1981-03

An introduction to a reliability shorthand

Repicky, John J., Jr.

Monterey, California; Naval Postgraduate School

https://hdl.handle.net/10945/20569

AN INTRODUCTION TO A RELIABILITY SHORTHAND

John J. Repicky, Jr.

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN INTRODUCTION TO A RELIABILITY SHORTHAND

by

John J. Repicky, Jr.

March 1981

Thesis Advisor: James D. Esary

Approved for public release, distribution unlimited

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	
2. GOVT ACCESSION NO.	
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle)	An Introduction to a Reliability Shorthand
5. TYPE OF REPORT & PERIOD COVERED	Master's Thesis March 1981
6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s)	John Joseph Repicky, Jr.
8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS	Naval Postgraduate School Monterey, California 93940
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
11. CONTROLLING OFFICE NAME AND ADDRESS	Naval Postgraduate School Monterey, California 93940
12. REPORT DATE	March 1981
13. NUMBER OF PAGES	44
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	
15. SECURITY CLASS. (of this report)	Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

reliability
survival function
reliability shorthand
system survival
systems
redundant systems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The determination of a system's life distribution usually requires the synthesis of a mixture of system survival modes. In order to alleviate the normal non-trivial calculations, this paper presents the concept of a reliability shorthand.

After describing the possible ways a system can survive a mission, the practitioner of this shorthand can use stock formulas to obtain a system's survival function. Then simple insertion of the failure rates of the system's components into the known equations results in the system's reliability.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601 |

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

em 20, continued:

Simple examples show the convenience of this shorthand. The Ti-59 is monstrated to be a useful tool, adequate to implement the methodology.

An Introduction to a Reliability Shorthand

by

John J. Repicky, Jr.
Lieutenant Commander, United States Navy
B.S., United States Naval Academy, 1971

Submitted in partial fulfillment of the
requirement for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1981

ABSTRACT

The determination of a system's life distribution usually requires the synthesis of a mixture of system survival modes. In order to alleviate the normal non-trivial calculations, this paper presents the concept of a reliability shorthand.

After describing the possible ways a system can survive a mission, the practitioner of this shorthand can use stock formulas to obtain a system's survival function. Then simple insertion of the failure rates of the system's components into the known equations results in the system's reliability.

Simple examples show the convenience of this shorthand. The Ti-59 is demonstrated to be a useful tool, adequate to implement the methodology.

TABLE OF CONTENTS

<u>LIST OF FIGURES</u>

I. INTRODUCTION

It is generally accepted that the reliability of a system is the probability that the system will operate adequately for a given period of time in its intended application. The determination of a system's life distribution usually requires the synthesis of a mixture of modes in which the system can survive. One can assuredly state that the calculations can be non-trivial.

This paper will present the concept of a reliability shorthand which can greatly simplify the degree of mathematical difficulty usually encountered in determining the reliability of a system. After describing the possible ways a system can survive a mission, the practitioner of this reliability shorthand methodology can specialize a standard formula to obtain a system's survival function. Then simple insertion of the failure rates of the system's components into known preformulated equations results in the system's reliability.

The convenience of this methodology is demonstrated through several simple examples. The reliability shorthand for many systems is catalogued in Appendix A as a ready reference. In Appendix B is a Ti-59 program which allows for the easy calculation of a system's reliability for two general cases of the shorthand methodology.

The concept of a reliability shorthand was first introduced in the Operations Research course 0A4662, 'Reliability and Weapons System Effectiveness Measurement'. The concept has evolved with each presentation of the course. It is hoped this paper will be a beneficial tutorial aid for the students of that course, and act as an introduction to the topic for the interested reader.

II. RELIABILITY SHORTHAND

As a convenient shorthand we will use the convention that the expression $EXP(\lambda_1) + EXP(\lambda_2)$ denotes the distribution for a random variable $T_1 + T_2$, where T_1, T_2 are independent, T_1 has an $EXP(\lambda_1)$ distribution, and T_2 has an $EXP(\lambda_2)$ distribution. The life distribution of many systems can be usefully described using this shorthand.

In the following examples we typically suppose that the components of the systems fail independently and have exponential life distributions.

A. A SYSTEM HAVING TWO ACTIVE COMPONENTS IN SERIES

A two component series system functions if, and only if, both active components, A_1 and A_2, function. The life of the system, T, would be the minimum of the two component lives, $T = \min(T_1, T_2)$.

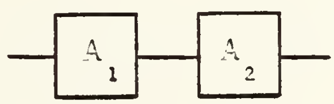


FIGURE 1: TWO ACTIVE COMPONENTS IN SERIES

We will assume $T_1 \sim EXP(\lambda_1)$, $T_2 \sim EXP(\lambda_2)$, and T_1, T_2 are independent. The system's survival function is

$$\begin{aligned}\bar{F}(t) &= P(T>t) \\ &= P(\min(T_1, T_2)>t) \\ &= P(T_1>t, T_2>t).\end{aligned}$$

Using the assumptions of independence and components having exponential life distributions we obtain

$$\begin{aligned}\bar{F}(t) &= P(T_1 > t)\, P(T_2 > t) \\ &= \bar{F}_1(t)\, \bar{F}_2(t) \\ &= e^{-\lambda_1 t} e^{-\lambda_2 t} \\ &= e^{-(\lambda_1 + \lambda_2)t}.\end{aligned}$$

The life distribution of the system is $T \sim EXP(\lambda_1 + \lambda_2)$.

When $\lambda_1 = \lambda_2 = \lambda$, then $\bar{F}(t) = e^{-2\lambda t}$ and $T \sim EXP(2\lambda)$. Our shorthand notation $EXP(2\lambda)$ represents the life distribution of a system where two identical components must both function for the system to survive.

B. A STANDBY SYSTEM HAVING ONE ACTIVE AND ONE SPARE COMPONENT

An active component, A, is to complete a mission of duration t. A spare component, S, is available to automatically replace the active component should it fail. The life of the active component is T_1. The life of the spare component is T_2. The life of the system is $T = T_1 + T_2$.

In determining the survival function of this system, we first describe how the system can survive to successfully complete a mission of duration t. Component A can live to time t with the spare never being utilized, or component A can fail at some intermediate time s. Then the spare component automatically replaces the failed component, and component S must live from time s to time t to successfully complete the mission.

With T_1, T_2 independent, the survival function of the system can be represented as:

$$\bar{F}(t) = \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s)\, f_1(s)ds,$$

where $\bar{F}_1(t)$ is the probability of component A living to time t, $f_1(s)$ is the likelihood that component A fails at some time s, and $\bar{F}_2(t-s)$ is the probability that component S lives from time s to time t.

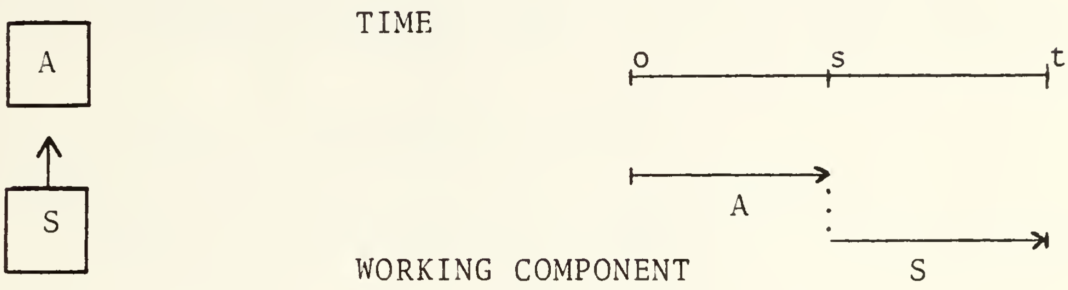


FIGURE 2: A SINGLE ACTIVE COMPONENT WITH ONE SPARE COMPONENT

1. Identical Components

If the active and spare components are identical, then $T_1 \sim EXP(\lambda)$, $T_2 \sim EXP(\lambda)$, T_1, T_2 are independent, and $T = T_1 + T_2$. The survival function is now expressed as

$$\begin{aligned}\bar{F}(t) &= e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)}(\lambda e^{-\lambda s})ds \\ &= e^{-\lambda t} + \int_0^t e^{-\lambda t} e^{\lambda s} \lambda e^{-\lambda s} ds \\ &= e^{-\lambda t} + e^{-\lambda t}\int_0^t \lambda\, ds \\ &= e^{-\lambda t} + e^{-\lambda t}(\lambda t) \\ &= (1 + \lambda t)e^{-\lambda t}.\end{aligned}$$

The shorthand notation for this survival function is EXP(λ) + EXP(λ). Visualize this as a system having one EXP (λ) component, and upon that component's failure the system

has a completely new and identical EXP(λ) component because of the spare.

2. Dissimilar Components

If the active and spare components are dissimilar, then $T_1 \sim EXP(\lambda_1)$, $T_2 \sim EXP(\lambda_2)$, T_1, T_2 are independent, and $T = T_1 + T_2$. The formulation of the survival function for this system is identical to the case of similar components, except for the change in failure rates. The survival function is

$$\begin{aligned}
\bar{F}(t) &= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_1 e^{-\lambda_1 s} ds \\
&= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 t} e^{\lambda_2 s} \lambda_1 e^{-\lambda_1 s} ds \\
&= e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} \int_0^t e^{-(\lambda_1-\lambda_2)s} ds \\
&= e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} \left(\frac{1}{\lambda_1-\lambda_2}\right) \int_0^t (\lambda_1-\lambda_2) e^{-(\lambda_1-\lambda_2)s} ds \\
&= \frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_2} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2 t} (1-e^{-(\lambda_1-\lambda_2)t}) \\
&= \frac{(\lambda_1-\lambda_2) e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} (1-e^{-(\lambda_1-\lambda_2)t})}{\lambda_1-\lambda_2} \\
&= \frac{\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_2 t} e^{-(\lambda_1-\lambda_2)t}}{\lambda_1-\lambda_2} \\
&= \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1-\lambda_2} \quad .
\end{aligned}$$

The shorthand notation for this survival function is $EXP(\lambda_1) + EXP(\lambda_2)$. As the active component fails a new component takes its place to complete the same task, however, the new component has a different failure rate than that of the initial component.

C. A SYSTEM HAVING TWO ACTIVE COMPONENTS IN PARALLEL

A two component parallel system functions if, and only if, at least one component functions. The life of the system, T, would be the maximum of the two component lives, $T = \max(T_1, T_2)$.

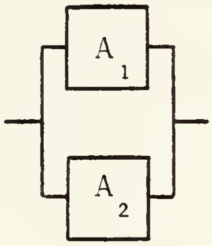


FIGURE 3: TWO ACTIVE COMPONENTS IN PARALLEL

Now assume $T_1 \sim EXP(\lambda_1)$, $T_2 \sim EXP(\lambda_2)$, and T_1, T_2 are independent. The survival function of the parallel system is

$$\begin{aligned}\bar{F}(t) &= P(\max(T_1, T_2) > t) \\ &= 1 - P(\max(T_1, T_2) \le t) \\ &= 1 - P(T_1 \le t, T_2 \le t).\end{aligned}$$

Using the assumption of independence

$$\begin{aligned}\bar{F}(t) &= 1 - [P(T_1 \le t)\, P(T_2 \le t)] \\ &= 1 - [(1 - \bar{F}_1(t))\, (1 - \bar{F}_2(t))] \\ &= 1 - [1 - \bar{F}_1(t) - \bar{F}_2(t) + \bar{F}_1(t)\bar{F}_2(t)] \\ &= \bar{F}_1(t) + \bar{F}_2(t) - \bar{F}_1(t)\, \bar{F}_2(t).\end{aligned}$$

Using the assumption that the components have exponential life distributions, the resulting life distribution is

$$\bar{F}(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}.$$

When $\lambda_1 = \lambda_2 = \lambda$, the survival function is

$$\bar{F}(t) = e^{-\lambda t} + e^{-\lambda t} - e^{-(\lambda + \lambda)t} = 2e^{-\lambda t} - e^{-(\lambda_1 + \lambda_2)t}.$$

The life of the parallel system begins with both active components functioning together for system survival. The time until one of the components fails has the distribution EXP(λ). When one of the components fails, the memoryless property of the exponential distribution provides that the surviving component has an additional EXP(λ) life with which to complete the mission. The shorthand notation for the survival function of the simple parallel system of identical components is EXP(2λ) + EXP(λ).

Now we will demonstrate the ease of using the reliability shorthand, compared to alternative calculations for determining a system's reliability. Recall that EXP(λ_1) + EXP(λ_2) is the shorthand notation for the survival function

$$\bar{F}(t) = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1 - \lambda_2}.$$

Noting that the parallel system is described by EXP(2λ) + EXP(λ), we can see the simplicity of substituting 2λ for λ_1 and λ for λ_2 into the known survival function equation. The resulting survival function is

$$\bar{F}(t) = \frac{(2\lambda)e^{-(\lambda)t} - (\lambda)e^{-(2\lambda)t}}{(2\lambda) - (\lambda)}$$

$$= \frac{\lambda(2e^{-\lambda t} - e^{-2\lambda t})}{\lambda}$$

$$= 2\, e^{-\lambda t} - e^{-2\lambda t}.$$

The survival functions are equivalent using either method, however, the shorthand methodology uses merely substitution and simple mathematics.

D. A STANDBY SYSTEM HAVING TWO ACTIVE COMPONENTS IN SERIES WITH ONE SPARE COMPONENT

Consider a system which has two identical components in series with a similar component as a standby spare which automatically replaces the first component that fails.

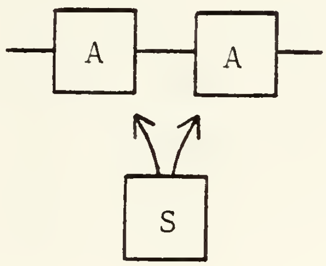


FIGURE 4: TWO ACTIVE COMPONENTS IN SERIES WITH ONE SPARE COMPONENT

The system has component times to failure $T_1 \sim EXP(\lambda)$, $T_2 \sim EXP(\lambda)$ and spare component time to failure $T_3 \sim EXP(\lambda)$, with T_1, T_2, T_3 independent. This system can complete its mission of duration t in two possible ways. It can survive if the original components live to time t and the spare component is never needed. Alternatively, one of the active components could fail at some intermediate time s, causing the system to fail. At that time the surviving component is like new and the spare component replaces the failed component creating a brand new series system to complete the mission from time s to time t.

In determining the system's survival function using reliability shorthand, we recall that a two component series system has an EXP(2λ) life distribution. With the spare component replacement the system accomplishes the task as if it had two independent series systems to function consecutively. The shorthand notation is simply EXP(2λ) + EXP(2λ).

Recall that the shorthand notation EXP(λ) + EXP(λ) represents the life distribution where the survival function is $\overline{F}(t) = (1 + \lambda t)e^{-\lambda t}$. To determine the survival function of EXP(2λ) + EXP(2λ) we substitute 2λ for λ into the known formula and obtain

$$\overline{F}(t) = (1 + 2\lambda t)e^{-2\lambda t}.$$

The usual method of determining the survival function is slightly more involved. The system can survive if the original series system lives to time t with no spare required. If one of the original components fails at some intermediate time s, then the spare component and the surviving component combine as a new series system. Both of the components of the new series system must live from time s to time t for the system to complete the mission. We formulate the survival function as

$$\begin{aligned}\overline{F}(t) &= e^{-2\lambda t} + \int_0^t e^{-\lambda(t-s)}e^{-\lambda(t-s)}2\lambda e^{-2\lambda s}ds \\ &= e^{-2\lambda t} + \int_0^t e^{-\lambda t}e^{\lambda s}e^{-\lambda t}e^{\lambda s}2\lambda e^{-2\lambda s}ds \\ &= e^{-2\lambda t} + e^{-2\lambda t}\int_0^t 2\lambda\, ds \\ &= e^{-2\lambda t} + e^{-2\lambda t}(2\lambda t) \\ &= (1 + 2\lambda t)e^{-2\lambda t}\ .\end{aligned}$$

The results are identical but the difference in mathematical difficulty is obvious. To easily determine a system's life distribution one need only be able to describe how the system successfully survives a mission, and then take advantage of the simple reliability shorthand methodology. In the next chapter we will expand this notation to include mixing of distributions.

III. MIXING DISTRIBUTIONS

In previous cases of systems utilizing spare components we assumed that those spare components would automatically and successfully replace failed components. Successful replacement occurred with probability equal to one. Perfect equipment in real life does not exist. We will assume that switchover and replacement by a spare component occurs with probability p, where $0<p<1$. No transfer occurs with probability $1-p$.

A. MIX NOTATION

For general application let D_1 and D_2 represent the probability distributions of the independent random times to failure T_1 and T_2. Let $D_1 + D_2$ stand for the distribution of the sum $T_1 + T_2$. Now let the notation

$$MIX(p_1 D_1, p_2 D_2)$$

denote the mixture of the distributions D_1 and D_2 with respect to the mixing probabilities p_1 and p_2, where $p_1 + p_2 = 1$. This mixture of distributions has the survival function

$$\overline{F}(t) = p_1\overline{F}_1(t) + p_2\overline{F}_2(t),$$

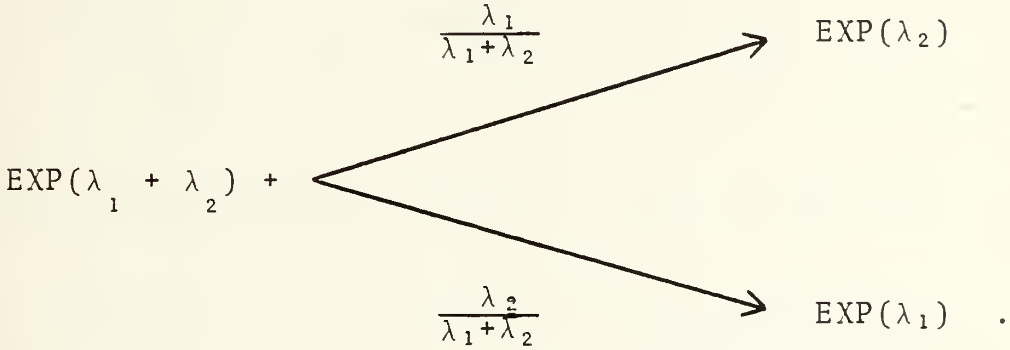
where $\overline{F}_1(t)$ and $\overline{F}_2(t)$ are the survival functions for D_1 and D_2.

1. A System having two Active Components in Parallel

A simple parallel system continues to survive as long as either active component still functions, regardless of the

order in which they fail. Assume component A_1 has life $T_1 \sim EXP(\lambda_1)$, component A_2 has life $T_2 \sim EXP(\lambda_2)$, T_1, T_2 are independent, and T = maximum (T_1, T_2).

From what we know of parallel systems, the life distribution is $EXP(\lambda_1 + \lambda_2) + EXP(\lambda_1)$ if component A_2 is the first to fail, or it is $EXP(\lambda_1 + \lambda_2) + EXP(\lambda_2)$ if component A_1 fails first. The probability that A_1 fails before A_2 is $\frac{\lambda_1}{\lambda_1+\lambda_2}$, and that A_2 fails before A_1 is $\frac{\lambda_2}{\lambda_1+\lambda_2}$. The system life distribution is described by the branching representation



Using the MIX notation this life distribution is
$EXP(\lambda_1 + \lambda_2) + MIX[(\frac{\lambda_1}{\lambda_1+\lambda_2})EXP(\lambda_2), (\frac{\lambda_2}{\lambda_1+\lambda_2})EXP(\lambda_1)]$.
The survival function for this distribution is

$$\bar{F}(t) = e^{-(\lambda_1+\lambda_2)t} + \int_0^t [\frac{\lambda_1}{\lambda_1+\lambda_2} e^{-\lambda_2(t-s)} + \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\lambda_1(t-s)}] [(\lambda_1+\lambda_2) e^{-(\lambda_1+\lambda_2)s}]ds.$$

Applying techniques used previously the survival function becomes

$$\begin{aligned}\bar{F}(t) &= e^{-(\lambda_1+\lambda_2)t} + e^{-\lambda_2 t}\int_0^t \lambda_1 e^{-\lambda_1 s}ds + e^{-\lambda_1 t}\int_0^t \lambda_2 e^{-\lambda_2 s}ds \\ &= e^{-(\lambda_1+\lambda_2)t} + e^{-\lambda_2 t}(1-e^{-\lambda_1 t}) + e^{-\lambda_1 t}(1-e^{-\lambda_2 t}) \\ &= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}.\end{aligned}$$

This demonstrates that this MIX notation does represent the parallel system's survival function

$\bar{F}(t) = \bar{F}_1(t) + \bar{F}_2(t) - \bar{F}_1(t)\bar{F}_2(t)$.

2. <u>Distributive Property</u>

The MIX notation has an algebraic distributive property. Notationally we have

$$D_3 + \text{MIX}(p_1 D_1,\ p_2 D_2) = \text{MIX}[p_1(D_1 + D_3),\ p_2(D_2 + D_3)].$$

A graphic representation of the distributive property is shown in figure 5.

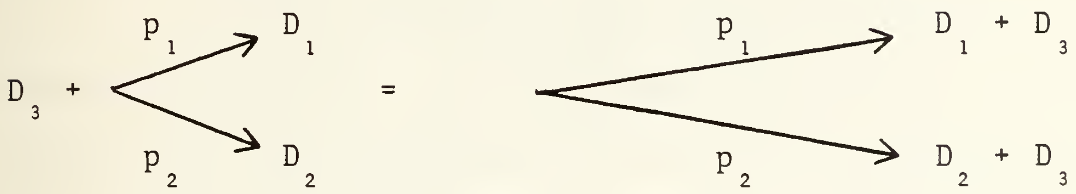


FIGURE 5: DISTRIBUTIVE PROPERTY OF THE MIX NOTATION

For our parallel system example note that

$D_1 = \text{EXP}(\lambda_2)$, $D_2 = \text{EXP}(\lambda_1)$, $p_1 = \frac{\lambda_1}{\lambda_1+\lambda_2}$, $p_2 = \frac{\lambda_2}{\lambda_1+\lambda_2}$,

and $D_3 = \text{EXP}(\lambda_1+\lambda_2)$. Using these values we see that

$\text{EXP}(\lambda_1+\lambda_2) + \text{MIX}[(\frac{\lambda_1}{\lambda_1+\lambda_2})\text{EXP}(\lambda_2),\ (\frac{\lambda_2}{\lambda_1+\lambda_2})\text{EXP}(\lambda_1)]$ is equivalent to

$\text{MIX}[(\frac{\lambda_1}{\lambda_1+\lambda_2})[\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_2)],\ (\frac{\lambda_2}{\lambda_1+\lambda_2})[\text{EXP}(\lambda_1+\lambda_2)$ $+\text{EXP}(\lambda_1)]]$. Note that this is of the form $\text{MIX}(p_1 D_1,$ $p_2 D_2)$.

The latter MIX notation, which combines known distributions, is easier to use computationally than the MIX notation previously given. Utilizing the known distribution of $\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2)$, which has the survival function

$$\bar{F}(t) = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1 - \lambda_2},$$

we can easily convert our MIX notation to determine the parallel system's life distribution.

Substituting this survival function into our MIX notation for the parallel system we obtain

$$\bar{F}(t) = \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)\left[\frac{(\lambda_1+\lambda_2)e^{-(\lambda_2)t} - (\lambda_2)e^{-(\lambda_1+\lambda_2)t}}{(\lambda_1+\lambda_2) - (\lambda_2)}\right]$$

$$+ \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)\left[\frac{(\lambda_1+\lambda_2)e^{-(\lambda_1)t} - (\lambda_1)e^{-(\lambda_1+\lambda_2)t}}{(\lambda_1+\lambda_2) - (\lambda_1)}\right].$$

By cancelling the λ_1's in the first term and the λ_2's in the second term, then dividing both terms by the denominator, $(\lambda_1+\lambda_2)$, the survival function reduces to that of the simple parallel system

$$\bar{F}(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}.$$

As seen in the previous section the alternative method of determining the system's survival function takes the general form

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t p_1 \bar{F}_1(t-s) f_3(s)ds + \int_0^t p_2 \bar{F}_2(t-s) f_3(s)ds.$$

The reliability shorthand methodology would appear to be preferable.

3. A Standby System Having one Active and a Possible Spare Component

An active component, A, is replaced when it fails by a spare component, S, with probability p. The system has an active component time to failure $T_1 \sim EXP(\lambda)$, a spare component time to failure $T_2 \sim EXP(\lambda)$, and T_1, T_2 are independent.

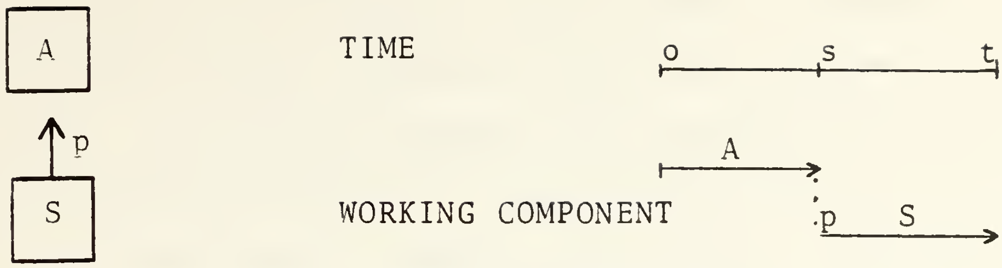


FIGURE 6: A SINGLE ACTIVE COMPONENT POSSIBLY HAVING ONE SPARE COMPONENT

The life of the system is $T = T_1$ with probability 1-p, or it is $T = T_1 + T_2$ with probability p. The shorthand method of determining the system's life distribution is to view the survival function as a combination of two possible distributions. If no switchover occurs the life T could be T_1 having $\bar{F}_1(t) = e^{-\lambda t}$, or if switchover occurs T could be $T_1 + T_2$ having $\bar{F}_2(t) = (1 + \lambda t)e^{-\lambda t}$. The survival functions $\bar{F}_1(t)$ and $\bar{F}_2(t)$ occur with probabilities 1-p and p, respectively. The life distribution is a mixture of the possible distributions where

$$\bar{F}(t) = (1 - p)\,\bar{F}_1(t) + p\bar{F}_2(t).$$

Thus the system's survival function is

$$\begin{aligned}\bar{F}(t) &= (1-p)e^{-\lambda t} + p(1+\lambda t)e^{-\lambda t} \\ &= e^{-\lambda t} - pe^{-\lambda t} + pe^{-\lambda t} + p\lambda te^{-\lambda t} \\ &= e^{-\lambda t} + p\lambda te^{-\lambda t} \\ &= (1+p\lambda t)e^{-\lambda t}.\end{aligned}$$

The alternate method of determining the life distribution of the system is to derive its survival function in

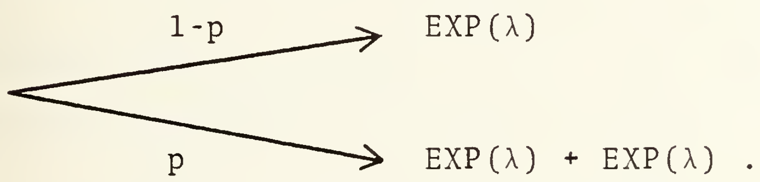
terms of its possible ways of mission success. The original component could survive to time t with no spare component required, or the original component could fail at some intermediate time s. The spare component then replaces the original component with probability p, and it must live from time s to time t to successfully complete the mission. The system's survival function is then

$$
\begin{aligned}
\bar{F}(t) &= e^{-\lambda t} + \int_0^t p e^{-\lambda(t-s)} \lambda e^{-\lambda s} ds \\
&= e^{-\lambda t} + \int_0^t p e^{-\lambda t} e^{\lambda s} \lambda e^{-\lambda s} ds \\
&= e^{-\lambda t} + p e^{-\lambda t} \int_0^t \lambda \, ds \\
&= e^{-\lambda t} + p e^{-\lambda t} (\lambda t) \\
&= (1 + p\lambda t) e^{-\lambda t}.
\end{aligned}
$$

Using the MIX notation we need only write

MIX[(1-p)EXP(λ), p(EXP(λ) + EXP(λ))].

the graphic representation is



The convenience of the shorthand methodology is again demonstrated.

B. DEGENERACY AT ZERO

Let ZERO be the name for the distribution of a random variable that is degenerate at zero. If $p[T_0=0] = 1$, then we say T_0 has the distribution ZERO, or $T_0 \sim$ ZERO. The survival function for T_0 is as shown in Figure 7.

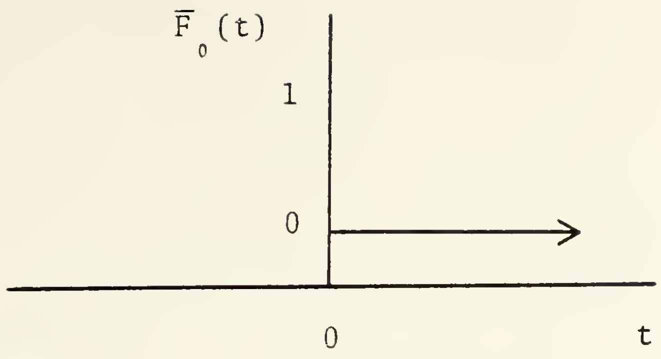


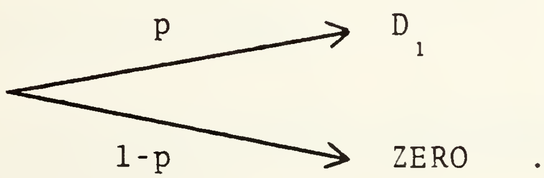
FIGURE 7: SURVIVAL FUNCTION OF THE ZERO DISTRIBUTION

The notion of a ZERO distribution compliments the MIX notation. Assume D_1 is the distribution of a nonnegative random variable T_1 which has the survival function $\overline{F}_1(t)$ and the density $f_1(t)$, where $t \geq 0$. We can then visualize the survival of a component as a combination of $\overline{F}_1(t)$ and $\overline{F}_0(t)$. The survival function is

$$\overline{F}(t) = \overline{F}_1(t) + \int_0^t \overline{F}_0(t-s) f_1(s)\, ds.$$

Since $\overline{F}_0(t-s) = 0$ for the ZERO distribution, the survival function, $\overline{F}(t)$, is simply $\overline{F}_1(t)$. The ZERO distribution adds nothing to another distribution's density, $D_1 + \text{ZERO} = D_1$.

In the MIX notation we could have MIX(pD_1, $(1-p)$ZERO) represent the survival function of a distribution. This would be graphically represented as



The survival function for this notation is

$$\bar{F}(t) = p\,\bar{F}_1(t) + (1-p)\bar{F}_0(t)$$

$$= p\,\bar{F}_1(t) + (1-p)\,(0)$$

$$= p\,\bar{F}_1(t)$$

The probability p need not be 1 since a system may not work when it is turned on.

For an example of the ZERO distribution's utilization, let us take the standby system composed of a single active component having a spare component for replacement. In section II-A we saw that T was T_1 with probability 1-p, or T was $T_1 + T_2$ with probability p. In our MIX notation this would be

$$MIX(p[EXP(\lambda) + EXP(\lambda)],\ (1-p)[EXP(\lambda)]).$$

If it were not for the ZERO distribution our distributive property would not hold. With the ZERO distribution this MIX notation can be reexpressed as

$$EXP(\lambda) + MIX[pEXP(\lambda),\ (1-p)ZERO].$$

Figure 8 graphically represents this equivalence, keeping in mind that $EXP(\lambda) + ZERO = EXP(\lambda)$.

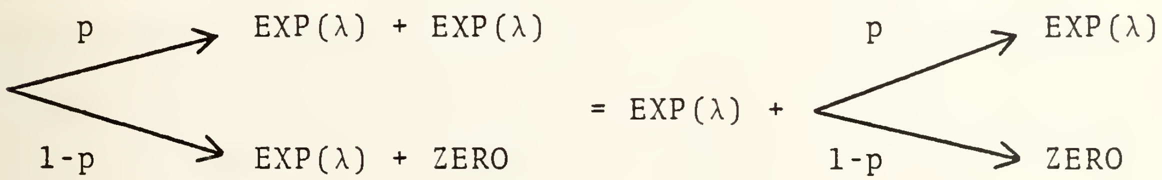


FIGURE 8: DISTRIBUTIVE PROPERTY INCORPORATING THE ZERO DISTRIBUTION

IV. SUMMARY

By learning a simple style of notation and applying it to the survivability of a system, the reliability practitioner can determine the life distribution of the system with non-calculus mathematics.

Appendix A is provided as a start for a ready reference catalogue of systems and their reliability shorthand.

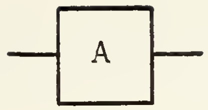
Through the use of computers we can reduce the burden of calculating the survival functions for many systems. Two of the general reliability shorthand cases have been programmed for the Ti-59 and they are presented in Appendix B. The examples provided in that section will demonstrate the computational convenience of the shorthand methodology.

The total scope and depth of the reliability shorthand methodology is yet to be investigated. Computationally, cases requiring the convolution of identical failure rates and distinct failure rates both have known survival function algorithms. Further study is required to determine if there is a useable algorithm which will permit the combination of both cases. This paper was designed to introduce this concept and its known properties to those already familiar with reliability. After seeing the convenience and benefit of the reliability shorthand methodology it is hoped the reader's interest will be further stimulated.

APPENDIX: A

This section contains several examples of the more common systems and their associated reliability shorthand. The format facilitates the addition of other systems in order to build a more thorough ready reference catalogue.

SYSTEM:



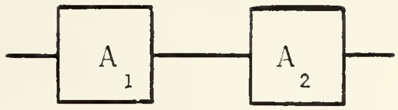
SHORTHAND: EXP(λ)

SURVIVAL FUCTION: $\overline{F}(t) = e^{-\lambda t}$

DESCRIPTION:

A single active component having an exponential life distribution.

SYSTEM:



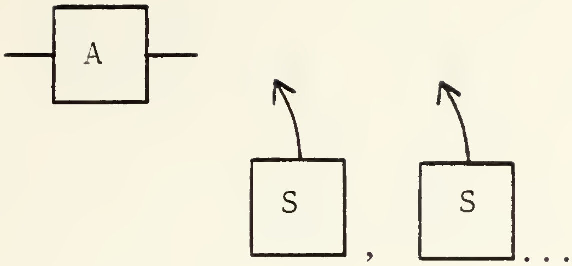
SHORTHAND: $EXP(\lambda_1 + \lambda_2)$

SURVIVAL FUNCTION: $\overline{F}(t) = e^{-(\lambda_1 + \lambda_2)t}$

DESCRIPTION:

A two component series system which requires both components to function for the system to survive.

SYSTEM:



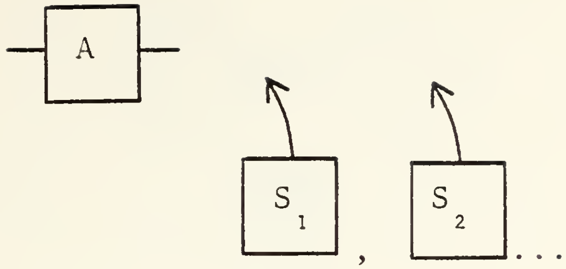
SHORTHAND: $EXP_1(\lambda) + EXP_2(\lambda) + \ldots + EXP_n(\lambda)$

SURVIVAL FUNCTION: $\bar{F}(t) = \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$

DESCRIPTION:

A single active component has n-1 identical spare components. As each component fails it is replaced by a new identical component which allows the system to survive. The system has n consecutive EXP(λ) lives.

SYSTEM:



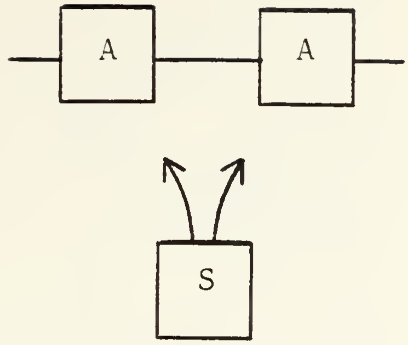
SHORTHAND: $EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n)$

SURVIVAL FUNCTION: $$\bar{F}(t) = \sum_{i=1}^{n} \frac{\Pi_{j\neq i}\,\lambda_j}{\Pi_{j\neq i}(\lambda_j - \lambda_i)}\, e^{-\lambda_i t}$$

DESCRIPTION:

A single active component has n-1 dissimilar spare components. As each component fails it is replaced by a new component which allows the system to survive. Each of the n components has a different failure rate, and the system has n consecutive $EXP(\lambda_i)$ lives.

SYSTEM:



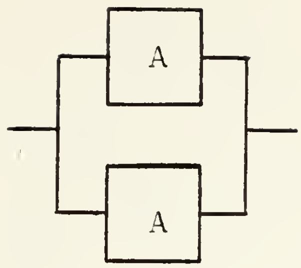
SHORTHAND: EXP(2λ) + EXP(2λ)

SURVIVAL FUNCTION: $\overline{F}(t) = (1 + 2\lambda t)e^{-2\lambda t}$

DESCRIPTION:

A series system composed of two identical active components has another identical component available as a spare. The original series system has a EXP(2λ) life. When either component fails and the spare takes its place, the system has a new EXP(2λ) life.

SYSTEM:



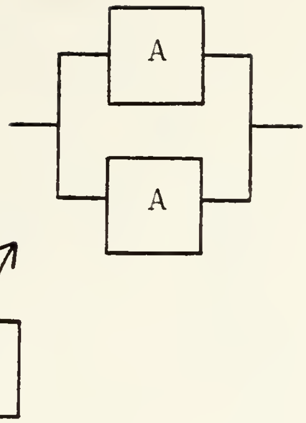
SHORTHAND: EXP(2λ) + EXP(λ)

SURVIVAL FUNCTION: $\overline{F}(t) = 2e^{-\lambda t} - e^{-2\lambda t}$

DESCRIPTION:

The parallel system has two identical active components functioning together with an EXP(2λ) life for system survival. When either component fails the surviving component is as if new with an EXP(λ) life. This new component alone functions for system survival.

SYSTEM:



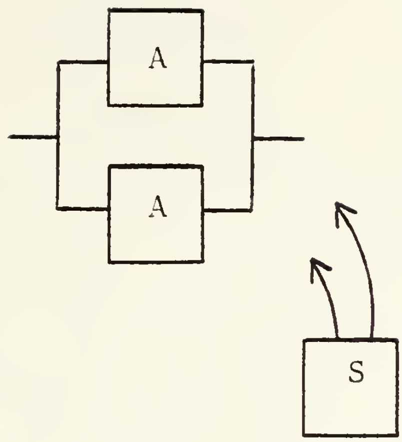
SHORTHAND: EXP(2λ) + EXP(2λ) + EXP(λ)

SURVIVAL FUNCTION: $\bar{F}(t) = 4e^{-\lambda t} - 3e^{-2\lambda t} - 2\lambda t e^{-2\lambda t}$

DESCRIPTION:

A parallel system composed of two identical components has a similar component as a spare which will replace the first component that fails. The original system functions with a EXP(2λ) life until a component fails. When it is replaced a new parallel system exists which has a life of EXP(2λ) + EXP(λ).

SYSTEM:



SHORTHAND: EXP(2λ) + EXP(λ) + EXP(λ)

SURVIVAL FUNCTION: $\bar{F}(t) = e^{-2\lambda t} + 2\lambda t e^{-\lambda t}$

DESCRIPTION:

A parallel system composed of two identical components has a similar component as a spare which will replace the last component that fails. The original system functions with an EXP(2λ) + EXP(λ) life until both components have failed. When the last component is replaced the system survives by the new component which has an EXP(λ) life.

APPENDIX: B

INTRODUCTION

There are two general cases in reliability shorthand where the aid of a programmable calculator greatly simplifies the tedious calculation of a system's survival function.

Case one in reliability shorthand is of the form $EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n)$, and each of the n failure rates are different. When the system description is of this form, $\sum_{i=1}^{n} EXP(\lambda_i)$, the survival function is $\bar{F}(t) = \sum_{i=1}^{n} \frac{\Pi_{j\neq i} \lambda_j}{\Pi_{j\neq i}(\lambda_j - \lambda_i)} e^{-\lambda_i t}$.

Case two in reliability shorthand is of the form $EXP(\lambda) + EXP_2(\lambda) + \ldots + EXP_n(\lambda)$, and each of the n failure rates are identical. When the system description is of this form, $\sum_{i=j}^{n} EXP_i(\lambda)$, the survival function is $\bar{F}(t) = \sum_{i=j}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$

USER PROCEDURES

1. Use any library module and read in side one of the magnetic card.
2. For case one the survival function is found using Label A. Enter t in R_{00}, n in R_{01}, and the n different failure rates in R_{13} through $R_{13+(n-1)}$. The order of the λ_i's does not matter. Press [A] for the system reliability.
3. For case two, the survival function is found using Label B. Enter t in R_{00}, n in R_{01}, and λ in R_{13}. Press [B] for the system reliability.

4. The maximum n for case two is not limited. The maximum for case one is limited to 47 due to the partioning 479.59. Using 9 2^{nd} OP 17 the maximum n can be increased to 77.

LABELS USED

A	A'	sin
B	B'	cos
C	C'	tan
D	D'	
	E'	

STORAGE REGISTER CONTENTS

00	t	08	used
01	n	09	used
02	used	10	used
03	used	11	used
04	used	12	used
06	used	13	λ
07	used	13-59	λ_i

EXAMPLE RUN TIMES

n	Case one - LBL A	Case two - LBL B
1	8 seconds	3 seconds
2	18 seconds	5 seconds
3	34 seconds	7 seconds
4	55 seconds	10 seconds
5	80 seconds	12 seconds

SAMPLE PROBLEMS

CASE ONE:

Reliability shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3)$

Longhand form:

$$\bar{F}(t) = \frac{\lambda_2\lambda_3}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)} e^{-\lambda_1 t} + \frac{\lambda_1\lambda_3}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)} e^{-\lambda_2 t} + \frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} e^{-\lambda_3 t}$$

Sample values: $t = 2$, $n = 3$, $\lambda_1 = .5$, $\lambda_2 = .6$, $\lambda_3 = .7$

Procedure:

1) Enter sample values, t=2 STO 00, n=3 STO 01, λ_1=.5 STO 13, λ_2=.6 STO 14, and λ_3=.7 STO 15.

2) Press [A] and $\bar{F}(t)$ is displayed. $\bar{F}(t)$=.88262530

CASE TWO:

Reliability shorthand: $EXP_1(\lambda)+EXP_2(\lambda)+EXP_3(\lambda)+EXP_4(\lambda)+EXP_5(\lambda)$

Longhand form:

$$\bar{F}(t) = \left(\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^4}{4!}\right)e^{-\lambda t}$$

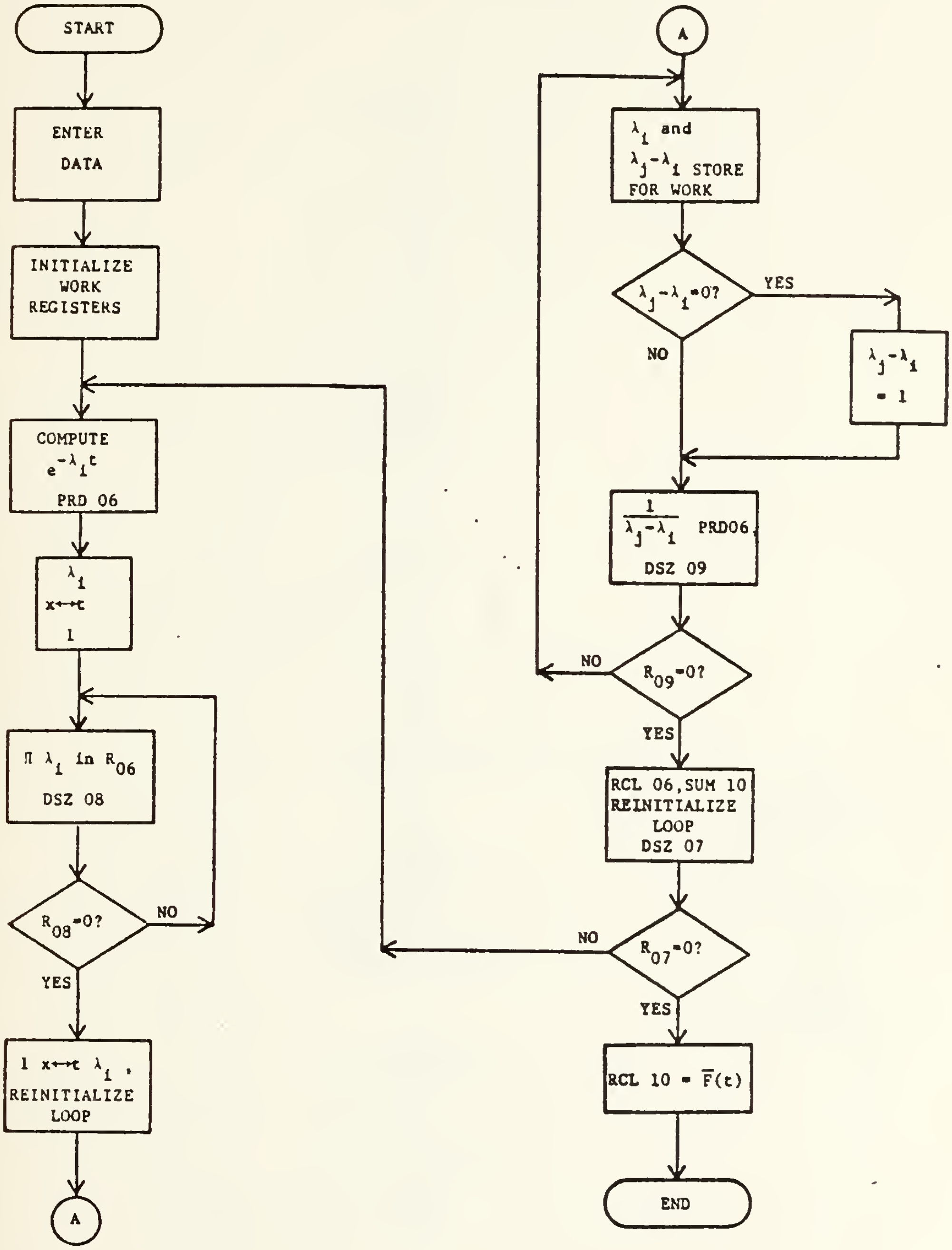
Sample values: $t=2$, $n=5$, and $\lambda=.5$

Procedure:

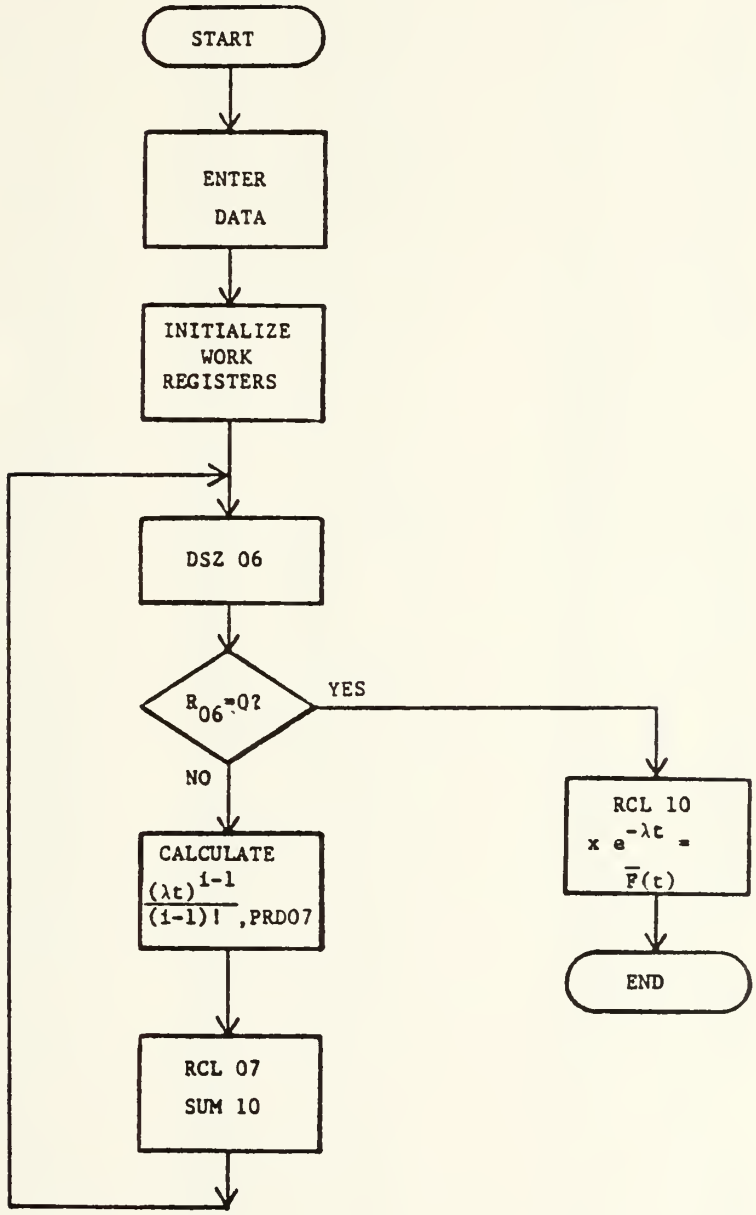
1) Enter sample value, t=2 STO 00, n=5 STO 01, and λ=.5 STO 13

2) Press [B] and $\bar{F}(t)$ is displayed. $\bar{F}(t)$=.9963401532

FLOWCHART: LABEL A, CASE 1



<u>FLOWCHART: LABEL B, CASE 2</u>



COMPUTER LISTING: LABEL A

000	76	LBL
001	11	A
002	01	1
003	22	INV
004	23	LNX
005	42	STO
006	02	02
007	01	1
008	42	STO
009	06	06
010	32	X:T
011	00	0
012	42	STO
013	10	10
014	42	STO
015	11	11
016	42	STO
017	12	12
018	43	RCL
019	01	01
020	42	STO
021	07	07
022	42	STO
023	08	08
024	42	STO
025	09	09
026	85	+
027	01	1
028	02	2
029	95	=
030	42	STO
031	03	03
032	42	STO
033	04	04
034	42	STO
035	05	05
036	76	LBL
037	16	A'
038	43	RCL
039	02	02
040	45	YX
041	53	(
042	73	RC*
043	03	03
044	94	+/-
045	65	×
046	43	RCL
047	00	00
048	54	)
049	95	=
050	49	PRD
051	06	06
052	73	RC*
053	03	03
054	32	X:T
055	01	1
056	72	ST*
057	03	03
058	76	LBL
059	17	B'
060	73	RC*
061	04	04
062	49	PRD
063	06	06
064	97	DSZ
065	04	04
066	38	SIN
067	76	LBL
068	38	SIN
069	97	DSZ
070	08	08
071	17	B'
072	43	RCL
073	01	01
074	42	STO
075	08	08
076	85	+
077	01	1
078	02	2
079	95	=
080	42	STO
081	04	04
082	00	0
083	32	X:T
084	72	ST*
085	03	03
086	42	STO
087	11	11
088	76	LBL
089	18	C'
090	73	RC*
091	05	05
092	75	-
093	43	RCL
094	11	11
095	95	=
096	42	STO
097	12	12
098	67	EQ
099	19	D'
100	76	LBL
101	10	E'
102	43	RCL
103	12	12
104	35	1/X
105	49	PRD
106	06	06
107	97	DSZ
108	05	05
109	39	COS
110	76	LBL
111	39	COS
112	97	DSZ
113	09	09
114	18	C'
115	43	RCL
116	01	01
117	42	STO
118	09	09
119	85	+
120	01	1
121	02	2
122	95	=
123	42	STO
124	05	05
125	43	RCL
126	06	06
127	44	SUM
128	10	10
129	01	1
130	42	STO
131	06	06
132	97	DSZ
133	03	03
134	30	TAN
135	76	LBL
136	30	TAN
137	97	DSZ
138	07	07
139	16	A'
140	43	RCL
141	10	10
142	92	RTN
143	76	LBL
144	19	D'
145	01	1
146	42	STO
147	12	12
148	61	GTO
149	10	E'
150	92	RTN

COMPUTER LISTING: LABEL B

```
151  76 LBL
152  12  B
153  01  1
154  22 INV
155  23 LNX
156  42 STO
157  02  02
158  01  1
159  42 STO
160  07  07
161  42 STO
162  10  10
163  43 RCL
164  01  01
165  42 STO
166  06  06
167  76 LBL
168  14  D
169  97 DSZ
170  06  06
171  13  C
172  43 RCL
173  10  10
174  95  =
175  65  ×
176  53  (
177  43 RCL
178  02  02
179  45 YX
180  53  (
181  43 RCL
182  13  13
183  94 +/-
184  65  ×
185  43 RCL
186  00  00
187  54  )
188  54  )
189  95  =
190  92 RTN
191  76 LBL
192  13  C
193  53  (
194  43 RCL
195  00  00
196  65  ×
197  43 RCL
198  13  13
199  54  )
200  55  ÷
201  53  (
202  43 RCL
203  01  01
204  75  -
205  43 RCL
206  06  06
207  54  )
208  95  =
209  49 PRD
210  07  07
211  43 RCL
212  07  07
213  44 SUM
214  10  10
215  61 GTO
216  14  D
217  92 RTN
```

BIBLIOGRAPHY

Esary, J.D., Course notes and problem sets for OA4662, Naval Postgraduate School, Monterey, CA., 1980.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3.	Department Chairman, Code 55 Department of Operations Research Naval Postgraduate School Monterey, California 93940	1
4.	Professor J.D. Esary, 55Ey Naval Postgraduate School Monterey, California 93940	1
5.	Professor R.N. Forrest, 55Fo Naval Postgraduate School Monterey, California 93940	1
6.	LCDR John J. Repicky, Jr. USN 17815 Brazil Road Cleveland, Ohio 44119	1

Book is due on the latest date stamped
Keep this card in the book pocket