Calhoun
Institutional Archive of the Naval Postgraduate School

Calhoun: The NPS Institutional Archive

DSpace Repository

NPS Scholarship | Theses

2009-06

Optimizing security force generation

Workman, Patrick E.

Monterey, California. Naval Postgraduate School

https://hdl.handle.net/10945/4708

Downloaded from NPS Archive: Calhoun

DUDLEY KNOX LIBRARY

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

OPTIMIZING SECURITY FORCE GENERATION
by
Patrick E. Workman
June 2009
Thesis Advisor: Robert F. Dell Second Reader: P. Lee Ewing

Approved for public release; distribution unlimited

THIS PAGE INTENTIONALLY LEFT BLANK

REPORT DOCUMENTATION PAGE	*Form Approved OMB No. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** June 2009	**3. REPORT TYPE AND DATES COVERED** Master's Thesis

4. TITLE AND SUBTITLE Optimizing Security Force Generation	**5. FUNDING NUMBERS**
6. AUTHOR(S) Patrick E. Workman	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) Center for Army Analysis	**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release, distribution unlimited	**12b. DISTRIBUTION CODE**

13. ABSTRACT (maximum 200 words)

Manpower modeling plays a significant role in the growth and management of today's militaries. Unfortunately, existing models do not properly address the challenges facing the growth of recently established indigenous security forces. This thesis develops a linear program to plan the generation of a recently established indigenous security force over an unknown (infinite) horizon. The Security Force Generation Model (SFGM) is different from standard personnel models in four ways: it combines the growth of the enlisted and officer corps into a single model; it plans force growth over an infinite horizon; it provides a variable-time planning horizon with monthly and annual fidelity; and it incorporates the growth of the force through standard recruitment, a legacy force, and enlisted accessions. SFGM prescribes monthly and annual promotion rates, recruitment goals, accessions from the enlisted corps, and inclusion of the preexisting security apparatus. We demonstrate SFGM using current data from the Afghan National Army (ANA), under scenarios focused on the recently announced need to grow it from 81,000 to 134,000 soldiers. Our analysis shows that the ANA is capable of reaching the desired end strength in 28 months, but this requires enlisted accessions as the primary means of filling the officer corps and inclusion of the legacy force. Without the legacy force, the officer corps will not reach its desired strength for five years.

14. SUBJECT TERMS Manpower Planning, Optimization, Infinite Horizon, Variable Time Model, Officer Management, Enlisted Management	**15. NUMBER OF PAGES** 89
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**19. SECURITY CLASSIFICATION OF ABSTRACT**	**20. LIMITATION OF ABSTRACT**
Unclassified	Unclassified	Unclassified	UU

NSN 7540-01-280-5500

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

THIS PAGE INTENTIONALLY LEFT BLANK

Approved for public release; distribution unlimited

OPTIMIZING SECURITY FORCE GENERATION

Patrick E. Workman
Major, United States Army
B.S., United States Military Academy, 1997

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
June 2009

Author: Patrick E. Workman

Approved by: Robert F. Dell
Thesis Advisor

LTC P. Lee Ewing
Second Reader

Robert F. Dell
Chairman, Department of Operations Research

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

Manpower modeling plays a significant role in the growth and management of today's militaries. Unfortunately, existing models do not properly address the challenges facing the growth of recently established indigenous security forces. This thesis develops a linear program to plan the generation of a recently established indigenous security force over an unknown (infinite) horizon. The Security Force Generation Model (SFGM) is different from standard personnel models in four ways: it combines the growth of the enlisted and officer corps into a single model; it plans force growth over an infinite horizon; it provides a variable-time planning horizon with monthly and annual fidelity; and it incorporates the growth of the force through standard recruitment, a legacy force, and enlisted accessions. SFGM prescribes monthly and annual promotion rates, recruitment goals, accessions from the enlisted corps, and inclusion of the preexisting security apparatus. We demonstrate SFGM using current data from the Afghan National Army (ANA), under scenarios focused on the recently announced need to grow it from 81,000 to 134,000 soldiers. Our analysis shows that the ANA is capable of reaching the desired end strength in 28 months, but this requires enlisted accessions as the primary means of filling the officer corps and inclusion of the legacy force. Without the legacy force, the officer corps will not reach its desired strength for five years.

THIS PAGE INTENTIONALLY LEFT BLANK

THESIS DISCLAIMER

The reader is cautioned that the computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF ACRONYMS AND ABBREVIATIONS

ANA	Afghan National Army
AGR EMPM	Active Guard Reserve Enlisted Manpower Projection Model
AREAFM	Army Reserve Enlisted Aggregate Flow Model
ASCAR	Accession Supply Costing and Requirements
AZ	Above Zone
BZ	Below Zone
CBC	Computational Infrastructure for Operations Research Branch and Cut
ELIM-COMPLIM	Enlisted Loss Inventory Model–Computation of Manpower Programs using Linear Programs
ETS	End Term of Service
GAMS	General Algebraic Modeling System
GAO	Government Accountability Office
IHMP	Infinite Horizon Manpower Planning
ISAF	International Security Assistance Force
MLRPS	Manpower Long-Range Planning System
NCO	Non-Commissioned Officer
SFGM	Security Force Generation Model
TACCOM	Total Army Competitive Category Optimization Model
TAPLIM	Total Army Personnel Lifecycle Model
TIG	Time in Grade
Rank 1	Soldier (Private to Corporal)
Rank 2	Sergeant (SGT)
Rank 3	Staff Sergeant (SSG)
Rank 4	Sergeant First Class (SFC)
Rank 5	Master Sergeant (MSG)
Rank 6	Command Sergeant Major (CSM)
Rank 7	Lieutenant (1^{st} and 2^{nd} LT)
Rank 8	Captain (CPT)
Rank 9	Major (MAJ)

Rank 10	Lieutenant Colonel (LTC)
Rank 11	Colonel (COL)
RCMOP	Requirements-Driven Costs-Cased Manpower Optimization
RCP	Retention Control Point

ACKNOWLEDGMENTS

I would like to take this opportunity to thank Professor Rob Dell for his tireless effort, mentorship, and guidance. I truly appreciated the time and energy he put in to making this thesis successful. I would also like to thank LTC Lee Ewing, his guidance and gentle nudges were instrumental in focusing me on the "big picture" questions that I needed to answer.

I would also like to thank my wife, Carrie, for her endless support and understanding. None of this would be possible without you. Finally, our three wonderful children, Jack, Kate, and Anna, it has been a joy watching the three of you grow, and getting hugs every night, no matter how late I got home. I love you all.

THIS PAGE INTENTIONALLY LEFT BLANK

EXECUTIVE SUMMARY

Manpower modeling plays a significant role in the growth and management of today's militaries. Unfortunately, existing models do not properly capture the challenges facing the growth of recently established indigenous security forces. This thesis develops a linear program to plan the generation of a recently established indigenous security force. The Security Force Generation Model (SFGM) prescribes monthly promotion rates, recruitment goals, accessions from the enlisted corps, and inclusion of the preexisting security apparatus if desired. SFGM is unique from standard personnel models in four ways: it combines the growth of the enlisted and officer corps into a single model; it plans force growth over an unknown (infinite) horizon; it provides a variable time planning horizon with monthly and annual fidelity; and it incorporates the growth of the force through standard recruitment, a legacy force, and enlisted accessions. Prior to the development of SFGM, planners only had heuristics that incompletely addressed the issues of how to grow an indigenous security force.

We demonstrate SFGM using the Afghan National Army (ANA) and many scenarios focused on the recently announced need to grow its force size from 81,000 to 134,000. We develop five primary scenarios. Scenario 1 grows the ANA using current recruitment capabilities, attrition rates, legacy force, and no lower bounds on promotion. Scenario 2 restricts monthly and yearly lower and upper bounds on promotions to control month-to-month and year-to-year fluctuations. Scenario 3 excludes the legacy force. Scenario 4 improves enlisted reenlistment rates from 50 percent to 70 percent at the beginning of the fourth year. Scenario 5 includes no enlisted accessions and grows the officer corps with only recruitment and the legacy force.

We find that at least 24 months are required for the ANA to reach its desired end strength under current recruiting capabilities even with the inclusion of the legacy force, accessions of enlisted soldiers to the officer corps, and large fluctuations in promotion rates. It requires four additional months with the elimination of the large fluctuations. We also find that the ANA must exceed the desired strength by up to 4,000 soldiers in

order to absorb losses that occur due to the large-scale enlistment in the first three years. The lieutenant end strength must also increase above current target levels to provide a stable promotion base for senior ranks.

Growth of the officer corps relies heavily on enlisted accessions and the inclusion of approximately 3,300 officers from the legacy force. The officer corps never reaches its required end state without enlisted accessions unless Lieutenant recruitment increases from 500 to 1,700 annually. With enlisted accessions but without the legacy force, it requires five years for the officer corps to reach the desired end strength (almost three years longer). Inclusion of the legacy force is especially useful in the top two ranks because it slows the rate of promotion and allows the officer corps to build strength in the lower three ranks. Unlike the officer corps, enlisted legacy force soldiers are unnecessary for the enlisted corps to reach its target strength.

I. INTRODUCTION

Si Vis Pacem, Para Bellum. (If you want peace, prepare for war.)
Flavius Vegetius Renatus, 390 A.D.

A. PURPOSE

As the United States enters the twenty-first century, its Army finds itself facing a new paradigm in military operations. Where the defeat of enemy formations was previously a primary focus, today creating indigenous security forces capable of assuming the fight and securing the population is its equal (Army 2008). This thesis develops a linear program to help plan the generation of a new indigenous security force. The Security Force Generation Model (SFGM) determines monthly promotion rates, recruitment goals, accessions from the enlisted corps, and inclusion of the preexisting security apparatus. SFGM provides the information necessary to assess the feasibility of the growth of an indigenous security force.

On 7 October 2001, the United States entered the first of two major theaters of war with Operation Enduring Freedom in Afghanistan. By May 2002, low intensity guerilla warfare replaced major combat operations against the Taliban and the United States Special Forces began the process of reconstructing the Afghan National Army (Jalali 2002). On 20 March 2003, the United States launched Operation Iraqi Freedom. By August of 2003, major combat operations ended and the generation of the new Iraqi Army was under way. Given the similarity between these two campaigns, our recent history suggests that future military engagements will resolve themselves to protracted low intensity conflicts, where the development of indigenous forces will be a decisive element in establishing security and winning the battle for the population (Army 2008).

Force generation refers to the combination of enlisted and officer personnel policies that manage the growth of the total force. An organization controls force generation through levels of recruitment for both officers and enlisted, promotion rates, and forced separation. Managing these allows an organization to attain the desired force size and composition. The major challenge facing a growing force is balancing these variables to create a rank structure that allows for upward mobility. Failure to manage a rapidly growing force will result in an excess of personnel at certain positions and will have a major impact on retention as promotion potential decreases.

This thesis provides a force generation model that assists in understanding the challenges in developing indigenous security forces. Given the desired size and time horizon to grow the force, SFGM provides the decision maker with a set of personnel management requirements to meet given goals. We demonstrate SFGM using current data from the ANA under scenarios focused on the recently announced need to grow its force size from 81,000 to 134,000.

B. MOTIVATION FOR BUILDING SFGM

With increasing global instability, there is a greater likelihood of collapsed or failed states. Whether the nexus of a state's failure is direct military action or internal political failure, the results are the same: a non-existent or weakened central government and security apparatus. These states pose several significant threats: they provide a safe haven for terrorists and other groups, and may create conflict, regional instability, and humanitarian emergencies. They also undermine efforts to promote democracy and good governance (Wyler 2007). Extremist organizations thrive in these conditions, as evidenced by Somalia and Afghanistan, and generally move quickly to fill the power vacuum left by the dissolution of the state. External powers such as the United States or the United Nations find it in their best interest to restore stability in these countries to counter the perceived threats that a failed state may create (Rotberg 2004).

While the use of external forces to stabilize a nation is initially unavoidable, the transfer to indigenous security forces is necessary to re-establish local governance and stability (Jones, et al., 2005). The time required to develop these indigenous forces drive economic, political and military decisions for the failed state and the external powers involved. The ability to identify recruiting needs and how to control the force's growth provides leaders with an understanding of the challenges facing a new Army.

The need to develop national security forces in Afghanistan became a major concern for the United States following the defeat of the Taliban in 2002 (Manuel and

Singer 2002). Initially, the United States intended to build an 18,000-man army in 18 months to relieve the strain from International Security Assistance Force (ISAF). By January of 2005, the ANA had grown to 17,800 soldiers with 3,000 in training. As the Taliban become more active in Afghanistan, ISAF revisited the size of the ANA and in December of 2008, the ANA reached 80,000 soldiers and plans to grow to 134,000 soldiers (Afghan National Army 2009).

To manage this growth, U.S. Army analysts developed a model to determine the promotion rates necessary for the sustained growth of the ANA (MacCalman and Benson 2008). This model has a narrow focus and allows for the manipulation of the current strength and desired end strength. SFGM, with the addition of features found in manning models such as Gibson (2008), Schrews (2002), and Clark (2009), significantly enhances this prior work.

More specifically, the work of MacCalman and Benson focuses force growth at a monthly level. Typically, models for officer growth such as Gibson (2007) and Yamada (2000) are year-based, focusing on annual growth and large personnel movements. This is similar to enlisted corps models where Ginther (2006) and Rodgers (1991) determine growth requirements on an annual basis. The level of fidelity required in developing a new army is much finer. Annual values do not provide sufficient information to manage month-to-month growth, promotion, and retention. For this reason, SFGM manages force growth using a monthly period initially and converting to a yearly period later in the planning horizon. This allows us to see the effects of the initial period of growth on long-term force development.

The officer corps development in SFGM is similarly unique. Where in most militaries the enlisted corps provides a small part of the officer corps, in an emerging state the pool for commissioned officers draws primarily from the enlisted ranks. This causes a significant impact on growth not only for the officer corps, but for the enlisted corps as well. Essentially, an additional separation factor is acting on the enlisted ranks, while the officers corps sustained growth is dependent on an uninterrupted flow of soldiers. This is not to say that recruitment does not occur, only that the recruitment numbers are far too low to independently fill and grow the officer corps. SFGM allows for this growth structure and accounts for it in the recruitment of enlisted soldiers.

The existence of a legacy force creates additional challenges in the growth of the ANA. The need for senior officers and NCOs to lead and manage a force are primary concerns. In both Afghanistan and Iraq one method of filing these positions was the use of the legacy force, which is defined as the nation's previous Army or members of organized militia groups. When creating a new Army, these soldiers and officers may play a valuable role in establishing the new force. SFGM allows for the inclusion of these personnel into the new force at some cost. This method is similar to Schrews (2002) who examined the growth of the reserve force and the ability to bring in soldiers at any rank.

A significant aspect of force growth is the control of enlisted and officer reenlistment to establish goals for the new force. In previous manpower models reenlistment is fixed in attrition data. This allows for reenlistment to occur as part of standard attrition at a specific level. SFGM incorporates reenlistment as a separate variable that allows the model to determine the optimal reenlistment rates within lower and upper bounds.

Addressing the problem of growing a new military for an emerging state is complicated by the short time available and the intricacies of its force generation. Dealing with these issues requires the combination of the enlisted personnel models and officer models and the implementation of new ideas for personnel growth. SFGM works to bring these aspects together to develop a coherent strategy for recruiting and growing the desired force.

C. PROBLEM STATEMENT AND THESIS OUTLINE

This thesis develops a linear program to assist in the development of national security forces over an unknown (infinite) time horizon. SFGM prescribes monthly and annual targets for recruitment, promotions, accessions, reenlistments, and retirements. Chapter II discusses a model that was previously used to advise on promotions for the ANA and for similar papers. Chapter III presents the SFGM. Chapter IV discusses

SFGM implementation and the results. Chapter V provides conclusions and areas of future research. Appendix A presents the primal equilibrium approximation for SFGM over an infinite horizon. Appendix B presents the dual equilibrium approximation for SFGM over an infinite horizon.

THIS PAGE INTENTIONALLY LEFT BLANK

II. SECURITY FORCE GROWTH

> *Establishing security involves domestic security, secure borders, and relatively accommodating neighbors. Of the three factors in achieving stabilization and reconstruction, domestic security is the most important and often the most difficult to achieve.*
>
> James Stephenson
> *Losing the Golden Hour: An Insider's View of Iraq's Reconstruction*

A. AN OVERVIEW OF MILITARY FORCE GENERATION

Military manpower planning has a long history in its impact on managing and growing armies. From the Roman Legions to today's militaries, some form of manpower planning was necessary for their growth and management. Manpower planning as it is known today was first reported following the second world war by Seal (1945) and Vadja (1947) in published works on hierarchical organizations. As the Allied victory over the Axis became an eventuality, the United Kingdom asked Seal and Vadja to develop plans for the reconstruction of the Royal Navy's technical and managerial civilian manpower structures. Seal's and Vadja's work became the foundation for modern manpower modeling. By 1960, manpower planning became a part of military force management as the United Kingdom and the U.S. shifted away from conscription armies (Smith and Bartholomew 1988).

Today, manpower models inform all aspects of force growth and development decisions in the U.S. Armed Services. The services use separate models to determine enlisted and officer recruiting requirements, promotion rates, retirement thresholds, and transitions within the organization. These models all have common attributes: hierarchical structures, fixed entry points, managed transitions and sustained attrition rates. In general, current military manpower models focus on either the enlisted force growth, such as Ginther (2006), Rodgers (1991) and Schrews (2002), or on officer management such as Clark (2009), Corbett (1995) and Gibson (2007). We were unable to identify any models that focus on the growth of the complete organization, as the separation of the models by rank is a natural result of the separate populations within the military.

Emerging states have a different set of force generation issues. The initial recruitment is problematic and the pool of qualified candidates to serve as officers is limited as seen in Iraq and Afghanistan (Haims, et al., 2008). There likely exists some previous military organization that the new government may not want to completely include in the new force, but does provide a pool from which to draw in limited quantities. Management of the force requires a high level of fidelity to allow planners to understand the near term needs of the force. This near-term focus must then transition to an extended horizon to determine the force's long-term needs to attain a desired end-state.

To answer this problem, U.S. Army analysts developed a heuristic model to determine the monthly promotion rates necessary to sustain force growth in the ANA (MacCalman and Benson 2008). This is an Excel-based, time-step model that seeks to find the optimal solution by conducting a binary search over the solution space. Recruitment and attrition targets are set and the model returns the size of the force at time p and the associated number of promotions for each rank r in that period. This model does not account for losses due to contract expiration or voluntary and forced retirement. The model allows a set number of gains from the legacy force as opposed to a variable rate dependent on the force's actual needs. It also does not account for the interaction between enlisted and officer growth.

In a more detailed account of ANA force generation, Benson (2008) discusses the specific aspects of developing the heuristic force generation model. Benson uses the Runge-Kutta 4 method to solve a dynamical system of equations and implements it in Excel. Runge-Kutta 4 is a step-based method for solving ordinary differential equations where it treats each step in the sequence in an identical manner to the previous step and does not use the prior behavior of the solution in the next step of the process (Press, et al., 1995). Benson's model provides monthly fidelity on promotion rates for the growing force; however, it maintains separate enlisted and officer personnel models that prevent observation of the interaction between their respective growths.

B. MANAGING FORCE GROWTH

We manage force growth at the personnel level and at the organizational level. At the personnel level, movement through an organization occurs in three phases: accession, promotion, and separation. Each of these phases provides force managers multiple methods that allow them to control the growth of the force. Organizationally, we manage the force with regard to the desired end-state by controlling the size of the final force and the required rank ratios. It is necessary to understand each of these levels and the second and third order effects that growth decisions may cause in the months or years to come.

1. Accessions

Accession is a military term used to describe recruitment of a person into the organization. Accession of enlisted soldiers occurs through basic training where recruits are selected from some pool of personnel that fill the needs of the force. In general, the pool of available recruits for the enlisted corps is large and able to meet the force demand. Recruitment of the enlisted force is constrained only by the size of the training facilities available to process new recruits. Those completing basic training and entering the force enter at the lowest grade.

We draw officer accessions from a smaller, more educated pool from the population at large and from the existing enlisted force. Afghanistan suffers from the lack of an educated class and is incapable of providing sufficient recruits to fill officer positions (Haims, et al., 2008). However, as the enlisted corps is established, enlisted soldiers who excel in leadership positions can be accessed to the officer corps. The maximum officer candidate training capacity constrains the total number of officers accessed to the maximum that can be trained in a period. Those completing officer training enter the force at the lowest officer rank.

SFGM may also access soldiers to both the officer and enlisted corps from a separate, previously existing pool allowing senior level positions to be filled without rapid promotion of junior officers (Chan 2009). The legacy force provides accessions directly into any rank within the new forces structure. Depending on the political climate surrounding the growth of the new force, these legacy elements might not be welcomed.

2. Promotions

Promotions are the advancement in rank for a soldier from their current grade to the next higher grade. SFGM incorporates upper and lower bounds on promotion rates to ensure that promotions fall within reasonable rates according to force size. Where Gibson (2008) incorporates the Below Zone (BZ) and Above Zone (AZ) decision variable, SFGM allows promotion of a soldier between the minimum required time in grade and their retention control point.

3. Separations

Separation refers to soldiers leaving the force. SFGM allows soldiers to leave in a number of different ways: attrition, End of Term of Service (ETS), retention control, and retirement. We use attrition to describe the loss of soldiers during their period of service to any number of reasons such as administrative separation, desertion, or combat losses. These unprogrammed losses occur independent of force growth and control measures. We measure attrition as a percentage of the force per period. Both the Iraqi Army and the Afghan Army endured periods of significantly high attrition, near 50 percent during the initial growth of their organization. Over time, the attrition rates fell to approximately 3 percent annually for both (Metz and Millen 2005; Government Accountability Office (GAO) 2005).

The next method of separation is when a soldier reaches their ETS. Afghan soldiers enlist for a tour of duty of three years for enlisted soldiers and five years for NCOs; promotions from one grade to another incur a new tour of duty. When a soldier reaches the expiration of their contract, they reenlist and serve another tour of duty or they choose to leave the service. SFGM's lower and upper bounds on reenlistment reflect the current reenlistment rates in Afghanistan (GAO 2008).

Retention control is the involuntary separation of a soldier who has surpassed the authorized time in grade. Soldiers in this category depart the force inventory in the next period.

Retirement allows for the removal of senior officers and NCOs from the inventory to allow junior officers and NCOs continued forward progression.

4. Organizational

The simplest control factor at the organizational level is the desired end strength. This factor provides the goal against which we measure solutions to determine their sufficiency. The second organizational control factor is the force ratio. Each rank has a target force ratio, which is that rank's percentage of the force's end strength. Failure to manage force ratio could result in a large inventory of enlisted coupled with a small inventory of officers that would create a poorly managed, ineffective organization. Together end strength and the force ratio decisions drive promotion, accession, and separation decisions.

C. RELATED LITERATURE

The SFGM is different from standard personnel models in four ways: it combines the growth of the enlisted and officer corps into a single model; it plans force growth over an infinite horizon; it provides a variable time planning horizon with monthly and annual fidelity; and it incorporates the growth of the force through standard recruitment, a legacy force, and enlisted accessions. We focus on three areas of past, related manpower research: hierarchical models, personnel management, and infinite time horizons.

1. Hierarchical Models

Most organizations are hierarchical in structure and the military is no exception. An extensive literature exists with focus on managing manpower planning in organization or on specific aspects of manpower planning in the hierarchy. Edwards (1983) provides a useful survey of manpower models and discusses the three needs of a manpower model: data on current stock, data on wastage (attrition), and data on inflow recruits. Hierarchical organizations exhibit two separate forms of growth: cohort and renewal. Vadja (1978) discusses the growth of organizations using a cohort model and Markovian transition matrices. Members enter the organization as groups, advance through the rank structure, and attrite according to some survival function. Renewal is a recruitment theory that dictates growth to maintain a current state. The attrition of the current force and their promotion drives the inflow of the organization (Bartholomew, Forbes and McClean 1991). The structure of SFGM results in recruiting that follows both forms, as SFGM initially generates the force and then stabilizes it at its desired end strength.

Mehlmann (1980) uses a Markov chain to optimize recruitment and grade transition. Mehlmann shows that using dynamic programming over a finite horizon provides optimal strategies based on a present state. Morgan (1979) shows that optimal recruiting levels may be detrimental to overall organizational health. Morgan argues that to achieve some desired steady state promotion rate, sub-optimal recruiting may be necessary to prevent bulges in manpower that affect upward mobility. This is a significant insight into tracking the promotion and recruitment in SFGM. SFGM initially focuses on maximizing recruiting to reach the desired end strength, similar to Mehlmann, and on promotion constraints to provide a stable long-term force, similar to Morgan.

The Army Manpower Long-Range Planning System (MLRPS) is a long horizon manpower planning model that determines promotions, accessions, losses and reclassification for the Army (Gass, et al., 1988). MLRPS determines manpower goals over a 7 to 20 year planning horizon. Unlike most models, MLRPS handles both officer and enlisted growth, however, it models each separately. MLRPS minimizes a weighted sum of the deviations from the target values as an objective function. Gass finds that achieving objectives for promotions, separations, and accessions conflict with each other, and without elastic variables would be infeasible (Gass, et al., 1988).

2. Personnel Management

We typically model personnel management separately for officers or enlisted. We examine the literature and models used with each.

a. Officer Management

Gibson (2007) develops the Total Army Competitive Category Optimization Model (TACCOM), a linear program, to determine accession requirements and promotion rates over a forty-year period. TACCOM specifically focuses on below the zone, primary zone and above zone promotions and their impact on officer strength. SFGM extends the above zone and below zone promotions developed in Gibson and incoporates them as piece-wise linear functions.

Corbett (1995) develops a linear program to determine the number of officer accessions required to manage the U.S. Army's junior and mid-level officer needs. Corbett specifically focuses on the management of officers between the combat arms and the combat support branches of the U.S. Army. His model optimizes the assignment of officers between the branches to provide the number of officers necessary at each grade. The purpose of the model is to identify the number of accessions and branch details necessary to manage the mid-level officers required later in the planning horizon.

Clark (2009) develops the Requirements-Driven Costs-Based Manpower Optimization (RCMOP) linear program to determine monthly values for promotion, inventory, accessions, natural losses and forced losses by minimizing the total penalty for deviations from manpower requirements. RCMOP optimizes over a two year time horizon on a monthly basis to meet the fiscal requirements of the U.S. Navy. Other than SFGN, RCMOP's is the only other example of a monthly fidelty military manpower linear program that we encountered.

b. Enlisted Management

The U.S. Army's initial efforts to model the growth of the force used the Enlisted Loss Inventory Model—Computation of Manpower Programs using Linear Programs (ELIM-COMPLIM). ELIM-COMPLIM is a linear program that forecasts strengths, gains, and losses over a seven-year period (Holz and Wroth 1980). ELIM focuses on the expected losses from the enlisted inventory and then determines the necessary accessions to maintain the desired force strength. The Accession Supply

Costing and Requirements (ASCAR) model is a successor to ELIM that forecasts the costs of optimizing the accession of new soldiers of different types into the force (Collins, Gass and Rosendahl 1983). Where ELIM is a near-term policy model that determines personnel objectives, ASCAR models the long-term impact of personnel requirements, personnel qualification, and types of recruits.

Another model developed to manage the growth of a hierarchical organization is the Total Army Personnel Lifecycle Model (TAPLIM). It is a linear program that develops near term strategic personnel planning by minimizing the total absolute deviation from a desired target (Durso and Donahue 1995). TAPLIM managed the downbuild of the U.S. Army following the end of the Cold War and determined the necessary mixture of active duty and reserve soldiers to meet the Army's strategic objectives.

Rodgers (1991) uses a multi-objective linear program to manage enlisted strength planning for the Navy. The model determines monthly inventories, advancement, and recruiting goals over a multi year period. Rodger's model incorporates budgetary and organizational constraints and seeks to minimize the total deviation from the desired end-state.

Ginther (2006) develops the Army Reserve Enlisted Aggregate Flow Model (AREAFM), which is a Markov growth model. AREAFM provides specific recruiting requirements based on aggregate accession, attrition, and retention rates. Rodgers and Ginther provided insight into enlisted manpower modeling and management of retention rates in SFGM.

Schrews (2002) develops the Active Guard Reserve Enlisted Manpower Projection Model (AGR EMPM), an enlisted reserve manpower linear program to determine the long term effects of preventing critical career fields from leaving the reserves, and of accessing senior enlisted soldiers directly into the force. Specifically, the model determines the impact on accessions and promotions that these policies would have. Schrews' model incorporates accessions directly into the ranks of E7 and below from a recruiting base or from the active Army. This differs from most military manpower models where the force pyramid is entered solely from the base. AGR EMPM was initially conceived to optimize the manpower decisions on a monthly basis over a seven year horizon. However, Schrews found the model size intractable and switched to model decisions on a yearly basis.

3. Infinite Time Horizons

When dealing with multi-period optimization, one area of concern is the end of the planning horizon. Manpower models are specifically designed for some short period of time, such as Holz (1980) which planned over a seven year horizon, but are actually modeling a system that extends out to some unknown (infinite) horizon. The artificial horizon, or truncation, may have effects on the optimal solution as it ignores variables that would continue to influence the solution beyond the planning horizon. These variations are end effects first discussed explicitly in Grinold (1983). Grinold suggests four methods for dealing with end effects: truncation, salvage, and primal and dual equilibrium. Truncation ignores the stable phase following the predetermined horizon, while the salvage technique places some value on the decisions from the period prior to the final period into the future. Finally, the primal and dual equilibrium impose an equilibrium constraint on the stable phase that accounts for penalties that would accrue over the infinite horizon.

Schochetman (1989) proves that primal and dual equilibrium converge to an infinite horizon where optimality is assured. Additionally, Schochetman discusses optimal solution sets and the stopping criteria for infinite horizon problems and determines that policy based stopping criteria are preferred over convergence, as convergence in value tends to be slow. SFGM incorporates a policy-based stopping criterion at period 50 rather than at convergence; convergence of the SFGM primal, dual, and truncation occurs near period 160 and is beyond any reasonable planning period.

Walker (1995) provides methods for implementing infinite horizon optimization for linear and integer programs where similar decisions need to be made repeatedly over many successive periods, and specifically discusses implementation of primal and dual equilibrium with respect to personnel models. In particular, Walker extended the horizon of the Army Manpower Model, TAPLIM, using both dual and primal equilibrium models to establish bounds on the optimal infinite time horizon solution. Using these bounding methods Walker generates a tight bound, within one percent, on the optimal solution.

Yamada (2000) develops an Infinite Horizon Manpower Planning model (IHMP) to determine the annual number of accession, promotions, and separations necessary for the Army to manage the officer corps. IHMP is a convex quadratic program that uses both the dual and primal equilibrium to bind the optimal solution to the Army's officer growth problem. Walker (1995) and Yamada (2000) provide the basis for the SFGM infinite horizon model; SFGM differs in that it incorporates two time period indices as opposed to one.

III. MODEL FORMULATION

A. MODELING ASSUMPTIONS

In the immortal words of George Box "*All models are wrong, but some are useful.*" We develop three SFGM models to account for end effects (Grinold 1983). Here we present the truncation model as it is the basis for both the dual and the primal equilibrium models. The following are assumptions for all three models.

1. Promotions result in incurring a service obligation for NCOs of five years. This assumption simplifies reenlistment.
2. To allow for flexibility, many SFGM constraints are elastic. An elastic constraint allows for violation at a cost per unit violation. We show elastic constraints with $\overset{o}{\le}$ (e.g., Brown, Dell and Wood 1997).
3. SFGM assumes that force managers desire smooth promotions, recruitment, and accessions. This smoothing prevents large fluctuations from period to period.
4. SFGM incorporates two separate time epochs. Monthly periods for the first three critical years of growth and annual periods at the start of the fourth year. These two epochs allow detailed monthly modeling of the short term, rapid growth of the ANA during the first three years and then avoid unnecessary monthly detail beyond year three.

Figures 1, 2 and 3 illustrate the flow of enlisted and officers through their career progression.

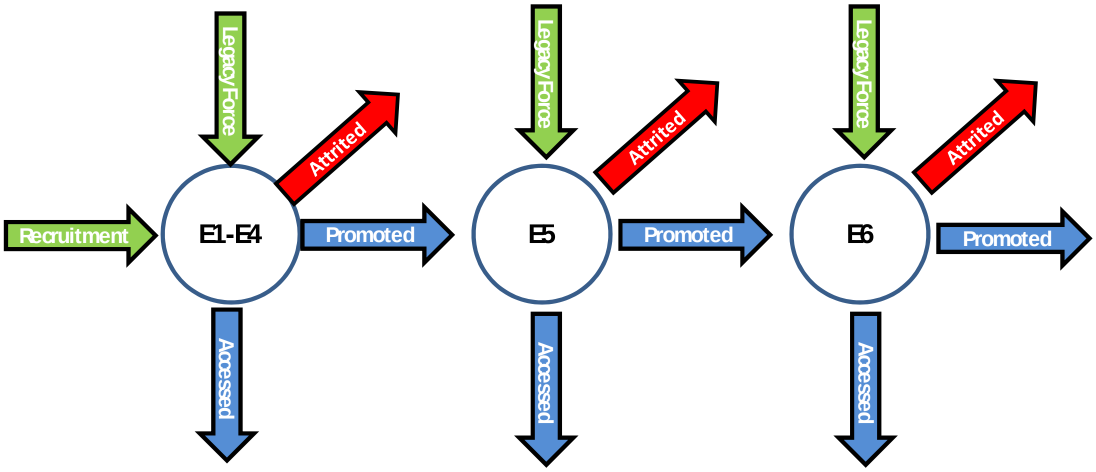

Figure 1. Enlisted Career Path Shown as a Network (E1 to E6)

Figure 1 shows the career path progression of an enlisted soldier. We group E1s through E4s as skill level one soldiers. Recruitment for the force occurs at this grade as does the first opportunity for legacy force soldiers to join the current force. Soldiers depart each node by promotion, separation, or accession to the officer corps. We limit accessions to the first seven grades (E1–E7) of the enlisted ranks.

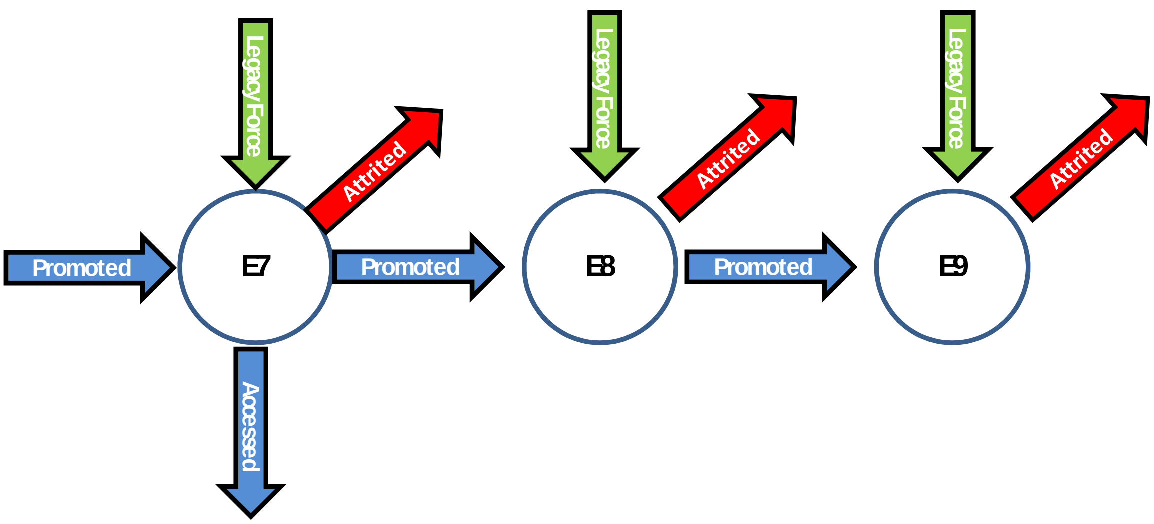

Figure 2. Enlisted Career Path Shown as a Network (E7 to E9)

Figure 2 shows the career path progressions for soldiers in the rank of E7 to E9. Soldiers enter each rank in one of two ways: promotion, or from the legacy force. The rank of E7 is the final rank where a soldier is available for accession to the officer corps. Soldiers depart the ranks through either separation or promotion, with the exception of E9 where there are no further promotions.

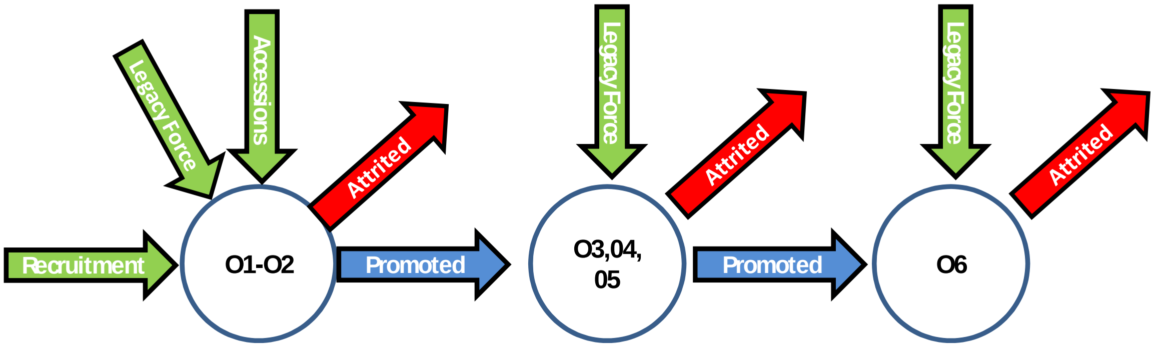

Figure 3. Officer Career Progression in SFGM Shown through a Network

Figure 3 shows the career progression of officers in SFGM from Lieutenant to Captain. 2nd and 1st Lieutenants are combined into one grade as the progression from one to another is typically dependent solely on time in service. All officer Grades from O3 to O6 have equivalent force flows with the exception that we do not promote officers out of the rank of O6. Legacy force soldiers are available for accession into all grades as needed to fill force requirement. Officer recruits and accessions from the enlisted corps enter through the grade of lieutenant. Officers depart their current grade through either promotion or separation.

B. INPUT PARAMETERS

1. Indices

r, r'	rank {1,2,…,11}
p	periods in planning horizon {1,2,…,P}
t, t'	Time in Grade (TIG) in period {1,2,…,T}

2. Sets

$t' \in epoch_{p,t}$	set of TIG measurement from one period length to another (months to years)
$t \in pw_{r,p}$	set of TIG promotion periods for rank *r* at period *p*, changes from TIG in months to TIG in years at transition point
$t \in rw_{r,p}$	TIG reenlistment window for rank *r* in period *p*

3. Data

$\alpha_{r,p}$	discount rate for rank *r* at period *p* [UNITS]
$access_{r,p}$	maximum fraction of officer accession per rank *r* in period *p* $\left[\frac{\text{Soldiers Accessed}}{\text{Total Soldiers}}\right]$
$attrite_{r,p}$	fraction of rank *r* attriting at the beginning of period *p* $\left[\frac{\text{Soldiers } r \text{ Attrited in Period } p}{\text{Total Soldiers } r \text{ in Period } p}\right]$
$current_{r,t}$	initial number of soldiers in rank *r* at time in grade *t* [soldiers]
$existing_r$	number of soldiers in rank *r* available from legacy force [soldiers]
$\underline{frac}_{r,p}, \overline{frac}_{r,p}$	fraction of soldiers of rank *r* desired at period *p* $\left[\frac{\text{Soldiers of rank } r}{\text{Total Soldiers}}\right]$
$fracL_{r,p}$	fraction of legacy force soldiers allowed per rank *r* per period *p* [soldiers]
otc_p	officer training capacity for period p [soldiers]
$\underline{prom}_{r,p}, \overline{prom}_{r,p}$	minimum and maximum promotion for rank *r* at the beginning of period *p* [number of soldiers]
$\underline{rec}_{r,p}, \overline{rec}_{r,p}$	minimum and maximum number of recruits of rank *r* at the beginning of month *p* [soldiers]

$\overline{retire}_{r,p}$	maximum retirement for rank *r* in period *p* [soldiers]
$\underline{reup}_{r,p}, \overline{reup}_{r,p}$	minimum and maximum fraction of reenlistment per rank *r* $\left[\frac{\text{Soldiers who Reenlist}}{\text{Soldiers Eligible to Reenlist}}\right]$
$\underline{sab}_{r,p}, \overline{sab}_{r,p}$	minimum and maximum fraction of accession for each rank *r* between periods *p*
$\underline{spb}_{r,p}, \overline{spb}_{r,p}$	minimum and maximum fraction promotions for each rank *r* between periods *p*
$\underline{srb}_{r,p}, \overline{srb}_{r,p}$	minimum and maximum fraction recruitment for each rank *r* between periods *p*
$\overline{target}_r, \underline{target}_r$	targeted number of soldiers in rank *r*
$\underline{tig}_{r,p}, \overline{tig}_{r,p}$	minimum and maximum time in grade per rank *r* for promotion in period *p*[periods]
tgt	Point at which time shifts from monthly to yearly
propoint	Desired point where promotion occurs

C. VARIABLES

$ACS_{r,p,t}$	accessed Soldier for rank *r 1 to 5* in TIG *t* at the beginning of period *p* [soldiers]
$ETS_{r,p,t}$	end time of service for rank *r* in TIG *t* at the beginning of period *p* [soldiers]

$HDO_{r,p}$	number of legacy soldiers in rank r to add to the force at the beginning of period p [soldiers]
$PRO_{r,p,t}$	soldiers promoted from rank r at the beginning of period p at time in grade t [soldiers]
$REC_{r,p}$	number of recruits of rank r added at the beginning of period p [soldiers]
$RET_{r,p,t}$	retirement number for rank r at the beginning of period p at time in grade t [soldiers]
$X_{r,p,t}$	number of rank r at the beginning of period p at time in grade t [soldiers]

D. MODEL FORMULATION EQUATIONS

1. Balance of Flow Equations

$$X_{r,p,t} = current_{r,t} + HDO_{r,p} \qquad \forall\ r, p = 1, t \qquad (1.1)$$

$$X_{r,p,t} = REC_{r,p} + HDO_{r,p} \qquad \forall\ r = 1, p > 1, t = 1 \qquad (1.2)$$

$$X_{r,p,t} = \sum_{r'<6} \sum_{t'} ACS_{r',p-1,t'} + HDO_{r-1,p-1} + REC_{r-1,p-1} \qquad \forall\ r = 7, p > 1, t = 1 \qquad (1.3)$$

$$\begin{aligned} X_{r,p,t} = & \left(1 - attrite_{r,p}\right) \sum_{t' \in epoch_{p,t}} X_{r,p-1,t'-1} \\ & - ACS_{r,p-1,t-1} - PRO_{r,p-1,t-1}\Big|_{t \in pw_{r,p}} \\ & - ETS_{r,p-1,t-1}\Big|_{t \in rw_{r,p}} - RET_{r,p-1,t-1} \end{aligned} \qquad (1.4)$$

$$\forall\ r \neq 6 \text{ or } 11, p > 1, 1 < t \leq \overline{tig}_{r,p}$$

$$X_{r,p,t} = \left(1 - attrite_{r,p}\right) \sum_{t' \in epoch_{p,t}} X_{r,p-1,t'-1} - RET_{r,p-1,t-1} \qquad \forall\ r = 6 \text{ or } 11,\ p > 1, 1 < t \le \overline{tig}_{r,p} \tag{1.5}$$

$$X_{r,p,t} = \sum_{t' \in pw_{r,p}} PRO_{r-1,p-1,t'-1} + HDO_{r-1,p-1} \qquad \forall\ r \ne 1 \text{ or } 7,\ p > 1, t = 1 \tag{1.6}$$

2. Constraints

a. Accession Constraints

$$ACS_{r,p,t} \overset{\circ}{\le} access_{r,p} \sum_{r'<6} \sum_{t \le \overline{tig}_{r,p}} X_{r',p,t} \qquad \forall\ r < 6,\ p > 1 \tag{1.7}$$

$$\sum_{r} \sum_{t \le \overline{tig}_{r,p}} ACS_{r,p,t} \overset{\circ}{\le} otc_{p} \qquad \forall\ p \tag{1.8}$$

$$\underline{sab}_{r,p} \sum_{t \le \overline{tig}_{r,p}} ACS_{r,p-1,t-1} \overset{\circ}{\le} \sum_{t \le \overline{tig}_{r,p}} ACS_{r,p,t} \overset{\circ}{\le} \overline{sab}_{r,p} \sum_{t \le \overline{tig}_{r,p}} ACS_{r,p-1,t-1} \qquad \forall\ r,\ p > 1 \text{ and } p \ne tgt + 1 \tag{1.9}$$

b. Promotion Constraints

$$\left(\underline{prom}_{r,p}\right) \sum_{t \in pw_{r,p}} X_{r,p,t} \le \sum_{t \in pw_{r,p}} PRO_{r,p,t} \le \left(\overline{prom}_{r,p}\right) \sum_{t \in pw_{r,p}} X_{r,p,t} \qquad \forall\ r \ne 6 \text{ or } 11,\ p \tag{1.10}$$

$$\underline{spb}_{r,p} \sum_{t \le \overline{tig}_{r,p}} PRO_{r,p-1,t-1} \overset{\circ}{\le} \sum_{t \le \overline{tig}_{r,p}} PRO_{r,p,t} \overset{\circ}{\le} \overline{spb}_{r,p} \sum_{t \le \overline{tig}_{r,p}} PRO_{r,p-1,t-1} \qquad \forall\ r,\ p > 1 \text{ and } p \ne tgt \tag{1.11}$$

c. Force Growth Constraints

$$\sum_{p} HDO_{r,p} \leq existing_{r} \qquad \forall\, r \qquad (1.12)$$

$$HDO_{r,p} \leq fracL_{r,p} \sum_{t<\overline{tig}_{r,p}} PRO_{r,p,t} \qquad \forall r > 1, p, t \qquad (1.13)$$

$$\underline{rec}_{r,p} \leq REC_{r,p} \leq \overline{rec}_{r,p} \qquad \forall\, r, p \qquad (1.14)$$

$$\begin{aligned} &\underline{srb}_{r,p} \sum_{t\leq\overline{tig}_{r,p}} REC_{r,p-1,t-1} \overset{o}{\leq} \sum_{t\leq\overline{tig}_{r,p}} REC_{r,p,t} \\ &\overset{o}{\leq} \overline{srb}_{r,p} \sum_{t\leq\overline{tig}_{r,p}} REC_{r,p-1,t-1} \qquad \forall\, r, p > 1 \text{ and } p \neq tgt+1 \end{aligned} \qquad (1.15)$$

$$RET_{r,p} \leq \overline{retire}_{r,p} \qquad \forall\, r = 6 \text{ or } 11, p \qquad (1.16)$$

$$\begin{aligned} &(1-\underline{reup}_{r,p})X_{r,p,t} \leq ETS_{r,p,t} \\ &\overset{o}{\leq} (1-\overline{reup}_{r,p})X_{r,p,t} \qquad \forall\, 1 \leq r < 7, p, t \in rw_{r,p} \end{aligned} \qquad (1.17)$$

d. Force Sizing Constraints

$$\underline{target}_{r} \overset{o}{\leq} \sum_{t\leq\overline{tig}_{r,p}} X_{r,p,t} \overset{o}{\leq} \overline{target}_{r} \qquad \forall\, r, p > 1 \qquad (1.18)$$

$$\underline{frac}_{r,p} \sum_{r'} \sum_{t\leq\overline{tig}_{r,p}} X_{r',p,t} \overset{o}{\leq} \sum_{t\leq\overline{tig}_{r,p}} X_{r,p,t} \overset{o}{\leq} \overline{frac}_{r,p} \sum_{r'} \sum_{t\leq\overline{tig}_{r,p}} X_{r',p,t} \qquad \forall\, r, p > 1 \qquad (1.19)$$

3. Objective Function

minimize *penalties* (1.20)

Where the priorities are:

- Meet the required number of soldiers at each rank
- Meet the required proportion for each rank
- Penalize early and late promotion of soldiers
- Smooth the rate of promotion, recruitment, and accessions
- Focus on building new forces as opposed to incorporating older forces.

E. MODEL EXPLANATION

1. Objective Function

The objective function (1.20) seeks to minimize the total weighted deviation from the desired force size at the given time horizon. We discount the penalties to provide greater importance to decisions made earlier in the planning horizon.

2. Constraints

a. Balance of Flow

The balance of flow equations track the initial size and the total growth of the overall force. Constraints (1.1) establish the initial force size of the ANA and distribute the existing force by rank and TIG.

Constraints (1.2) and (1.3) track the creation of new soldiers and officers respectively. Constraints (1.2) are for soldiers of rank one with a TIG of one (the sum of recruits and legacy force soldiers brought into the ANA). Constraints (1.3) track officers of rank seven with TIG one (soldiers from the previous period who were accessed, officers who were recruited, and officers from the legacy force).

Constraints (1.4) track soldiers, NCOs and officers as they progress by rank and TIG from one period to the next by accession, promotions, and ETS.

Constraints (1.5) are a version of constraints (1.4) for senior NCOs and COLs whose monthly strength is a function of only normal attrition and retirement.

Constraints (1.6) track the NCOs and officers entering a new grade with a time in grade of one.

b. Accession Constraints

To control accessions we use constraint sets (1.7), (1.8) and (1.9). We track the total number of accessions that occur in a single period with constraints (1.7). Constraints (1.8) limit the total number of accessions in a period to the training slots available. Constraints (1.9) restrict upward swings in accession rates.

c. Promotions

Constraint sets (1.10) and (1.11) constrain promotion. Constraints (1.10) allow the total number of promotions to fall between upper and lower bounds by rank and period. Constraints (1.11) restrict unpenalized promotion rates.

d. Force Growth

Constraints (1.12) limit the total number of soldiers available from the legacy force. Constraints (1.13) restrict the number of soldiers from the legacy force for a given period to a percentage of the number promoted into that rank in the same period.

Constraints (1.14) ensure that the number of recruits in a period falls within a minimum and maximum. Constraints (1.15) prevent large shifts in recruitment.

Constraint sets (1.16) and (1.17) constrain losses. Constraints (1.16) restrict the number of retirements that occur in a period. Constraints (1.17) control the rate at which soldiers ETS from the ANA.

e. Force Sizing

Constraints (1.18) constrain the size of the force to an upper and lower limit. Constraints (1.19) constrain the ratio of each rank to the total force.

3. Elastic Implementation

SFGM incorporates elastic variables to allow for flexibility in growing the ANA. Elastic variables allow for violation of constraints at a penalty per unit violation. We present the penalties, elastic variables (with constraint identification), and their incorporation in the objective function.

a. Penalty Parameters

$pexisting_{r,p}$	penalty for using legacy soldiers of rank *r* in period *p*
$pvar_r$	Penalty for varying from the desired rank ratio for soldiers of rank *r*
$prec_r$	penalty for exceeding the desired number of recruits for soldiers of rank *r*
$pacc_r$	penalty for exceeding the desired accession rate for soldiers of rank *r*
$psA_{r,p}$	penalty for varying accession from the smoothing function for rank *r* in period *p*
$psP_{r,p}$	penalty for varying promotion from the smoothing function for rank *r* in period *p*
$psR_{r,p}$	penalty for varying recruitment from the smoothing function for rank *r* in period *p*
$pgoal$	penalty for violating desired Target Values
$ppro_{r,p,t}$	penalty for promotions in rank *r* in period *p* TIG *t* occuring below minimum TIG or above the desired TIG

b. ***Elastic Variables***

ACC_p	number of officers over the maximum number of accessions allowed in a period *p* [soldiers] Equation (1.7)
$DA_{r,p}$	number of accession of rank *r* at the beginning of period *p* over the maximum allowed [soldiers] Equation (1.8)
$DGU_{r,p}$	number of rank *r* over the desired level at the beginning of period *p* [soldiers] Equation (1.18)
$DGD_{r,p}$	number of rank *r* below the desired level at the beginning of period *p* [soldiers] Equation (1.18)
$DREC_{r,p}$	number of recruits of rank *r* over the maximum allowed at the beginning of period *p* [soldiers] Equation (1.14)
$DOWN_{r,p}$	number of soldiers rank *r* below the required ratio at the beginning of period *p* [soldiers] Equation (1.19)
$UP_{r,p}$	number of soldiers rank *r* above the required rank ratio at the beginning of period *p* [soldiers] Equation (1.19)

$SA_{r,p}$ — number of accessions for soldiers in rank *r* over period *p* greater than or less than the minimum or maximum number of accessions from the previous period[soldiers]

Equation (1.9)

$SP_{r,p}$ — number of promotions for soldiers in rank *r* over period *p* greater than or less than the minimum or maximum number of promotions from the previous period[soldiers]

Equation (1.11)

$SR_{r,p}$ — number of recruits in rank *r* over period *p* greater than or less than the minimum or maximum number of recruits from the previous period[soldiers]

Equation (1.15)

c. *Objective Function*

We now present the objective function that explicitly shows the elastic variables and penalties.

$$\min \sum_r \sum_p \alpha_{r,p} \begin{pmatrix} pvar_r \left(UP_{r,p} + DOWN_{r,p}\right) \\ + prec_{r,p} * DREC_{r,p} \\ + \sum_t ppro_{r,p,t} PRO_{r,p,t} \\ + pacc_{r,p} \left(DA_{r,p} + ACC_p\right) \\ + pexisting_{r,p} HDO_{r,p} \\ + pgoal(DGU_{r,p} + DGD_{r,p}) \\ + psA_{r,p} SA_{r,p} \\ + psP_{r,p} SP_{r,p} \\ + psR_{r,p} SR_{r,p} \end{pmatrix} \qquad (1.21)$$

THIS PAGE INTENTIONALLY LEFT BLANK

IV. ANALYSIS OF MODEL RESULTS

We implement SFGM using a 2.4 GHz Dual Core windows based personal computer running the General Algebraic Modeling System (GAMS) (GAMS Development Corporation 2001) and the free, open source Computational Infrastructure for Operations Research Branch and Cut (CBC) solver (Forrest 2005). GAMS is interfaced through Microsoft Excel, which provides an input platform for data.

We implement the SFGM Dual Equilibrium to solve the growth of the ANA. A typical instance is 100,000 variables, 50,000 constraints with 1.8 million non-zero elements. Total solution time is 10 minutes for a 15-year planning horizon (36 monthly periods followed by 12 annual periods).

A. MODEL IMPLEMENTATION

SFGM uses a number of parameters (Table 1) allowing for manipulation of a planning scenario in many different ways. We define parameters separately for each rank to provide greater control of force management. Each rate in Table is at a monthly level with integer values indicating personnel numbers, such as current and end strength, or specific times such as maximum and minimum TIG. Table 1 provides the baseline values for SFGM from which we draw each succeeding scenario.

MONTHLY PARAMETERS	RANK										
	1	2	3	4	5	6	7	8	9	10	11
Attrition Rate	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0005	0.0005	0.0005	0.0005	0.0005
Accession Rate	0.017	0.017	0.017	0.017	0.017	0	0	0	0	0	0
Current Strength	40635	15410	8155	5584	1900	580	2978	3271	2620	1440	422
End Strength	65604	24880	13166	9015	3067	936	4808	5281	4231	2325	682
Rank Ratio	0.490	0.186	0.098	0.067	0.023	0.007	0.036	0.039	0.032	0.017	0.005
Legacy Force	0	0	0	0	0	0	500	700	500	400	50
Minimum Promotion Rate	0.005	0.005	0.005	0.005	0.005	N/A	0.005	0.005	0.005	0.005	N/A
Maximum Promotion Rate	0.03	0.03	0.03	0.03	0.03	N/A	0.03	0.03	0.03	0.03	N/A
Minimum TIG (Promotion)	24	24	36	36	48	0	24	36	36	48	0
Promotion Point	30	30	40	40	50	0	30	40	40	50	0
Maximum TIG	120	120	120	120	120	120	120	120	120	120	120
Minimum Recruitment	1400	N/A	N/A	N/A	N/A	N/A	20	N/A	N/A	N/A	N/A
Maximum Recruitment	2400	N/A	N/A	N/A	N/A	N/A	50	N/A	N/A	N/A	N/A
Retirements	0	0	0	0.005	0.01	0.01	0	0	0.005	0.01	0.01
Minimun Reenlistment Rate	0.2	0.2	0.2	0.2	0.3	1	0.6	0.6	0.7	0.7	1
Maximum Reenlistment Rate	0.5	0.57	0.57	0.57	0.57	1	0.7	0.7	0.8	0.9	1

Table 1. SFGM Parameters

Table 1 shows the default parameters for SFGM. We display all parameters in monthly values.

1. Data

a. Attrition Rates

SFGM attrites the force according to a fixed rate. We set this to a monthly enlisted attrition rate of 0.13 percent (annual 1.56 percent) and monthly officer attrition rate of 0.05 percent (annual 0.6 percent)(Radin 2009).

b. Accession Rates

There is no specific information regarding historic accession rates (number of enlisted soldiers available for accession in a given period); we set the upper bound to 1.7 percent (annual 20 percent) of the available enlisted corps, with a lower bound of 0.

c. Current Strength

The current strength of the force is the starting point from which the force is grown. We initialize the force strength at the current manning level of the ANA, 82,000. The total number by rank in the current force is available (Table 1) but the distribution of these soldiers in TIG is not as well defined. SFGM assumes a linear distribution of soldiers across time in grade, with the largest number in TIG 1 to no soldiers beyond seven years TIG because 2002 was the first year of recruitment into the ANA (Jalali 2002).

d. End Strength

The target inventory in SFGM is the stated goal of an ANA end strength of 134,000 (Carden 2009) with its distribution by ranks shown in Table 1. A secondary aspect of target inventory is the rank ratio for the force. We determine rank ratio by dividing the end strength of each rank over the total end strength of the force.

e. Legacy Force

The legacy force (holdovers) is the total number of soldiers available from either militias or the previous army for each rank. There is no complete accounting for the number of former mujahedin or militia fighters, we establish a total upper bound of 8,000 and establish limits by rank (Table 1).

f. Promotion Rates

Promotion rates establish the upper and lower limit on forward advancement for each rank. Promotion becomes more difficult as soldiers move up the organization due to the decreasing size of the force. We establish a monthly lower bound of zero and a monthly upper bound of three percent of each rank for promotions per period. Promotions only occur without penalty for soldiers within a prescribed zone of promotion. Each rank has a lower and upper TIG requirement on promotion and an ideal point, the "promotion point," at which promotions should occur.

Implementation of the promotion point creates a BZ, primary zone, and AZ region for each rank. For example, Table 1 shows that the TIG required before a soldier in Rank 1 is eligible for promotion is between 24 months and 120 months. We use a desired promotion point of 30 months. SFGM allows early promotion of a soldier (TIG of 1 to 29 months) but the promotion incurs a penalty. SFGM allows for a three-year promotion window (TIG between 30 and 66), where there is no penalty for promotion. Finally, a soldier promoted after TIG 66 would incur an increasing penalty the further they are from TIG 66.

g. Recruitment Levels

We bound recruitment levels to the present monthly throughput of the Kabul Military Training Center (KMTC). The maximum number of soldiers produced monthly is 2,400 with a minimum of 1,400. The maximum training throughput annually, with a surge capacity, is 40,000 with an average of 28,000 (Davis 2008). SFGM recruits soldiers only into either Rank 1 (Soldier) or Rank 7 (Lieutenant).

h. Forced Retirement

Retirement allows the force to open higher ranks to prevent stagnation. At most, 1 percent monthly (annually 12 percent) of the officers or NCOs in the highest three ranks are available for forced retirement. A soldier is available for forced retirement at any TIG.

i. Reenlistment Rates

We bound reenlistment rates according to the most current upper and lower observed reenlistments in the ANA. Presently, the upper reenlistment rate for enlisted soldiers is 50 percent with a lower bound of 20 percent Table 1 shows reenlistment rates for NCOs and officers. We set the minimum ETS rate at 50 percent and a maximum rate of 80 percent. We implement this each period by assessing soldiers who meet the ETS parameter for their rank. For enlisted soldiers, the ETS variable exists when they are at a multiple of their 36-month TIG in the monthly portion or three years TIG in the annual portion.

j. Time-in-Grade Maximums

TIG maximum is the absolute longest a soldier may remain in a single rank. Once a soldier reaches that point, they depart the force. SFGM sets an upper bound of 10 years in any one rank.

k. Time Horizons

SFGM incorporates two separate time epochs; monthly periods for the first three critical years of growth and annual periods at the start of the fourth year. These two epochs allow detailed monthly modeling of the short term to manage the rapid growth of the ANA during the first three years and then avoid unnecessary monthly detail beyond year three.

l. Discount Rate

To account for the importance of decisions made in the near term and reduce the impact of decisions late in the planning horizon, we use a discount factor of 0.95 for all computations reported in this chapter. Using a discount rate of 0.95, our decisions at year 14 have a 50 percent weight, meaning a violation of a constraint at year 14 incurs half the cost. We explore the effect of a lower discount factor (such as a discount factor of 0.8 where violations of constraints after the fourth year have a 50 percent weight) and find there is no significant impact on the optimal solutions.

B. TRUNCATION, PRIMAL AND DUAL EQUILIBRIUM

Implementation of a linear program, which seeks to optimize over part of an infinite horizon, can result in end effects as described by Grinold (1983) and Walker (1995). The Truncation ignores all events that occur past its planning horizon, is a relaxation of the infinite horizon, and therefore provides a lower bound. To overcome these effects we develop the Primal and Dual equilibrium. The Primal and Dual equilibrium seek to approximate the infinite horizon that occurs at the end of the finite planning horizon. The Primal equilibrium accomplishes this by fixing the values of the final decision for every period beyond a truncated planning horizon. This structure is a restriction on the infinite horizon problem, resulting in the Primal equilibrium providing an upper bound for a given period. Appendix A provides the mathematical formulation of the Primal equilibrium. The Dual equilibrium aggregates the decisions made beyond a truncated planning horizon into an infinite sum and is a relaxation. Appendix B provides the mathematical formulation of the Dual equilibrium and an example of the infinite sum.

Implementation of the three models provides a basis for determining the horizon that provides results not influenced by end effects. Due to the shift from a monthly to yearly structure, we evaluate the three models after all variables are present after period 40. Figure 4 shows the convergence of the Truncation, Primal, and Dual equilibriums over 120 periods (40 to 160). From Figure 4 we identify which function closes on the optimal infinite horizon objective function at the earliest period. Here, the Dual equilibrium closes to within 0.2 percent of the optimal value at period 50. The Primal

Equilibrium reaches three percent of the optimal value at period 160, while at period 100 the Truncation is within one percent of the optimal value. From this information, the Dual equilibrium is the preferred model for SFGM when executed out to period 50. The results provided over 50 periods from the Dual equilibrium provide information for 48 planning periods. Periods 49 and 50 contain variables that represent an infinite sum.

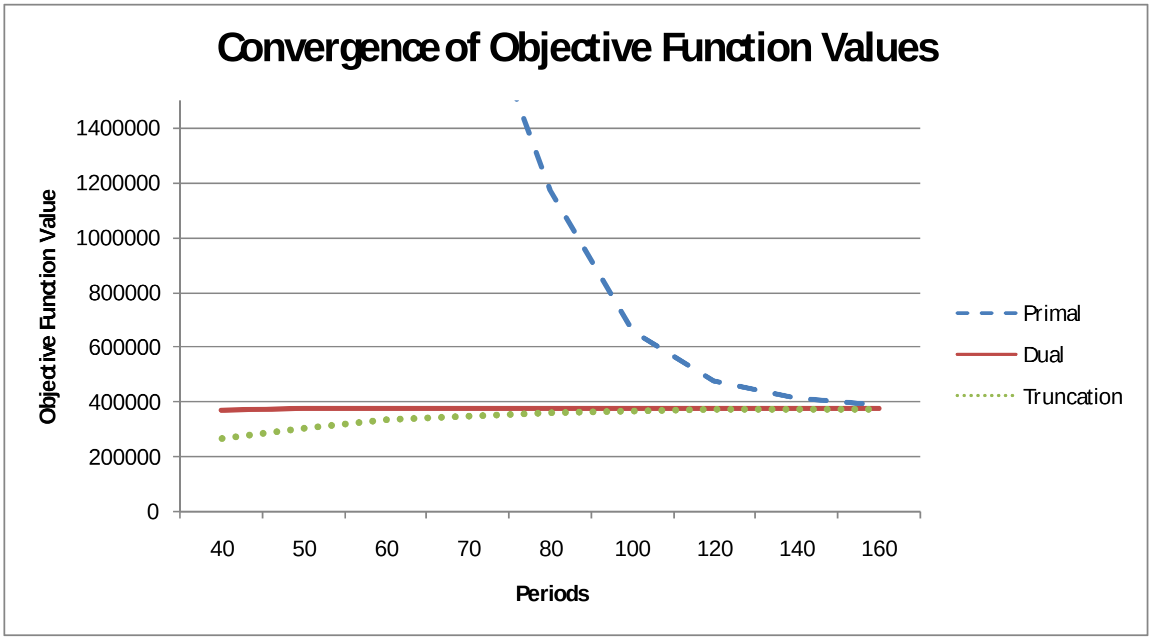

Figure 4. Convergence of the Objective Function Values for the Truncation, Primal and Dual Equilibriums

Convergence of the objective function values for the Truncation, Primal and Dual equilibrium. We see that the Primal equilibrium is converging from infinity and at period 160 it is within three percent of the Dual equilibrium. The truncation begins rising to converge with the Dual equilibrium and closes to within one percent at Period 100. The Dual equilibrium is within 0.2 percent of its optimal value at period 50.

C. SCENARIOS AND RESULTS

We develop five primary scenarios to demonstrate SFGM. Scenario 1 grows the ANA using the current recruitment capabilities, attrition rates, and legacy force. Scenario 2 restricts lower and upper bounds on promotions based on the means and standard deviations found in Scenario 1. Scenario 3 excludes the legacy force. Scenario 4 improves enlisted reenlistment rates from 50 percent to 70 percent at the beginning of the fourth year. Scenario 5 includes no enlisted accessions and grows the officer corps with only recruitment and the legacy force.

1. Overview of Results

We present an overview of the results for each scenario with subsequent sections providing more details. Scenario 1 indicates that maximum recruiting is necessary for two years to establish the enlisted corps promotion base. The rapid promotions that accompany this growth are evident later as Rank 3 (Staff Sergeant) and Rank 4 (Sergeant First Class) experience sustained decreasing promotion rates. In general, the promotion rates during the three-year buildup phase have large variations and would be nearly impractical to implement. Despite this, both the enlisted and officer corps reach the desired end strength in 24 months.

Scenario 2 indicates that greater control over promotion rates slows the total force growth by four months (the ANA meets its desired end strength in 28 months). Scenario 2 also indicates that the current size of the officer corps does not support long-term stability. The number of officers in Rank 7 (Lieutenants), Rank 8 (Captains), and Rank 9 (Majors) is effectively equal which results in an extremely low separation rate for Ranks 7 and 8 (essentially no voluntary separation at either Rank 7 or Rank 8).

Scenario 3 excludes legacy force soldiers, and we find it takes five years (two more than Scenario 1 to grow the officer corps). An important aspect of this is the slow growth of the senior levels of the officer corps. Inclusion of the legacy force injects officers directly into the three senior ranks of the ANA that reduces rapid promotions within the officer corps and provides additional stability at each rank.

Scenario 4 shows that improvement of retention rates for enlisted soldiers has two significant effects: lowered promotion rates and lowered recruiting needs. Promotion rates decreases because fewer departing soldiers require fewer replacements. We make a similar argument for the resulting reduction in recruitment requirements. Essentially, increased retention rates reduce the recruiting demand on the ANA by approximately 4,000 soldiers annually.

By eliminating enlisted accessions, Scenario 5 shows the necessary increase in officer recruitment that occurs to meet the desired end strength. At the present recruiting rate of 600 officers annually, the ANA does not meet its end strength without enlisted accessions. By reducing officer promotion rates and increasing the initial recruitment to 2,400 annually for two years and 1200 annually after that point, the ANA reaches its officer end strength in 28 months.

2. Meeting Current ANA End Strength Requirements

Our first scenario considers the growth of the ANA from its current strength of 82,000 soldiers to its stated goal of 134,000 soldiers. We establish our parameters based on the most current information available regarding the ANA (Table 1). We report 48 (15 years) of 50 periods, the first 36 periods are monthly, followed by 12 annual. We omit the last two periods as they contain variables that represent an infinite sum.

a. Analysis of Scenario 1

Analysis of the results provides insight into the challenges facing the growth of the ANA. Given the current recruitment capabilities, the ANA can meet the desired end strength in less than three years. Figure 5 shows the total force exceeds its end strength and slowly descend until it reaches a stable strength at year 10. The enlisted corps achieves its desired strength in slightly less than two years, the officer corps will reach its end strength eight months after the enlisted force. To achieve this, large scale recruitment is necessary during the initial two-year buildup. Figure 6 and Figure 7 indicate the recruitment and accessions necessary to meet and maintiain the desired end strength for the ANA force of 134,000.

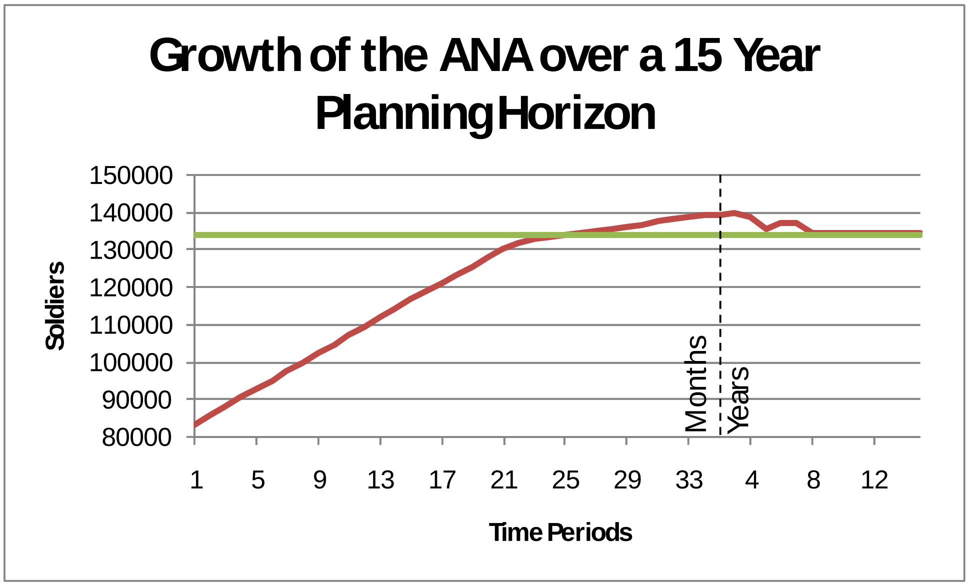

Figure 5. Scenario 1 15-year Growth of the ANA

Figure 5 shows the growth of the ANA over a 15-year planning horizon. We see that the force achieves the desired enlisted strength in 23 months with the officers reaching their end strength in 31 months.

Figure 5 indicates that the initial growth of the ANA enlisted corps requires KMTC to operate at its maximum of 2,400 recruits per month for two years, followed by approximately 18,000 soldiers annually. Figure 5 provides insight into the recruitment needs to effectively grow the officer corps. Recruitment for the officer corps remains at its maximum level throughout the planning horizon. During the initial build up, the recruitment levels are high but fail to provide a sufficient number of officers for reasonable growth. To fill this short fall, enlisted accessions during the initial year of growth are approximately double the number of officers recruited. Accessions remain high as the officer corps stabilizes, in the annual period we see that enlisted accessions remain near 500 annually.

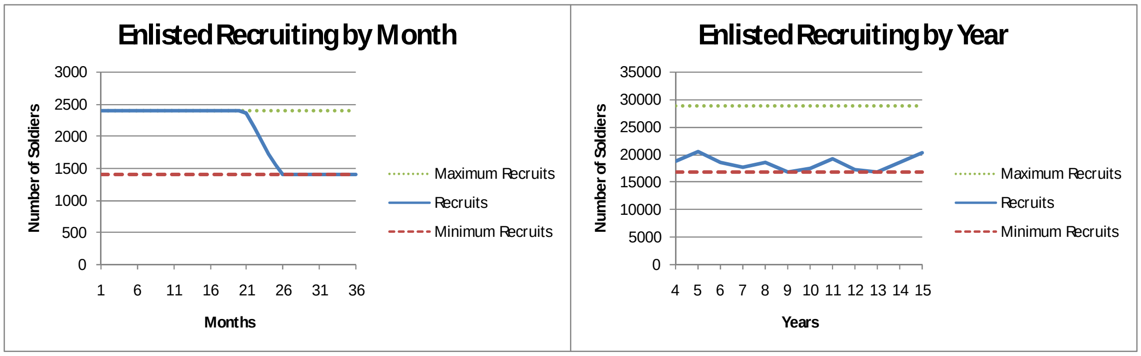

Figure 6. Scenario 1 Enlisted Recruiting Requirements

Figure 6 shows the recruiting needs for the ANA during the monthly build up (years 1 to 3) and the annual periods (years 4 to 15) following. We see that during the initial two years, the ANA requires maximum number of recruits possible to allow for its growth. In the annual period, we see that an average of 18,000 recruits is required.

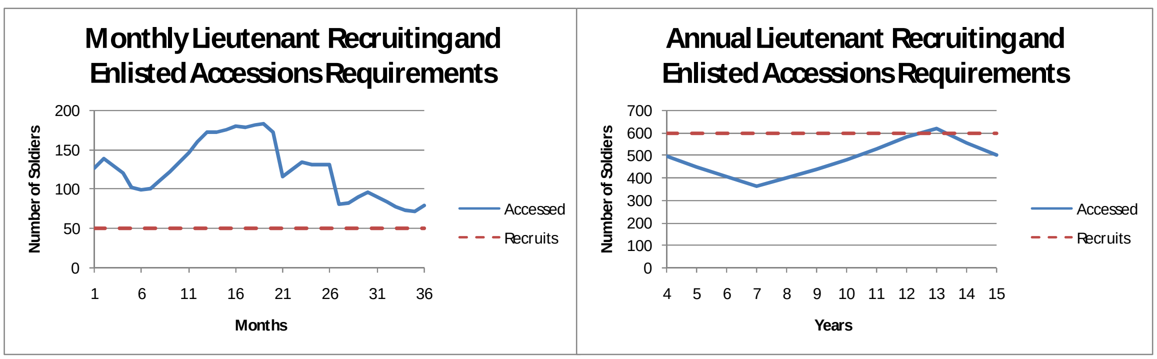

Figure 7. Scenario 1 Lieutenant Recruiting Requirements

Figure 7 shows the monthly and annual recruiting requirements for the ANA officer corps. The recruitment level for both the monthly (years 1 to 3) and annual periods (years 4 to 15) are at their maximum value, while accessions fluctuate significantly during the buildup and reach a high of 600 a month during the annual period.

Promotions are similarly affected by the initial rapid growth of the force. A result of this rapid growth is shifting promotion rates that impacts advancement opportunities into the future. This shift is an example of the impacts of optimal recruiting on organizational growth as reported in Morgan (1979). In this instance, the ANA's rapid growth results in a force with unequal distribution across TIG. The heavy concentration of soldiers in lower level ranks reduces promotion opportunities as the force ages at the same rate. The rapid promotions create a manpower bulge that slowly progresses through each rank limiting the number of promotions in and out of each rank. We see this specifically in Ranks 3 and 4 in the annual period.

Here we consider the percentage of soldiers promoted in one rank with regard to the total number of soldiers in that rank. Figure 8 shows the promotion rates for Ranks 1 through 5 over the planning horizon; all display similar patterns during the initial buildup as the force attempts to grow to meet the manning and force ratio requirements. For soldiers in Rank 1 (Soldier), 2 (Sergeant), and 3 (SSG) a large surge in promotions with rates up to three percent per month during the first year preceeds a lull as the population of soldiers availabe for promotion is exhausted and promotion rates fall to between one percent and 0.5 percent per month. Rank 4 and 5 (Master Sergeant) promotion rates grow slowly during the initial monthly buildup and then slowly decrease to 0.5 percent. In the annual period, Ranks 3 and 4 show decreasing promotion rates; from 17 percent to 10 percent for Rank 3 and 11 percent to six percent for Rank 4. Ranks 1, 2 and 5 show relativley stable promotion rates throughout the annual period with Rank 1 approximately seven percent, Rank 2's promotion rate is stable at 12 percent, and Rank 5 is stable at 0.6 percent. This is a direct result of the large growth in the previous three years. The two Ranks with decreasing promotion rates, Ranks 3 and 4, received a large number of promotions in the first two years from Ranks 2 and 3.

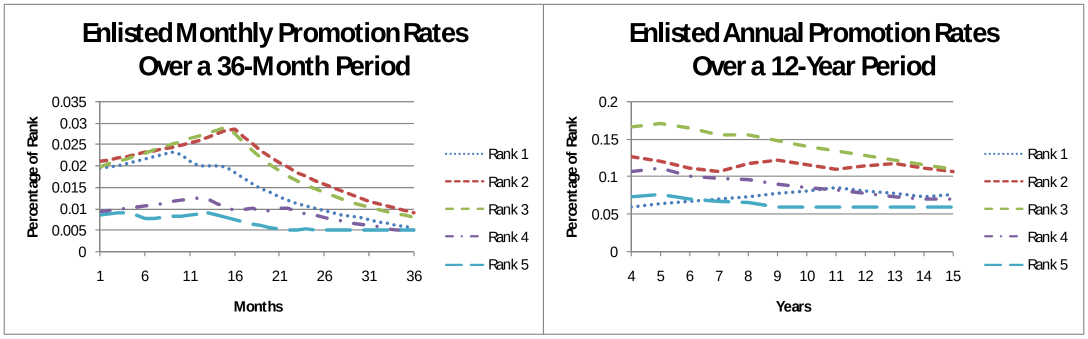

Figure 8. Scenario 1 Enlisted Promotion Rates

Figure 8 shows the monthly and annual promotion rates for enlisted soldiers. In the monthly period (years 1 to 3), we see an increase in promotion in all Ranks up to month 16 then a decline to reach their steady state values. In the annual period (years 4 to 15), Ranks 3 and 4 are the only Ranks that experience a sustained decrease in promotions.

Figure 9 indicates that officer promotion rates start at high levels, drop at one year and then rise again at two years before dropping to their annual average. Rank 7 begins with promotions rates at their maximum level of three percent per month and experiences a sharp drop in promotions at the 28-month mark with a promotion rate of two percent per month. This drop corresponds to exhausting the population of officers available for promotion at the end of year two. Rank 10 shows a constant promotion rate of 0.5 percent throughout the monthly period.

There is no long-term impact on officer promotion rates following the initial three year build up. Due to the size of the officer corps and the rate of promotions, the force is able to maintain a constant promotion rate after the initial buildup. Only Rank 7 shows an increase in promotion rates over time as a result of a larger number of officers reaching their ETS or RCP beginning in year ten.

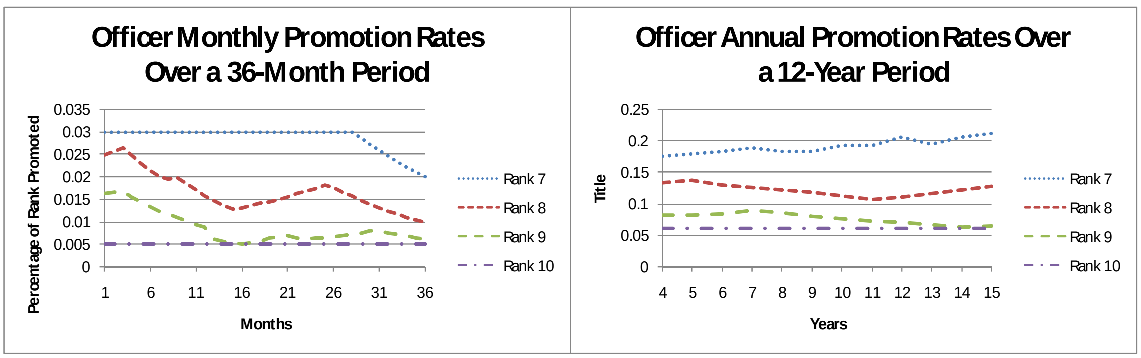

Figure 9. Scenario 1 Officer Promotion Rates

> Figure 9 show the officer promotion rates over the monthly (years 1 to 3) and annual periods (years 4 to 15). During the monthly period, Rank 7 shows constant maximum promotions for a year and a half, while Ranks 2 and 3 show high promotion rates that decline, rise, and decline again. This cycle is a function of the officer corps promotion pool being refilled with the officers that entered the prior year. The annual period shows that the officer promotion rates are effectively constant with only Rank 1 showing sustained growth. Rank 10 retains a constant promotion rate throughout the planning horizon.

These results reflect an assumed rate of reenlistment of 50 percent for enlisted and 57 percent for NCOs. These values are the average of the current ANA reenlistment rates. ETS, retirement, and TIG separation in year one through ten result in an annual loss of approximately 20,000 soldiers across the Force (Figure 10). The highest separation occurs at Rank 1, approximately 12,000 soldiers departing the service annually with a high in year four of 16,000 and low in year nine of 10,000. Both Ranks 5 and 6 show similarly high separation rates that are at 23 percent in year four and drop to approximately 15 percent by year 15. The remaining Ranks 2, 3, and 4 oscillate from years four to eight before reaching a sustainable rate at approximately year 13.

The officer corps also experiences varying separation but not to the same extent as the enlisted ranks. Rank 11 maintains a steady, high separation rate at 20 percent. As the officer corps is a much smaller pool with a much higher reenlistment rate than the enlisted corps, progress through the ranks is only achieved by constant turnover at the highest rank. The remaining officer ranks vary between 0 percent and 10 percent separation initially and settle to approximately 5 percent.

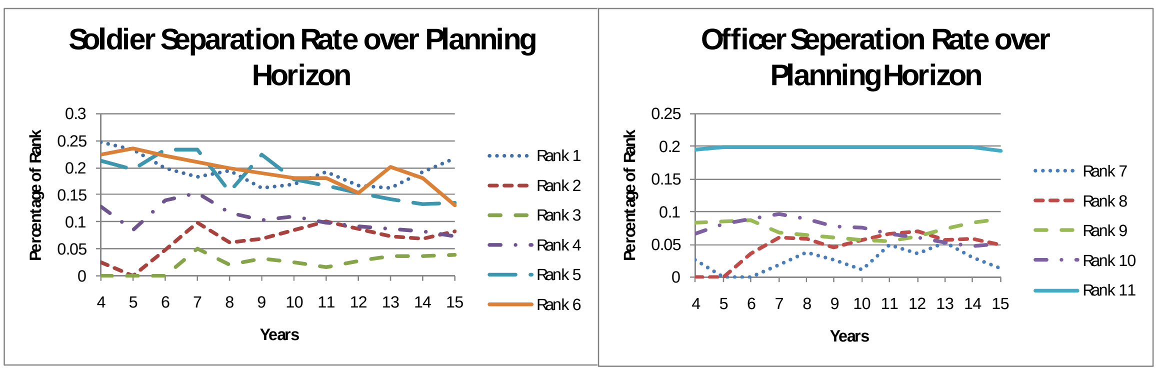

Figure 10. Scenario 1 Annual Separation Rate

Figure 10 show the separation rates for officers and enlisted from planning year four to planning year 15. For the enlisted we see a relatively unstable separation rate for the initial 10 years of the planning horizon before the rates reach the steady state rates. The officer corps reaches its steady state in year seven with only Rank 7 showing oscillations.

b. Conclusions for Scenario 1

SFGM indicates that it is possible to achieve the desired end strength, with the appropriate rank distribution, in under three years. To reach this goal, KMTC must provide 29,000 soldiers annually for two years and approximately 18,000 to 20,000 annually afterwards. Initial results indicate that promotion rates must fluctuate greatly in the first three years to allow for appropriate force growth over the long term. There is minimal impact on long-term promotion potential across the force with one exception in Rank 3, which experiences a 7 percent drop over the planning horizon. We expect separation rates for officers and enlisted to stabilize by year ten as the ANA reaches a steady state operation.

3. Increasing the Minimum Promotion Rate

Scenario 1 shows that large swings in promotion rates are necessary to reach the desired end strength for the ANA as quickly as possible. Such variable promotion rates are difficult to implement effectively in an organization and a more consistent promotion policy is preferable. We establish upper and lower limits based on the Scenario 1 optimal promotion rates. Using the promotions rates from the 36 monthly periods, we find the mean and standard deviation for each rank and set the lower (upper) bound to the mean minus (plus) one standard deviation. For example, the monthly promotion mean for Rank 1 is 0.0153 with a standard deviation of 0.006. We subtract the standard deviation from the mean and find a lower promotion bound of 0.0093. We extend this to all ranks as shown in Table 2.

Promotion Rates												
Period		Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8	Rank 9	Rank 10	Rank 11
Monthly	Minimum	0.0093	0.0156	0.0138	0.0074	0.0046	0.0000	0.0272	0.0113	0.0035	0.0050	0.0000
	Maximum	0.0213	0.0276	0.0271	0.0120	0.0079	0.0000	0.0328	0.0200	0.0106	0.0050	0.0000
Annual	Minimum	0.1115	0.1877	0.1651	0.0893	0.0554	0.0000	0.3263	0.1355	0.0425	0.0600	0.0000
	Maximum	0.2551	0.3313	0.3247	0.1434	0.0951	0.0000	0.3937	0.2403	0.1275	0.0600	0.0000

Table 2. Scenario 2 Promotion Rates

Table 2 provides the monthly (years 1 to 3) and annual (years 4 to 15) promotion rates established for Scenario 2. Each minimum is the mean promotion rate for that rank minus its standard deviation from the results of Scenario 1, while the maximum is its mean plus its standard deviation.

a. Analysis of Scenario 2

As expected, establishing controlled lower and upper bounds stabilized the promotion rates for each Rank in the monthly and annual period. Separation rates shifted Figure 11 to correspond with the stable promotion rates and present a realistic expectation of annual losses. Of most concern is the expected separation rate for Rank 7 of 0 percent. One possible explanation is the similar size of Rank 7 and Rank 8. Because Rank 8 is slightly larger than Rank 7, Rank 7 must maintain its maximum level of promotion to support the growth of the total force and cannot suffer un-programmed separation. The growth of the officer corps is rapid and results in the ANA achieving its desired end strength in approximately 28 months (four more months than Scenario 1) as shown in Figure 12.

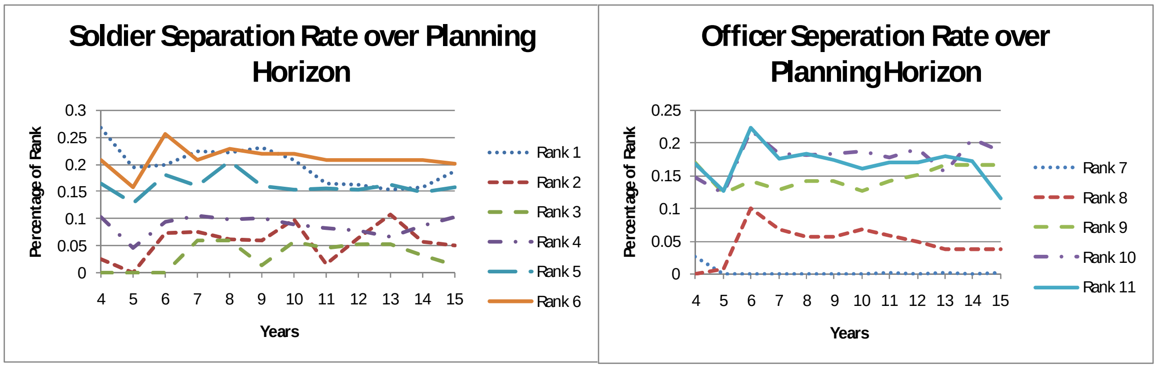

Figure 11. Scenario 2 Enlisted and Officer Separation

Figure 11 shows the expected separation rates for both enlisted and officers. Unlike Scenario 1 we see stabilization in separation for both enlisted and officers beginning in year seven. These separation rates are acceptable showing high separation for the entry-level soldiers and soldiers at the senior levels of the organization with low separation in the mid level ranks. The officer corps shows a different type of separation with no separation for Rank 7 and high separation for Ranks 9 –11.

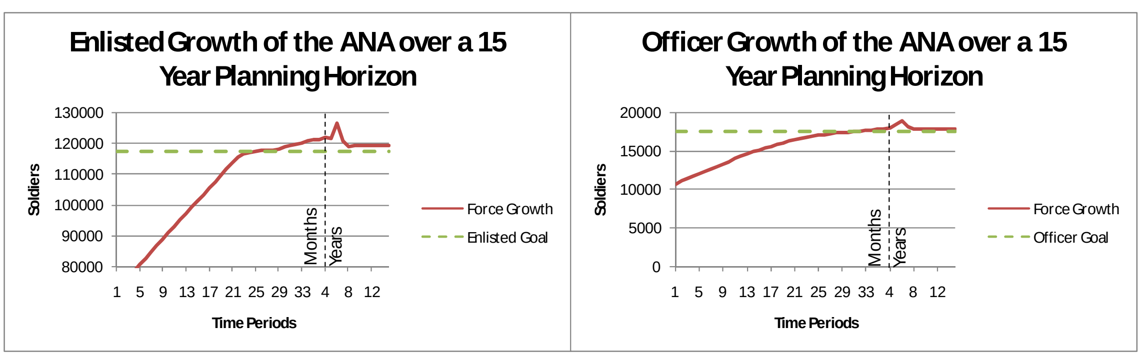

Figure 12. Scenario 2 Force Growth

Figure 12 shows the growth of the enlisted and officer corps of the ANA over the planning horizon. The enlisted force reaches end strength at approximately 20 months while the officer's reach their end strength at approximately 28 months.

b. Conclusion for Scenario 2

Close management of promotions controls the separation that occurs within the force and allows for sustained growth. Present force structure requires an unsustainable separation rate for Rank 7 (0 percent separation from un-programmed losses is unlikely to occur and suggest that a larger force structure is necessary for Rank 7).

4 Exclusion of the Legacy Force

Both Scenario 1 and Scenario 2 include the use of the Legacy Force for the officer corps. We build Scenario 3 from the promotion rates determined in Scenario 2 and exclude legacy force officers during its growth.

a. Analysis of Scenario 3

Figure 13 shows the impact that exclusion has on the growth of the ANA. Exclusion of the legacy force results in higher enlisted accessions to sustain the officer corps growth. Despite the increase in enlisted accessions, we see that the officer corps growth slows and reaches the desired end strength at year five. This is an addition of two years onto the growth of the ANA.

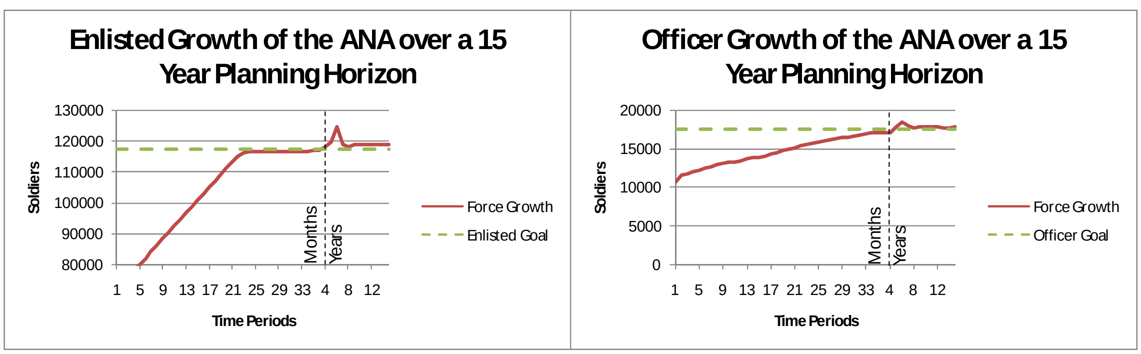

Figure 13. Scenario 3 ANA Growth

Figure 13 shows the growth of the officer and enlisted corps over the course of the planning horizon in Scenario 3. Exclusion of the legacy force results in slower growth of the officer corps with it reaching its end strength in five years (two more years when compared to Scenario 2). The enlisted forces growth is affected slightly as it increases its accessions to meet the new demand for officers.

b. Conclusion for Scenario 3

Exclusion of the legacy force results in an increased draw from the enlisted corps ,and the ANA reaching its desired end strength two years later than with the legacy force.

5. Improvement of Reenlistment Rates after Three Years

Experience in Iraq indicates that reenlistment rates improve as the army establishes itself and becomes a more effective and legitimate organization. We extend this to the ANA following the third year of its build up. Using Scenario 2 as our basis, we set reenlistment rates with a minimum reenlistment of 60 percent of the force and a maximum of 90 percent of the force. Table 3 shows the reenlistment rates for each rank.

Reenlistment Rates											
Period	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8	Rank 9	Rank 10	Rank 11
Minimum	0.60	0.70	0.75	0.80	0.80	1.00	0.60	0.60	0.70	0.70	1.00
Maximum	0.70	0.80	0.90	0.90	0.90	1.00	0.70	0.70	0.80	0.90	1.00

Table 3. Scenario 4 Reenlistment Rates

a. Analysis of Scenario 4

Increasing enlisted reenlistment rates at year three results in reductions in both promotions and recruitment. Promotion rates for most ranks fall below 10 percent annually due to the high number of soldiers remaining in the force. As shown in Figure 14 only Rank 3 maintains its original promotion rate of 16 percent. The large number of soldiers remaining in the force affects the number of annual recruits necessary to sustain the force. Figure 15 shows the change in recruiting requirements for the ANA over the planning horizon. Here we see that the total number of recruits falls from 19,000 annually to 15,000 annually. This reduction results in a monthly recruitment goal of approximately 1,250 soldiers.

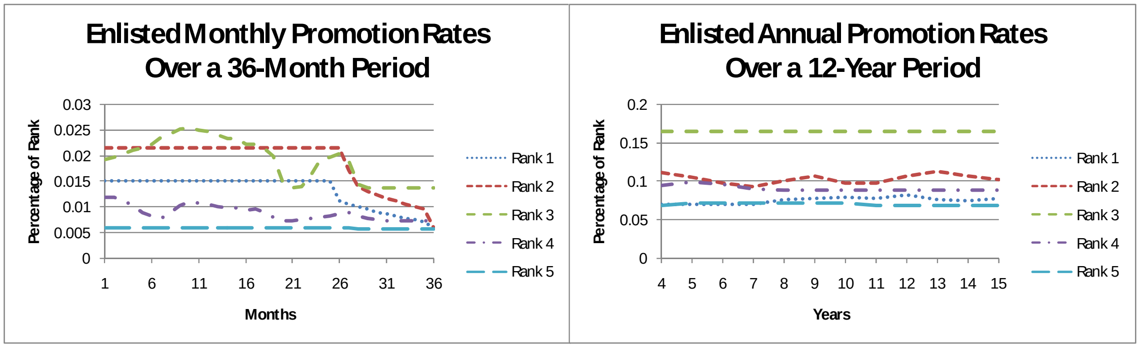

Figure 14. Scenario 4 Enlisted Promotion Rates

Figure 14 shows the impact that the increased retention has on promotion rates for enlisted soldiers. While the buildup period is similar to previous results, the annual period shows a downward shift in all ranks except Rank 3. A possible explanation for this is that as more soldiers remain in a rank they reduce the number of positions the lower rank can fill.

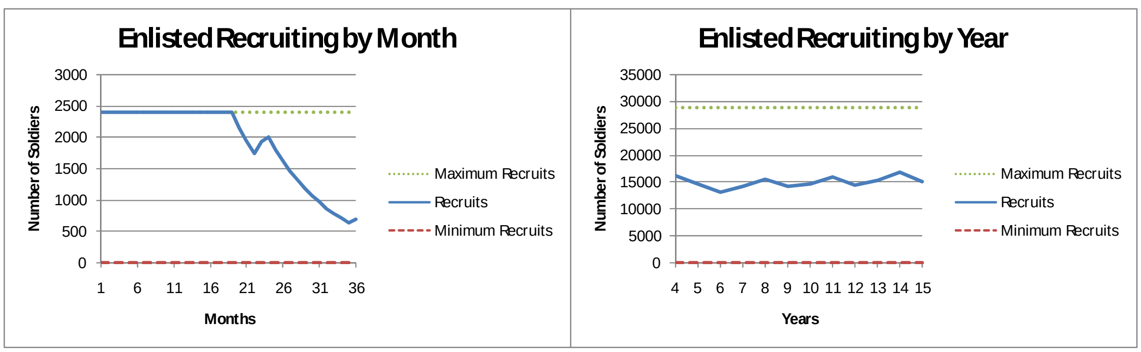

Figure 15. Scenario 4 Enlisted Recruiting

Figure 15 shows the impact of increased retention on recruiting. During the initial build-up there is no effect, in the annual period recruitment needs drop by up to 3,000 soldiers annually.

b. Conclusion for Scenario 4

The ability to improve reenlistment has two major effects: lowering promotion rates and decreasing recruitment needs. While we assume that the lower promotion rates will not have an effect on reenlistment, this may not be the case as the soldiers TIG increase with limited promotion potential.

6. No Enlisted Accessions

We now evaluate the impact of enlisted accessions on the growth of the officer corps. We return to Scenario 2, which bounds promotions and incorporates the Legacy force in the growth of the ANA. We lower the promotion bounds for the officer corps to 0.005 percent and prevent the accession of enlisted soldiers. We set the recruitment level for officers to a monthly minimum of 20 and a maximum of 50 with the ability to exceed this maximum value if necessary.

a. Analysis of Scenario 5

Figure 16 shows the growth of the ANA subject to these new constraints. While the growth of the enlisted corps is unchanged, we see a dramatic change in the growth of the officer corps. The officer corps fails to reach its desired end strength despite inclusion of the legacy force with its current limit of 600 recruits annually. To meet the desired end strength, officer recruiting must exceed its maximum limit by approximately 200 per month for the first three years followed by an annual recruiting requirement of 1,200 officers. Figure 17 shows the recruiting needs for the force over the planning horizon. Figure 18 shows the impact this increased recruiting has on the time to reach the desired end strength. Similar to Scenario 2, the officer corps reaches its end strength at 28 months with maximum recruiting.

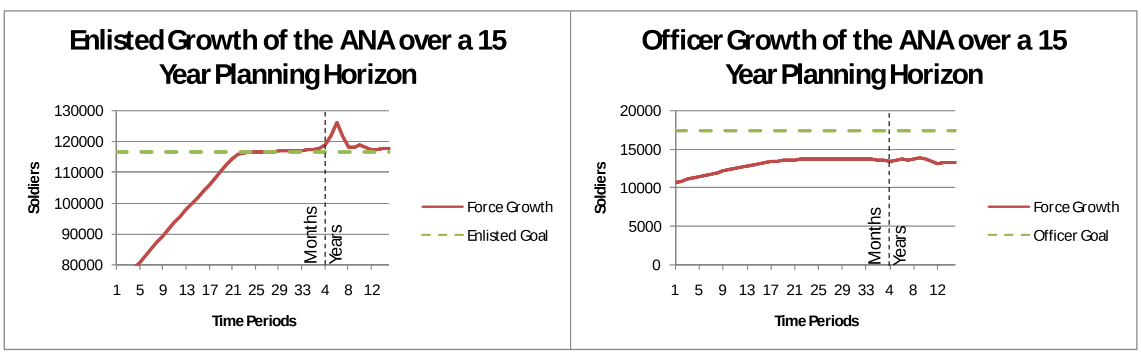

Figure 16. Scenario 5 Growth of the ANA 1

Figure 16 shows the impact of no enlisted accessions on the growth of the officer corps with the current officer recruitment bounds. We see that the officer corps is unable to meet its end strength requirements.

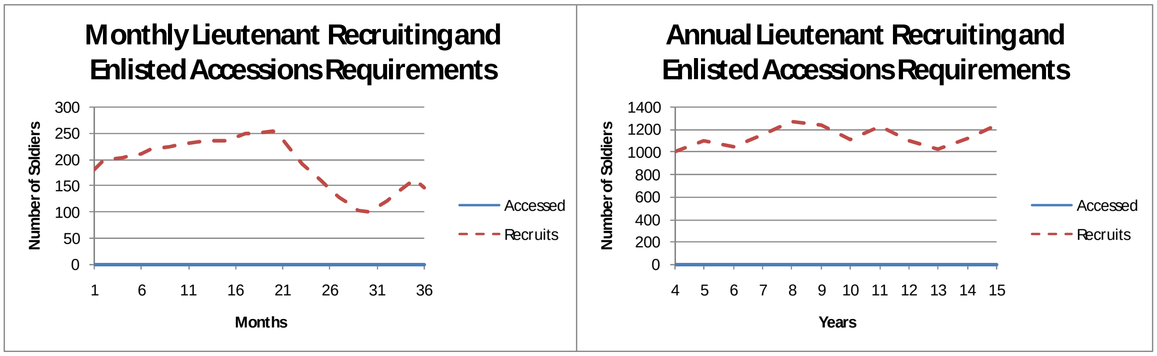

Figure 17. Scenario 5 Officer Recruiting

Figure 17 shows the recruiting requirements for the ANA to meet its officer strength needs. Approximately 200 officers are necessary monthly for the officer corps to reach a stable strength, followed by an annual recruiting requirement of 1,200 officers. These recruits are separate from the legacy force officers that join the force monthly during the ANA's initial growth.

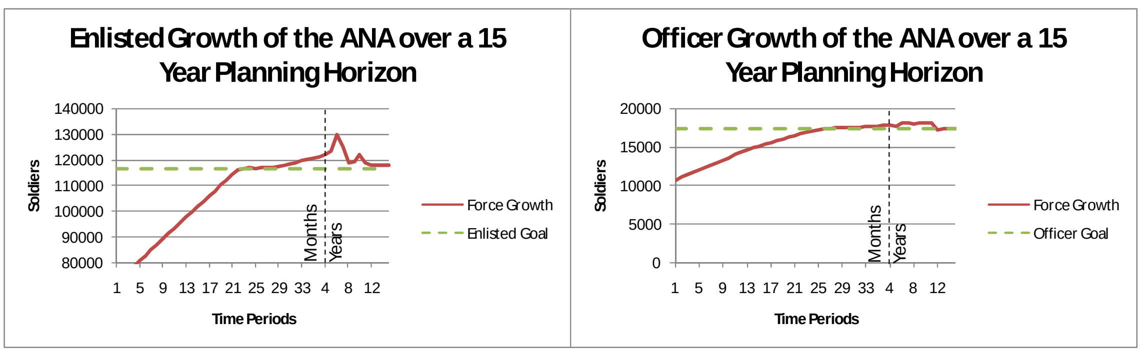

Figure 18. Scenario 5 Growth of the ANA 2

Figure 18 shows the growth of the officer and enlisted corps with no enlisted accessions and double the officer recruiting rates. The officer corps reaches its end strength in 28 months while the enlisted corps reaches its end strength in 20 months.

b. Conclusions for Scenario 5

Enlisted accessions are critical to the growth of the ANA during this initial buildup and into its steady state operations. In order to reduce dependence on enlisted accessions, the ANA must recruit 2,400 officers for two years followed by a steady state recruitment of approximately 1,200 annually.

V. CONCLUSIONS

Manpower modeling plays a significant role in the growth and management of today's militaries. Unfortunately, existing models do not properly capture the challenges facing the growth of recently established indigenous security forces. SFGM is the first linear program specifically designed to grow an indigenous security force. SFGM's ability to provide high fidelity prescriptions over a short-term planning horizon and indications of the impact that those prescriptions have over an unknown (infinite) horizon is unique. We demonstrate SFGM for the current effort to grow the Afghan National Army (ANA).

SFGM simultaneously prescribes enlisted accessions, legacy force inclusions, reenlistment levels, and the resulting separation information, to provide a quantitative assessment of the impact of manpower decisions. We find it requires at least 24 months for the ANA to reach its desired end strength under the current recruiting capabilities even with the inclusion of the legacy force, accessions of enlisted soldiers to the officer corps, and large fluctuations in promotion rates. It requires four additional months when eliminating the large fluctuations. We also find that the ANA must exceed the desired strength by up to 4,000 soldiers in order to absorb losses that occur due to the large-scale enlistment in the first three years. The lieutenant end strength must also increase above current desired levels to provide a stable promotion base for senior ranks.

SFGM demonstrates that it is capable of providing prescriptions for the growth of a force from an existing level to an end strength through our ANA scenarios. SFGM is also capable of providing prescriptions for generating an army where none exists. The SFGM flexibility allows planners to determine how quickly a force can grow and what policy decisions are necessary to facilitate that growth. SFGM's ability to provide a detailed overview of the entire force's growth, recruitment, promotion, and separation over an unknown (infinite) horizon is a powerful tool in generating new indigenous security forces.

Areas for future research involve managing the growth of separate branches and the mixture of religious or ethnic factors in the force. With regard to branches, there is specific interest in the competition that occurs at the recruitment base and its impact on the organization meeting its end requirements. Finally, the inclusion of religious or ethnic groups and their impact on growth, recruitment, and promotion as the force attempts to manage equal representation as it reaches its end strength.

APPENDIX A: PRIMAL EQUILIBRIUM FORMULATION

The primal equilibrium asserts that the model reaches its final equilibrium at period P. In SFGM both T and P are locked as the staircase structure of the flow balance equation incorporates the two time indices. This results in $X_{r,p,t} = X_{r,P,T}$ for all $p \geq P$. For each constraint that we derive from the previous period and TIG, we add a constraint that replicates it over an infinite horizon. For example:

$$\begin{aligned} X_{r,p,t} &= (1 - Attrite) X_{r,p-1,t-1} \\ X_{r,p+1,t+1} &= (1 - Attrite) \overline{X}_{r,P,T} \\ \overline{X}_{r,P,T} &= (1 - Attrite) \overline{X}_{r,P,T} \end{aligned}$$

The result is that all future decisions assume the value from period P. To capture this infinite sum of penalties, we modify the objective function to provide the desired weight to the decisions over the infinite horizon (Grinold 1983). We accomplish this by knowing that:

$$\sum_{i=T}^{\infty} \alpha^i = \frac{\alpha^T}{1-\alpha}$$

Which results in the primal objective function:

$$\min \sum_r \sum_{p<P} \alpha_{r,p} \times Penalties + \sum_r \frac{\alpha_{r,P}}{1-\alpha} \times Penalties$$

Where the denominator, $1-\alpha$, is the discount rate for the infinite horizon, in SFGM this is $(1-0.95) = 0.05$. This results in a large penalty in the final period for failing to achieve a sustainable equilibrium. We now present the primal equilibrium model.

1. Balance of Flow Equations

$$X_{r,p,t} = \sum_{r'<6} \sum_{t'} ACS_{r',p-1,t'-1} + HDO_{r,p} + REC_{r,p} \qquad \forall\ r = 7, 1 < p < P, t = 1 \qquad (1.3)$$

$$X_{r,P,T} = \sum_{r'<6} ACS_{r',P,T} + REC_{r,P} \qquad \forall\ r = 7,\ p \geq P, t = 1 \quad (1.3P)$$

$$\begin{aligned} X_{r,p,t} = & \left(1 - attrite_{r,p}\right) \sum_{t' \in epoch_{p,t}} X_{r,p-1,t'-1} \\ & - ACS_{r,p-1,t-1} - PRO_{r,p-1,t-1}\Big|_{t \in pw_{r,p}} \\ & - ETS_{r,p-1,t-1}\Big|_{t \in rw_{r,p}} - RET_{r,p-1,t-1} \end{aligned} \quad (1.4)$$

$$\forall\ r \neq 6 \text{ or } 11, p > 1, 1 < t \leq \overline{tig}_{r,p}$$

$$\begin{aligned} X_{r,P,T} = & \left(1 - attrite_{r,P}\right) \sum_{t' \in epoch_{p,t}} X_{r,P,T'} \\ & - ACS_{r,P,T} - PRO_{r,P,T}\Big|_{t \in pw_{r,p}} \\ & - ETS_{r,P,T}\Big|_{t \in rw_{r,p}} - RET_{r,P,T} \end{aligned} \quad (1.4P)$$

$$\forall\ r \neq 6 \text{ or } 11, p \geq P, 1 < t \leq \overline{tig}_{r,p}$$

$$\begin{aligned} X_{r,p,t} = & \left(1 - attrite_{r,p}\right) \sum_{t' \in epoch_{p,t}} X_{r,p-1,t'-1} \\ & - RET_{r,p-1,t-1} \qquad \forall\ r = 6 \text{ or } 11, 1 < p < P, 1 < t \leq \overline{tig}_{r,p} \end{aligned} \quad (1.5)$$

$$\begin{aligned} X_{r,P,T} = & \left(1 - attrite_{r,P}\right) \sum_{t' \in epoch_{p,t}} X_{r,P,T'} \\ & - RET_{r,P,T} \qquad \forall\ r = 6 \text{ or } 11, p \geq P, 1 < t \leq \overline{tig}_{r,p} \end{aligned} \quad (1.5P)$$

$$X_{r,p,t} = \sum_{t' \in pw_{r,p}} PRO_{r-1,p-1,t'-1} + HDO_{r,p} \qquad r \neq 1 \text{ or } 7, 1 < p < P, t = 1 \quad (1.6)$$

$$X_{r,P,T} = \sum_{t' \in pw_{r,p}} PRO_{r-1,P,T'} \qquad \forall\ r \neq 1 \text{ or } 7, p \geq P, t = 1 \quad (1.6P)$$

2. Constraints

Constraints (1.1) and (1.2) are included in the primal formulation but not after *P>p*. All remaining constraints for *P>p* are equivalent to the truncation with *p=P*.

3. Objective Function

$$\min \sum_{r} \sum_{p<P} \alpha_{r,p} \times Penalties + \sum_{r} \frac{\alpha_{r,p}}{1-\alpha} \times Penalties \qquad (1.21P)$$

THIS PAGE INTENTIONALLY LEFT BLANK

APPENDIX B. DUAL EQUILIBRIUM FORMULATION

The dual equilibrium aggregates constraints for $p>P$ to represent the penalties accrued for failing to reach equilibrium.

$$\overline{X}_{r,P,t} = \sum_{p=P}^{\infty} (1-\alpha)\alpha^{p-P} X_{r,p,t}$$

First we show that the weight applied $(1-\alpha)\alpha^{t-T}$ sums to 1:

$$\sum_{p=P}^{\infty}(1-\alpha)\alpha^{p-P} \rightarrow (1-\alpha)\sum_{p=P}^{\infty}\alpha^{p-P} \rightarrow (1-\alpha)\frac{1}{1-\alpha} \rightarrow 1$$

And

$$\overline{X}_{r,P,t} = (1-\alpha)A_{r,p-1,t} + \alpha(1-\alpha)A_{r,p,t} + \alpha^2(1-\alpha)A_{r,p+1,t}\cdots$$

$$\overline{X}_{r,P,t} = (1-\alpha)A_{r,p-1,t} + \alpha(1-\alpha)\sum_{p=P}^{\infty}\alpha^{p-P}A_{r,P,t}$$

$$\overline{X}_{r,P,t} = (1-\alpha)A_{r,p-1,t} + \alpha\overline{A}_{r,P,t}$$

1. Balance of Flow Equations

$$X_{r,p,t} = \sum_{r'<6}\sum_{t'} ACS_{r',p-1,t'-1} + HDO_{r,p} + REC_{r,p} \qquad \forall\ r = 7, 1 < p < P, t = 1 \qquad (1.3)$$

$$\begin{aligned}\overline{X}_{r,P,t} &= (1-\alpha)\sum_{r'<6}\sum_{t'} ACS_{r',P-1,t'-1} \\ &+\alpha\sum_{r'<6}\sum_{t'}\overline{ACS}_{r',P,t'-1} + \overline{REC}_{r,P} \qquad \forall\ r = 7, p \geq P, t = 1\end{aligned} \qquad (1.3D)$$

$$
\begin{aligned}
X_{r,p,t} = &\left(1 - attrite_{r,p}\right) \sum_{t' \in epoch_{p,t}} X_{r,p-1,t'-1} \\
&- ACS_{r,p-1,t-1} - PRO_{r,p-1,t-1}\Big|_{t \in pw_{r,p}} \\
&- ETS_{r,p-1,t-1}\Big|_{t \in rw_{r,p}} - RET_{r,p-1,t-1} \\
&\qquad \forall\ r \neq 6 \text{ or } 11,\ p > 1, 1 < t \leq \overline{tig}_{r,p}
\end{aligned}
\tag{1.4}
$$

$$
\begin{aligned}
\overline{X}_{r,P,t} = &\left(1-\alpha\right)\begin{bmatrix} \left(1 - attrite_{r,P-1}\right) \sum_{t' \in epoch_{p,t}} X_{r,P-1,t'-1} \\ - ACS_{r,P-1,t-1} - PRO_{r,P-1,t-1}\Big|_{t \in pw_{r,p}} \\ - ETS_{r,P-1,t-1}\Big|_{t \in rw_{r,p}} - RET_{r,P-1,t-1} \end{bmatrix} \\
+ &\left(\alpha\right)\begin{bmatrix} \left(1 - attrite_{r,P}\right) \sum_{t' \in epoch_{p,t}} \overline{X}_{r,P,t'-1} \\ -\overline{ACS}_{r,P,t-1} \\ -\overline{PRO}_{r,P,t-1}\Big|_{t \in pw_{r,p}} \\ -\overline{ETS}_{r,P,t-1}\Big|_{t \in rw_{r,p}} \\ -\overline{RET}_{r,P,t-1} \end{bmatrix} \quad \forall\ r \neq 6 \text{ or } 11,\ p \geq P, 1 < t \leq \overline{tig}_{r,p}
\end{aligned}
\tag{1.4D}
$$

$$
\begin{aligned}
X_{r,p,t} = &\left(1 - attrite_{r,p}\right) \sum_{t' \in epoch_{p,t}} X_{r,p-1,t'-1} \\
&- RET_{r,p-1,t-1} \qquad \forall\ r = 6 \text{ or } 11, 1 < p < P, 1 < t \leq \overline{tig}_{r,p}
\end{aligned}
\tag{1.5}
$$

$$
\begin{aligned}
\overline{X}_{r,P,t} = &\left(1-\alpha\right)\begin{bmatrix} \left(1 - attrite_{r,P-1}\right) \sum_{t' \in epoch_{p,t}} X_{r,P-1,t'-1} \\ - RET_{r,P-1,t-1} \end{bmatrix} \\
+ &\left(\alpha\right)\begin{bmatrix} \left(1 - attrite_{r,P}\right) \sum_{t' \in epoch_{p,t}} \overline{X}_{r,P,t'-1} \\ -\overline{RET}_{r,P,t-1} \end{bmatrix} \qquad \forall\ r = 6 \text{ or } 11,\ p \geq P, 1 < t \leq \overline{tig}_{r,p}
\end{aligned}
\tag{1.5D}
$$

$$X_{r,p,t} = \sum_{t' \in pw_{r,p}} PRO_{r-1,p-1,t'-1} + HDO_{r,p} \qquad r \neq 1 \text{ or } 7, 1 < p < P, t = 1 \quad (1.6)$$

$$\begin{aligned} \overline{X}_{r,p,t} &= (1-\alpha) \sum_{t' \in pw_{r,p}} PRO_{r-1,P-1,t'-1} \\ &+ (\alpha) \sum_{t' \in pw_{r,p}} \overline{PRO}_{r-1,P,t'-1} \qquad \forall\ r \neq 1 \text{ or } 7, p \geq P, t = 1 \end{aligned} \quad (1.6D)$$

2. Constraints

Constraints (1.1) and (1.2) are included in the dual formulation but not after *P>p.* All remaining constraints for *P>p* are equivalent to the truncation with *p=P*.

3. Objective Function

$$\min \sum_{r} \sum_{p<P} \alpha_{r,p} \times Penalties + \sum_{r} \frac{\alpha_{r,p}}{1-\alpha} \times Penalties \qquad (1.21D)$$

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF REFERENCES

Afghan National Army. 2009. http://www.mod.gov.af/ (accessed February 26, 2009).

Army, U.S. *Stability Operations, FM3–07.* Washington, D.C.: Headquarters, Department of the Army, 2008.

Bartholomew, D.J., A.F. Forbes, and S.I. McClean. *Statitical Techniques for Manpower Planning.* New York: John Wiley & Sons, 1991.

Benson, Kirk C. *Security Force Generation Requirements for Emerging States.* Carlisle Barracks: U.S. Army War College, 2008.

Brown, Gerald G., Robert F. Dell, and R. Kevin Wood. "Optimization and Persistence." *Interfaces*, 1997: 15–37.

Carden, Michael J. *WWW.ARMY.MIL*. May 27, 2009. http://www.army.mil/-news/2009/03/27/18870-president-unveils-afghan-pakistan-strategy-review/ (accessed April 23, 2009).

Chan, Samuel. "Sentinels of Afghan Democracy: The Afghan National Army." *Military Review*, 2009: 25–40.
'

Clark, David T. *Navy Officer Manpower Optimization Incorporating Budgetary Constraints.* Operations Research Master's Thesis, Monterey: Navel Postgraduate School, 2009.

Collins, Roger W., Saul I. Gass, and Edward E. Rosendahl. "The ASCAR Model for Evaluating Military Manpower Policy." *Interfaces*, 1983: 44–53.

Corbett, J. C. *Military Mapower Planing: Optimization Modeling for the Army Officer Accession/Branch Detail Program.* Operations Research Masters Thesis, Monterey: Naval Postgraduate School, 1995.

Davis, Anthony. *Jane's Defense Weekly*. December 12, 2008. http://www.defence.pk/forums/military-forum/17579-home-grown-army-afghan-national-army.html (accessed April 23, 2009).

Durso, Anthony, and Scott F. Donahue. "An Analytical Approach to Reshaping the United States Army." *Interfaces*, 1995: 109–133.

Edwards, John S. "A survey of Manpower Planning Models and Their Application." *The Journal of the Operational Research Society*, 1983: 1031–1040.

Forrest, John. *CBC User Guide.* 2005. http://www.coin-or.org/Cbc/cbcuserguide.html (accessed March 1, 2009).

GAMS Development Corporation. "General Algebraic Modeling System (GAMS) Rev 23." Washintgon, D.C., 2001.

GAO. *Afghanistan Security.* GAO-05-0575, Washington, D.C.: Government Accountability Office, 2005.

GAO. *Afghanistan Security.* GAO-08-661, Washington, D.C.: Government Accountability Office, 2008.

Gass, Saul I, Roger W Collins, Craing W Meinhardt, Douglas M Lemon, and Marcia D Gillette. "The Army Manpower Long-Range Planning System." *Operations Research*, 1988: 5–17.

Gibson, Hise O. *The Total Army Competitive Category Optmization Model: Analysis of U.S. Army Officer Accessions and Promotions.* Operations Research Masters Thesis, Monterey: Naval Postgraduate School, 2007.

Ginther, Tricia A. *Army Reserve Enlisted Aggregate Flow Model.* Operations Research Masters Thesis, Monterey: Naval Postgraduate School, 2006.

Grinold, Richard C. "Model Building Techniques for the Correction of End Effects in Multistage Convex Programs." *Operations Research*, 1983: 407–431.

Haims, Marla C., David C. Gompert, Gregory F. Treverton, and Brooke K. Stearns. *Breaking the Failed-State Cycle.* Santa Monica: Rand Corporation, 2008.

Holz, Betty W., and James M. Wroth. "Improving Strength Forecasts: Support for Army Manpower management." *Interfaces*, 1980: 37–52.

Jalali, Ali A. "Rebuilding Afghanistan's National Army." *Parameters*, August 2002: 72–86.

Jones, Seth G., Jeremy M. Wilson, Andrew Rathmell, and K. Jack Riley. *Establishing Law and Order After Conflict.* Santa Monica: Rand Corporation, 2005.

MacCalman, Alex, and Kirk Benson. "ANAFORGEN." Excel Model, 2008.

Manuel, Anja, and P.W. Singer. "A New Model Aghan Army." *Foreign Affairs*, 2002.

Mehlmann, Alexander. "An Approach to Optimal Recruitment and Transition Strategies for Manpower Systems using Dynamic Programming." *The Journal of the Operations Research Society*, 1980: 1009–1115.

Metz, Steven, and Raymond Millen. "Intervention, Stabilization, and Transformation Operations: The Role Of Landpower in the New Strategic Environmant." *Parameters*, 2005: 51–52.

Morgan, R.W. "Some Models for a Hierarchical Manpower System." *The Journal of the Operational Research Society*, 1979: 727–736.

Press, William H., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. *Numerical Recipes in C, Second Edition.* Cambridge: Cambridge University Press, 1995.

Radin, CJ. *The Long War Journal.* February 23, 2009. http://www.longwarjournal.org/archives/2010/02/afghan_national_army_1.php (accessed February 26, 2009).

Rodgers, Philip D. *A Linear Programming Based Decision Support Aid for Navy Enlisted Strength Planning.* Opeartions Research Masters Thesis, Monterey: Naval Postgraduate School, 1991.

Rotberg, Robert I. *When States Fail: Causes and Consequences.* Princeton: Princeton University Press, 2004.

Schochetman, Irwin E., and Robert L. Smith. "Infinite Horizon Optimization." *Mathematics of Operations Research*, 1989: 559–574.

Schrews, Alan K. *Optimizing Active Guard Reserve Enlisted Manpower.* Operations Research Masters Thesis, Monterey: Naval Postgraduate School, 2002.

Seal, H.L. "The Mathematics of a Population Composed of k Stationary Strata Each Recruited from the Stratum Below and Supported at the Lowest Level by a Uniform Number of Annual Entrants." *Biometrika*, 1945: 226–230.

Smith, A.R., and D.J. Bartholomew. "Manpower Planning in the United Kingdom: An Historical Review." *The Journal of the Orperational Research Society*, 1988: 235–248.

Vadja, S. *Mathematics of Manpower Planning.* New York: John Wiley & Sons, 1978.

Vajda, S. "The Stratified Semi-Stationary Population." *Biometrika*, 1947: 243–254.

Walker, Steven C. *Evaluating End Effects for Linear and Integer Programs Using Infinite-Horizon Linear Programming.* PhD Thesis, Monterey: Naval Post Graduate School, 1995.

Wyler, Liana S. "Weak and Failing States: Evolving Security Threats and U.S. Policy." Congressional Research Service Report for Congress, Foreign Affairs, Defense, and Trade Division, Internation Crimes and Narcotics, 2007.

Yamada, Wade S. *An Infinite Horizon Military Manpower Model.* Operations Research Masters Thesis, Monterey: Naval Postgraduate School, 2000.

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Center for Army Analysis
   Ft. Belvoir, Virginia

3. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California

4. Robert F. Dell
   Naval Postgraduate School
   Monterey, California

5. Lieutenant Colonel Paul L. Ewing
   Naval Postgraduate School
   Monterey, California

6. Major Patrick Workman
   TRAC-WSMR
   White Sands, New Mexico