is observed, the a posteriori probability assigns non-zero values to the two assumed mixtures y_2 and y_3. In this situation all I need estimate is that II's mixture of courses of action lies in a sector either to the left or right of a mixture $y*$ lying between y_2 and y_3. The side to which it lies is determined by the conjectured mixture, y_2 or y_3, having the greatest a posteriori probability.

We shall see in the discussion of expected return to I that if the quantity $\frac{a+1}{a+b}$ lies in the interval (0,1), I need only estimate whether Y is greater than or less than $\frac{a+1}{a+b}$ in order to maximize

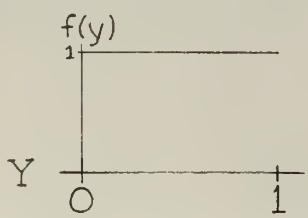
[illegible]

the expected return to him. If $\frac{a+1}{a+b}$ lies outside this interval, I's course of action or mixture of courses of action which maximizes this expected return is uniquely determined.

When Y is continuous we consider all possible values in the interval (0,1) and associate with this interval a probability density function, f(y). Nation I's initial a priori estimate when Y is continuous is f(y). After I observes II's courses of action one or more times with the results, say, b_1 occurring i times and b_2 occurring j times, the resulting a posteriori probability density function is given by Bayes formula to be:

$$(2.2) \qquad f_{ij}(y) = \frac{f(y)\, y^i (1-y)^j}{\int_0^1 f(y)\, y^i (1-y)^j\, dy} \quad \text{for } 0 \le y \le 1 \text{ and zero elsewhere.}$$

Assuming stochastic independence of successive observations, the a posteriori probability density function after any set of observations becomes the a priori estimate for the next observation. Nation I's initial a priori estimate for the case where Y is continuous may be designated as $f_{0,0}(y) = f(y)$. As an example, if I regards all values of Y as equally likely, then f(y) is uniform, i.e. f(y) = 1, and may be shown as in Fig. 6.

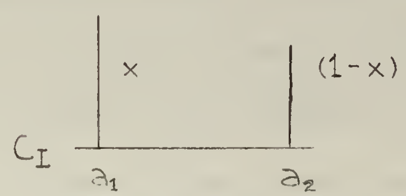


Uniform density function associated with Y

Figure 6

A numerical example of this case is worked out in Chapter III using an initial a priori density function $f(y)$ which is uniform. The general results are that the a priori probability density function is changed after each observation and the resulting a posteriori probability density function is biased in favor of the value of Y associated with II's course of action, b_1 or b_2, observed the greater number of times.

Up to this point we have been concerned with I's view of the set, C_{II}, of courses of action, b_1 and b_2, which I considers II is employing with probabilities Y and $(1-Y)$, respectively. Let us now turn our attention to the courses of action (a_1 and a_2) available to I. We shall say that I has an <u>action space</u> consisting of a_1 and a_2 which we shall identify as $C_I = (a_1, a_2)$. Nation I will employ either of his courses of action a_1 and a_2, or a mixture of his courses of action, selecting that course of action or mixture of courses of action which is "optimal" relevant to his estimate of II's behavior. In this general formulation of the conflict situation and the numerical examples which follow we shall use the criterion that I will maximize his national interests, that is to say, I will maximize the expected (average) "value" of the return. We may identify, C_I, as represented in Fig. 7, where x denotes the relative frequency with which I uses a_1 and $(1-x)$ the relative frequency with which he uses a_2, x lies in the interval $(0, 1)$.



I's action space showing the relative frequency of use of his two courses of action

Figure 7

We use a little x in this case since we are not dealing with a stochastic variable. The quantity x is equal to $\frac{b+1}{a+b}$ if a mixture of courses of action by I is indicated and 0 or 1 otherwise, depending on I's <u>a posteriori</u> estimate of Y. These three values of x will be discussed in the discussion on the expected "value" of the return to I.

Our matrix representation of the conflict situation, Fig. 1, may now be represented as in Fig. 8.

C_I \ C_{II}	Y b_1	(1-Y) b_2
x a_1	+1	a
(x-1) a_2	b	-1

Conflict Situation Matrix showing "values" assigned by I and Relative Frequencies of C_I and C_{II}.

Figure 8

The expected (average) value of the return to I is a function of the courses of action available to I and II which we shall designate by the symbol $E\left[K(C_I, C_{II})\right]$. $E\left[K(C_I, C_{II})\right]$ has the meaning that considering all the possible situations (outcomes) that can occur and the relative frequency with which they may occur, the expected "value" of the return to I is the sum over all possible situations which can occur with the value of each possible situation multiplied by the frequency of occurrence of that situation. The term "expected"

[illegible]

[illegible]

[illegible]

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[illegible]

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[illegible]

value of the return to I is <u>not</u> to be interpreted as the value that will necessarily obtain for a single occurrence of the conflict situations in which I uses a particular course of action against his particular estimate of the course of action II is pursuing, but rather may be interpreted as a "value" of the return to I due to the uncertainty as to which situation will prevail. As an example, consider the conflict situation represented by Fig. 8, its expected value is given by equation (2.3).

$$(2.3) \qquad E[K(C_I, C_{II})] = 1 \times Y + ax(1-Y) + b(1-x)Y + (-1)(1-x)(1-Y)$$

$$= -(a+b)\left(x - \frac{b+1}{a+b}\right)\left(Y - \frac{a+1}{a+b}\right) + \frac{(b+1)(a+1)}{a+b} - 1$$

The quantity E in equation (2.3) is the function that I will maximize with respect to x for his estimated value of Y. Fig. 9 is a pictorial representation over the a, b plane showing the values for x for which $E\left[K(C_I, C_{II})\right]$ is maximized for the situation represented in Fig. 8. The Fig. 9 results from a detailed analysis of the "gains" to I which result from all possible combinations of values of <u>a</u> and <u>b</u>. From equation (2.3) and Fig. 9 we see that the value of x which maximizes $E\left[K(C_I, C_{II})\right]$ is determined not from the specific value of Y estimated, but rather the relationship of the estimated Y to the quantity $\frac{a+1}{a+b}$. The values, a and b, it will be recalled, are the quantities assigned by I to the situations (C, H) and (H, C) and reflect his preferences for these outcomes of the conflict situation.

Note that if $Y = \frac{a+1}{a+b}$, indicated by the vertical dotted

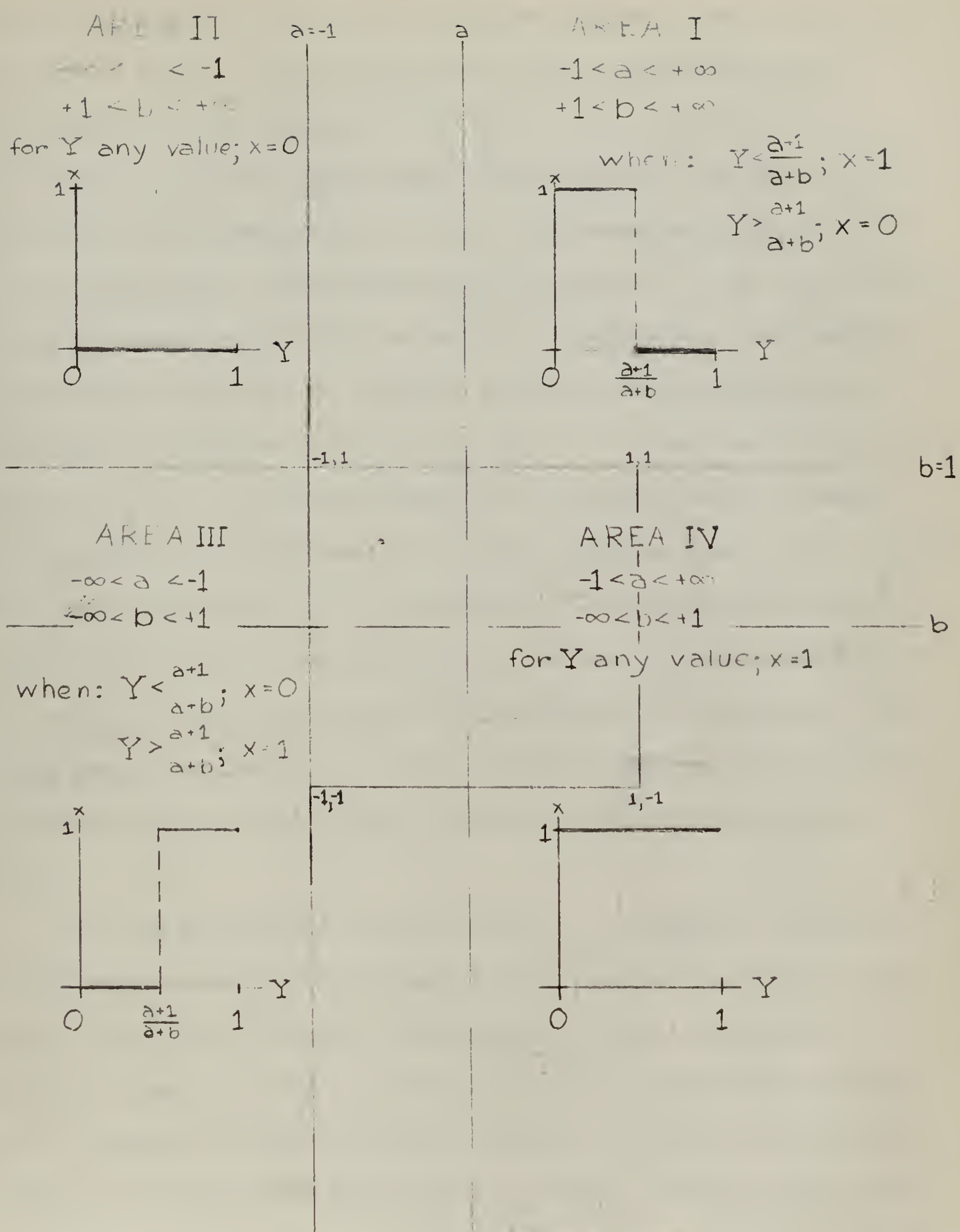
[illegible]

line in Fig. 9, x can be set equal to any value in the interval $(0, 1)$ and the expected "value" of the return to I is the same for all values of x. A corresponding phenomenon occurs when I utilizes a mixture of his courses of action such that $x = \frac{b+1}{a+b}$, and the expected value of the return to I is the same as when $Y = \frac{a+1}{a+b}$. This provides I with a means of coping with a situation which arises when an action is required of I prior to his estimation of the course of action II is pursuing, for if $\frac{b+1}{a+b}$ is in the interval $(0,1)$ he employs the mixture indicated by $x = \frac{b+1}{a+b}$ and the expected value of the return to I is the same no matter which course of action II is pursuing. It will be recalled from Chapter I that to use this mixture I must select that course of action a_1 or a_2 by means of some random device, such as a spinner with the circumference partitioned in proportion to the relative frequencies of utilization of a_1 and a_2.

Fig. 9 is also a pictorial representation of I's decision rules for the use of his own courses of action, once he has enumerated his own preference ordering, for it is noted that a particular preference ordering is associated with only one Area of the a, b plane. For example, in Area I only the following preference orderings are represented:

1. $(H, C) > (C, C) > (C, H) > (H, H)$
2. $(H, C) > (C, H) > (C, C) > (H, H)$
3. $(H, C) > (C, H) \sim (C, C) > (H, H)$
4. $(H, C) \sim (C, H) > (C, C) > (H, H)$
5. $(C, H) > (H, C) > (C, C) > (H, H)$

where the symbol $\sim$ has the meaning that the situation to the left of the



Values of x for which $E\left[K(C_I, C_{II})\right]$ and decision rules for I.

Figure 9

symbol is indifferent (I has equal preference for) to the situation to the right. Areas II, III and IV have a similar preference pattern lists, but different preference ordering.

From Fig. 9, we see that except for the special case $Y = \frac{a+1}{a+b}$, the action to be taken by I is a_1 or a_2 corresponding to $x = 1$ or $x = 0$ respectively. The decision by I in favor of a_1 or a_2 is based on the knowledge of the "true" value of Y which Nation I presumably does not know. Nation I is therefore confronted with estimating Y, and with an estimate of Y he may use Fig. 9 to reach a decision in favor of a_1 or a_2. In the language of statistical decision theory, I's estimate of Y represents the terminal decision space, D^t; the terminal decision reached by I is then used in the game theoretic analysis summarized in Fig. 9 to reach a point in the action space consisting of a_1 and a_2. We will now turn to a consideration of I's problem of deciding upon an estimate of Y. The rules which will be developed below represent the statistical decision function of statistical decision theory.

Let us first consider the case where Y is discrete. Suppose that a sufficient number of observations on II's courses of action have been made so that the a posteriori probability $f_{ij}(y_n)$ associated with a particular discrete value y_n is greater than or equal to some value, say .9, which I considers sufficient for him to decide that the particular y_n is indeed the "true" value of Y. Then I uses the value of the particular y_n for Y to enter Fig. 9 and comes to a decision in favor of a_1 or a_2. This case in the language of statistical decision theory is the case of making a point estimate of Y.

The decision procedure just described requires that a sufficient number of observations have been made of II's actions so that one of the values of Y, say y_n, has achieved a "sufficient" probability, say .9, to lead I to a point estimate of Y. This procedure will not be satisfactory in two situations. The first of these is for the case where Y is continuous. The second situation is the case where Y is discrete but the requirement for a decision by I to use a_1 or a_2 arises before a sufficient number of observations have been obtained to allow I to arrive at a point estimate of Y as described above. For these cases an alternate criterion is suggested. It is based on the maximization of the expected "value" of the return to I and the fact that I need only estimate whether Y is less than or greater than   in order to select a_1 or a_2 so as to maximize the expected "value" of the return to him. In the discrete case I computes the expected "value" of the return to him for all y_n if he uses a_1, and if he uses a_2, and employs his course of action which gives him the greater expected return. This expectation is computed with respect to the a posteriori probabilities. We shall symbolize this as:

(2.4) Choose a_k for which $\sum_{n=1}^{N} E(a_k, y_n) f_{ij}(y_n)$ is a maximum for $k = 1, 2$.

In the continuous case I follows a similar procedure to that described for the discrete case except that (2.4) becomes:

(2.5) Choose a_k for which $\int E(a_k, y) f_{ij}(y)\, dy$ is a maximum for $k=1,2$.

Note that this alternate criterion differs from the procedure of making a point estimate of Y in that it allows a decision to be made at any

time, i.e. before any observations have been made on II's courses of action or after any number of observations have been made.

A more sophisticated method I may use in deciding upon an estimate Y is a complete analog of the Statistical Decision Problem considered by Abraham Wald [4] in which the cost of experimentation and the cost of a wrong decision must be considered after each observation when determining whether to act now or take another observation. The analogous components for I to consider in his problem could be the "cost" of observation, this "cost" being either real or conjectured, and the "gain" which would accrue to him if he took another observation. When the "cost" of making another observation is greater than the "gain" to I, he makes his estimate of Y. This case requires that "cost" and "gain" be commensurable with a and b. It will not be considered further in this thesis.

CHAPTER III

NUMERICAL EXAMPLES

General Situation

Let us suppose that a conflict of interest situation has arisen between Nation I and Nation II wherein let us say Nation II is offering to supply arms to the Beta States. Additionally, let us suppose that these same Beta States are engaged in open hostilities with their neighbor Gamma. Nation I, which has declined to supply arms to either side, considers that it is vital to its interests to prevent the supply of arms to the Beta nations. In this situation let us consider that Nation I has two courses of action available to it, namely:

(1) attempt to gain its objective without resort to open hostilities. For example, this might be through the use of diplomatic channels only or the United Nations. This course of action we shall call a_1, which is equivalent to a cold war course of action.

(2) attempt to gain its objective by resorting to open hostilities. For example, this might be through physically preventing the receipt of such shipments by the Beta States. This course of action we shall call a_2, which is equivalent to a hot war course of action.

Similarly Nation I conjectures that Nation II has two courses of action, namely:

(1) attempt to gain its objective without resort to open hostilities. This course of action we shall call b_1, which is equivalent to a cold war course of action.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(2) attempt to gain its objective through resort to open hostilities. This course of action we shall call b_2, which is equivalent to a hot war course of action.

Having set the stage let us now proceed to examine the various cases discussed in the general formulation of the problem. First the case where Y is discrete and the three cases enumerated under it.

<u>Case (1)</u>

N = 2; Y can assume only two values, y_1 and y_2; $y_1 = 0$ and $y_2 = 1$

Assumptions:

(1) Nation I's preference ordering:

$$(a_1, b_1) > (a_2, b_1) > (a_2, b_2) > (a_1, b_2)$$

or equivalently

$$(C, C) > (H, C) > (H, H) > (C, H)$$

(2) Quantification of Nation I's preference ordering:

$$+1 > 1/2 > -1 > -2$$

(3) Conjectured a priori probability:

$$f_{0,0}(y_1) = \Pr(Y = y_1) = 3/4; \qquad y_1 = 0$$
$$f_{0,0}(y_2) = \Pr(Y = y_2) = 1/4; \qquad y_2 = 1$$

Let us suppose that the first observation on Nation II's courses of action results in b_2. The a posteriori estimate is by Bayes formula:

$$(2.1) \qquad f_{ij}(y_n) = \frac{f(y_n)\, y_n^{\,i}(1-y_n)^j}{\sum_{n=1}^{N} f(y_n)\, y_n^{\,i}(1-y_n)^j}$$

$$f_{0,1}(y_1) = \frac{(3/4)(1)}{(3/4)(1) + (1/4)(0)} = 1$$

$$f_{0,1}(y_2) = \frac{(1/4)(0)}{(3/4)(1) + (1/4)(0)} = 0$$

We see that after one observation, Nation I is enabled to estimate that $Y = 0$ and that Nation II is pursuing its hot war course of action. It is interesting to note that all the a posteriori probability would be on y_1 for an observation of b_2 regardless of the a priori probability assumed and on y_2 for an observation of b_1.

Examination of Fig.9 shows that Nation I's preference pattern lies in Area III. In this area the decision rules for Nation I are:

(1) $Y < \frac{a+1}{a+b}$; select a_2, the hot war course of action.

(2) $Y > \frac{a+b}{a+1}$; select a_1, the cold war course of action.

In our case the quantity $\frac{a+1}{a+b} = 2/3$, and since Nation I has estimated that Nation II's course of action is $b_2(Y = 0)$, the appropriate course of action for Nation I is a_2. The expected "value" of the return to Nation I from equation (2.3) is $E\left[K\ (C_I, C_{II})\right] = 3/2\ (x + 1) \cdot (0 - 2/3) + 1 - 1$, which for $x = 0$ is equal to -1, and for $x = 1$ is equal to -2. From this we also see that Nation I's course of action a_2 maximizes its expected return.

Case (2)

$N = 3$; Y can assume only three values y_1, y_2, and y_3; $y_1 = 0$, y_2 where $0 < y_2 < 1$, and $y_3 = 1$.

Assumptions:

(1) Nation I's preference ordering:

$$(a_1, b_1) > (a_2, b_1) > (a_2, b_2) > (a_1, b_2)$$

or equivalently

$$(C, C) > (H, C) > (H, H) > (C, H)$$

(2) Quantification of Nation I's preference ordering:

$$1 > 1/2 > -1 > -2$$

(3) Nation I conjectures that Nation II may be employing the mixture of its courses of action represented by $y_2 = 1/3$.

(4) Conjectured a priori probability:

$$f_{0,0}(y_1) = Pr(Y = y_1) = 1/3; \qquad y_1 = 0$$

$$f_{0,0}(y_2) = Pr(Y = y_2) = 1/6; \qquad y_2 = 1/3$$

$$f_{0,0}(y_3) = Pr(Y = y_3) = 1/2; \qquad y_3 = 1$$

(5) Predetermined value which the a posteriori probability must reach on a course of action or mixture of courses of action before Nation I is willing to estimate that Nation II is following that particular course of action or mixture of courses of action is taken as .9.

Let us suppose that the first observation on Nation II's courses of action results in b_2. The a posteriori estimate becomes:

$$f_{0,1}(y_1) = \frac{(1/3)(1)}{(1/3)(1)+(1/6)(2/3)+(1/2)(0)} = 3/4$$

$$f_{0,1}(y_2) = \frac{(1/6)(2/3)}{(1/3)(1)+(1/6)(2/3)+(1/2)(0)} = 1/4$$

$$f_{0,1}(y_3) = \frac{(1/2)(0)}{(1/3)(1)+(1/6)(2/3)+(1/2)(0)} = 0$$

We see that from one observation Nation I is able to estimate that Nation II is pursuing either its course of action b_2 or the mixture represented by $y_2 = 1/3$. Had the first observation resulted in b_1, Nation I would have been able to estimate that Nation II was pursuing either its course of action b_1 or the mixture represented by $y_2 = 1/3$.

Inasmuch as the a posteriori probability, that associated with y_1, is less than .9, Nation I makes another observation. Suppose that this observation again results in b_2. The a posteriori probability after this second observation is:

$$f_{0,2}(y_1) = 9/11$$

$$f_{0,2}(y_2) = 2/11$$

$$f_{0,2}(y_3) = 0$$

Since the a posteriori probability associated with y_1 is again less than .9, Nation I makes another observation and will continue to make observations until one of the a posteriori probabilities reaches .9. As long as subsequent observations do not result in b_1, the a posteriori probabilities remain distributed over y_1 and y_2 with the probability over y_1 increasing after each observation. The total number of consecutive observations of b_2 required such that $f(y_1) \geqq .9$,

is four. Therefore, Nation I, after four observations, makes a point estimate of $Y = 0$ corresponding to II's course of action b_2. However, as soon as b_1 is observed, the a posteriori probability of one attaches to y_2. For example supposing the third observation on the actions of Nation II resulted in b_1, the resulting a posteriori probability after this observation would be:

$$f_{1,2}(y_1) = 0$$
$$f_{1,2}(y_2) = 1$$
$$f_{1,2}(y_3) = 0$$

Application of Bayes theorem will show that as soon as the opposite course of action from the one originally observed is noted, the a posteriori probability of one attaches itself to y_2. In this instance we see that Nation I estimates that Nation II is pursuing a mixture of its courses of action.

Examination of Fig. 9 shows that Nation I's preference pattern lies in Area III. In this area the decision rules for Nation I are:

(1) $Y < \frac{a+1}{a+b}$; select a_2, the hot war course of action.

(2) $Y > \frac{a+1}{a+b}$; select a_1, the cold war course of action.

In our case the quantity $\frac{a+1}{a+b} = 2/3$. Therefore, for the instance where Nation I estimates that Nation II is following a course of action corresponding to $Y = 0$, the appropriate course of action for Nation I is a_2. The expected value of the return to Nation I from equation (2.3) for $x = 0$ is -1 and for $x = 1$ is -2. From this we also see that Nation I's course of action a_2 maximizes its expected return.

In the instance where Nation I estimates that Nation II is following a mixture of courses of action corresponding to $Y = 1/3$, the

[illegible]

appropriate course of action for Nation I is a_2. The expected value of the return to Nation I from equation (2.3) for $x = 0$ is -1/2 and for $x = 1$ is -1. From this we also see that Nation I's course of action a_2, maximizes its expected return.

Case (3)

$N = 4$; Y can assume only 4 values y_1, y_2, y_3, and y_4;

$y_1 = 0$, y_2 and y_3 such that $0 < y_2 < y_3 < 1$, and $y_4 = 1$.

Assumptions:

(1) Nation I's preference ordering:

$$(a_1, b_1) > (a_2, b_2) > (a_2, b_1) > (a_1, b_2)$$

or equivalently

$$(C, C) > (H, H) > (H, C) > (C, H)$$

(2) Quantification of Nation I's preference ordering:

$$+1 > -1 > -2 > -3$$

(3) Nation I conjectures that Nation II may be employing the mixtures of its courses of action represented by $y_2 = 2/7$ and $y_3 = 6/7$.

(4) Predetermined value which the a posteriori probability must reach on a course of action or mixture of courses of action before Nation I is willing to estimate that Nation II is following that particular course of action or mixture of courses of action is taken as .9.

(5) Conjectured a priori probability:

$$f_{0,0}(y_1) = Pr(Y = y_1) = 1/4; \quad y_1 = 0$$
$$f_{0,0}(y_2) = Pr(Y = y_2) = 1/4; \quad y_2 = 2/7$$
$$f_{0,0}(y_3) = Pr(Y = y_3) = 1/4; \quad y_3 = 6/7$$
$$f_{0,0}(y_4) = Pr(Y = y_4) = 1/4; \quad y_4 = 1.$$

Let us suppose that the first observation on Nation II's courses of action results in b_1. The a posteriori probability after this first observation is:

[illegible]

$$f_{1,0}(y_1) = 0$$
$$f_{1,0}(y_2) = 2/15$$
$$f_{1,0}(y_3) = 6/15$$
$$f_{1,0}(y_4) = 7/15$$

We see that from one observation Nation I is able to estimate that Nation II is pursuing either its course of action b_1 or a mixture of its courses of action.

Inasmuch as the greatest a posteriori probability, that associated with y_4, is less than .9, Nation I makes another observation. Suppose that this observation results in b_2. The a posteriori probability after this second observation is:

$$f_{1,1}(y_1) = 0$$
$$f_{1,1}(y_2) = 5/8$$
$$f_{1,1}(y_3) = 3/8$$
$$f_{1,1}(y_4) = 0.$$

We see that when both of the courses of action b_1 and b_2 have been observed, the a posteriori probability assigns zero probabilities to $y_1 = 0$ and $y_4 = 1$. However, had the second and all subsequent observations resulted in b_1, the a posteriori probabilities would have remained distributed over y_2, y_3 and y_4 with the maximum probability on y_4.

Let us suppose that after two observations Nation I is required to implement one of its courses of action and suppose the observations have been b_1 and b_2. Since insufficient observations have been obtained for Nation I to arrive at a point estimate of Y on the basis of an _a posteriori_ probability greater than or equal to .9, I uses an alternate procedure in order to select the appropriate course of action. This alter-

nate procedure is defined as follows:

(2.4) choose a_k for which $\sum_{n=1}^{N} E(a_k, y_n) f_{ij}(y_n)$ is a maximum for $k = 1, 2$.

In the example under consideration we see that:

$$\sum_{n=1}^{N} E(a_1, y_n) f_{1,1}(y_n) = E(a_1, y_1)(0) + E(a_1, y_2)(5/8) + E(a_1, y_3)(3/8) + E(a_1, y_4)(0)$$

$$= \left[-(a+b)\left(x - \frac{b+1}{a+b}\right)\left(y_2 - \frac{a+1}{a+b}\right) + \frac{(b+1)(a+1)}{a+b} - 1\right](5/8) +$$

$$\left[-(a+b)\left(x - \frac{b+1}{a+b}\right)\left(y_3 - \frac{a+1}{a+b}\right) + \frac{(b+1)(a+1)}{a+b} - 1\right](3/8)$$

$$= [+5(1-1/5)(2/7-2/5) - 2/5 - 1](5/8) +$$

$$[+5(1-1/5)(6/7-2/5) - 2/5 - 1](3/8)$$

$$= -65/56 + 9/56$$

$$= -1$$

and

$$\sum_{n=1}^{N} E(a_2, y_n) f_{1,1}(y_n) = E(a_2, y_1)(0) + E(a_2, y_2)(5/8) + E(a_2, y_3)(3/8) + E(a_2, y_4)$$

$$= -84/56$$

Since $\sum_{n=1}^{N} E(a_1, y_n) f_{ij}(y_n)$ is greater than $\sum_{n=1}^{N} E(a_2, y_n) f_{ij}(y_n)$, Nation I employs his course of action a_1.

In this case we see that II's mixture of courses of action having the greatest a posteriori probability, y_2, lies in the sector to the left of y^* and also this same mixture is less than $\frac{a+1}{a+b}$ which is equal to 2/5. Under the simple criterion of selecting a point estimate y_n for Y for which the associated a posteriori probability $f_{ij}(y_n)$ is greatest, I would have been lead to select action a_2. Under the criterion of maximizing the expected value of return, Nation I is lead to selecting a_1. Note, however, that the a posteriori probability on y_2 is 5/8, which is less than .9. Had the a posteriori probability on

y_2 been greater than or equal to .9, Nation I would have also been lead to select action a_2.

Case (4)

Y is continuous

Assumptions:

(1) Nation I's preference ordering:

$$(a_2, b_1) > (a_1, b_1) > (a_1, b_2) > (a_2, b_2)$$

or equivalently

$$(H, C) > (C, C) > (C, H) > (H, H)$$

(2) Quantification of Nation I's preference ordering:

$$+5 > +1 > 0 > -1$$

(3) Conjectured a priori probability density function,

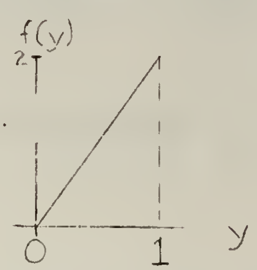
$f(y)$, uniform, i.e. $f(y) = 1$ for $0 \leq y \leq 1$, and is zero elsewhere.

Let us suppose that the first observation on Nation II's courses of action results in b_1. The a posteriori probability density function is by Bayes formula,

(2.2) $$f_{ij}(y) = \frac{f(y)\,(y)^i\,(1-y)^j}{\int_0^1 f(y)\,(y)^i\,(1-y)^j\,dy} \quad \text{for } 0 \leq y \leq 1 \text{ and zero elsewhere}$$

$$f_{1,0}(y) = \frac{(1)\,y^1\,(1-y)^0}{\int_0^1 (1)\,y^1\,(1-y)^0\,dy} = 2y \quad \text{for } 0 \leq y \leq 1 \text{ and zero elsewhere}$$

This a posteriori probability density function may be pictured as follows:



A Posteriori Probability Density Function of Y after observation of b_1

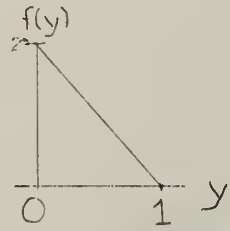
Figure 10

Note that as a result of the single observation, the conjectured a priori probability density function is changed to an a posteriori probability density function biased in favor of the value of 1. Had the observation been b_2 the a posteriori probability density function would have been biased in favor of zero.

In that case:

$$f_{0,1}(y) = \frac{(1)(y)^0(1-y)^1}{\int_0^1 (1)(y)^0(1-y)^1 dy} = 2-2y \quad \text{for } 0 \leq y \leq 1 \text{ and zero elsewhere}$$

and this a posteriori probability density function may be pictured as follows:



A Posteriori Probability Density Function
of Y after observation of b_2

Figure 11

Let us suppose that Nation I after one observation is required to implement one of its courses of action. Nation I uses the alternate procedure to decide which of its courses of action to utilize. The alternate procedure for the continuous case is:

(2.5) Choose a_k for which $\int_0^1 E(a_k, y)\, f_{ij}(y)\,dy$ is a maximum for $k = 1, 2$.

In the example that we are considering we see that:

$$\int_0^1 E(a_1,y)f_{1,0}(y)dy = \int_0^1 \left[-(a+b)\left(x-\frac{b+1}{a+b}\right)\left(y-\frac{a+1}{a+b}\right)+\frac{(b+1)(a+1)}{a+b}-1\right]2y\,dy$$

$$= \int_0^1 [-5(1-6/5)(y-1/5)+6/5-1]2y\,dy$$

$$= \int_0^1 [(y-1/5)+6/5-1]2y\,dy$$

$$= \int_0^1 2y^2dy$$

$$= 2/3$$

and

$$\int_0^1 E(a_2,y)f_{1,0}(y)dy = \int_0^1 [-5(0-6/5)(y-1/5)+6/5-1]2y\,dy = 3$$

Since $\int_0^1 E(a_2,y)f_{1,0}(y)dy$ is greater than $\int_0^1 E(a_1,y)f_{1,0}(y)dy$, Nation I employs his course of action a_2.

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