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Endogenous Split Awards as a Bid Protest and Procurement Management Tool

18 July 2012

by

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Abstract

When competitors for a federal government contract feel that the contract has been unfairly or unlawfully awarded, they can protest the contract award. A protest stops a contract award while the protest is evaluated for merit and remedy, as appropriate. While legitimate award protests improve procurement efficiency, integrity, and accountability, frivolous award protests create significant contract delays and cost growth. There is increasing interest in mechanisms that reduce frivolous contract award protests, while retaining the integrity of the process for meritorious protests.

This research explores split procurement awards as a tool to rationalize the bid protest process and potentially improve the general procurement process. It discusses split procurement as an award protest and procurement management tool, and models bidding incentives and outcomes with both fixed or exogenous split awards and variable or endogenous split awards (where the split depends on the relative competitiveness of the vendors' bids). Endogenous split awards can increase the competitiveness of vendor bids relative to fixed-split awards, while reducing the incentives for frivolous contract protests. Endogenously split contract awards can also improve the general acquisition process, particularly if retaining competition is important for follow-on procurement actions.

Keywords: Contract protests, bid protests, split procurement, contract mechanism design.

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About the Authors

Dr. Peter J. Coughlan is an associate research professor of business strategy and managerial economics at the Naval Postgraduate School in Monterey, CA. At NPS since 2004, Professor Coughlan teaches graduate courses in the two fields of economics and strategic management in both the MBA and Executive MBA programs as well as in the Navy Senior Leadership Seminar. He was recently awarded the Louis D. Liskin Award for Teaching Excellence, being recognized by two graduating classes as the most outstanding professor in the Graduate School of Business and Public Policy. In addition to his work in the classroom, Dr. Coughlan conducts research on the topic of mechanism and market design as well as in the area of innovation and technology strategy.

Prior to his arrival at NPS, Dr. Coughlan served six years as a professor in the Strategy Unit at the Harvard Business School, where he taught several core strategy courses in the MBA, PhD, and Executive Education programs. While at Harvard, Professor Coughlan also designed and taught his own original advanced strategy course focusing on competitive dynamics. In addition, Dr. Coughlan researched, wrote, and published more than 30 business case studies during his time at Harvard, concentrating primarily on issues of strategy and innovation.

In addition to his academic experience, Dr. Coughlan spent several years as a management consultant with Booz Allen Hamilton, specializing in government clients. He earned his MS and PhD degrees in economics from the California Institute of Technology, specializing in game theory and behavioral economics, and his BA degree in economics and mathematics from the University of Virginia.

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Dr. William (Bill) Gates was appointed the Dean of the Graduate School of Business and Public Policy (GSBPP) on February 1, 2009. A graduate of Yale University (PhD) and UC San Diego, Dr. Gates joined the GSBPP faculty in 1988. Prior to joining NPS, he was an economist at the Jet Propulsion Laboratory.

Dean Gates has been widely recognized for teaching excellence, receiving the Administrative Sciences Department Teaching Excellence award (1992), the RADM John Jay Schiefflin award (1999), the Allen Griffin award (2000), and the Louis D. Liskin award (2008). His teaching interests include Cost Benefit Analysis, Business Modeling, and Basic Quantitative Methods in Management.

Dean Gates' research focuses on game theory and mechanism design applied to military manpower, defense acquisition and defense alliances. Military manpower applications include retention and voluntary separation bonus auctions, using purely monetary incentives or individualized combinations of monetary and non-monetary incentives, and matching mechanisms for placing service members in hard-to-fill billets. Defense acquisition applications focus on incentive contracts, procurement auctions, contractor protests, and technology transfer. Past research involved cost-benefit analysis, public policy analysis, and federal R&D and technology policy.

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Introduction

Whenever competitors or potential competitors for a federal government contract feel that the contract has been, or is about to be, unfairly or unlawfully awarded, or that they have been unfairly denied the opportunity to compete for a contract, they can protest the contract award. This is referred to as a bid protest (GAO, 2009). A bid protest has the potential to stop a contract award while the protest is evaluated for merit and remedy, as appropriate. While legitimate bid protests improve government contracting by encouraging procurement competition, integrity, transparency, and accountability, frivolous bid protests can create significant contract delays, cost growth, and, potentially, operational performance gaps. Furthermore, the mere threat of frivolous protests may discourage some potential bidders, reducing competition, or may be used strategically by losing bidders to extract concessions from the winning bidder or procuring agency (e.g., subcontracts; Melese et al., 2010, pp. 16–21).

Contract award protests have recently received increasing attention (Castelli, 2009; Censer, 2010). This reflects a few high-profile contract award protest cases, including the Air Force's new KC-X aerial refueling tanker (Gates, 2008; O'Rourke, 2009; GAO, 2008a, 2008b), as well as perceptions that protests have been increasing in frequency and in the dollar value of the impacted contracts (Gansler & Lucyshyn, 2009; Melese et al., 2010). As a result, there is increasing interest in mechanisms that reduce frivolous contract award protests, while retaining the integrity of the process for meritorious protests.

This research explores split procurements as a management tool to help rationalize the bid protest process and potentially improve the procurement process in general. The analysis adopts the paradigm that potential protestors decide whether to file a bid protest based on the expected costs and benefits of their actions. Unsuccessful bidders are more likely to file protests when the potential benefits are higher and/or the costs are lower, and less likely to file protests when the benefits are lower and/or the costs are higher. By awarding the less competitive bidders a portion of the contract, splitting contract awards across two or more vendors can reduce the benefits of a protest. The mechanism ultimately proposed in this research ties the contract splits (or shares) to the relative competitiveness of the winning and losing bids.

It is important to recognize that the goal is not to minimize the number of bid protests, but to "right size" the number of such protests. The process should encourage protests that correct "significant" procurement mistakes, whether they be honest mistakes (e.g., information limitations, bounded rationality, etc.) or dishonest mistakes (e.g., government decision-maker bias or fraud, etc.), while discouraging frivolous or inconsequential protests. Split procurements could provide one lever to appropriately filter out many frivolous or inefficient bid protests while preserving incentives for meritorious and valuable protests.

This paper reviews the bid protest process and past data (protests filed and upheld), discusses split procurement as a bid protest and procurement management tool, models bidding incentives and outcomes with fixed or exogenous split awards (splits determined *ex ante*), and, finally, models bidding incentives and outcomes with endogenous split awards (splits determined *ex post*). Model results demonstrate that endogenous split awards can increase the competitiveness of vendor bids relative to fixed-split awards, while also reducing the incentives for frivolous contract protests. Endogenously split contract awards can also improve the acquisition process in general, particularly if retaining competition is important for follow-on procurement actions.

Bid Protest Rules and Historic Data

The Federal Acquisition Regulation (FAR; 2005) defines a bid protest as follows:

> Protest means a written objection by an interested party to any of the following: (1) A solicitation or other request by an agency for offers for a contract for the procurement of property or services, (2) The cancellation of the solicitation or other request, (3) An award or proposed award of the contract, (4) A termination or cancellation of an award of the contract if the written objection contains an allegation that the termination or cancellation is based in whole or part on improprieties concerning the award of the contract. (Subpart 33.101; see also Competition in Contracting Act, 1984, § 3551[1])

Recognizing that bid protests can provide oversight against bias, fraud, and unintended errors in the procurement process, thereby fostering competition and increasing efficiency, timeliness, and operational performance, the Competition in Contracting Act (CICA) was enacted in 1984 (Title 31 of the *U.S. Code*, Sections 3551–3556),[1] supplementing the FAR (Parts 5, 10, 12–15, and 33). The CICA codified the current bid protest process and the Government Accountability Office's (GAO) authority to adjudicate bid protests. However, law also allows dissatisfied bidders to file protests with the procuring agency or the Court of Federal Claims. Between 2001 and 2007, 9,281 federal bid protests were filed with GAO, while only 424 were filed with the Court of Federal Claims (4.4% of the total; Schaengold, Guiffré, & Gill, 2009, p. 255); there is no comprehensive data on protests filed with the procuring agency. This analysis focuses exclusively on GAO protests.[2]

A GAO protest begins when an unsuccessful bidder files a formal objection (Figure 1). The GAO adjudication process basically involves two steps. First, the

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[1] For further discussion, see Melese et al. (2010). As reported in Melese et al., GAO and CFC statistics were drawn from Schaengold, Guiffré, and Gill (2009, p. 255).

[2] The protest process is discussed in Melese et al. (2010, pp. 55–63, and Appendix A).

GAO determines if the protest has merit; if not, it is dismissed prior to the cumbersome discovery and review process. Protests found to have merit are then reviewed and either sustained or not. If sustained, some remedial action is imposed (e.g., a new contract solicitation). The GAO tries to balance expediency with accuracy in resolving contract disputes. Expediency favors a quick resolution, while accuracy may require a more deliberative process. The CICA requires that the GAO resolve all bid protests within 100 calendar days, a statutory deadline the GAO has never failed to meet (GAO, 2009, p. 4). In fact, the GAO reports that over 50% of DoD protests are closed within 30 days, typically cases that are found to lack merit (GAO, 2009, p. 10).

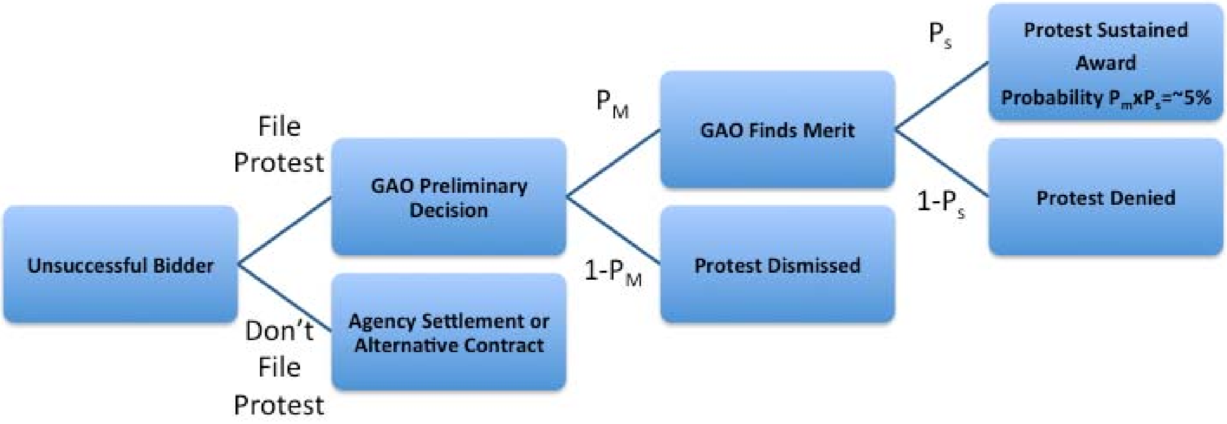

Figure 1. Stylized Bid Protest Process

Note. This figure was adapted from Melese et al. (2010, p. 62).

For reference, the number of DoD protests filed with the GAO that were closed between 2001 and 2008 is shown in Figure 2. Figure 2 also shows the percent of closed protests found to have merit and the percent sustained. Over this period, the protests found to have merit averaged approximately 24% of protests closed, and the protests sustained averaged approximately 5% of protests closed.

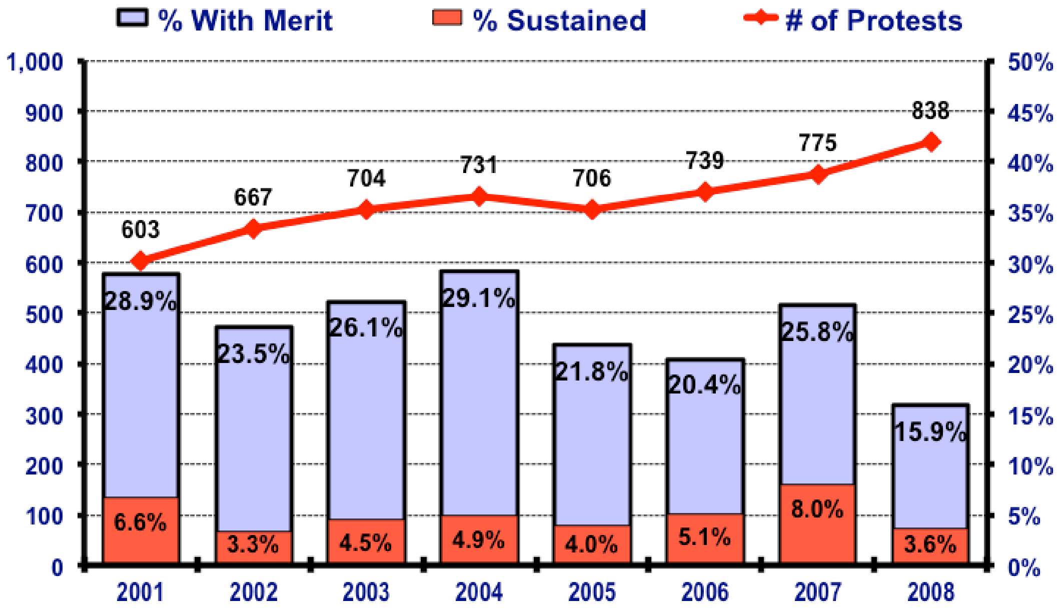

Figure 2. DoD Bid Protests Filed With the GAO, Merit and Sustainment Determination Rates
(Gansler & Lucyshyn, 2009)

Despite the relatively low number of meritorious and sustained protests, there is concern that these often involve large acquisition programs; mere numbers may understate the relative impact of bid protests. Melese et al. (2010) observed that only three U.S. Air Force (USAF) protests were sustained in 2001, out of 98 protests closed that year; there were only two sustained protests out of 112 closed in 2008. From this perspective, bid protests appear to be insignificant and stable or even declining.

In terms of dollar value, the three USAF protests sustained in 2001 involved contracts worth roughly $260 million, impacting around 1% of the total 2001 competitive procurement dollars. The two USAF protests sustained in 2008 involved contracts worth over $36 billion, impacting nearly 97% of USAF competitive procurement dollars in 2008. While the number of sustained protests may be low, bid protests can have impacts far exceeding their relatively small volume.

According to Gansler and Lucyshyn (2009), the DoD completed 7,711,596 contract actions between 2001 and 2008. Only 61 contract awards exceeded $1 billion in value, but 53 of those contracts faced sustained protests. Gansler and Lucyshyn (2009) concluded that bid protests are not a large problem relative to all DoD contract awards, but they appear problematic for complex, high-value contracts. As they observe, “there is no disincentive to try for another bite at the apple” (Gansler & Lucyshyn, 2009, slide 32).

Managing Bid Protests

As noted previously, bid protests are intended to correct mistakes in contract awards and terminations, whether honest (imperfect information) or dishonest (bias or fraud). The goal is to “manage,” not minimize, protests. The DoD should encourage protests that correct more significant mistakes, while discouraging protests that are frivolous or address relatively insignificant corrections.

This discussion does not distinguish between inadvertent and unintentional mistakes or more malicious and intentional mistakes. A contract award or termination mistake will have the same impact whether unintentional or otherwise. Furthermore, this discussion does not assume that the current incidence of bid protests is too high or too low. Bid protest management requires creating larger incentives to file protests when the potential mistakes are larger, while reducing the incentive to file protests when potential mistakes are smaller.

For reference, Gansler and Lucyshyn (2009) examined justifications for sustained DoD bid protests between 2001 and 2008. They found that agency misevaluation was the most common justification, including alleged errors in source selection, past performance evaluation, technical evaluation, cost evaluation, and information dissemination (meaningful discussions). Other common justifications included inconsistencies in evaluation criteria and the Request for Proposals (RFP) terms, contracts awarded to bidders who are outside the terms of the RFP, and other (small business concerns not addressed, contractors eliminated from consideration for no apparent good reason, awards made to contracts at prices exceeding the RFP requirements).

Recognizing these justifications, recommendations to improve acquisition decisions and reduce bid protests frequently center on more standardized reporting and proposal evaluation processes, better defense acquisition workforce training, more standardized cost-evaluation techniques, better ethics training and oversight, and so forth. These efforts can be characterized as reducing the likelihood of a successful protest by reducing the probability of mistakes in contract awards, hence reducing the probability that a protest will be meritorious and sustained.

Taking a slightly different approach, this research models bid protest decisions as occurring within a benefit-cost framework. Dissatisfied bidders will file a bid protest when they expect the potential benefit from protesting to exceed the expected protest cost. Protest costs include filing and documentation costs, legal fees, potential reputation costs, and opportunity costs associated with the resources committed to the protest; they could also include any penalties imposed on losing or frivolous protestors, though none are currently imposed. The benefits from a protest include the expected profits to be gained through the protest; the expected profit is the gain if the protest is successful multiplied by the probability of success. A dissatisfied bidder will file a protest if the expected benefits exceed the expected costs.

This characterization highlights three potential levers to manage bid protests: protest costs, the probability of success, and the gain if successful. The more traditional efforts to manage (typically reduce) bid protests, as described previously, effectively focus on reducing the probability of a successful protest by reducing both honest and dishonest mistakes. Increasing protest costs and decreasing the gains from protests have received less attention.[3] This research addresses the potential gain from a protest as a bid protest management tool.

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[3] Gansler and Lucyshyn (2009) do suggest that the DoD should examine disincentives to filing protests. Much like National Football League coaches who lose a timeout if they protest a call and the call is upheld, Gansler and Lucyshyn suggest the DoD consider “punishing” contractors that file protests that lack merit or are not sustained by including that information as part of the contractor’s past history in future source selection processes.

Modeling Bid Protests

Consider a model driven by imperfect *ex ante* information regarding contractors' actual performance (costs, technical performance, schedule, and the tradeoffs between these elements). This model presumes that small mistakes are more likely than big mistakes, whether the mistake is inadvertent or intentional (intentional mistakes due to bias or favoritism are more transparent as information becomes more perfect and distributed). This characterization accurately reflects past empirical results. Imperfect information is consistent with agency misevaluation and other protest justifications cited by Gansler and Lucyshyn (2009); the hypothesis that small mistakes are more common than large mistakes is consistent with the small fraction of protests found to be meritorious and sustained.

For simplicity, consider a model with two potential vendors and one buyer. The vendors both supply the same product, but offer different prices.[4] Each vendor faces a different cost to supply the item in question. Vendors submit proposed prices in a competitive solicitation, though the buyer has some uncertainty regarding the terms of pricing in the proposal. The buyer selects the winning vendor based on their perceived prices and awards the contract. The losing contractor decides whether to file a protest. If the protest is filed, the contract award is canceled and the protestor and buyer incur protest costs; the submissions are reevaluated and prices are determined with certainty. The contract is re-awarded, either to the original contractor or the protesting contractor, depending on the reevaluation results. Model notation and a process diagram are provided in Figure 3.

Model Notation:

C_i	Cost for vendor i

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[4] This model can generalize to multiple vendors and products with multiple attributes. For ease of discussion, we explore the more simplified case here.

P_i	Perceived bid price for vendor i
P_i^T	True bid price for vendor i (revealed by protest or contract completion)
K_p	Cost to losing vendor of filing protest (research, legal fees, reputational penalty, opportunity cost)
K_b	Protest costs incurred by buyer (reevaluation)
X	Quantity purchased
$X \times P_i^T$	Revenues for vendor i (i.e., buyer's payment to vendor i)
$X \times C_i$	Production costs for vendor i
$\mathrm{Prob}(P_j^T<P_i^T \vert P_j>P_i)$	Probability of sustained protest (probability $P_j^T<P_i^T$ when $P_j>P_i$)

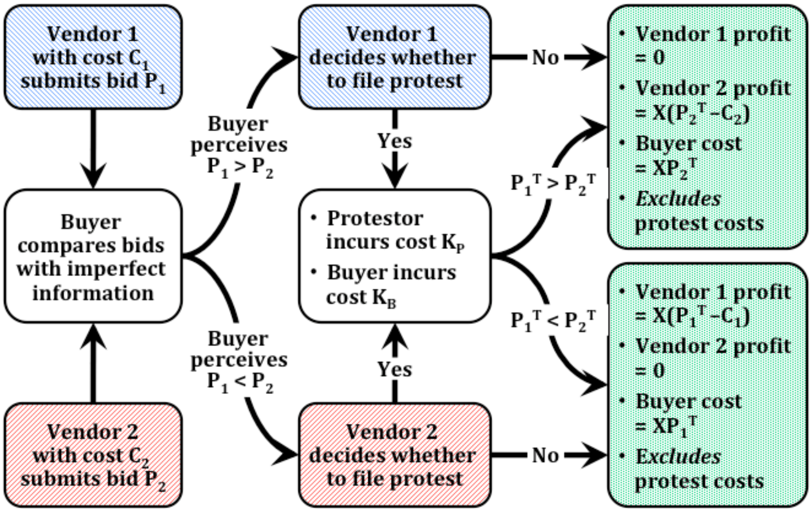

Figure 3. Bid and Protest Process Flow

Note that the expected cost of filing—and pursuing to completion—a bid protest includes the vendor's research fees, legal fees, and reputational penalty, as well as opportunity costs, and is given by K_p. Meanwhile, the probability of a successful protest by a losing vendor j is the probability that $P_j^T < P_i^T$ when $P_j > P_i$, or $\mathrm{Prob}(P_j^T < P_i^T \vert P_j > P_i)$. Thus, the expected benefits are the vendor's profits if selected multiplied by the probability of a successful protest, or $X(P_j^T - C_j) \times \mathrm{Prob}(P_j^T < P_i^T \vert P_j > P_i)$.

Since the expected profit from filing a protest is equal to the expected benefits minus the expected costs, expected profit is given by the following equation:

$$\text{Prob}(P_j^T < P_i^T | P_j > P_i) \times X(P_j^T - C_j) - K_p \quad (1)$$

The losing vendor will thus file a protest if and only if

$$\text{Prob}(P_j^T < P_i^T | P_j > P_i) \times X(P_j^T - C_j) > K_p. \quad (2)$$

Recall that the goal in managing bid protests is to encourage protests that address potentially significant mistakes while discouraging protests where the mistakes are insignificant or frivolous. From Equation 2, there are three control levers to manage bid protests:

1. $\text{Prob}(P_j^T < P_i^T | P_j > P_i)$, the probability of success;
2. K_p, protest costs; or
3. $X(P_j^T - C_j)$, the gains from a protest.

As described previously, bid protest “remedies” commonly address the first lever, the probability of success, by improving the initial assessment accuracy. There has been some limited attention on influencing protest costs, particularly by imposing a financial or reputational penalty on unsuccessful protestors. There has been little attention paid, however, to the third lever, the gain from a successful protest.

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Managing Protest Incentives With Split Award Contracts

One mechanism for adjusting and controlling the potential gain from a protest is the utilization of split award contracts. In a winner-takes-all contract award, the potential gain in procurement quantity from a successful protest is X, the entire quantity purchased. If the losing bidder receives a share of the procurement quantity, the gain from a successful protest is limited to the increase in the protestor’s share as a result of the protest (in other words, the difference between the winner’s share and the loser’s share, which is less than the full quantity, X). If the procurement quantity is 100, the losing vendor can increase its sales by 100 in a winner-takes-all contract with a successful protest. If, however, the winning and losing contractor split the procurement 70/30, for example (with the winning vendor receiving the larger share), the gains from a successful protest are limited to an increase in sales of 40 (the winning share of 70 minus the losing share of 30).

While there does exist a collection of economic literature investigating split award procurement, previous investigations have not considered such split awards in the context of potential bid protests. Moreover, existing investigations focus almost exclusively on scenarios in which the magnitude of award splits (the shares accruing to the winning and losing vendor) are fixed and determined *ex ante* (or exogenously), before bids have even been submitted.[5] Even when previous models of split award procurements have allowed for variable splits, they generally only explore scenarios in which vendors submit a different bid for each potential split, creating strong incentives for manipulation by the bidders (a vendor can essentially

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[5] This discussion could easily generalize to multiple vendors where the contract is split over the most attractive two or more bids. For simplicity in presentation, we will restrict the discussion here to two vendors, and refer to them as the winning and losing vendor, as opposed to the winning vendor and the second best.

“veto” any split it does not want by submitting exorbitantly priced bids for the undesirable splits.

In the next section, we first examine bidding and protest incentives with traditional fixed award splits. Then, after explaining the disadvantages of such fixed splits (in terms of buyer cost), we explore the potential benefits of endogenous (or *ex post* determined) award splits, in which the ratio of the winning and losing shares depends on the relative prices of vendor bids.

Modeling Bid Protests With Split Awards

Let us understand how incorporation of split awards alters our model of the bid protest process and incentives, using the simpler case of fixed award splits. Model notation and a process diagram for this scenario are provided in Figure 4. The notation is the same as presented previously, with the addition of the definitions for contract splits.

Model Notation:

C_i	Cost for vendor i
P_i	Perceived bid price for vendor i
P_i^T	True bid price for vendor i (revealed by protest or contract completion)
K_p	Cost to losing vendor of filing protest (research fees, legal fees, reputational penalty, opportunity costs)
K_b	Protest costs incurred by buyer (reevaluation)
X	Quantity purchased
$X \times P_i^T$	Revenues for vendor i (i.e., buyer's payment to vendor i)
$X \times C_i$	Production costs for vendor i
S_L	Contract share or split awarded to the low-price vendor
S_H	Contract share or split awarded to the high-price vendor (where $S_L + S_H = 1$, $0 \leq S_H \leq ½$, and $½ \leq S_L \leq 1$)

With split shares, if the buyer perceives that $P_1 < P_2$, vendor 1 is awarded a contract to produce S_L * X units and vendor 2 is awarded a contract to produce S_H * X units; the shares are reversed if the buyer perceives that $P_2 < P_1$.

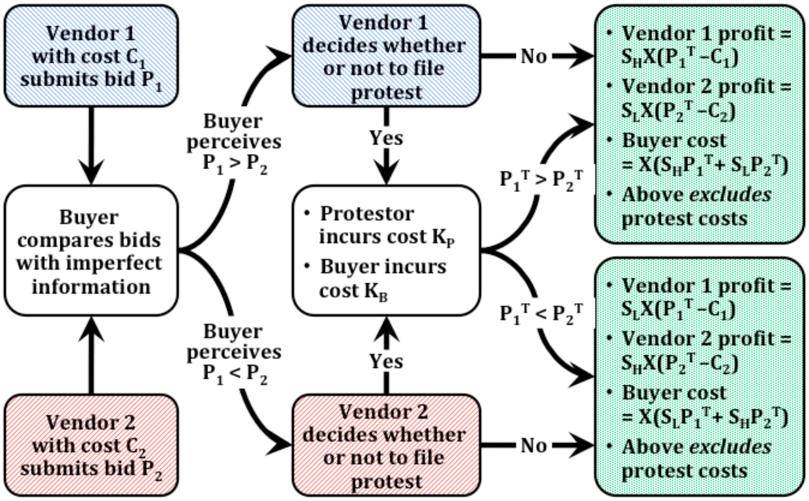

Figure 4. Bid and Protest Process Flow With *Ex Ante* (Exogenous) Split Procurement Shares

Recall, as derived previously, that the losing vendor will file a protest in the presence of winner-take-all contract awards if and only if

$$\mathrm{Prob}(P_j^T < P_i^T | P_j > P_i) \times X(P_j^T - C_j) > K_{p.} \tag{3}$$

Following a similar logic, the losing vendor will file a protest under this split award scenario if and only if:

$$\mathrm{Prob}(P_j^T < P_i^T | P_j > P_i) \times X(P_j^T - C_j) \times (S_L - S_H) > K_{p.} \tag{4}$$

As a result, split awards raise the hurdle for filing bid protests. The higher the contract split accruing to the losing vendor, the higher the hurdle to file a protest. At the extreme, if contractors split the purchase quantity 50/50 (i.e., $S_L = S_H$), there is no incentive to protest (i.e., no gain).

More precisely, substituting $S_L = 1 - S_H$, the expected profit of a protest is given by Equation 5:

$$E(\pi) = \mathrm{Prob}(P_j^T < P_i^T | P_j > P_i) \times X(P_j^T - C_j) \times (1 - 2S_H) - K_{p.} \tag{5}$$

Taking the partial derivative with respect to S_H shows the change in expected profit as S_H changes:

$$\frac{\partial E(\pi)}{\partial S_H} = -2 \times \text{Prob}(P_j^T < P_i^T | P_j > P_i) \times X(P_j^T - C_j) < 0. \qquad (6)$$

Because this derivative is less than zero, the incentive to file a protest decreases as S_H increases.

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Split Awards and Buyer Cost

While split awards have the advantage of reducing the bid protest incentive, they also have the disadvantage of raising buyer costs. This happens in two ways: (1) By awarding a share of the procurement quantity to the higher priced vendor and (2) by creating an incentive for vendors to increase their bids even further above cost. In this section, we focus on the first cost effect (purchasing from the more expensive vendor) and temporarily ignore the second cost effect (bid inflation). Thus, we assume for now that vendors submit the same bids under split award competition as under winner-take-all competition.

Thus, assuming the buyer correctly chooses the low-priced vendor, the buyer's total cost in the winner-takes-all (abbreviated in the following equations as WTA) case is:

$$TC_{WTA} = X \times P_L^T \tag{7}$$

Meanwhile, given our momentary assumption that vendor bids are the same under each form of competition, the buyer's total cost with split awards (SAs) is given by Equation 8:

$$TC_{SA} = X \times S_H \times P_H^T + X \times (1 - S_H) \times P_L^T = X \times P_L^T + X \times S_H \times (P_H^T - P_L^T). \tag{8}$$

Hence, the direct cost increase the buyer incurs (when moving from winner-take-all to split award contracting) is the quantity awarded to the higher priced vendor ($X \times S_H$) multiplied by the price difference between the lower and higher priced vendor ($P_H^T - P_L^T$). This additional buyer cost is given by

$$TC_{SA} - TC_{WTA} = X \times S_H \times (P_H^T - P_L^T). \tag{9}$$

Thus, the larger the split given to the higher priced bidder (S_H), or the larger the difference between the two submitted bids ($P_H^T - P_L^T$), the greater the increase in buyer cost under split award contracting.

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Identifying the “Right” Contract Splits

While we illustrated in the previous section how split awards would increase initial procurement costs, it is important to note that such awards could ultimately decrease buyer cost by (1) reducing the buyer costs associated with protests and (2) preserving viable competition for potential follow-on competitive procurements. Dynamic considerations of such follow-on procurements, as well as learning effects and innovation over time, will be a topic of future research.

In the end, the buyer would want to identify the optimal contract split that maximizes total buyer utility, factoring in initial procurement cost, follow-on procurement cost, and expected bid protest cost. In terms of managing bid protests, the buyer might want to identify the particular contract split that sets the protest hurdle at the “right” level—high enough to avoid insignificant, frivolous protests, but low enough to encourage protests correcting significant mistakes. Note that it is not the intent of the current research to identify or characterize such an optimal fixed award split, but instead to explore the potential benefit of contract splits (both fixed and endogenous) as a tool for managing the procurement process in the context of potential bid protests.

Operationally, it is important to note that *de facto* split procurement awards are already being used as a bid protest management tool, for example, in the Air Force contract for an aerial refueling tanker (Johnsson, 2009). Contractors filing or threatening to file bid protests are often rewarded (appeased) with subcontracts, alternative contracts in other procurements, consulting fees, or other agency settlements. This phenomenon is referred to as “Fed-mail,” particularly when the protest appears frivolous. “Buy-offs,” a related phenomenon, involve similar appeasements initiated by procurement officials to offset unintentional or intentional errors or omissions in the procurement process (GAO, 2009; GAO Office of the General Counsel, 2004; Marshall, Meurer, & Richard, 1991; Melese et al., 2010, pp. 16–21; United States Senate, 1994). Split procurements simply institutionalize this practice, which is already occurring, and make it an explicit policy tool subject to more formal analysis and transparency.

Bid Inflation Under Fixed Award Splits

So far we have seen that although split awards can help manage the bid protest process, they also increase buyer costs as the contract share awarded to the higher priced vendor. We also mentioned a second, as-yet-unaddressed, buyer cost impact of split awards: bid inflation. Under split award competition, vendors must consider whether it is better to receive a lower profit margin on a larger procurement share, or a larger profit margin on a smaller procurement share. As all contractors feel some incentive to pursue a larger profit margin on a smaller procurement share, the buyer's costs can increase significantly. This is especially true under fixed award splits.

To simplify our illustration of bid inflation under fixed award splits, it is helpful to focus exclusively on the competitive bidding competition without incorporating the information or incentive issues that are uniquely related to the bid protest process. This is not only helpful but also important because, as we illustrate in this report, the bid inflation effect is completely independent of whether protest is possible.

Thus, let us assume for the moment that, when submitting their bids, vendors ignore the "continuation value" of the bid protest process illustrated in Figure 4. In other words, suppose the vendors assume that the buyer's initial award determination will be the final award determination. From the buyer's perspective, let us similarly assume for the moment that $P_H = P_H^T$ and that $P_L = P_L^T$ or, in other words, that the buyer knows each vendor's prices with certainty and does not intentionally misrepresent the vendors' prices (i.e., there are no contract award mistakes, either unintentional or intentional). Hence, in the remainder of our analysis, we will dispense with the superscript T and designate vendor prices (or bids), both true and perceived, as simply P_H and P_L.

With those simplifying assumptions in place, consider for illustration the case of two vendors with random unit costs uniformly distributed over the range from zero to 100.[6] Let each vendor j follow an expected-profit maximizing bidding strategy, given by λ(C_i), which maps the vendor j's unit cost C_i into an optimal bid within the range zero to 100.

Analysis in Appendix A derives the symmetric Nash equilibrium bidding strategy for both vendors in this simplified case as a function of the fixed award splits, S_L and S_H. As shown therein, the optimal symmetric bidding strategy for this simplified case is given by Equation 10:

$$\lambda(C_i) = \frac{10{,}000 - (S_L - S_H)C_j^2}{200S_L - 2C_j(S_L - S_H)} \qquad (10)$$

Figure 5 graphically illustrates this equilibrium bidding strategy for various fixed award splits. As shown, the profit-maximizing bid increases as the contract split awarded to the higher priced bidder increases. Under a winner-takes-all procurement, the vendor's profit maximizing bids will vary from 50 to 100 as the vendor's prices vary from zero to 100. At the other extreme, with equal procurement shares accruing to both the high- and low-priced bidders, neither vendor has any incentive to bid below the maximum of 100, regardless of cost.

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[6] Bid inflation can be clearly illustrated with this simplified procurement scenario. These results could be generalized to multiple bidders and more complicated cost functions. However, additional complication would not change the primary result: Exogenously determined award shares encourage bid inflation.

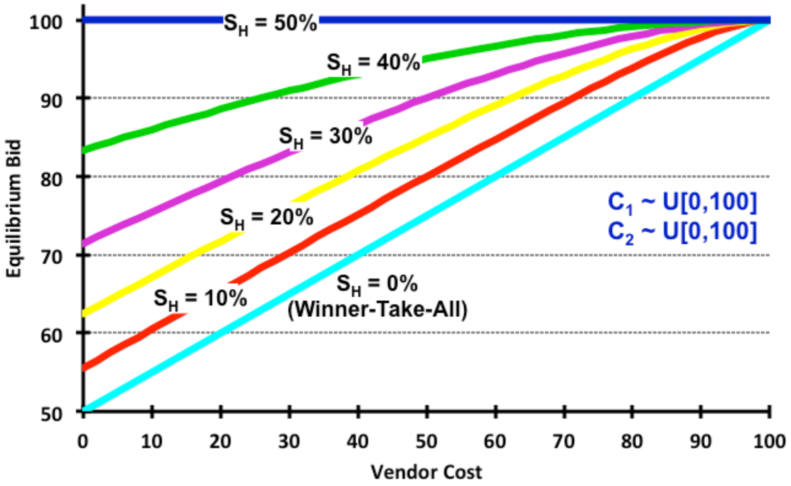

Figure 5. Equilibrium Bidding Strategy With Fixed Award Split

With random costs distributed uniformly over the range from zero to 100, the expected price for the low-priced vendor is 33 and 67 for the high-priced vendor. The buyer's expected cost can be calculated under each fixed award share scenario using these vendor expected prices and their associated profit-maximizing bids. The resulting buyer costs are depicted in Figure 6. As shown, the buyer's average unit cost increases from 67 to 100 as the contract split awarded to the high-priced vendor increases from zero (winner-take-all) to 50% (equal procurement shares).

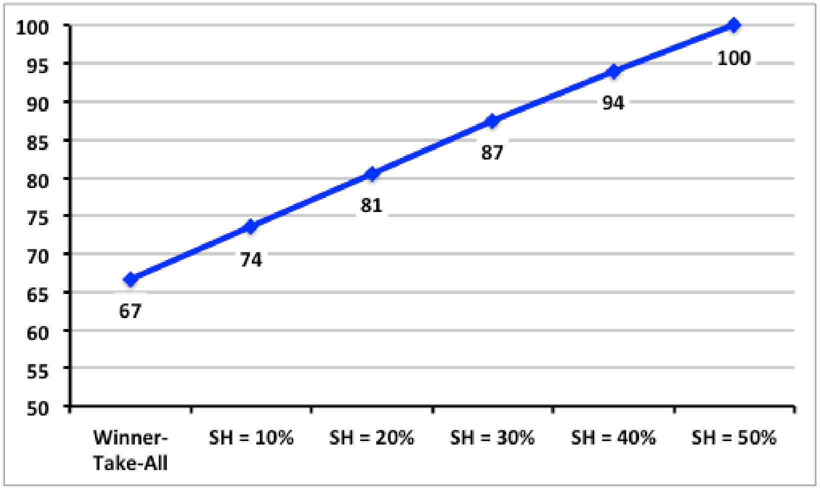

Figure 6. Expected Buyer's Cost Relative to the Contract Award Split

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The Promise of Endogenous (*Ex Post*) Award Splits

To this point, we have seen that split procurement awards help manage bid protests by reducing the dissatisfied vendor's expected gain from a protest, but fixed award splits increase the buyer's total cost in two ways. First, there is a direct additional cost as a share of the procurement quantity is shifted to a higher priced vendor. Second, there is also bid inflation among all vendors as fixed award splits introduce a strategic incentive to increase one's bid compared to the optimum bid under winner-take-all competition.

This raises the question whether award splits can be designed and utilized in such a way as to maintain their bid protest management characteristics while mitigating the negative impact on buyer's cost. One possibility is linking the size of the contract award split to the size of the difference between the low and high vendor bids, such that the share captured by the higher priced vendor decreases as the difference between the low- and high-priced bids increases. Under such an arrangement, the procurement award split is determined endogenously by the vendor bids.

In an endogenous split award contract competition, the size of the split awarded to the higher priced vendor, S_H, is determined *ex post*, after vendor bids P_H and P_L have already been submitted. Moreover, the size of the split S_H decreases as the difference between the two bids ($P_H – P_L$) increases. The closer the two bids are together, the greater the split S_H awarded to the higher priced vendor. The further apart the two bids are, the smaller the split S_H.

The primary advantage of endogenous split awards (over fixed splits) is mitigation of the increased buyer cost associated with split award contract competition, which was discussed in previous sections. Before formally illustrating how endogenous splits mitigate this increased buyer cost, however, let us first consider the basic intuition behind the cost-reduction benefit of endogenous versus fixed contract splits. To do this, recall that the increased buyer cost from split award contract competition (relative to winner-take-all competition) has two components:

1. The direct cost of purchasing a portion of the quantity desired from the higher priced vendor, and
2. The indirect cost of increasing the incentive for bid inflation.

As noted previously, the magnitude of the first additional cost component above is given by $X \times S_H \times (P_H – P_L)$. With endogenous split awards, however, the larger the difference between the two bids $(P_H – P_L)$, the smaller the split S_H. Hence, the use of endogenous splits essentially limits or regulates the magnitude of the direct additional cost, $X \times S_H \times (P_H – P_L)$.

Furthermore, with endogenous split awards, the incentive for bid inflation (the second additional cost component noted previously) is also reduced. To understand this, recognize that each vendor maximizes its profit by continuing to increase its bid (above cost) as long as the marginal benefit of doing so exceeds the marginal cost. The marginal benefit of increasing one’s bid is given by the additional margin ($P_j – C_j$ for vendor j) received on each unit sold. Meanwhile, the marginal cost of increasing one’s bid has two elements. First, any increase in a vendor’s bid increases that vendor’s likelihood of going from being the low-priced bidder (and ultimately receiving the majority split) to being the high-priced bidder (and ultimately receiving the minority split). This first marginal cost is present under both fixed and endogenous split awards.

With endogenous split awards, however, there is an additional marginal cost associated with increasing one’s bid that is not present under fixed splits: Each increase in a vendor’s bid directly reduces the split that this particular vendor can expect to be awarded (even if submitting a higher bid does not change whether or not this vendor is the higher priced or lower priced bidder). Hence, increasing one’s bid under fixed or endogenous split awards essentially provides the same marginal benefit, but there is an additional marginal cost of doing so under endogenous split awards that is not present under fixed-split awards. For this reason, the incentive for bid inflation is also reduced if contract splits are determined *ex post* and endogenously based on the bids submitted.

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An Example Endogenous (*Ex Post*) Award Split Function

To illustrate the cost benefits of endogenous split awards more formally, we introduce and investigate the strategic implications of one particular functional form for determining contract splits. In future research we will examine alternative split award functions more generally to better identify the optimal policy. The discussion that follows, therefore, should be viewed as a proof of concept for endogenous split awards.

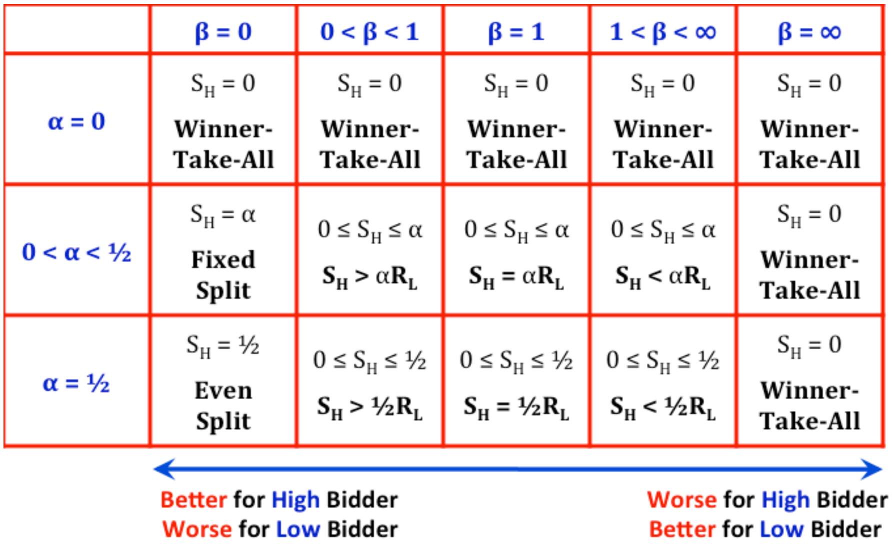
To derive a potential endogenous split award function, let us define $R_L = P_L/P_H$ (so that $0 \leq R_L \leq 1$). Let S_H, the higher priced vendor's contract split, then be expressed as a function of R_L, such that $S_H = F(R_L)$, where $0 \leq F(R_L) \leq ½$, so that the contract share awarded to the higher priced vendor cannot exceed 50%. Furthermore, it is necessary that $F(R_L)$ be increasing in R_L, so that the contract share awarded to the higher priced vendor increases as the higher and lower vendor prices converge.

One potential specification for $F(R_L)$ that satisfies these conditions is given by $S_H = \alpha R_L^{\beta}$, where α is the maximum share awarded the high priced vendor ($0 \leq \alpha \leq ½$), and $\beta \geq 0$. In this case, S_H is increasing in α and R_L, and decreasing in β. To summarize this example endogenous split award function, we have

$R_L = P_L/P_H$
$S_H = F(R_L) = \alpha R_L^{\beta}$ where $0 \leq \alpha \leq ½$ and $\beta \geq 0$.

The versatility and potential for this example endogenous split award function can be first illustrated by looking at scenarios with various alternative values for α and β, as shown in Table 1. One feature of this functional form is that it is generalizable enough to represent, given the appropriate values for α and β, several common split award rules. As indicated in the table, this endogenous split award function equates to winner-take-all competition if either $\alpha = 0$ or $\beta = \infty$, and it equates to fixed award splits if $\beta = 0$. As a result, this award split function can represent both extreme cases, as well as intermediate cases. Furthermore, adjusting the values for α and β will influence vendor bidding strategies and (therefore) buyer costs, so the buyer must choose α and β to maximize utility (or, alternatively, minimize cost), inclusive of the costs and benefits of potential bid protests, follow-on procurements, and so on.

Table 1. Endogenous Split Award Scenarios With $S_H = \alpha R_L^{\beta}$



	$\beta = 0$	$0 < \beta < 1$	$\beta = 1$	$1 < \beta < \infty$	$\beta = \infty$
$\alpha = 0$	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**
$0 < \alpha < ½$	$S_H = \alpha$ **Fixed Split**	$0 \le S_H \le \alpha$ $\mathbf{S_H > \alpha R_L}$	$0 \le S_H \le \alpha$ $\mathbf{S_H = \alpha R_L}$	$0 \le S_H \le \alpha$ $\mathbf{S_H < \alpha R_L}$	$S_H = 0$ **Winner-Take-All**
$\alpha = ½$	$S_H = ½$ **Even Split**	$0 \le S_H \le ½$ $\mathbf{S_H > ½R_L}$	$0 \le S_H \le ½$ $\mathbf{S_H = ½R_L}$	$0 \le S_H \le ½$ $\mathbf{S_H < ½R_L}$	$S_H = 0$ **Winner-Take-All**

To further illustrate the dynamics of this functional form for determining endogenous award splits, Figure 7 shows how the award split accruing to the high-priced vendor varies for different values of both β and $R_L = P_L/P_H$, when $\alpha = 0.4$ (meaning that 40% is the maximum share awarded to the high-priced vendor).

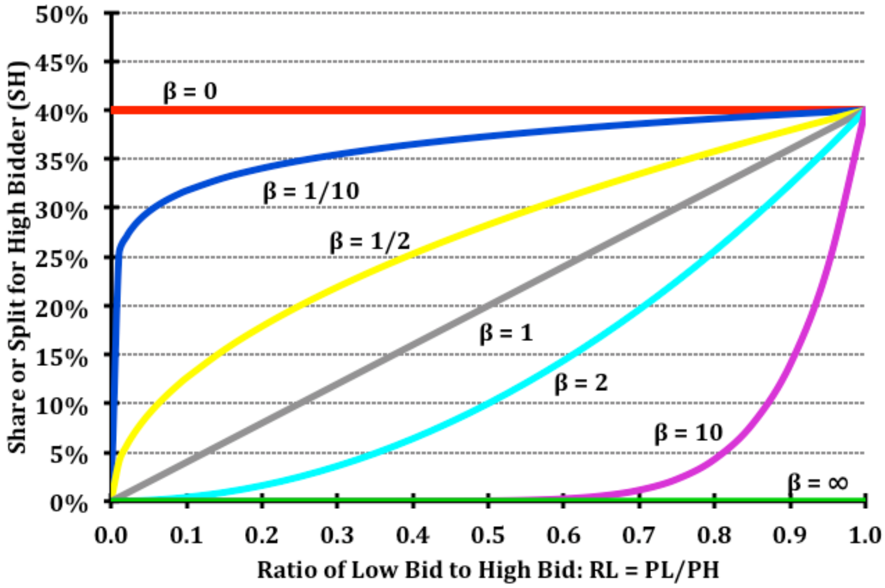


Figure 7. Split Award Scenarios With α = 0.4 and $S_H = 0.4R_L^{\beta}$

As illustrated in Figure 7, when β = 0, S_H becomes a fixed-split amount at the maximum share allowed under this scheme (40% when α = 0.4). When β **=** ∞, S_H equals zero in all cases, yielding a winner-take-all scheme. As P_L/P_H approaches one, where vendors have similar prices, the high-priced vendor’s share approaches its maximum possible value (again 40% when α = 0.4). On the other hand, as P_L/P_H approaches zero, signifying that the low-priced vendor is significantly less expensive than the high-priced vendor, the high-priced vendor’s share approaches zero.

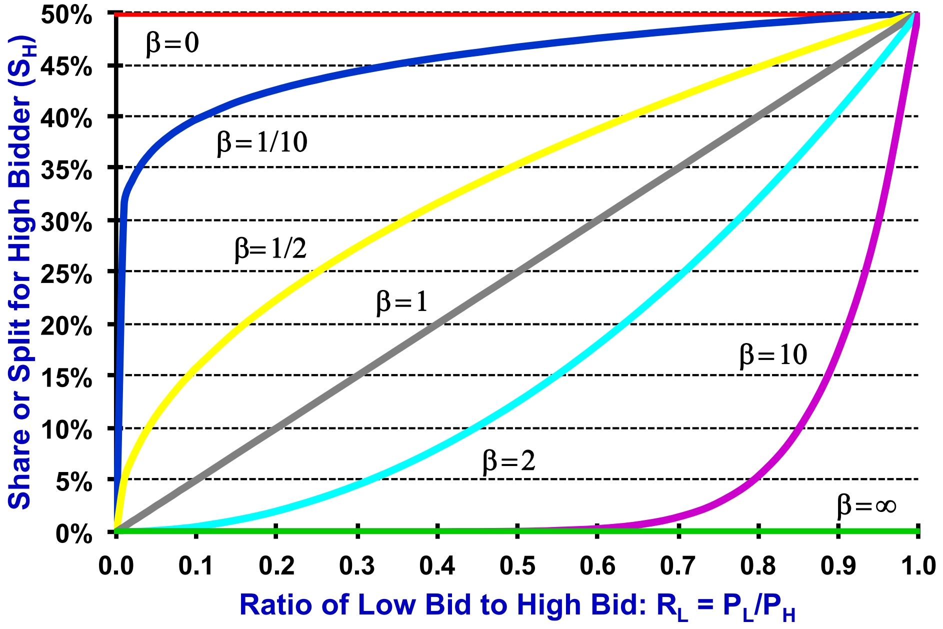


Figure 8. Split Award Scenarios With $\alpha = 0.5$ and $S_H = 0.5R_L^{\beta}$

Similar patterns emerge as α assumes values between 0 and 0.5, but the maximum value for S_H, depicted on the vertical axis, adjusts accordingly. For example, Figure 8 shows how the award split accruing to the high-priced vendor varies for different values of both β and $R_L = P_L/P_H$, when $\alpha = 0.5$ (meaning that 50% is the maximum share awarded to the high-priced vendor). The same dynamics are illustrated as in Figure 7, only this time the maximum value for SH is 50% instead of 40%.

Identifying Comparable Fixed and Endogenous Split Rules

So how would vendor bids and buyer cost differ under the example endogenous split rule ($S_H = \alpha R_L^{\beta}$) from bids and cost under a comparable fixed-split rule? We have already begun to answer this question qualitatively, using the intuition described in previous sections. In order to derive a more precise answer, however, the first challenge is to identify a fairly “comparable” pair of endogenous versus fixed-split rules. In other words, we must be certain we are making a reasonable “apples-to-apples” comparison between the two approaches.

For the fixed-split half of our comparison, let us choose *ex ante* splits of $S_H = 0.4$ and $S_L = 0.6$. Recall that, with $C_1, C_2 \sim U[0,100]$, the equilibrium bidding strategy under fixed splits is given by Equation 11:

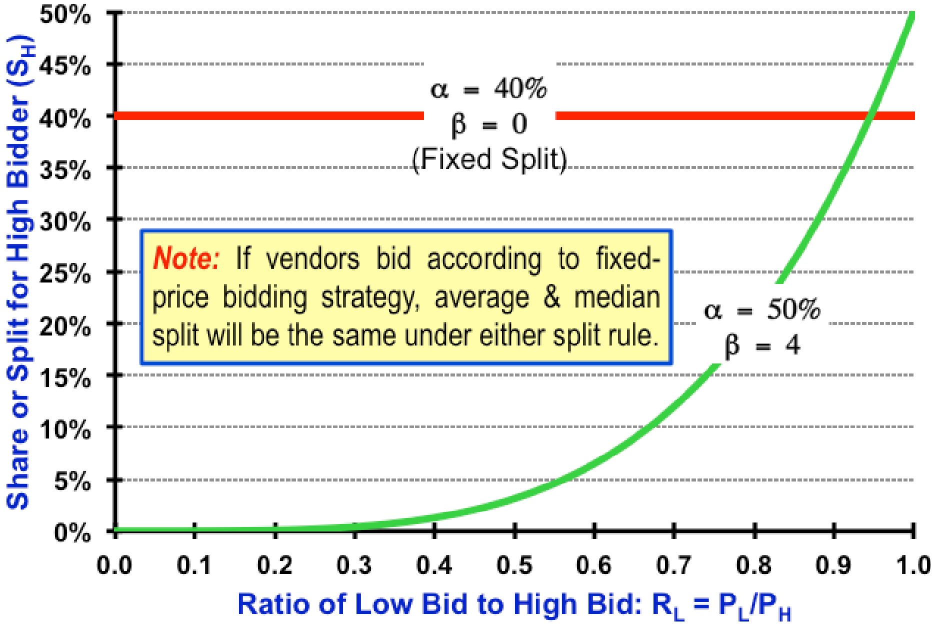
$$\lambda(C_i) = \frac{10{,}000 - (S_L - S_H)C_i^2}{200S_L - 2C_i(S_L - S_H)}. \qquad (11)$$

With $S_H = 0.4$ and $S_L = 0.6$, this equilibrium strategy for vendor bids becomes

$$\lambda(C_i) = \frac{10{,}000 - 0.2C_i^2}{120 - 0.4C_i}. \qquad (12)$$

Moreover, note that the equilibrium average buyer cost under such fixed splits was 94 per unit, as shown in Figure 6. Now we must only determine an endogenous split rule which is fairly comparable to fixed splits of $S_H = 0.4$ and $S_L = 0.6$.

In particular, what values of α and β will generate an endogenous split award function, $S_H = \alpha R_L^{\beta}$, which we can reasonably and fairly compare to our candidate fixed-split rule? We propose that the values $\alpha = ½$ and $\beta = 4$, yielding a split award function of $S_H = ½R_L^4$, provide just such a fair comparison. Figure 9 provides a graphical illustration of how award splits are determined under each rule for various values of $R_L = P_L/P_H$.



Note: If vendors bid according to fixed-price bidding strategy, average & median split will be the same under either split rule.

Figure 9. Split Award Scenarios With S_H = 40% vs. $S_H = ½R_L^4$

These rules certainly look a lot different in Figure 9, so how do we justify the validity of comparing outcomes under these two rules? To do so, we must show that, given the same vendor bidding under each approach, procurement outcomes (and thus costs) are the same or similar. In particular, using fixed- and endogenous split rules, which yield similar outcomes (and costs) given similar vendor bidding, allows us to isolate the differences between the two approaches that are specifically attributable to the differing effects on vendor bidding incentives (and not due to our specific choice of fixed- or endogenous split rules).

With that in mind, Appendix B illustrates the similarity (and comparability) of the fixed-split rule $S_H = 0.4$ and the endogenous split rule $S_H = ½R_L^4$. In particular, if both vendors continue to bid according to the fixed-split equilibrium bidding strategy given by Equation 11 our endogenous split rule $S_H = ½R_L^4$ will still yield both an average split for the high-priced vendor (i.e., an average value of $S_H = ½R_L^4$) equal to 0.4 and a median split for the high-priced vendor (i.e., a median value of $S_H = \frac{1}{2} R_L^4$) equal to 0.4 as well. Hence, our two split rules do indeed provide a reasonable apples-to-apples comparison between fixed and endogenous award splits, as they yield similar realized splits (and thus similar buyer costs) given similar vendor bids.

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Equilibrium Bidding and Cost Under Endogenous Award Splits

So how can we expect vendors to actually bid under the endogenous split function $S_H = ½R_L^4$? In the previous section, we demonstrated that, if vendors continue to follow the fixed-split bidding strategy for $S_H = 0.4$ under this endogenous split rule, the expected (average) and median values of the resulting split would still be $S_H = 0.4$. But is this same bidding strategy still optimal when splits are endogenous?

Unfortunately, determining a closed-form solution for the Nash equilibrium bidding strategy under the example endogenous split rule $S_H = \alpha R_L^\beta$ is particularly complicated and remains an objective for future research. That said, given particular values for α and β, we can calculate the equilibrium bidding strategy computationally (or numerically). We do this using an "iterative best-response" approach.

This iterative equilibrium calculation proceeds via the following steps:

1. Start by assuming vendor 1 follows the equilibrium bidding strategy under the fixed split SH = 0.4, which was given by Equation 12. Label this bidding strategy for vendor 1 as $\lambda_{1,1}(C)$, where the first subscript designates the vendor and the second subscript designates the iteration or step.
2. Next, for each possible vendor 2 cost (C_2) between 0 and 100 (using increments as small as computationally possible), compute vendor 2's optimal bid (or "best response") given vendor 1's bidding strategy $\lambda_{1,1}(C)$ and the endogenous split rule $S_H = ½R_L^4$. Combining all such vendor 2 optimal bids yields a best-response bidding strategy for vendor 2. Label this bidding strategy $\lambda_{2,2}(C)$.
3. Next, for each possible vendor 1 cost (C_1) between 0 and 100, compute vendor 1's optimal bid (or "best response") given vendor 2's bidding strategy $\lambda_{2,2}(C)$ and the endogenous split rule $S_H = ½R_L^4$. Combining all such vendor 1 optimal bids yields a best-response bidding strategy for vendor 1. Label this bidding strategy $\lambda_{1,3}(C)$.

⋮

N. Next, for each possible vendor 2 cost (C_2) between 0 and 100, compute vendor 2's optimal bid (or "best response") given vendor 1's bidding strategy $\lambda_{1,N-1}(C)$ and the endogenous split rule $S_H = ½R_L^4$. Combining all such vendor 2 optimal bids yields a best-response bidding strategy for vendor 2. Label this bidding strategy $\lambda_{2,N}(C)$.

N+1. Next, for each possible vendor 1 cost (C_1) between 0 and 100, compute vendor 1's optimal bid (or "best response") given vendor 2's bidding strategy $\lambda_{2,N}(C)$ and the endogenous split rule $S_H = ½R_L^4$. Combining all such vendor 1 optimal bids yields a best-response bidding strategy for vendor 1. Label this bidding strategy $\lambda_{1,N+1}(C)$.

We continue this iterative process until reaching a "fixed-point" best-response bidding strategy. In other words, the equilibrium bidding strategy is identified when $\lambda_{i,n-1}(C) \approx \lambda_{j,n}(C) \approx \lambda_{i,n+1}(C)$ for some n > 1.

The equilibrium outcome of this iterative best-response computation process for our chosen endogenous split rule $S_H = ½R_L^4$ is illustrated by the blue line in Figure 10. This figure shows that the equilibrium bids under this endogenous split rule are significantly lower than the equilibrium bids under the comparable fixed-split rule $S_H = 0.4$ (red line in Figure 10). Moreover, the equilibrium bids under this endogenous split rule are actually closer (for most vendor costs) to the equilibrium bids under winner-take-all competition (green line in Figure 10), which is the least-cost alternative (in terms of initial procurement cost).

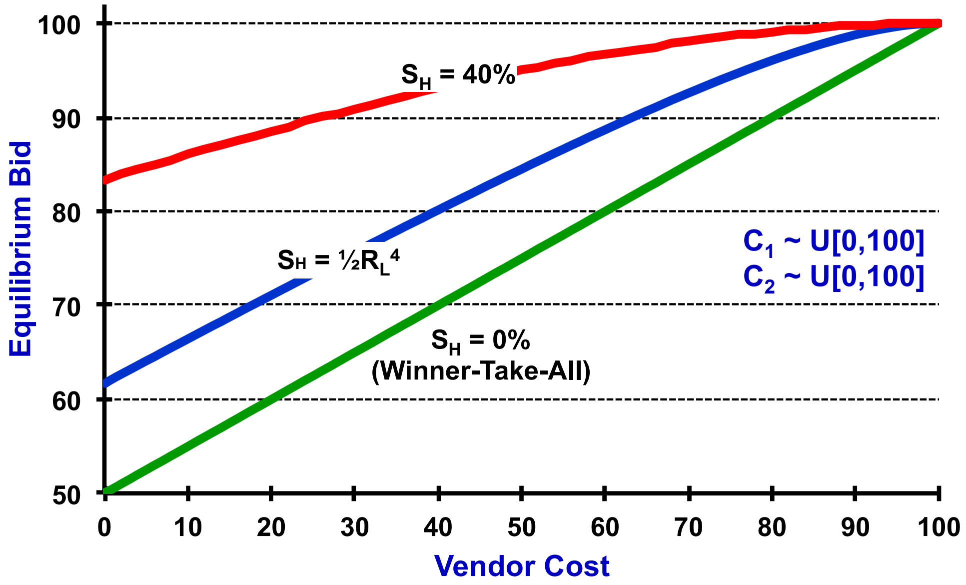

Figure 10. Equilibrium Bidding Strategies With S_H = 0% vs. S_H = 40% vs. S_H = ½R_L^4

With the equilibrium bidding strategy under the rule S_H = ½R_L^4 thus derived, we can now calculate the average cost per unit for the buyer under this endogenous split rule. The results (and comparisons to fixed-split and winner-take-all competition) are shown in Figure 11.

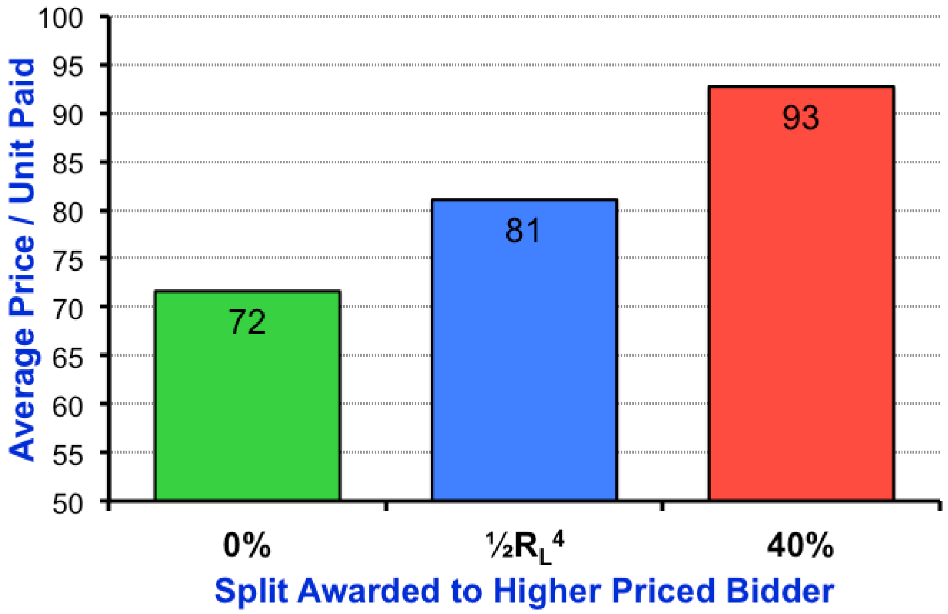

Figure 11. Average Price per Unit Under Various Award Splits

We have previously discussed (see Figure 6) how the buyer's average unit cost was calculated for fixed splits and for winner-take-all competition. For the case of the endogenous split rule $S_H = ½R_L^4$, the average cost was calculated via simulation (or computation).

The essential elements of this simulation were as follows:

1. Costs for vendor 1 and vendor 2 (C_1 and C_2) were randomly drawn from a uniform distribution over the range 0 to 100.
2. The computed equilibrium bidding function (illustrated by the blue line in Figure 10) was then used to determine the combination of bids (P_1 and P_2) associated with the two vendor costs drawn in Step 1.
3. From the vendor bids calculated in Step 2, the award split was determined using the rule $S_H = ½R_L$, where $R_L = P_L/P_H$.
4. Average buyer cost was then calculated using the following formula: Average Buyer Cost = $S_H \times P_H + S_L \times P_L = S_H \times P_H + (1 – S_H) \times P_L = P_L + (P_H - P_L) \times S_H$

Repeating Steps 1 through 4 for hundreds of iterations produces an average buyer cost of approximately 81 per unit under the endogenous split rule $S_H = ½R_L$.

Contrast this with the average cost per unit of 94 under the fixed-split rule $S_H = 0.4$ and the average cost per unit of 67 under winner-take-all competition.

In sum, moving from winner-take-all competition to split procurement with fixed splits increases initial procurement cost for the buyer. Moving from fixed award splits to endogenous award splits, however, eliminates about half of this increased buyer cost, while retaining the other benefits of split award procurement, such as mitigating or filtering bid protests, retaining competition for follow-on procurements, and so on.

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Conclusions

This research has applied game-theoretic modeling and computational analysis to investigate two common elements of the procurement process: bid protests and split-award contracts. More directly, we have explored the interaction between these two procurement features, asking how split-award contracts might be used to manage bid protests.

On the topic of bid protests in general, it is important to emphasize that the objective of any procurement reform in this area should be to manage, not minimize, the number and nature of bid protests. In particular, the purchasing organization or agency would like to encourage efficient protests that serve to identify, address, and potentially correct significant procurement mistakes, while discouraging those "frivolous" or inefficient bid protests that do not offer the potential to make significant, valuable corrections.

This research has further shown that split-award contracts offer one potential level for managing bid protests. The use of split awards, as opposed to winner-take-all contract awards, reduces the magnitude of potential gain from successful protest. Hence, with split-award contracts, the losing bidder has less incentive to file a protest, and will, therefore, only do so when the probability of prevailing in the protest process is very high. In other words, the presence of split awards helps to discourage those wasteful protests having little chance of success, while preserving the incentive to file efficient protests in which it seems very clear (at least to the losing bidder) that a significant procurement mistake has been made. This is exactly the type of protest "filtering" that we would like to see from any proposed reform.

Thus, split-award contracts may offer benefits in terms of protest management. This is in addition to other potential benefits previously demonstrated in the literature, including reducing the buyer's dependence on any particular vendor during the initial procurement, preserving competition for follow-on procurement, and attracting more vendors to compete for the contract.

That said, we also demonstrated as part of the research that split-award contracts have a disadvantage (relative to “winner-take-all” contracts), which is higher total procurement cost for the buyer. The buyer’s cost under split-award contracts is higher for two reasons: (1) A portion of the contract is procured from a higher priced vendor, and (2) the split awarded to a higher price vendor increases the incentive for vendors to inflate their bids. In the case of fixed-award splits (i.e., where the splits awarded to the low-price and high-price vendor are determined *ex ante*), we showed that the additional cost incurred by the buyer (relative to winner-take-all contract competition) can be significant.

Fortunately, however, our research uncovered and illustrated a potential (partial) solution to this problem of increased buyer cost: Endogenous award splits, which are determined *ex post*, based on the actual bids submitted, with the split awarded to the higher priced vendor being larger the more competitive this “losing” vendor’s bid is. When award splits are determined in this fashion, we demonstrated that the increase in buyer cost (over winner-take-all competition) is significantly mitigated.

In sum, split-award contracts offer a potential lever for efficient management of bid protests, as well as other potential benefits. Moreover, while split-awards do involve higher buyer costs, this disadvantage can be effectively mitigated through the use of endogenously determined award splits.

Research Agenda Moving Forward

Our research agenda moving forward on this project first and foremost includes several extensions or generalizations to our model of the bid protest and contract competition process. For example, we hope to expand the number of competing vendors in the model as well as expanding the number of contract splits available to the competitors. In particular, it would be helpful to investigate scenarios in which the number of competitors exceeds the number of contract splits available, to see whether this mitigates the bid-inflation effect.

Expanding the number of contract splits beyond two will, of course, require investigation of more complex split functions, which endogenously determine how a contract would be divided among three or more winning vendors, based on their submitted bids. Moreover, the split function might even endogenously determine the number of contract splits, such that whether there will be two, three, four, or more splits is not known until after bids have been submitted.

Another natural extension to the model would be the inclusion of economies of scale. For many procured products or services, vendors will experience significant economies of scale in production, primarily due to the necessity of incurring significant fixed costs that are independent of the volume produced. Such economies of scale actually create another important cost consideration (and disadvantage) for split-award contracts: A single vendor could produce the entire procurement quantity at a lower cost than could multiple vendors each producing only a portion of this total quantity. It is natural and important, therefore, to incorporate such economies of scale in any model investigating split-award contracting. The easiest way to do so would be to incorporate a fixed-cost element into each vendor's cost structure. For example, vendor j's total cost to produce a quantity X could be given by $C_j(X) = F_j + X \times V_j$, where F_j represents vendor j's fixed production cost and V_j represent vendor j's variable production cost per unit.

Vendors could then submit two-dimensional bids, consisting of a fixed-bid component and a variable-bid component.

A final, but critical, model extension would be to allow for repeated procurements. Considering that one of the primary arguments in favor of split-award contracts is the preservation of competition for follow-on contracts, it is again both natural and important to incorporate repeated competitive procurements involving the same set of (potential) vendors. Any such repeated game model of procurement would need to include inter-temporal effects such as experience or learning (production costs declining over time as cumulative historical production increases) as well as possible innovation (production costs declining due to innovation investment and/or random shocks). A repeated procurement model would further incorporate a discount factor, representing the lower net present value of future costs and revenues, and/or the likelihood that procurement of that particular product or service could terminate after that particular iteration of the game. With such discounting of future procurements, the entire repeated game could presumably be solved via recursive economic methods.

With our model of the bid protest and contract competition process extended and/or generalized as described here, a number of important research questions become more validly addressed. First and foremost, we would like to determine or provide some guidance as to (1) under what conditions split-award procurement is preferred to winner-take-all procurement competition; and (2) when split-award procurement is indeed preferred, what is the optimal endogenous split function, taking into account (a) the buyer's objective to maximize long-term expected utility (or minimize cost), (b) including the cost of bid protests as well as the corrective benefits of such protests, and (c) also including the impact of other benefits of split-award contract competition. We also intend to explore model dynamics, comparative statics, and sensitivity analyses in which we determine the impact of changes in key variables, such as vendor and buyer information, the costs of protest, and so on.

We hope to explore all of these questions with the unique combination of research methodologies we utilize within our overall research agenda of applying the field of mechanism design to the context of defense procurement. These methodologies include game-theoretic analysis and dynamics, numerical computation and simulation, as well as laboratory experimentation.

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Appendix A. Optimal Bidding Strategy With Fixed-Split Awards

Assume that the procurement being considered is a one-shot procurement with no follow-on options. Also, assume that the buyer is perfectly informed regarding the vendors' prices P_1 and P_2 (i.e., $P_i = P_i^T$). With perfect information, there is no need for bid protests. The bidding strategy model can be written as follows:

C_i	Cost for vendor i
P_i	True bid price for vendor i
X	Quantity purchased
$X(P_i)$	Revenues for vendor i; buyer's cost
$X(C_i)$	Production costs for vendor i
S_L	Contract share or split awarded the low-price vendor
S_H	Contract share or split awarded the high-price vendor
	where: $S_L + S_H = 1;\ 0 \leq S_H \leq ½;\ ½ \leq S_L \leq 1$

For simplicity, assume two vendors, though the model can be easily generalized to multiple vendors. A vendor's expected profit becomes

$$
\begin{aligned}
E\Pi_1(P_1) \quad &= X(P_1 - C_1)(\mathrm{Prob}(P_1 > P_2)S_H + \mathrm{Prob}(P_1 < P_2)S_L) \\
&= X(P_1 - C_1)(\mathrm{Prob}(P_1 > P_2)S_H + (1 - \mathrm{Pr}(P_1 > P_2))S_L) \\
&= X(P_1 - C_1)(S_L + \mathrm{Prob}(P_1 > P_2)(S_H - S_L)) \\
&= X(P_1 - C_1)(S_L - \mathrm{Prob}(P_1 > P_2)(S_L - S_H)).
\end{aligned}
$$

Assume C_1 and C_2 are identically and independently distributed over the interval [0,M], with distribution function F(C) and density function $f(C) \equiv F'(C)$. Also, assume both vendors follow symmetric bidding strategies $\lambda(C)$, where

$$
\begin{aligned}
&\lambda(C): [0,M] \rightarrow [0,M] \\
&\lambda(M) = M.
\end{aligned}
$$

The optimal bid for vendor 1, assuming vendor 1 has cost C_1 and vendor 2 is bidding according to strategy $\lambda(C_2)$, can be calculated as follows:

$\mathrm{Prob}(P_2 < P_1) = \mathrm{Prob}(\lambda(C_2) < P_1) = \mathrm{Prob}(C_2 < \lambda^{-1}(P_1)) = F(\lambda^{-1}(P_1))$.

$$\begin{aligned} E\Pi 1(P1) &= X(P_1 - C_1)(S_L - \mathrm{Prob}(P_1 > P_2)(S_L - S_H)) \\ &= X(P_1 - C_1)(S_L - F(\lambda^{-1}(P_1))(S_L - S_H)). \end{aligned}$$

Using the Chain Rule and the inverse derivate theorem yields

$$\begin{aligned} \partial E\Pi 1/\partial P1 &= X(S_L - F(\lambda^{-1}(P_1))(S_L - S_H)) \\ &= X(P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1)) / \lambda'(\lambda^{-1}(P_1)). \end{aligned}$$

The first-order condition for profit maximization is

$$\begin{aligned} &S_L - F(\lambda^{-1}(P_1))(S_L - S_H) = (P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1))/ \lambda'(\lambda^{-1}(P_1)) \\ &\lambda'(\lambda^{-1}(P_1))[S_L - F(\lambda^{-1}(P_1))(S_L - S_H)] = (P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1)) \end{aligned}$$

Assuming a symmetric equilibrium, $P_1 = \lambda(C_1) \Rightarrow \lambda^{-1}(P_1) = C_1$, thus

$$\begin{aligned} &\lambda'(C_1)[S_L - F(C_1)(S_L - S_H)] = (\lambda(C_1) - C_1)(S_L - S_H)f(C_1) \\ &S_L\lambda'(C_1) = (S_L - S_H))[F(C_1)\lambda'(C_1) + \lambda(C_1)f(C_1) - C_1f(C_1)] \\ &(S_L - S_H)[F(C_1)\lambda'(C_1) + f(C_1)\,\lambda(C_1)] = S_L\lambda'(C_1) + C_1(S_L - S_H)f(C_1) \\ &(S_L - S_H)\frac{\partial}{\partial C_1})[F(C_1)\lambda(C_1) = S_L\lambda'(C_1) + C_1(S_L - S_H)f(C_1). \end{aligned}$$

Integrating both sides of this equation over the definite interval $[C_1,M]$ yields

$$(S_L - S_H)\int_{C_1}^{M}\frac{\partial}{\partial C_1}\left[F(C_1)\lambda(C_1)\right]dC_1 = S_L\int_{C_1}^{M}\lambda'(C_1)dC_1 + (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1$$

$$(S_L - S_H)\left[F(M)\lambda(M) - F(C_1)\lambda(C_1)\right] = S_L\left[\lambda(M) - \lambda(C_1)\right] + (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1$$

$$(S_L - S_H)\left[M - F(C_1)\lambda(C_1)\right] = S_L\left[M - \lambda(C_1)\right] + (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1$$

$$S_LM - S_HM - (S_L - S_H)F(C_1)\lambda(C_1) = S_LM - S_L\lambda(C_1) + (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1$$

$$-S_HM - (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1 = \left(S_L - S_H\right)F(C_1)\lambda(C_1) - S_L\lambda(C_1)$$

$$S_HM + (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1 = S_L\lambda(C_1) - \left(S_L - S_H\right)F(C_1)\lambda(C_1)$$

$$S_HM + (S_L - S_H)\int_{C_1}^{M}C_1f(C_1)dC_1 = \lambda(C_1)\left[S_L\left(1 - F(C_1)\right) + S_HF(C_1)\right]$$

$$S_H M + (S_L - S_H)\left(1 - F(C_1)\right)\frac{\int_{C_1}^{M} C_1 f(C_1)\,dC_1}{1 - F(C_1)} = \lambda(C_1)\left[S_L - S_L F(C_1) + S_H F(C_1)\right]$$

$$S_H M + (S_L - S_H)\left(1 - F(C_1)\right)E\left(C_2 \middle| C_2 > C_1\right) = \lambda(C_1)\left[S_L - \left(S_L - S_H\right)F(C_1)\right]$$

$$\lambda(C_1) = \frac{S_H M + (S_L - S_H)\left(1 - F(C_1)\right)E\left(C_2 \middle| C_2 > C_1\right)}{S_L - \left(S_L - S_H\right)F(C_1)}$$

λ(C_1) provides the symmetric equilibrium bidding strategy whenever vendor costs C_1 and C_2 are identically and independently distributed over some interval [0,M].

As further illustration and exploration, consider the case when vendor unit costs are uniformly distributed over the range from zero to 100. In this case, the equilibrium bidding strategy in equation (5) becomes:

$$\lambda(C_1) = \frac{S_H M + (S_L - S_H)\left(1 - F(C_1)\right)E\left(C_2 \middle| C_2 > C_1\right)}{S_L - (S_L - S_H)F(C_1)}$$

$$\lambda(C_1) = \frac{100S_H + (S_L - S_H)\left(1 - \tfrac{1}{100}C_1\right)\tfrac{1}{2}\left(C_1 + 100\right)}{S_L - \tfrac{1}{100}(S_L - S_H)C_1}$$

$$\lambda(C_1) = \frac{20{,}000S_H + (S_L - S_H)\left(100 - C_1\right)\left(C_1 + 100\right)}{200S_L - 2(S_L - S_H)C_1}$$

$$\lambda(C_1) = \frac{20{,}000S_H + \left(1 - 2S_H\right)\left(10{,}000 - C_1^2\right)}{200S_L - 2C_1(S_L - S_H)}$$

$$\lambda(C_1) = \frac{20{,}000S_H + 10{,}000 - C_1^2 - 20{,}000S_H + 2S_H C_1^2}{200S_L - 2C_1(S_L - S_H)}$$

$$\lambda(C_1) = \frac{10{,}000 + (2S_H - 1)C_1^2}{200S_L - 2C_1(S_L - S_H)} = \frac{10{,}000 - (S_L - S_H)C_1^2}{200S_L - 2C_1(S_L - S_H)}$$

Thus, the symmetric Nash equilibrium bidding strategy for vendor j (j = 1 or 2) as a function of the fixed award splits S_L and S_H in this simplified case, is given by:

$$\lambda(C_j) = \frac{10{,}000 - (S_L - S_H)C_j^2}{200S_L - 2C_j(S_L - S_H)} \qquad (6)$$

Q.E.D.

Appendix B. Optimal Bidding Strategy With Fixed-Split Awards

In this appendix, we explain how we determined that the endogenous split rule $S_H = ½R_L^4$ was fairly comparable to the fixed split rule $S_H = 0.40$. In particular, to provide a “fair” comparison, we wanted a endogenous split rule that would yield similar results to the fixed split rule, under identical bidding strategies.

Because the fixed split rule always yields a predetermined split, $S_H = 0.40$ in our example here, we wanted to verify that our chosen endogenous split rule would generate similar splits given identical bidding strategies. However, an endogenous split rule will generate a different split for different sets of bids. Therefore, the best measure of comparability was to calculate the median and/or average split generated by the chosen endogenous split rule, assuming identical bidding to the derived equilibrium bid strategy with a fixed split of $S_H = 0.40$.

Thus, we looked at all possible combinations of vendor costs C_1 and C_2 (for all integer values between 0 and 100) and then determined the combination of equilibrium bids (or prices) using the previously calculated symmetric equilibrium bidding strategy for a fixed split of $S_H = 0.40$ as follows:

$$P_1 = \lambda(C_1) = \frac{10{,}000 - 0.2C_1^2}{120 - 0.4C_1} \quad \text{and} \quad P_2 = \lambda(C_2) = \text{ß}\frac{10{,}000 - 0.2C_1^2}{120 - 0.4C_1}$$

For each possible combination of prices generated this way, we then calculated the ratio of low bid price (P_L) to high bid price (P_H) given by $R_L = P_L/P_H$. With this ratio, we then calculated the endogenous split $S_H = ½R_L^4$ for all possible combinations of vendor costs C_1 and C_2 (for all integer values between 0 and 100).

Looking at the distribution of realized splits under the rule $S_H = ½R_L^4$ for all these cost combinations, we find an average value of S_H equal to 0.40 and a median value of S_H equal to 0.40. Thus, we conclude that the endogenous split rule $S_H = \frac{1}{2}R_L^4$ provides a reasonable comparison for the fixed split rule $S_H = 0.40$.

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