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THE FUNDAMENTALS OF SALVO WARFARE

by

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Thesis Advisor: Wayne P. Hughes, Jr.

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The Fundamentals of
Salvo Warfare

by

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ABSTRACT

This thesis presents a detailed study of the fundamentals of modern naval surface missile combat and, through the vehicles of combat modeling, simulation, and quantitative analysis, describes a method of evaluating tactics. It establishes three basic laws of naval combat, tests the theory that undergirds the laws against a data set, and provides a thorough analysis of the results.

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I. INTRODUCTION

There currently exist no naval combat models that adequately describe modern naval warfare. This fact has consequences that reach far beyond the academic use of such models. In all areas of naval operations analysis, including analysis in support of procurement, planning, logistics, and, most important, tactical decisions made during conflict, there is a compelling need for a coherent, useful analytical tool that brings the fundamentals of this process to light. In the absence of such a tool, analysis continues to be diffuse, independent, and disconnected.

This thesis presents a detailed study of the fundamentals of modern naval surface warfare, and through modeling, simulation, and quantitative analysis, provides a useful method of tactical analysis. By developing a theory of warfare based on salvo exchanges, testing this theory against a robust data set, and dissecting the constituent elements of the process, this thesis seeks to establish a global framework for further study and application.

Specifically, it is the aim of this thesis to describe the characteristics of modern salvo warfare, the tactical implications of these characteristics, the components of salvo exchanges, and the variables associated with the application of lethal force at sea. In addition, it seeks to summarize the tactics of modern naval combat with a single model and with a single measure of effectiveness.

Given that these goals are accomplished, the contents of this thesis will aid in such decisions as determining future weapons and platform characteristics, selection of operational doctrine, and logistical planning requirements. Ultimately, it will provide the means to determine which assets a fleet commander must give his group commander to accomplish a certain mission, what uncertainties the group commander may experience in the application of these assets, and how to avoid making tactically inefficient decisions.

II. THEORY

A. TERMINOLOGY

The first step in discussing naval combat models is to establish a framework of terminology. The following terms and their definitions suggest a relationship to physical systems that helps to describe "the dynamics of physical bodies that warriors apply to the processes of combat" [Ref. 1].

1. Combat Energy

Combat Energy is a characteristic of a participant that has some lethal value in combat, e.g., missiles engender lethal energy and, therefore, combat energy within a guided missile ship.

2. Combat Potential

Combat potential is stored combat energy. Combat potential resides in the missiles stored in the launchers and magazines of a guided-missile ship, for example.

3. Combat Power

Combat power is the expenditure rate of combat energy by one participant against another during conflict. The number of missiles a ship shoots at another in one salvo is a delivered pulse of combat power.

4. Effective Combat Power

Effective combat power is combat energy applied to a participant as a result of physical interaction between participants. The number of missiles (after accounting for the effect of the defense) which actually hit a ship and cause damage is a measure of effective combat power.

5. Staying Power

Staying power is a measure of the amount of enemy combat energy a ship can absorb before its own combat energy is extinguished. The number of missile hits a ship can sustain until it is of no remaining value in combat is a measure of the staying power of that ship.

It is important to note the relationships between these terms. Combat power erodes staying power, while staying power determines the value of combat potential. Moreover, a ship must have combat potential before it can expend combat power. Staying power is often an evaluation made in relation to a "mission kill", i.e., how much damage must be done to render a ship useless for current combat purposes (not necessarily the amount of high explosive required to actually sink it). Therefore, combat potential and combat power can only be evaluated with respect to the staying power of a specific enemy target ship. This ship may usefully be defined as a notional, or benchmark, ship (a device not used in this thesis).

B. SQUARE LAW THEORY

The "Lanchester" square law is important in any study of naval combat models, but particularly useful here since it is the point of departure for the theory presented in this thesis.

1. Background

Naval combat models had their inception in 1902 when J. V. Chase, a lieutenant at the Naval War College, devised the classic square law model as a way to mathematically express naval combat as force-on-force attrition [Ref. 2]. Unfortunately, his work was classified until 1972, thus denying him the credit given to F. W. Lanchester [Ref. 3] and M. Osipov [Ref. 4], who published their square law land combat models independently in 1914 and 1915, respectively.

2. Square Law Equations

Chase's basic square law equation gives the surviving combat power of side M at time t after an inferior side N has been annihilated:

$$(a_1 m)_t = \sqrt{(a_1 m)_0^2 - \frac{b_2 a_1}{b_1 a_2}(a_2 n)_0^2}$$

Where:

m: number of ships on side M

n: number of ships on side N

a_1:   staying power of side M, per ship

a_2:   staying power of side N, per ship

b_1:   combat power of side M, per ship

b_2:   combat power of side N, per ship

The model shows the effect of combat power on enemy staying power during a time interval (0,t]. In an era when heavily armored hulls and gunnery were the determir-nts of naval combat, their interaction was well described by Chase's equations.

C. FISKE'S SALVO METHOD

Also of note in the early development of naval combat models is Rear Admiral (then Commander) B. A. Fiske's numerical description of the square law effect, which appeared in his 1905 prize-winning essay "American Naval Policy". Fiske, not privy to Chase's application of calculus, portrayed naval combat as salvoes inflicted by the participants over discrete periods of time. The square law phenomenon is still evident even though he only employed elementary mathematics in his computations. [Ref. 5]

D. SALVO WARFARE THEORY

Profound technological advances in weapons systems have dated the description presented by Chase and Fiske of naval combat as a gradual erosion of one force by the other. These vast changes have resulted in systems that deliver great doses of combat power over long ranges with nearly simultaneous arrival at the target of the entire combat power of a participant. In addition, prior to the advent of modern missile exchanges, the only defense was a ship's staying power. Current technology provides for an active defense (e.g., anti-cruise missile missiles and guns) and a passive defense (e.g., chaff and electronic countermeasures). Thus, ships may have both offensive and defensive combat power as well as staying power. In effect, the entire character of naval combat has cnanged and the process is more aptly called **salvo warfare**.

1. The First Law of Salvo Warfare

Near-instantaneous attrition has thus replaced incremental attrition as the fundamental concept of naval warfare. Time is no longer integral to the process and, therefore, square law theory does not apply. The elimination of time implies that new theories may be "event-stepped" (the salvo being the event) instead of "time-stepped". This, then, is the point of departure for a new theory of naval combat, dictating the first law of salvo warfare:

> **Salvo exchanges are interactions of pulses of combat power and therefore event-stepped phenomena rather than continuous processes of attrition.**

2. The Taylor Model

There are currently only two salvo warfare models resident in the literature. The first, by T. C. Taylor [Ref. 6], describes force effectiveness as:

$$F_{AR} = 1 - \{E_{OB} \times (1 - E_{DA})\}$$

Where:

F_{AR}: fraction of side A's combat effectiveness remaining after the salvo

E_{OB}: fraction of side A's total tactical capability destroyed by side B's salvo in the absence of defensive measures

E_{DA}: fraction of E_{OB} annulled by A's defense

Although Taylor's equation addresses the fundamentals of salvo warfare, it is computationally misleading because the variables express effectiveness as fractions. Adding to this conceptual confusion, tactical inputs are implied rather than directly represented. Moreover, the effects of overkill and scouting are not discussed. Appendix A contains sample calculations that show the distortions of Taylor's model in greater detail.

Further development by Taylor leads to a measure of effectiveness which describes the outcome of a salvo exchange as the difference of fractions of combat power remaining. This result is difficult to reach, however, without a more tangible method of computing combat effectiveness.

3. The Hughes Model

A better approach is found in Chapter 10 of Hughes, Fleet Tactics [Ref. 7]. The theory behind his model is:

$$\text{Losses to } A = \frac{\text{Effective Offensive Combat Power of } B}{\text{Staying Power of } A\text{, Per Ship}}$$

Where:

Effective Offensive Combat Power of B =

Offensive Combat Power of B - Defensive Combat Power of A

The appealing features of Hughes' theory are that the basic computations are contained in the model and that staying power is directly represented. In addition, the inputs are readily determined by tactical evaluation and the output is easily applied to mission-specific goals. Based upon the intuitively engaging approach, concise formulation, and tactically meaningful framework, Hughes' theory is adopted as the Second Law of Salvo Warfare:

Effective combat power is the attacker's pulse minus the defender's actions, inflicting damage proportional to the ratio of effective combat power to staying power.

The second law may be expressed mathematically as:

$$\Delta A = -\left(\frac{\sigma_B \beta B - a_1 A}{a_2}\right)$$

Where:

A: number of ships on side A

B: number of ships on side B

β: offensive combat power of side B, per ship

a_1: defensive combat power of side A, per ship

a_2: staying power of side A, per ship

σ_B: scouting effectiveness of side B

Henceforth, this will be referred to as the <u>second law model</u>. The inclusion of the dimensionless variable σ_B, the scouting effectiveness of side B, will be discussed below.

Dimensional analysis of the model yields some useful results. Defining "hits" as the units of measure for combat power drives the following analysis.

Staying power, a_2, is the number of hits sustained over a defending ship's combat life and is therefore measured in "hits/ship".

If combat potential is viewed as the total combat energy that may be transformed into combat power to do work (specifically, to erode the defender's staying power) then by dividing the amount of combat energy employed (combat power) by the amount of work to do (staying power) results in units of hits per hits per ship, or "ships". Since the concept of combat potential is highly useful in tactical planning, it is thus convenient to value the combat energy stored in a ship's missile magazine, for example, as the number of notional ships they are capable of destroying.

Damage is measured, according to the second law model, as:

$$\frac{hits - hits}{\frac{hits}{ship}} = \frac{hits}{\frac{hits}{ship}} = ships,$$

which makes both computational and tactical common sense. In addition, whether combat potential is viewed as the capacity to erode staying power or as the ability to do damage, the dimensions will still be units of notional ships. These results constitute the third law of salvo warfare:

Combat power is measures in units of hits, staying power in units of hits per ship, and combat potential and damage in units of ships.

4. Salvo Attrition

Further evidence for rejecting square law models follows from the fact that they consist of coupled differential equations based on simultaneous attrition where one side dominates the other to extinction. The full square law effect only appears when the battle is fought to the annihilation of one of the sides, producing results that are of little use in modeling exchanges that inflict only partial damage on an enemy.

Moreover, square law models do not address the various cases of salvo attrition. Salvo warfare permits different types of interaction during combat. Because of longer weapons ranges and the attendant scouting problems, side A may shoot at side B without side B returning fire, side B may shoot at side A without A returning fire, or A and B may exchange fire.

Also, one side may surprise the other with a salvo, which, in turn responds with a salvo of its own (if it has any surviving combat power). The second law model allows for modeling each side independently in all of these exchange variations, i.e., each equation describing ΔA has a companion equation describing ΔB. Measuring the process independently after each event satisfies the need to describe exchanges where annihilation and simultaneous attrition do not occur.

5. Scouting Effectiveness

In Chapters 4 and 10 of Fleet Tactics, Hughes shows that the use of pulse power weapons makes scouting as crucial as the weapons themselves, concluding that the fundamental maxim of modern naval warfare is "fire effectively first". [Ref. 8] In the second law model, σ_B, a dimensionless force degrader with range [0,1], illustrates the impact of firing an effective salvo before an opponent can. A value of zero means that side B has useless or no scouting information about side A, either because it is surprised by A, or because A has used countermeasures ("anti-scouting", in Fleet Tactics). A value of one means that side B has perfectly scouted side A. [Ref. 9] Thus, surprising or confusing an enemy nullifies his ability to do damage, whereas having perfect scouting is the deterministic starting point for evaluating the damage a ship will do with its effective combat power.

III. CONDUCT OF THE EXPERIMENT

Theories, once presented, must be tested against either historical or experimental data. Since there remains a dearth of historical data from modern (Post World War II) naval combat, it was necessary to design an experiment which would provide a data set to test the theory of salvo warfare presented above. The best way short of war to generate such a data set is through a high-resolution wargame simulation. Of the numerous wargaming assets at the Naval Postgraduate School the one that best serves this purpose is the Naval Tactical Gaming System (NAVTAG). The following is a description of this system, the design of the experiment, and a discussion of the data that were collected.

A. THE NAVTAG SYSTEM

NAVTAG, used by the fleet since 1982, is primarily a medium for training surface warfare officers to make tactical decisions. The current version operates on a network of three personal computers and has an extensive data base containing air, surface, and subsurface platforms from the United States and Soviet Union orders of battle, as well as those from many other countries.

The most attractive aspect of NAVTAG is that it conducts simulations at a very detailed level, allowing the operator to order movement, process sensor information, and employ weapons from an individual ship commander's tactical perspective. It also retains all orders, contacts, weapons interactions, and damage in memory for post-game analysis. This was crucial during the analysis, since it permitted pairing causes and effects.

B. DESIGN OF EXPERIMENT

Simulation of salvo warfare necessarily involves many variables and a significant amount of variation among trials. It was therefore important to eliminate as many variables as possible by developing simple scenarios that retained the essence of salvo warfare yet produced analytically meaningful results. It was also equally important to conduct many iterations of the same scenario to reduce the effect of the variation between trials for the final analysis. Prudent selection of ships, scenarios, and rules of engagement (ROE's) helped reduce the number of variables during the simulation.

1. Ship Selection

Many different ship classes were considered, but based on the simplicity of the weapon systems and its familiarity to the operator, the Knox-class frigate (FF-1052) was chosen. The NAVTAG representation of this platform has four surface-to-surface missiles (SSM's), a 20mm caliber gatling gun close-in weapon system (Mk 91 CIWS), and a five-inch caliber anti-air battery (5"/54 Mk 42). This platform was selected mainly because it limited the size of each ship's salvo to four SSM's (increasing computational efficiency during the analysis) and dealt only with point defenses (avoiding the confusion in evaluating the contributions of area defense systems).

The author had 39 months experience on this class of ship (24 months in the operations department and 12 months as Tactical Action Officer). Therefore, personal operator familiarity decreased the possibility of test errors resulting from operator inexperience.

2. Scenario Selection

The distance between sides was set at 50 nautical miles to incorporate targeting beyond the horizon. The spacing between ships on a side was set at 1000 yards and the ships were placed in a column perpendicular to the incoming SSM's. This helped reduce variability since a salvo could be aimed at the center of a formation and have a higher probability of hitting.

NAVTAG allows the operator to create a scenario, save it in memory, then re-use it repeatedly or update it as required. It was therefore determined that a logical experimental sequence would start with the most simple scenario (one FF surprising another), collect data until enough were obtained, and then add ships incrementally (or change tactics) after enough data were obtained, until a useful set of data was collected.

Of the six scenarios conducted, two were surprises and four were exchanges. Table 1 is a summary of the scenarios and the force compositions.

TABLE 1. SCENARIO SUMMARY

Scenario	# of Units on Side A	# of Units on Side B	Comments
I	1	1	A surprises B
II	1	2	A surprises B
III	1	1	A and B exchange fire
IV	1	2	A and B exchange fire
V	1	3	A and B exchange fire
VI	2	3	A and B exchange fire

3. ROE Selection.

Perfect scouting information was given to the surpriser in the surprise scenarios and to both sides in the exchange scenarios. A surprised ship was given no scouting information and was therefore unable to activate any active or passive defenses. Although airborne reconnaissance provided offensive targeting data, each ship had to use its own sensors for point defense assignments.

Each firing ship launched its SSM's in the active mode at a range and bearing dictated by the targeting data. All missiles were fired at a single point and arrived at the target simultaneously as a single pulse of power.

Defensively, CIWS would automatically acquire and engage a target which the NAVTAG system determined was within its firing parameters. The 5"/54 was fired in air barrage mode with a priority assignment described as follows:

a. The incoming missile closest in range and coming from the bearing closest to the threat axis was assigned first.

b. If no range information was available, the incoming missile coming from the bearing closest to the threat axis had assignment priority.

c. If no information was available, no assignment was made.

In each exchange scenario, chaff was continuously deployed as soon as an incoming missile was detected.

The operator was the same for every battle and in every scenario, providing the closest possible concord among decisions throughout the experiment while still allowing human interface.

As can be seen, every effort was made to keep the amount of noise in the experiment to a minimum while retaining a conceptually meaningful exchange. The important point is that although the Knox-class was represented in the experiment, the results should not be viewed as the actual results of Knox-on-Knox battles. The emphasis was to create scenarios that simulated naval combat generically, i.e., involved mobile platforms, significant pulses of power over long ranges, active and passive defenses, and units with staying power. For the purposes of the analysis, NAVTAG produced "real" naval combat in the absence of actual historical battle data.

C. DISCUSSION OF DATA SET

A total of 275 battles were fought in NAVTAG and 1900 missiles were exchanged between 700 ships. Appendix B, a sample data sheet, shows the type of information recorded immediately following each battle during the post-game analysis. Appendix C is a compilation of the raw data.

An important result determined during data collection was that invariably exactly two missile hits would destroy an FF. The staying power of the chosen platform, a_2 in the second law model, was therefore set at 2 hits/ship in the analysis (although not used in computations, the combat potential of each ship's 4 missiles was therefore evaluated as 2 ships). Table 2 presents a summary of the data.

Numerous iterations of each scenario were run so that the inputs for the model are averages across each scenario. Since NAVTAG computes the percent of damage to each ship, it was necessary to transform the damage figures into units of ships. This was done by simply multiplying the average percent damage to each side by the number of ships on that side. The result is the average number of ships lost for each side for each scenario.

TABLE 2. SUMMARY OF DATA

Scenario	Side	# of Ships	Avg. # of offensive hits	Avg # of defensive hits	Avg # of Ships lost
I	A	1	2.54	0.00	0.00
I	B	1	0.00	0.00	0.94
II	A	1	2.36	0.00	0.00
II	B	2	0.00	0.00	1.24
III	A	1	1.32	1.46	0.67
III	B	1	1.32	1.46	0.67
IV	A	1	0.96	1.40	1.00
IV	B	2	4.08	2.42	0.54
V	A	1	0.48	1.56	0.96
V	B	3	5.16	3.32	0.30
VI	A	2	1.64	2.96	1.61
VI	B	3	4.48	4.04	0.97

IV. CALCULATIONS

Chapter II is a compilation of possible starting points for calculations rather than a mere review of the literature. Although all of the models were in hand prior to the numerical analysis, the seemingly straightforward task of applying the data to them was hindered by the lack of an established theoretical framework. From which points to start was decided only after returning to first principles and defining the terms rigorously. Square law models could then be rejected on theoretical grounds and Taylor's model could be rejected on computational grounds. Only Hughes' model advanced past the second chapter to the detailed numerical analysis which follows.

A. SECOND LAW MODEL

The key theoretical assumption of the second law model is that the combat power of a side will increase linearly with the number of units on a side, thus βB (or, equally, αA) will describe the theoretical combat power of a side. Given that the ships in the experiment have a staying power of 2 hits/ship, the damage predicted from the model should equal the observed damage from NAVTAG.

1. Calculations

Applying the data to the second law model involved converting offensive and defensive hits to "probabilities of kill", p_k.

The offensive p_k was calculated by dividing the number of hits during the surprise scenarios by the total number of missiles shot during these scenarios. This assumed a binomial distribution, which gives the probability of y successes (defined, in this case, as a hit) out of a possible total of n in a trial where the fixed probability of an individual success is p. The observed proportion of hits, y/n, is an estimate, $\hat{p}$, of the probability p.[Ref. 20] A review of the data sheets showed that out of 400 missiles, 244 were hits, so $\hat{p}_k = 0.61$. This value was multiplied by 4 to determine the combat power during the salvo for each ship (2.44 hits).

The defensive p_k was calculated by dividing the number of missiles shot down during the exchange scenarios by the total number of missiles shot in these scenarios. Here, n = 1500, y =526, so $\hat{p}_k = 0.351$. This value was multiplied by 4 (the number of incoming targets per salvo) to get the defensive combat power for each ship (1.40 hits). Figure 1 shows the resulting calculations. Note that the defensive combat power of a surprised ship is 0 (by definition, the defenses are not alerted) and the defensive combat power of a surprising ship is 0 (no defense is required). Also note that losses in excess of the number of ships present are rounded to the number of

$$\Delta A = -\left\{\frac{\sigma_B \beta B - a_1 A}{a_2}\right\} \quad \text{and} \quad \Delta B = -\left\{\frac{\sigma_A \alpha A - b_1 B}{b_2}\right\}$$

$\alpha = \beta = 2.44\ hits \quad a_1 = b_1 = 1.40\ hits$

$a_2 = b_2 = 2\ \frac{hits}{ship}$

Scenario I: $A=1,\ B=1,\ \sigma_A=1,\ \sigma_B=0$

$\Delta A = 0$

$$\Delta B = -\left\{\frac{2.44-0}{2}\right\} = -1.22 = -1$$

Scenario II: $A=1,\ B=2,\ \sigma_A=1,\ \sigma_B=0$

$\Delta A = 0$

$$\Delta B = -\left\{\frac{2.44-0}{2}\right\} = -1.22$$

Scenario III: $A=1,\ B=1,\ \sigma_A=\sigma_B=1$

$$\Delta A = -\left\{\frac{2.44-1.40}{2}\right\} = -0.52$$

$$\Delta B = -\left\{\frac{2.44-1.40}{2}\right\} = -0.52$$

Scenario IV: $A=1,\ B=2,\ \sigma_A=\sigma_B=1$

$$\Delta A = -\left\{\frac{4.88-1.40}{2}\right\} = -1.74 = -1$$

$$\Delta B = -\left\{\frac{2.44-2.80}{2}\right\} = +0.18 = 0$$

Figure 1. Second Law Calculations

Scenario V: $A=1,\ B=3,\ \sigma_A=\sigma_B=1$

$$\Delta A = -\{\frac{7.32-1.40}{2}\} = -2.96 = -1$$

$$\Delta B = -\{\frac{2.44-4.20}{2}\} = +0.88 = 0$$

Scenario VI: $A=2,\ B=3,\ \sigma_A=\sigma_B=1$

$$\Delta A = -\{\frac{7.32-2.80}{2}\} = -2.26 = -2$$

$$\Delta B = -\{\frac{4.88-4.20}{2}\} = -0.34$$

Figure 1 (Continued). Second Law Calculations

TABLE 3. COMPARISON OF PREDICTIONS AND OBSERVATIONS (SECOND LAW MODEL)

Scenario	A	ΔA (Model)	ΔA (NAVTAG)	B	ΔB (Model)	ΔB (NAVTAG)
I	1	0.00	0.00	1	-1.00	-0.94
II	1	0.00	0.00	2	-1.22	-1.24
III	1	-0.52	-0.67	1	-0.52	-0.67
IV	1	-1.00	-1.00	2	0.00	-0.54
V	1	-1.00	-0.96	3	0.00	-0.30
VI	2	-2.00	-1.61	3	-0.34	-0.97

ships present and that positive loss values are rounded to zero to preclude a side from gaining ships during an exchange.

2. Discussion

Table 3 shows the comparison between second law model predictions and the experimental observations. Table 4 shows

TABLE 4. % DIFFERENCE IN PREDICTIONS AND OBSERVATIONS (SECOND LAW MODEL)

Scenario	% Difference: ΔA	% Difference: ΔB
I	0.00	6.00
II	0.00	1.00
III	15.00	15.00
IV	0.00	27.00
V	4.00	10.00
VI	19.50	21.00

the percent difference in lost ships between the predictions and the experimental observations.

There are definite patterns in these tables which led to some specific conclusions. First, the second law model accurately predicted the outcome of the surprise scenarios (Scenarios I and II). Second, the second law model accurately predicted the outcome of scenarios where an abundance of effective combat power, or "overkill", was a factor (Scenario IV, side A and Scenario V, side A). Third, the second law model inaccurately predicted the outcome of scenarios where both defenses were involved and overkill was not a factor (Scenario III, Scenario IV, side B, Scenario V, side B, and Scenario VI).

These conclusions suggest that the offensive combat power estimates were reasonable, but that the defensive combat power estimates were not. (Note that the offensive combat power estimates were derived from Scenarios I and II). Furthermore, linear aggregation of the combat power of the sides was a faulty assumption, since neither of the estimates produced accurate predictions as the number of ships on a side increased. It became evident that the second law model, useful as a theoretical tool, needed modification to be useful as a working model. It was not that the basic theoretical law failed, but that the mathematics of the equation were inaccurate in practice. New estimates of combat power, more sensitive to the Scenarios III through VI, were required.

B. THE FOUR-ELEMENT MODEL

Using the data in Table 2, a new method of aggregation was devised. Since it was not accurate to assume that the p_k's were the same for all scenarios, estimating the combat power values on a case-by-case basis logically appeared more accurate. The assumption is that combat power does not increase linearly with the number of ships on a side, but is unique to each side in each scenario. The only useful observed data available were effective offensive combat power and defensive combat power values. Offensive combat power, however, could be derived by adding defensive combat power to effective offensive combat power. The resulting model is:

$$\Delta A = -\left(\frac{\sigma_B \beta' - A_1}{a_2}\right)$$

Where σ_B and a_2 are as before, but:

β': estimated offensive combat power of B

A_1: observed defensive combat power of A

This model will hereafter be referred to as the <u>four-element model</u>. Clearly, the observed defensive combat power of A has been added to the observed effective offensive power of B with the intention of subtracting it again in the model. This additional step was chosen to adhere to the theoretical concepts and to test for the reliability of the model's predictive as well as descriptive abilities. Although it prompts the additional assumption that every incoming missile the defense shot at was going to hit, β' is still a more accurate estimate since it is unique to each scenario and directly derived from observed values. Obviously, A_1 is a better aggregate value than a_1A, since it is the observed value for each scenario.

1. Calculations

Table 6 lists the values of α', β', A_1, and B_1, and Figure 2 shows the four-element model calculations. Note that the offensive and defensive combat power values seem to be, in general, tied more to the target environment than the linear estimates, i.e., more targets produce more missile hits and fewer targets produce fewer missile hits.

TABLE 5. FOUR-ELEMENT MODEL VALUES

Scenario	A	α'	A_1	B	β'	B_1
I	1	2.54	0.00	1	0.00	0.00
II	1	2.36	0.00	2	0.00	0.00
III	1	2.78	1.46	1	2.78	1.46
IV	1	3.38	1.40	2	5.48	2.42
V	1	3.80	1.56	3	6.72	3.32
VI	2	5.68	2.96	3	7.44	4.04

$$\Delta A = -\left(\frac{\sigma_B \beta' - A_1}{a_2}\right) \quad and \quad \Delta B = -\left(\frac{\sigma_A \alpha' - B_1}{b_2}\right)$$

$$a_2 = b_2 = 2 = \frac{Hits}{Ship}$$

Scenario I: $\sigma_A = 1,\ \sigma_B = 0$

$$\Delta A = 0$$

$$\Delta B = -\left(\frac{2.54-0}{2}\right) = -1.27 = -1$$

Scenario II: $\sigma_A = 1,\ \sigma_B = 0$

$$\Delta A = 0$$

$$\Delta B = -\left(\frac{2.36-0}{2}\right) = -1.18$$

Scenario III: $\sigma_A = \sigma_B = 1$

$$\Delta A = -\left(\frac{2.78-1.46}{2}\right) = -0.66$$

$$\Delta B = -\left(\frac{2.78-1.46}{2}\right) = -0.66$$

Scenario IV: $\sigma_A = \sigma_B = 1$

$$\Delta A = -\left(\frac{5.48-1.40}{2}\right) = -2.04 = -1$$

$$\Delta B = -\left(\frac{3.38-2.42}{2}\right) = -0.48$$

Scenario V: $\sigma_A = \sigma_B = 1$

$$\Delta A = -\left(\frac{6.72-1.56}{2}\right) = -2.58 = -1$$

$$\Delta B = -\left(\frac{3.80-3.32}{2}\right) = -0.24$$

Figure 2. Four-Element Model Calculations

Scenario VI: $\sigma_A=\sigma_B=1$

$$\Delta A = -\left(\frac{7.44-2.96}{2}\right) = -2.24 = -2$$

$$\Delta B = -\left(\frac{5.68-4.04}{2}\right) = -0.82$$

Figure 2 (Continued). Four-Element Model Calculations

2. Discussion

Table 6 shows the comparison of the four-element model predictions and the experimental observations. Table 7 shows the percent difference in lost ships between the predictions and the observations. Compared to the second law model, the four-element model was remarkably more accurate across all scenarios, with the exception of Scenario VI, side A. A number of conclusions were drawn from a review of these tables. First, the four-element model accurately predicted outcomes in all but one case. Second, Scenario VI, side A, was an anomaly, but warranted further investigation.

The results from side A in scenario VI suggest that side B is losing almost 20 percent of its combat potential. After accounting for the defense by the same means that provided convergence in every other case, in this one instance the model still did not predict how a significant portion of a side's combat energy would be transformed into damage. In pursuit of this lost potential the concept of combat entropy, a useful new method of quantifying tactics, was developed. The discussion of combat entropy requires its own chapter, which follows.

TABLE 6. COMPARISON OF PREDICTIONS AND OBSERVATIONS (FOUR-ELEMENT MODEL)

Scenario	A	ΔA (Model)	ΔA (NAVTAG)	B	ΔB (Model)	ΔB (NAVTAG)
I	1	0.00	0.00	1	-1.00	-0.94
II	1	0.00	0.00	2	-1.18	-1.24
III	1	-0.66	-0.67	1	-0.66	-0.67
IV	1	-1.00	-1.00	2	-0.48	-0.54
V	1	-1.00	-0.96	3	-0.24	-0.30
VI	2	-2.00	-1.61	3	-0.82	-0.97

TABLE 7. % DIFFERENCES IN PREDICTIONS AND OBSERVATIONS (FOUR-ELEMENT MODEL)

Scenario	% Difference ΔA	% Difference ΔB
I	0.00	6.00
II	0.00	3.00
III	1.00	1.00
IV	0.00	3.00
V	4.00	2.00
VI	19.50	5.00

V. COMBAT ENTROPY

Combat entropy, a new term coined to describe the difference between theoretical predictions and observed results, is defined as a measure of the loss in combat power by the attacker in a salvo exchange. It is the lost combat power as a fraction of the maximum combat power attainable. The term is borrowed from thermodynamics, where it is used to quantify energy unavailable for work [Ref. 11]. The analogy is incomplete, however. Combat is two-sided with inefficient, or "wasted", offensive and defensive combat power both possible. Negative losses, i.e., gains, are therefore possible with combat entropy, but not in thermodynamics, where work always entails wasted energy and positive entropy.

Although combat entropy is related to Clausewitz' notion of friction [Ref. 12], friction subsumes combat entropy, since friction takes other forms in addition to that associated with salvo exchanges. Combat entropy is the result of many factors, all of which are best identified by examining the set of salvo exchange possibilities.

A. THE SALVO EXCHANGE SET

The salvo exchange set, S, is the set of all engagement combinations possible during an exchange. By definition,

$$S \equiv [P \cap D] \cup [P \cap \bar{D}] \cup [\bar{P} \cap D] \cup [\bar{P} \cap \bar{D}],$$

where:

P: effective offensive shots

D: effective defensive shots

$\bar{P}$: ineffective offensive shots

$\bar{D}$: ineffective defensive shots

Each subset of the salvo exchange set contains tactical factors which contribute to combat entropy.

1. $P \cap D$

This subset contains all of the effective offensive shots which are shot down by the defense. Combat power is gained by the offense when the defense must expend more than one defensive shot to destroy each incoming missile. The case when two (or more) defenders engage the same incoming missile is called defensive "double-teaming" ("triple-teaming", etc).

2. $P \cap \bar{D}$

This subset contains all the effective shots which hit. Combat power is gained by the offense when the defense misses incoming missiles or double-teams incoming missiles and misses. Combat power is gained in this subset when the offense, firing into a target-rich environment, has a higher probability of hitting targets and thus, more hits.

3. $\bar{P} \cap D$

This subset contains all the badly aimed and ineffective offensive shots that are needlessly shot down by the defense. Combat power is gained by the offense when the defense shoots down or double-teams missiles that would have missed. Defensive combat power is lost in a target-poor defensive environment, since the defense, not stressed to the saturation threshold, does not realize its full potential. This situation is defined as "defensive overkill".

4. $\bar{P} \cap \bar{D}$

This subset contains all the badly aimed and ineffective offensive shots which the defender does not shoot down. Combat power is lost because of offensive overkill, the "missile-sump" effect, and when the offense shoots into a target-poor environment.

Offensive overkill, the application of more combat power to a target than is required, wastes shots.

The missile-sump effect occurs when a target absorbs more than its share of hits when other targets are in the environment. For example, 4 hits can be inflicted on 2 ships, each with a staying power of 2 hits/ship, in many different ways. The resulting damage ranges from 1 ship destroyed (all 4 hit the same ship and the sump effect is maximized) to 2 ships destroyed (2 hits on each ship and the sump effect does not occur). When the missile-sump effect occurs, a hit is a "wasted" shot and thus lost combat power.

Combat power is also lost when the offense fires into a target-poor environment. Each missile's probability of acquiring a target decreases, resulting in fewer hits than expected.

5. Types of Combat Entropy

As shown above, there are many causes of combat entropy. For analytical purposes, two distinct types can be defined. First, combat entropy may be caused by factors affecting the quantity of hits inflicted. This is defined as "engagement-induced" combat entropy. Second, combat entropy may be caused by factors affecting the quality of the hits inflicted. This is defined as "scenario-induced" combat entropy. The following two sections will dissect the data set and quantitatively describe the contributions of each type to the total fractional loss in combat power.

B. ENGAGEMENT-INDUCED COMBAT ENTROPY

Salvo warfare is such a highly interactive process that there are often an enormous number of different engagement combinations in each exchange. This huge source of variability is one reason that the second law model does not hold in practice. Taking the observed values with the four-element model in effect gives a much more accurate accounting of factors which influence the quantity of hits inflicted. These factors are:

a. the defense double-teams

b. the defense misses shots that hit

c. the defense double-teams, then misses, shots that hit

d. the defense hits shots that would have missed

e. the defense double-teams, then misses, shots that would have missed

f. defensive overkill

g. a target-poor environment for the offense

h. a target-rich environment for the offense

1. Calculations

The best way to measure these gains and losses is to gauge all calculations relative to the simplest case. For offensive combat power, the one-on-one surprise value from Scenario I will be used. This scenario will be considered an "offensive firing range" case, thus the theoretical combat power for each ship in all calculations will be based on a departure from the baseline of 2.54 hits. For defensive combat power, the one-on-one exchange value from Scenario III will be used. This scenario will be considered a "defensive firing range" case, thus the theoretical combat power for each ship in all calculations will be based on a linear extrapolation from 1.46 hits. The resulting fractional loss of combat power is computed by first calculating the lost hits from the formula:

$$Lost\ Hits = \Delta H = \alpha A - b_1 B - \alpha'_0 + B_1$$

where:

αA: predicted offensive combat power of A

$b_1 B$: predicted defensive combat power of B

α'_0: observed effective offensive combat power of A

B_1: observed defensive combat power of B

Lost hits are then converted to a fraction of lost combat power by :

$$\textit{Fraction of Lost Combat Power} = 1 - \left\{\frac{H_T - \Delta H}{H_T}\right\}$$

Where:

H_T: theoretical combat power of side A

Tables 8 and 9 list the lost hits, total theoretical combat power, fraction of lost combat power, and, for comparison, damage inflicted for sides A and B, respectively. These values reflect the influence of tactical efficiency on the quantity of hits inflicted.

2. Discussion

The most striking figures are those from Scenarios I and II. The surprisers realize an almost 60 and 110 percent increase in effectiveness, respectively. This is the result of not having to waste combat power overcoming the defense.

Another important result from these tables is evident when comparing the exchange scenarios. In Table 8, with one offender the loss of combat power decreases as defenders increase from one to three. This implies that the influence of an increasingly stronger defense and richer target environment

TABLE 8. ENGAGEMENT-INDUCED COMBAT ENTROPY FOR SIDE A

Scenario	ΔH	H_T	Fraction of H_T lost	ΔB
I	-1.46	2.54	-0.57	-0.94
II	-2.74	2.54	-1.08	-1.24
III	1.22	2.54	0.48	-0.67
IV	1.08	2.54	0.43	-0.54
V	1.00	2.54	0.39	-0.30
VI	3.10	5.08	0.61	-0.97

makes the scenario increasingly more deterministic. Contrasting these values with the damage inflicted shows that although less combat power is wasted, less damage is done, since the dominating effect is the stronger defense.

As the attackers increase in number from one to two, entropy almost doubles, punctuating the increase in variability by adding just one ship to the offense. Damage, however, almost triples, showing the net effect of doubling the salvo size for the same size defense.

In Table 9, the surprised ships obviously lose combat power since they do not fire. In the exchange scenarios, an interesting result can be inferred from comparing Scenarios III and IV. If the salvo size is doubled for the same size defense, this time in a target poor environment, combat entropy decreases by a factor of 2.5, but damage only increases by 33 percent. This shows that not all entropy is

TABLE 9. ENGAGEMENT-INDUCED COMBAT ENTROPY FOR SIDE B

Scenario	ΔH	H_T	Fraction of H_T lost	ΔA
I	1.08	2.54	0.43	-0.00
II	3.62	5.08	0.71	-0.00
III	1.22	2.54	0.48	-0.67
IV	0.94	5.08	0.19	-1.00
V	2.56	7.62	0.36	-0.96
VI	3.18	7.62	0.42	-1.61

the result of the quantity of hits, but their quality. As with Scenario V, where entropy and damage only slightly increase but the salvo size is 50 percent larger, this is caused by overkill.

In Scenario VI, where the defense has one more ship than Scenario V but not much more combat power is wasted by the offense, a richer target environment and less overkill results in more hits. The damage, however, is almost 20 percent less than the theoretical damage, showing the profound influence of the missile-sump effect.

C. SCENARIO-INDUCED COMBAT ENTROPY

Analyses of Tables 8 and 9 show how tactical efficiencies affect the number of hits an offense can expect. They do not show how the quality of these hits is affected by overkill and the missile-sump effect. The following analysis will develop a method of quantifying these significant causes of combat entropy.

1. Overkill

Overkill is a subjective concept. It is never clear how much overkill is wasted combat power and how much is increasing the certainty of destroying the defender. It therefore remains the province of an individual commander to decide how much force is required to ensure the success of the mission. Although the need for increased certainty is apparent to a current mission, the need to set aside a reserve for future action is often compromised as a result.

The theoretical calculations of Figure 1, pages 22-23, show that overkill was evident in Scenarios I, IV, V, and VI. Table 10 lists the amount of overkill in each scenario in numbers of hits and the resulting fraction of combat power lost as a result.

The overkill in Scenario I seems to be a comfortable margin, since the probability of all four missiles hitting is only $0.635^4 = 0.165$.

Comparing the overkill with the damage inflicted shows that there is not much difference in damage between Scenarios IV and V, although the amount of entropy increases. The significance of this is that if Side A in Scenario V fired a salvo only two-thirds the size, it would only overkill by 0.74

hits instead of 1.96, decreasing combat entropy from 0.257 to 0.097. The tactical implications are greater than just the savings in lost combat power. The 4 SSM's held in reserve account for 33 percent of the total theoretical combat potential. That side would be able to engage one subsequent defender and still inflict damage of 0.67 ships (67 percent of the total) while sustaining damage of only 0.30 ships (10 percent of the total).

TABLE 10. OVERKILL BY SCENARIO

Scenario	Overkill (Hits)	Fraction of h_T lost	Damage Inflicted
I	0.22	0.087	-0.94
IV	0.74	0.146	-1.00
V	1.96	0.257	0.96
VI	0.26	0.034	2.27

Scenario VI, however, shows that planning for overkill is not sufficient. Even with an excess of 0.26 hits, the resulting damage is still almost 20 percent lower than predicted. The missile-sump effect is still the dominating cause of combat entropy in this scenario.

2. Missile-sump Effect

In Scenario VI the predictions and observations differ by 19.5 percent. After accounting for all other sources of waste, this difference remains. It is clear that the only remaining source of combat entropy is the missile-sump effect.

This 19.5 percent difference in damage equates to 0.39 ships' worth of damage out of a total of 2 and a waste in hits of 0.78. More than three-quarters of an SSM were lost during the exchange for no other reason than there were 2 targets and 12 missiles. These 0.78 hits correspond to an additional fractional loss of combat power of 0.10.

Given that the two sides were 50 nautical miles apart and the ship spacing was only 1000 yards, this is a strong argument for massing to augment defenses. Massing is tactically sound not only for the ability to increase the attacker's combat entropy but also to strengthen the defense.

In addition, the missile-sump effect emphasizes the variability in outcomes, since Side A, expecting overkill by 0.26 missiles, actually, has its effective hits reduced by 0.78, a total of more than 1 SSM.

Thus, both overkill and the missile-sump effect have a significant influence on salvo exchanges, making them an important consideration in tactical planning.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The lack of conformity among analysts concerning modern naval combat models compounded the initial difficulty experienced while analyzing the data set. Inasmuch as the analytical pursuit prompted a deeper theoretical effort, the conclusions will remain directed more at promoting the constituent elements of modern salvo warfare than presenting the results of the experiment as universal. Accepting the following postulates as maxims in salvo warfare theory will finally allow naval professionals and military operations analysts to put rudders over and steer a common course.

First, salvo exchanges are event-stepped phenomena rather than time-stepped processes of attrition. This is offered above as the First Law of Salvo Warfare.

Second, theoretical concepts must be unified as follows: damage to a defender results from the effective combat power of the attacker and is computed by calculating the ratio of effective combat power to staying power. This is stated above as the Second Law of Salvo Warfare. Modelers must adopt the second law model as clearly portraying this relationship while illustrating the importance of scouting effectiveness in translating combat energy into ship damage. In addition, the

Four-Element model is a practical application of this theory with significant utility for the refined computations is and essential to tactical planning and simulation analysis.

Third, adherence to a unified dimensional analysis of the problem is crucial. Logically, combat power must be in units of hits, staying power must be in units of hits per ship, and combat potential and damage must be in units of ships. This is stated above as the Third Law of Salvo Warfare. Plainly and simply, this avoids confusion and keeps the discussion in clear, tactically meaningful parlance.

Finally, the study of both the forms and causes of combat entropy is the most instructive way to divine the sources of wasted combat energy and is thus the key to developing effective tactics. Especially in a salvo exchange where the forces are close to parity, a clearer understanding of not only what could go wrong but also what could go right may mean the difference between victory and defeat.

B. RECOMMENDATIONS

Although historical research is a highly valuable method of estimating staying power estimates, computer simulation with many iterations is the only way short of extensive firing range tests to estimate the model's remaining parameters. In addition, computer simulation is replaceable only by war in determining the value of entropic parameters and fueling the attendant tactical discourse (including the presentation of addition evidence in support of the above conclusions). Henceforth, wargaming must be a major focus of the further research of this topic. Although the resources exist at the academic level, the fleet remains ignorant of this alternative use of a system as readily available as NAVTAG. Gaming for research and tactical experimentation should be encouraged at the group, squadron, and ship level in pursuit of a greater understanding of salvo warfare phenomena.

C. FUTURE RESEARCH

As the scope of the research began to narrow, it became evident that there were major areas for future work left untouched by this analysis. The following topics are among those encountered but (unfortunately) bypassed in the effort to codify the basic tenets of salvo warfare:

a. the use of another class of ship in an isometric exchange scenario.

b. the introduction of heterogenous forces and a refined aggregation methodology.

c. the effect of different ship formations and spacing on the missile-sump effect.

d. an investigation of the relative effects of area AAW vs. point defense on defensive combat power.

e. a determination of nominal values for force scouting effectiveness.

f. a collection of more accurate firing range values.

g. an investigation into event-stepping the exchange to annihilation.

h. an investigation into range-stepping the exchange to account for the effect of dissimilar weapons ranges on tactics and damage predictions.

i. a collection of more accurate values for combat entropy and a sensitivity analysis of the effect of entropic parameters on battle outcome ,e.g., studying the effect of the ratio of ship intervals to force separation on the magnitude of the sump effect.

j. the development of a tactical tutorial for the fleet that will bring everyone up to speed and into the debate.

As an epilogue, considering that a seemingly benign data set evoked some unexpected seminal insights into not so much unexplored but uncharted seas, it is hoped that ensuing discussion and research within the framework presented above may have even greater results. Agreeing on the framework, however, is the imperative first principle.

APPENDIX A

TAYLOR MODEL SAMPLE CALCULATIONS

A. VARIABLE DEFINITION

According to the Taylor salvo model (see discussion, page 8), let:

E_{OA}: the raw offensive effectiveness of side A, i.e., the fraction of side B's total tactical capabilities which would be destroyed by side A's salvo in the absence of defensive actions by side B.

E_{DB}: the fraction of E_{OA} which is nullified by a successful, active defense.

F_{SB}: the fraction of side B's capabilities surviving.

Since additional variables will be needed for the calculations, let:

α: Number of missiles launched in each salvo by side A

b_1: Number of missiles side B's defense can shoot down in each salvo

b_2: Number of missiles side B can absorb before its total tactical capability is destroyed

B. CALCULATIONS

1. Taylor Equation

$$F_{SB} = 1 - (E_{OA} \times (1 - E_{DB}))$$

2. **Case I:**

$\alpha = 2,\ b_1 = 1,\ b_2 = 2$

$E_{OA} = 1.00$

E_{DB} = 1/2 of the effect of 2 missiles nullified =

$(1/2) \times 1.00 = 0.50$

$F_{SB} = 1 - (1.00 \times (1 - 0.50)) = 0.50$

The model produces the expected result: 1 missile penetrates the defense and causes 50% damage. The fraction remaining is 0.50.

3. **Case II:**

$\alpha = 3,\ b_1 = 1,\ b_2 = 2$

$E_{OA} = 1.00$

E_{DB} = 1/3 of the effect of 3 missiles nullified =

$(1/3) \times 1.00 = 0.333$

$F_{SB} = 1 - (1.00 \times (1 - 0.333)) = 0.333$

The model does not produce the expected results: 2 missiles penetrate the defense and cause 100% damage. The fraction remaining should be 0.00, not 0.333.

4. **Case III:**

$\alpha = 4,\ b_1 = 2,\ b_2 = 3$

$E_{OA} = 1.00$

E_{DB} = 2/4 of the effect of 4 missiles nullified =

$(2/4) \times 1.00 = 0.50$

$F_{SB} = 1 - (1.00 \times (1 - 0.50)) = 0.50$

The model does not produce the expected results: 2 missiles penetrate the defense and cause 100% damage. The fraction remaining should be 0.00, not 0.50.

5. Case IV:

$\alpha = 8,\ b_1 = 4,\ b_2 = 2$

$E_{OA} = 1.00$

E_{DB} = 4/8 of the effect of 8 missiles nullified =

$(4/8) \times 1.00 = 0.50$

$F_{SB} = 1 - (1.00 \times (1 - 0.50)) = 0.50$

The model does not produce the expected results: 4 missiles penetrate the defense and cause 100% damage. The fraction remaining should be 0.00, not 0.50.

6. Case V:

$\alpha = 2,\ b_1 = 1,\ b_2 = 3$

$E_{OA} = 0.667$

E_{DB} = 1/2 of the effect of 2 missiles nullified =

$(1/2) \times 0.667 = 0.333$

$F_{SB} = 1 - (0.667 \times (1 - 0.333)) = 0.556$

The model does not produce the expected results: 1 missile penetrates the defense and causes 33% damage. The fraction remaining should be 0.667, not 0.556.

7. **Case VI:**

$\alpha = 8,\ b_1 = 2,\ b_2 = 2$

$E_{OA} = 1.00$

E_{DB} = (2/8) of the effect of 8 missiles nullified =

(2/8) x 1.00 = 0.25

$F_{SB} = 1 - (1.00 \times (1 - 0.25)) = 0.25$

The model does not produce the expected results: 6 missiles penetrate the defense and cause 100% damage. The fraction remaining should be 0.00, not 0.25.

C. DISCUSSION

The Taylor model held in the first sample case, presented to show that it does work in some cases. The other five cases are presented to show that it does not hold in general.

Note that overkill is completely lost in computing E_{OA}. Furthermore, it is necessary to invent additional variables to do the calculations, whereas the Hughes model is self-contained. In addition, an explicit expression of staying power is central to the Hughes model, but lost in the Taylor Model.

APPENDIX B

SAMPLE LAB REPORT

LAB REPORT
THESIS DATA COLLECTION
Jeffrey R. Cares, LT, USN

Trial #:____

Scenario: NX12 / Red (1 FF) vs Blue (2 FF's) / Turn 149

Date:___/___/___

Losses to A:____% (Blue)

Losses to B:____% (Red)

SSM Engagement:

Blue:
- Shot At By:
- Hit By:
- Shot Down:

Red:
- Shot At By:
- Hit By:
- Shot Down:

Remarks:

Red fires on defended Blue w/ 1 salvo of 4 SSM's at 50 NM. Ship spacing 1000 yds.

____
Jeffrey R. Cares, LT,USN

APPENDIX C

COMPILATION OF RAW DATA

A. SCENARIO I

Side A (1 FF) surprises side B (1 FF) with a salvo of 4 SSM's at a range of 50NM. Data in each cell are "hits, percent damage inflicted".

3,100	2,100	3,100	4,100	3,100
2,100	1,63	3,100	3,100	2,100
4,100	2,100	0,0	2,100	2,100
2,100	3,100	3,100	3,100	3,100
3,100	1,63	2,100	2,100	3,100
2,100	4,100	3,100	4,100	2,100
1,63	2,100	4,100	2,100	3,100
4,100	4,100	2,100	3,100	3,100
2,100	1,63	4,100	3,100	3,100
1,63	2,100	2,100	2,100	3,100

Average number of hits: 2.54

Standard deviation of hits: 0.952

Average percent damage: 94.3

Standard deviation of percent damage: 17.62

B. SCENARIO II

Side A (1 FF) surprises side B (2 FF's) with 1 salvo of 4 SSM's at a range of 50NM. Data in each cell are "hits, percent damage inflicted".

4,100	0,0	2,100	4,81.5	3,81.5
2,63	3,81.5	3,81.5	1,31.5	2,63
3,81.5	3,81.5	4,100	1,31.5	1,31.5
3,81.5	3,50	3,81.5	2,50	1,31.5
3,81.5	2,63	3,81.5	1,31.5	1,31.5
2,63	3,50	2,50	1,31.5	3,81.5
3,81.5	4,81.5	1,31.5	4,50	4,81.5
2,63	3,81.5	3,50	1,31.5	2,63
3,81.5	3,81.5	3,81.5	1,31.5	2,63
2,63	3,81.5	2,63	2,50	1,31.5

Average number of hits: 2.36

Standard deviation of hits: 1.0253

Average percent damage: 62.22

Standard deviation of percent damage: 23.3472

C. SCENARIO III

Side A (1 FF) exchanges salvoes with side B (1 FF) at 50NM. Salvo size: 4 SSM's. Both defenses active. Data in each cell are "hits, percent damage inflicted, number of missiles shot down by opposing side's defense".

2,100,1	2,100,2	0,0,2	1,63,2	3,100,1
2,100,2	3,100,0	2,100,1	2,100,1	1,63,2
1,63,2	2,100,1	1,63,1	1,63,1	1,63,2
2,100,2	2,100,1	1,63,2	1,63,2	1,63,2
0,0,1	3,100,0	1,63,2	0,0,2	2,100,1
0,0,2	1,63,2	2,100,0	1,63,2	2,100,2
0,0,2	2,100,2	0,0,2	1,63,1	0,0,2
3,100,0	1,63,2	2,100,2	1,63,1	2,100,2
2,100,2	0,0,2	2,100,2	1,63,1	2,100,2
1,63,2	1,63,2	1,63,1	2,100,0	1,63,1

Average number of hits: 1.32

Standard deviation of hits: 0.8676

Average percent damage: 66.74

Standard deviation of percent damage: 37.15

Average number shot down by defense: 1.46

Standard deviation of number shot down by defense: 0.7068

D. SCENARIO IV

Side A (1 FF) exchanges salvoes with side B (2 FF's) at a range of 50NM. Salvo size per ship: 4 SSM's. All defenses active. Data in each cell are "hits, percent damage inflicted, number of missiles shot down by opposing side's defense".

1. Side A

0,0,3	2,50,2	0,0,4	0,0,3	1,31.5,1
1,31.5,2	1,31.5,3	0,0,2	0,0,4	0,0,3
1,31.5,2	0,0,4	1,31.5,3	1,31.5,2	0,0,0
3,81.5,1	2,50,2	1,31.5.3	2,50,2	1,31.5,3
0,0,4	1,31.5,2	1,31.5,2	1,31.5,2	0,0,4
1,31.5,3	2,50,2	3,81.5,1	1,31.5,3	2,50,2
3,81.5,0	2,63,2	1,31.5,1	0,0,4	1,31.5,2
1,31.5,3	2,63,2	1,31.5,2	0,0,4	2,50,2
1,31.5,3	1,31.5,3	0,0,4	1,31.5,3	0,0,1
1,31.5,3	1,31.5,2	0,0,4	1,31.5,2	0,0,0

Average number of hits: 0.92

Standard deviation of hits: 0.8041

Average percent damage: 27.09

Standard deviation of percent damage: 23.37

Average number shot down by defense: 2.42

Standard deviation of number shot down by defense: 1.0897

2. Side B

3,100,1	4,100,2	5,100,1	4,100,1	4,100,2
5,100,0	2,100,2	4,100,2	3,100,2	6,100,0
4,100,1	5,100,1	4,100,1	2,100,3	4,100,0
4,100,2	2,100,4	5,100,1	4,100,2	6,100,2
3,100,1	7,100,0	3,100,2	5,100,2	4,100,0

Average number of hits: 4.08

Standard deviation of hits: 1.2557

Average percent damage: 100

Standard deviation of damage: 0.00

Average number shot down by defense: 1.40

Standard deviation of number shot down by defense: 1.00

E. SCENARIO V

Side A (1 FF) exchanges salvoes with side B (3 FF's) at 50NM. Salvo size per ship: 4 SSM's. All defenses active. Data in each cell are "hits, percent damage inflicted, number of missiles shot down by opposing side's defense".

1. Side A

0,0,3	0,0,4	2,42,1	0,0,4	0,0,4
0,0,4	2,42,2	0,0,4	0,0,4	1,21,3
0,0,4	1,21,3	1,12,3	0,0,4	1,21,3
1,21,3	1,21,3	0,0,3	1,21,3	0,0,4
1,21,3	0,0,3	0,0,4	0,0,4	0,0,4

Average number of hits: 0.48

Standard deviation of hits: 0.7071

Average percent damage: 10.08

Standard deviation of percent damage: 13.7171

Average number shot down by defense: 3.32

Standard deviation of number shot down by defense: 0.7483

2. Side B

0,0,3	4,100,2	6,100,1	5,100,1	5,100,1
5,100,2	4,100,2	2,100,3	3,100,0	6,100,1
5,100,2	8,100,2	4,100,2	9,100,2	3,100,2
4,100,1	8,100,1	7,100,1	3,100,2	5,100,0
7,100,1	6,100,3	5,100,3	5,100,1	10,100,0

Average number of hits: 5.16

Standard deviation of hits: 2.2301

Average percent damage: 96

Standard deviation of damage: 20

Average number shot down by defense: 1.44

Standard deviation of number shot down by defense: 0.9165

F. SCENARIO VI

Side A (2 FF's) exchange salvoes with side B (3 FF's) at 50NM. Salvo size per ship: 4 SSM's. All defenses active. Data in each cell are "hits, percent damage inflicted, number of missiles shot down by opposing side's defense".

1. Side A

4,54.3,2	0,0,5	1,21,5	1,21,6	2,42,3
1,21,3	1,21,5	1,21,4	2,42,4	1,21,5
3,54.3,2	2,42,3	1,21,5	2,42,4	3,63,4
1,21,3	2,42,5	2,42,4	2,42,5	0,0,5
3,54.3,1	1,21,5	2,42,4	0,0,4	3,54.3,5

Average number of hits: 1.64

Standard deviation of hits: 1.036

Average percent damage: 32.21

Standard deviation of percent damage: 18.161

Average number shot down by defense: 4.04

Standard deviation of number shot down by defense: 1.2069

2. Side B

2,50,5	5,100,2	3,81.5,4	5,100,2	1,31.5,3
5,50,3	3,81.5,4	6,100,3	4,50,3	3,81.5,4
5,100,4	7,100,2	6,100,1	8,100,2	5,81.5,4
3,81.5,4	7,100,3	4,100,1	5,100,2	3,50,4
6,81.5,1	4,81.5,2	7,100,3	3,50,4	2,63,4

Average number of hits: 4.48

Standard deviation of hits: 1.806

Average percent damage: 80.6

Standard deviation of percent damage: 21.44

Average number shot down by defense: 2.96

Standard deviation of number shot down by defense: 1.1358

APPENDIX D

GLOSSARY

1. Combat Energy (E)

Combat energy is a characteristic of a participant that has some lethal value in combat.

2. Combat Entropy (ΔH)

Combat entropy is the gain (or loss) of combat power due to tactical efficiencies (or inefficiencies).

3. Combat Potential (P)

Combat potential is stored combat energy.

4. Combat Power (H)

Combat power is the expenditure rate of combat energy by one participant against another during conflict.

5. Combat Work (ΔS)

Combat work is the result of transforming combat potential to effective combat power and eroding an opponent's staying power.

6. Damage (ΔN)

Damage is the loss in notional ship units to a force of N notional ships.

7. Effective Combat Power (H_{eff})

Effective combat power is combat energy applied to a participant as a result of physical interaction between participants.

8. Missile-Sump Effect

The missile-sump effect occurs when targets absorb proportionally more enemy combat energy than other targets in the target environment.

9. Mission Kill

A mission kill is a determination of how much damage must be done to an opponent to render it useless for current combat purposes.

10. Overkill

Overkill is the overabundance of effective offensive combat power.

11. Scouting Effectiveness (σ_N)

The scouting effectiveness of side N is a dimensionless effective offensive combat power degrader of range [0,1].

12. Salvo

A salvo is combat power which arrives at the target in a single, instantaneous pulse.

13. Staying Power (S)

Staying power is the measure of the amount of enemy combat energy a participant can absorb before its own combat energy is extinguished.

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