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WITH INGOT TEMPERATURE
DISTRIBUTION LIMITING
WATER FLOW RATES
COEFFICIENTS OF
HEAT TRANSFER

A. S. CHAPMAN

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by

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I wish to express my appreciation for the suggestions and material aid given by Assistant Professor E. E. Drucker and Associate Professor T. E. Oberbeck of the United States Naval Postgraduate School in the preparation of this thesis.

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NOMENCLATURE

C_p	Specific heat of steel
D	Diameter of ingot
H	Heat of fusion of steel
h_i	Surface coefficient of heat transfer, ingot to mold
h_w	Surface coefficient of heat transfer, water side
I	Mold wall thickness
L	Mold length
q	Heat transferred, Btu/hr.,ft^2
Q	Heat transferred, Btu/hr.
r_1	radius of ingot
r_2	radius of mold, water side
t_1	average surface temperature of ingot within mold
t_2	average water temperature in mold
$\bar{T}$	mean temperature of ingot cross-section at mold exit
T_p	pouring temperature
U	Composite heat transfer coefficient
V_z	velocity of ingot
α	thermal diffusivity
γ	density of steel
$h_w D/k$	Nusselt number, water
DG/μ_f	Reynold's number, water
$c_p \mu_f / k_f$	Prandtl number, water

SUMMARY

Analysis of continuous casting involves many phases, two of which are described in this thesis. The first investigation consists of determining the temperature distribution in a moving ingot of metal which is solidified and cooled sufficiently in a water cooled mold to permit the continuous processing of the resulting billet. In the second phase there has been established a relationship for maximum water flow rates for heat transfer and their association with corresponding coefficients of heat transfer.

The analysis has shown that the rate of heat transfer is primarily a function of the surface coefficient of heat transfer between the ingot and the mold. To increase this rate some means must be provided to increase this coefficient. Addition of oil along the boundary has been suggested which would not only increase this surface coefficient but will also decrease the frictional drag.

The results, although only approximate, are provided as a guide for future work.

INTRODUCTION

Generally, the analytic mode of approach to the solution of a problem also requires experimentation. However, lack of facilities and time limitations precluded experimentation for verification of any results in this thesis.

The primary endeavor in this investigation is to assist the experimenter in his work by forming limits of practicability and providing mathmatical solutions to important phases of the problem. Results are only approximations, consistent with assumptions necessary to perform the operations, but they can be used as a guide for future work.

Treatment of the entire problem of continuous casting is very long and complex, and to the best knowledge of the writer no work has been done analytically along these lines, either in full or upon any phase. No information is available for many of the problems; however, with work being done at the present time on properties of metals at temperatures at and near the melting point, some hope is held that in the future information will be available concerning these properties.

The process involves pouring molten steel (with a minimum amount of superheat to allow good fluidity) into a vertical, water cooled, metal mold. At the start of the process the mold is plugged at the lower end until sufficient freezing takes place to allow adequate strength to permit pulling the plug and continuously processing the resulting billet.

An analysis involves investigation of various phases which are

outlined below and discussed later in this paper:

(a) Temperature distribution at any point in the ingot.

(b) Investigation of limiting water flow rates, and

(c) Prediction of accompanying coefficients of heat transfer between the ingot and the mold.

(d) Possibility of frictional drag exerted on the mold by the ingot.

(e) Maximum velocity of ingot travel.

(f) Load capacity of the ingot.

(g) Determination of ingot breaking point.

(h) Investigation of possible surface boiling on water side of the mold.

(i) Investigation of surface tension and surface film effects.

HISTORY

The greatest amount of work on continuous casting of steel has been done by Edward R. Williams and the Williams Engineering Company. His solutions to the problems have been of an experimental nature, and to the best knowledge of the writer, no attempt has been made toward an analytical investigation.

The Williams process accomplishes the necessary heat removal by providing a very thin mold wall of high conductivity material (commercial drawn copper or brass tubing) through which heat can rapidly pass from the ingot, and by removing this heat extremely rapidly from the outside of the mold wall by means of water or other cooling fluid passed at high velocity along the surface. By removing the heat rapidly from the outer surface of the mold wall, a high temperature gradient is produced in the mold wall, which permits fast removal of heat from the ingot. It is claimed that this instantly freezes the molten metal into a skin which shrinks away from frictional resistance and is sufficiently strong to prevent fracturing as it is moved along the mold wall.

The cooling water in the mold passes through passages which have a very small clearance, the velocity being quite high. It has been found that a relatively small amount of water is sufficient to satisfactorily remove the heat from a steel ingot to enable it to be poured and withdrawn at a speed of seven or more feet per minute.

As the ingot emerges from the bottom it passes through a series

of sizing rolls that prevent bulging of the walls by ferrostatic pressure from the molten interior. Water sprays surround these sizing rolls and the ingot is quite cool when it gets down to the pinch rolls. Williams frequently uses mold lubrication to increase speed of casting. The lubricant is introduced at the top along the mold wall, the quantity being controlled by the speed of the pinch rolls drawing the continuous billet out.

The Williams machine has been given no steady trial over a long run on steel; however, the experimental ingots produced have, for the most part, satisfactory structure.

A number of makers of carbon steel who habitually handle small heats have shown interest in the process. It is also conceivable that those makers of high alloy steels and certain types of stainless steels may see in this process a possible means for eliminating present high scrap losses and a more flexible method of handling small tonnage orders.

DETERMINATION OF THE TEMPERATURE AT ANY POINT IN THE INGOT

In the following investigation certain assumptions are made to simplify the problem and bring it within the realm of possible solution. These assumptions are as follows:

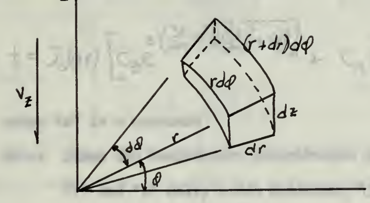
(a) Thermal conductivity does not vary with temperature.

(b) No recognition of the heat of fusion of steel is made in the temperature distribution in the ingot.

(c) Convection in the molten metal is neglected.

It is realized that the first two assumptions will cause appreciable error in the final results; however, unless these assumptions are made the problem would be beyond the scope of this paper. The assumption of the absence of convection in the molten steel is justified since it will become "mushy" upon entrance into the mold and resist any appreciable turbulence.

For purposes of this analysis a cylindrical ingot is used with all results in cylindrical coordinates. Consider first an elemental volume as follows:



The partial differential equation expressing the heat conduction for this problem is: (refer to Appendix "A" for derivation).

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} - \frac{V_z}{\alpha}\frac{\partial t}{\partial z} = 0$$

where α is the "thermal diffusivity" of the material.

$$\alpha = k/\rho c_p$$

The thermal diffusivity is a property of the substance in which heat is transferred and stored. It is the ratio of a quantity which is proportional to the heat conduction (numerator) and to a quantity which is proportional to the heat storage (denominator). Heat units cancel and the dimensions become ft^2/hr.

Assuming it is possible to express the solution of the above equation as the product of 3 functions, one of which is a function of "r" alone, one a function of "ϕ" alone, and the other a function of "z" alone, then

$$t = R(r)\,\Phi(\phi)\,Z(z)$$

The solution (as shown Appendix "B") is

$$t = J_0(ar)\left[C_3 e^{z\left(\frac{V_z}{2\alpha} + \sqrt{\left(\frac{V_z}{2\alpha}\right)^2 + 4a^2/4}\right)} + C_4 e^{z\left(\frac{V_z}{2\alpha} - \sqrt{\left(\frac{V_z}{2\alpha}\right)^2 + 4a^2/4}\right)}\right].$$

where "a" is a constant

Note: Since the temperature distribution is a function of "r" and "z" only, terms including "ϕ" drop out.

The ingot is considered to be a semi-infinite cylinder with the following boundary conditions:

1. $t = f(r)$, $z = 0$, $0 < r < R$
2. $t \neq \infty$, $z = \infty$, $r = 0$
3. $t = F(z)$, $0 < z < \infty$, $r = R$

From the second boundary condition it is seen that for $t \neq \infty$ at $z = \infty$, C_3 must be zero. The equation then reduces to

$$t = A J_0(ar)\, e^{z\left(\frac{V_z}{2\alpha} - \sqrt{\left(\frac{V_z}{2\alpha}\right)^2 + a^2}\right)}$$

Solution for the above is made by assuming that the temperature "t" is the sum of two solutions, that is

$t = t_1 + t_2$	where	$t_1 = f(r)$, $z = 0$, $0 \le r \le R$
$t_1 = t_1(z,r)$	such that when	$t_1 = 0$, $0 \le z < \infty$, $r = R$
$t_2 = t_2(z,r)$	such that	$t_2 = 0$, $z = 0$, $0 \le r \le R$
		$t_2 = F(z)$, $0 \le z < \infty$, $r = R$

Solutions for t_1 and t_2 are taken as modified forms of equations 31 and 32, pp 192, Carslaw and Jaeger, <u>Conduction of Heat in Solids</u> or

$$t_1 = \frac{2}{R^2} \sum_{n=1}^{\infty} \frac{J_0(ra_n)}{J_1^2(Ra_n)} e^{K_n z} \int_0^R r f(r)\, J_0(ra_n)\, dr \qquad (1)$$

$$t_2 = \frac{1}{R} \sum_{n=1}^{\infty} \frac{J_0(ra_n)}{J_1(Ra_n)} \int_0^{\infty} F_1(\beta) \left[e^{K_n|\beta - z|} - e^{K_n(\beta + z)} \right] d\beta \quad (2)$$

where

$$K_n = \frac{V_z}{2\alpha} - \sqrt{\left(\frac{V_z}{2\alpha}\right)^2 + a_n^2}$$

and the a_n are the positive roots of $J_0(Ra_n) = 0$.

The total solution then becomes the sum of equations (1) and (2) above.

Reducing the latter part of equation (2) to more usable form, we have,

$$\int_0^\infty F_1(\beta)\left[e^{K_n(\beta-z)} - e^{K_n(\beta+z)}\right]d\beta = \int_0^z F_1(\beta)e^{K_n(z-\beta)}d\beta + \int_z^\infty F_1(\beta)e^{K_n(\beta-z)}d\beta$$

$$+\int_0^z F_1(\beta)e^{K_n(\beta-z)}d\beta - \int_0^z F_1(\beta)e^{K_n(\beta-z)}d\beta - \int_0^\infty F_1(\beta)e^{K_n(\beta+z)}d\beta$$

This further reduces to

$$2\int_0^z F_1(\beta)\sinh K_n(z-\beta)\,d\beta + \int_0^\infty F_1(\beta)e^{K_n(\beta-z)}d\beta - \int_0^\infty F_1(\beta)e^{K_n(\beta+z)}d\beta$$

The temperature distribution in the ingot thus becomes

$$t = \frac{2}{R^2}\sum_{n=1}^{\infty}\frac{J_0(ra_n)}{J_1^2(Ra_n)}e^{K_n z}\int_0^R r f(r) J_0(ra_n)\,dr \; +$$

$$\frac{1}{R}\sum_{n=1}^{\infty}\frac{J_0(ra_n)}{J_1(Ra_n)}\left[2\int_0^z F_1(\beta)\sinh K_n(z-\beta)\,d\beta \; + \right.$$

$$\left.\int_0^\infty F_1(\beta)\,e^{K_n(\beta-z)}d\beta - \int_0^\infty F_1(\beta)\,e^{K_n(\beta+z)}d\beta\right]$$

For actual temperatures the functions f(r) and $F(\beta)$ must be supplied from physical aspects of the problem. Consider, for steady flow, the following assumptions:

$f(r) = T_p$ (pouring temperature)

$F(\ ) = T_p e^{-\beta/L}$ where L is the length of the mold.

Substituting these values and integrating we have

$$t = \frac{2T_p}{R^2}\sum_{n=1}^{\infty}\frac{J_0(ra_n)}{J_1^2(Ra_n)}e^{K_n z}\left[R^2\frac{J_1(Ra_n)}{Ra_n}\right] + \frac{1}{R}\sum_{n=1}^{\infty}\frac{J_0(ra_n)}{J_1(Ra_n)}\left[\frac{2T_pL^2K_n}{1-K_n^2L^2}\right\{$$

$$\left\{K_n e^{-z/L} + \frac{1}{K_nL}\sinh K_n z - \cosh K_n z\right\} - \frac{2T_pL}{1-K_nL}\sinh K_n z\Big]$$

with further simplification, this reduces to

$$t = \frac{2T_p}{R^2}\sum_{n=1}^{\infty}\frac{J_0(ra_n)}{J_1^2(Ra_n)}e^{K_n z}\left[\frac{R^2 J_1(Ra_n)}{Ra_n}\right] +$$

$$\frac{1}{R}\sum_{n=1}^{\infty}\frac{J_0(ra_n)}{J_1(Ra_n)}\left[\frac{2T_pL^2K_n}{1-K_n^2L^2}\left(e^{-z/L} - e^{K_n z}\right)\right]$$

Since K_n is negative for all values of a_n, this will converge very rapidly.

INVESTIGATION OF MAXIMUM COOLING WATER FLOW RATES, RELATION TO HEAT TRANSFER COEFFICIENT, AND EXPRESSION FOR MOLD LENGTH

For this phase of the problem consider the magnitude of the heat transferred from the ingot to the cooling water with the assumption that the velocity of the ingot is such as to cause final freezing at the instant the ingot emerges from the mold. Since the mass rate of flow of the metal is constant, the heat transferred in unit time for a mold of length L is equal to

$$Q = \dot{m}(h_{in} - h_{out}) \text{ Btu/sec}$$

where $\dot{m} = \frac{\gamma \pi D^2}{4} V_z$

V_z = velocity, ft/hr
D = diameter ingot
γ = density steel

now

$$h_{in} = H + C_p T_p$$
$$h_{out} = C_p \bar{T}$$

T_p = pouring temperature
H = heat of fusion
C_p = Specific heat
$\bar{T}$ = mean temperature of ingot cross-section at mold exit

Substituting these values into the above equation, the total heat transferred is

$$Q = \frac{\pi \gamma D^2 V_z}{4}\left[H + C_p(T_p - \bar{T})\right] \text{ Btu/hr}$$

Dividing through by πDL, we have

$$q = \frac{Q}{\pi D L} = \frac{\gamma D V_z}{4L}\left[H + C_p(T_p - \bar{T})\right] \text{ Btu/hr, ft}^2$$

Since "q" and "V_z" are independent there is obtained an expression for the length of the mold in terms of the heat transferred.

$$L = \frac{\gamma D V_z}{4q}\left[H + C_p(T_p - \bar{T})\right]$$

From the basic law of heat conduction for a hollow cylinder, the heat transferred from the ingot to the cooling water is

$$q = -U L 2\pi r \frac{dt}{dr}$$

and by integration within the limits r_1 and r_2

$$q = -U L 2\pi \frac{t_1 - t_2}{\ln r_1 - \ln r_2}$$

where

r_1 = radius of ingot
r_2 = radius of mold, water side
t_1 = average surface temperature of ingot within mold
t_2 = average water temperature in the mold

The term "U" is a composite or "overall" coefficient of heat transfer and is defined as follows:

$$\frac{1}{U} = \frac{1}{h_w} + \frac{l}{k_w} + \frac{1}{h_i}$$

where

h_w = surface coefficient of heat transfer, water side
k_w = thermal conductivity of mold
h_i = surface coefficient of heat transfer, ingot to mold
l = mold wall thickness

from this, it is seen that

$$U = \frac{h_w k_w h_i}{h_i k_w + l h_w h_i + k_w h_w}$$

Substituting this into the equation of heat conduction and solving for h_i , we have

$$h_i = \frac{-q\, k_w h_w (\ln r_1 - \ln r_2)}{q(k_w + l h_w)(\ln r_1 - \ln r_2) + h_w k_w l\, 2\pi (t_1 - t_2)}$$

For liquids with Reynold's numbers exceeding 10,000 the expression for h_w is given by the Colburn equation

(equation 4d, pp 168, McAdams, Heat Transmission)

$$\frac{h_w D}{k_f} = 0.023 \left(\frac{DG}{\mu_f}\right)^{0.8} \left(\frac{c_p \mu_f}{k}\right)^{1/3}$$

$$\left(\frac{h_w D}{k}\right) = \text{Nusselt Namber}$$

$$\left(\frac{DG}{\mu}\right) = \text{Reynolds Number}$$

$$\left(\frac{c_p \mu_f}{k}\right) = \text{Prandtl Number}$$

Applying this data to a specific problem, consider the following data:

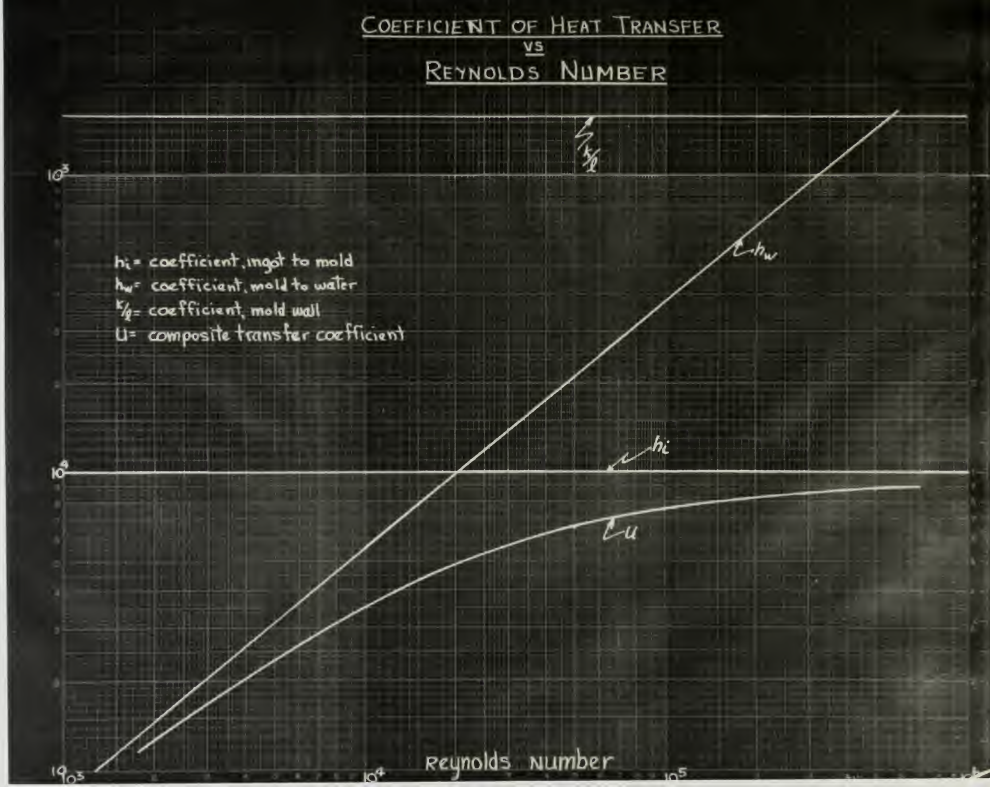
Diameter of ingot	4 inches
Pouring temperature, T	2800°F
Length of mold, L	3 ft.
Freezing temperature	2760°F
$\bar{T}$	2560°F
Heat of fusion of steel	65 Btu/#
Specific heat steel	0.1075 Btu/#
Mold thickness	0.5 inch
k_w/l	1650 B/hr.ft.F

From this

$$Q = 11.1 \times 10^5 \ \text{Btu/hr}$$
$$q = 3.54 \times 10^5 \ \text{Btu/hr,ft}^2$$

Using this value of "q" in the equation for the determination of the surface coefficient of heat transfer (ingot to mold), h_i can be determined. With this value and values for h_w and k_w/l, an "overall" coefficient for heat transfer for varying Reynold's numbers is calculated. Curves of h_i, h_w, k_w/l and U versus Reynolds numbers are shown plate I, pp 14.

The results of these curves show that the overall coefficient of heat transfer is dependent upon the surface coefficient between the ingot and the mold, and for this particular set of conditions, water flow rates in excess of Reynolds number 5×10^4 would be impractical. Such determinations can be made for varying sets of conditions giving a family of similar curves, and from these flow rates, mold length, and surface coefficients of heat transfer can be predicted. These results are not conclusive as much depends upon reasonably accurate determination of temperatures within the ingot. It does, however, show that maximum cooling water flow rates are dependent upon the amount of heat that can be transferred from the ingot to the mold, which in turn is dependent upon the corresponding surface coefficient.


COEFFICIENT OF HEAT TRANSFER
VS
REYNOLDS NUMBER
hi = coefficient, ingot to mold
hw = coefficient, mold to water
k/l = coefficient, mold wall
U = composite transfer coefficient
k/l
hw
hi
u
10³
10⁴
10³
10⁴
10⁵
Reynolds Number

CONCLUSIONS

1. As pointed out previously the solution for the temperature distribution is only approximate because the heat of fusion of the metal has been neglected in the derivation. The solution, however, is useful in heat transfer problems for determining temperatures in cylindrical bodies moving with constant velocity, and also in predicting temperature distributions in continuous casting problems. Solutions for different velocities can be modified by proper selection of the functions f(r) and $F(\beta)$ which can be fairly accurately predicted from experimental data.

2. In the investigation of maximum water flow rates, and their relation to surface coefficients of heat transfer between ingot and mold, it is shown that the over-all coefficient of heat transfer is, in the limit, a function of the surface coefficient between the billet and the mold. From the basic law of heat conduction and for a given size ingot, it is concluded that to increase the heat transferred some means must be provided to increase this surface coefficient. In the Williams process mold lubrication is used, which, in addition to reducing friction a resistance, increases the surface coefficient of heat transfer by making the heat transfer across the space more nearly that of pure conduction. It is recommended that for the lubricant an oil of high thermal conductivity be used - the optimum between the lubricating effect and the increased heat transfer coefficient is left for future investigation.

After determination of the maximum heat transfer rate, the optimum length of the mold can be determined from the relation

$$L = \frac{\gamma D V_z}{4q}\left[H + C_p\left(T_p - \bar{T}\right)\right]$$

It is to be noted here, however, that the above equation is based on the assumption that final freezing takes place just as the ingot emerges from the mold. More realistically, this final freezing point takes place some distance below the mold. That is, the mold length need be only sufficient to establish a "skin" around the molten metal which is strong enough to prevent rupture either from excessive load or from ferrostatic pressure from the molten interior. Investigation of minimum mold length is recommended for further work.

RECOMMENDATIONS FOR FURTHER WORK

Since the technique of continuous casting is complex, and since time limitations precluded analysis of all phases, the following recommendations are made for consideration:

1. Frictional drag exerted on the mold by the ingot

Although it is claimed that the molten metal instantaneously freezes upon entry into the mold and shrinks away from frictional resistance (The Iron Age, April 4 and April 11, 1940), it seems highly improbable. One of the laws of sliding friction is that the friction is proportional to the force pressing the surfaces together. This force is increased by the ferrostatic pressure from the molten interior, which, it is believed, will overcome the shrinking due to freezing of the metal. Adding a lubricant to decrease the coefficient of friction will increase the maximum load capacity and hence the maximum allowable velocity.

Investigation of the mechanical friction is simplified because, within limits, sliding friction is independent of the area of contact, and (except at start) of the speed of relative motion. The problem, therefore, resolves itself to a balance of forces between the ferrostatic pressure and the shrinking due to cooling.

2. Maximum Velocity of ingot travel.

The velocity of the ingot varies directly as the amount of heat which can be satisfactorily removed from the steel. With sufficient heat removal from the mold a skin will be developed such that

the ingot can withstand the load imposed by the pinch rolls. The maximum velocity will be determined by the maximum heat removal consistent with strength characteristics of the skin of the billet.

3. Load capacity of ingot and determination of ingot breaking point

The load capacity of the ingot is a function of the load capacity of the skin formed in the mold. The load will be caused by surface tension and surface film effects of the steel and frictional resistance to the passage of the ingot. This load capacity will reach its maximum when such velocity is reached that the maximum heat removal will be below that necessary to develop sufficient skin thickness and strength. Since the ferrostatic pressure increases down through the mold, the frictional resistance will also increase, and hence some point will be reached where the load exceeds the ultimate strength of the billet. Experimental evidence has shown that the breaking point will occur some distance down the mold, but the exact location has not been determined.

4. Investigation of possible surface boiling on the water side of the mold

With very high rates of heat transfer and high temperature gradients produced in the mold wall there are excellent possibilities of surface boiling in the cooling water. With high feed rates, a small fraction is vaporized, and the boiling-side coefficients may be roughly twice those predicted from straight water cooling. These increased coefficients may be due to increased turbulence in the film near the wall. Investigation along these lines can be very instruthe lower can withstand the load imposed by the given walls. The maximum velocity will be determined by the maximum heat removal consistent with strength characteristics of the skin of the billet.

3. Load capacity of crust and determination of upper breaking point.

The load capacity of the crust is a function of the load capacity of the skin formed in the mold. The load will be caused by surface tension and surface film effects of the metal and frictional resistance to the passage of the crust. This load capacity will reach its maximum when mold velocity is reduced that the maximum heat removal will be below that necessary to develop sufficient skin thickness and strength. Since the hydrostatic pressure increases down through the mold, the frictional resistance will also increase, and hence some point will be reached where the load exceeds the ultimate strength of the billet. Experimental evidence has shown that the breaking point will occur some distance into the mold, but the exact location has not been determined.

4. Investigation of possible surface boiling on the water side of the mold

With very high rates of heat transfer and high temperature gradients produced in the mold wall there are excellent possibilities of surface boiling in the cooling water. With high heat rates, a small fraction is converted, and the boiling-heat coefficients may be roughly twice those predicted from straight water cooling. These increased coefficients may be due to increased turbulence in the film near the wall. Investigation along these lines can be very instrucmental in increasing heat transfer rates.

5. Surface Tension and Surface Film Effects

A high value of surface tension has the effect of increasing the pressure required to cause the metal to enter and flow along a narrow channel; but the direct effect of the true surface tension is almost overshadowed by the effect of surface films, the influence of which is equivalent to that of a greatly increased surface tension. It has been found that surface oxide films affect the flowing power of metals with the apparent surface tension of some metals increased three times. Steel has a very high value of surface tension. (Briggs, Metallurgy of Steel Castings)

Both effects will add to the resistance to flow and decrease the maximum load capacity of the ingot. Although it is thought to be considerably small compared to frictional resistance, further investigation should be initiated to determine its effects.

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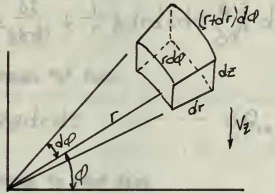
APPENDIX A

Equation of Conduction for a Moving Cylinder



Heat conducted through "r" face is

$$k \frac{\partial t}{\partial r} r\, d\varphi\, dz\, d\tau \text{ - - - - - } dQ_{1r}$$

Heat conducted through the "r + dr" face is

$$k\, d\varphi\, dz \frac{\partial t}{\partial r} r\, d\tau + k\, d\varphi\, dz \frac{\partial t}{\partial r} dr\, d\tau + k\, d\varphi\, dz\, r \frac{\partial^2 t}{\partial r^2} dr\, d\tau \text{ - - - } dQ_{2r}$$

Heat conducted through "φ" face

$$k \frac{\partial t}{\partial (r\varphi)} dr\, dz\, d\tau \text{ - - - - - } dQ_{1\varphi}$$

Note: Temperature gradient in φ direction is the change of temperature with respect to the distance represented by the angle φ which distance also depends on the radius r.

Heat conducted through " φ + dφ " face

$$k\, dr\, dz \frac{\partial t}{\partial (r\varphi)} d\tau + k\, dr\, dz \frac{\partial}{\partial (r\varphi)} \left[\frac{\partial t}{\partial (r\varphi)} r\, d\varphi \right] d\tau$$

$$\text{Since } \frac{\partial}{\partial(r\phi)}\left[\frac{\partial t}{\partial(r\phi)} r\right] = r\,\frac{\partial \frac{1}{r}\frac{\partial t}{\partial \phi}}{\partial(r\phi)} = \frac{r\,\frac{\partial \frac{1}{r}\frac{\partial t}{\partial\phi}}{\partial\phi}}{\frac{\partial(r\phi)}{r\phi}} = \frac{1}{r}\frac{\partial^2 t}{\partial\phi^2}$$

then, $dQ_{2\phi}$ =

$$k\,dr\,dz\,\frac{\partial t}{\partial(r\phi)} + \frac{1}{r}k\,dr\,dz\,d\phi\,\frac{\partial^2 t}{\partial\phi^2}\,d\tau \quad \cdots\cdots dQ_{2\phi}$$

Heat conducted through "z" face

$$k\frac{\partial t}{\partial z}r\,d\phi\,dr\,d\tau \quad \cdots\cdots dQ_{1z}$$

Heat conducted through "z +dz" face

$$k\frac{\partial t}{\partial z}r\,d\phi\,dr\,d\tau + k\,d\phi\,dr\,\frac{\partial}{\partial z}\left[\frac{\partial t}{\partial z}r\,dz\right]d\tau$$

$$\text{or}\quad k\frac{\partial t}{\partial z}r\,d\phi\,dr\,d\tau + k\,d\phi\,dr\,dz\,r\frac{\partial^2 t}{\partial z^2}d\tau \quad \cdots\cdots dQ_{2z}$$

Heat gained by the elemental volume due to conduction is equal to

$$\sum dQ_2 - \sum dQ_1$$

which equals

$$kr\frac{\partial^2 t}{\partial r^2}d\phi\,dr\,dz\,d\tau + k\frac{\partial t}{\partial r}d\phi\,dr\,dz\,d\tau + \frac{k}{r}\frac{\partial^2 t}{\partial\phi^2}d\phi\,dr\,dz\,d\tau + kr\frac{\partial^2 t}{\partial z^2}d\phi\,dr\,dz\,d\tau$$

Heat stored in the differential body

$$dQ_3 = \rho\,C_p\,(r\,d\phi\,dz\,dr)\,d\tau$$

However the temperature increment of a particle of the fluid in time interval $d\tau$ becomes

$$dt = \frac{\partial t}{\partial r}dr + \frac{1}{r}\frac{\partial t}{\partial\phi}d\phi + \frac{\partial t}{\partial z}dz + \frac{\partial t}{\partial\tau}d\tau$$

dividing through by $d\tau$

$$\frac{dt}{d\tau} = V_r \frac{\partial t}{\partial r} + \frac{1}{r} V_\phi \frac{\partial t}{\partial r} + V_z \frac{\partial t}{\partial z} + \frac{\partial t}{\partial \tau}$$

Since we are considering velocity in the z-direction and a condition of steady state, this reduces to

$$dt = V_z \frac{\partial t}{\partial z} d\tau$$

From this, then, the Heat stored in the body becomes

$$dQ_3 = \rho C_p (r d\phi dr dz) V_z \frac{\partial t}{\partial z} d\tau$$

Since no heat energy is being developed in the body, the law of the conservation of energy reduces to

Heat stored = heat in - heat out

$$dQ_3 = dQ_1 - dQ_2$$

Substituting and dividing through by $r d\phi dr dz d\tau$

$$\rho C_p V_z \frac{\partial t}{\partial z} = k \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} \right]$$

introducing the term "thermal diffusivity"

$$\alpha = k / \rho C_p$$

(The thermal diffusivity is a property of the substance in which heat is transferred and stored).

Substituting this value, the equation becomes

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} - \frac{V_z}{\alpha} \frac{\partial t}{\partial z} = 0$$

APPENDIX B

SOLUTION FOR THE EQUATION OF CONDUCTION - TEMPERATURE DISTRIBUTION IN INGOT

Assume it is possible to express the solution of the differential equation (Appendix A) as the product of three functions, one of which is a function of "r" alone, one a function of "ϕ" alone, and the other a function of "z" alone, then

$$t = R(r)\,\Phi(\phi)\,Z(z)$$

The equation takes the form,

$$Z\Phi R'' + \frac{1}{r} Z\Phi R' + \frac{1}{r^2} Z\Phi'' R + Z''\Phi R - \frac{V_z}{\alpha} R\Phi Z' = 0$$

dividing through by $R\Phi Z$, transposing and equating to a constant, we have

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Phi''}{\Phi} = -\frac{Z''}{Z} + \frac{V_z}{\alpha}\frac{Z'}{Z} = -a^2 \quad (1)$$

From the left side of the equation, write

$$r^2\frac{R''}{R} + r\frac{R'}{R} + r^2a^2 = -\frac{\Phi''}{\Phi} = p^2 \quad (2)$$

which gives

$$\Phi'' + p^2\Phi = 0 \quad (3)$$

the solution of which is

$$\Phi = C_1 \sin p\phi + C_2 \cos p\phi \quad (4)$$

Equation (2) also gives

$$r^2R'' + rR' + (r^2a^2 - p^2)R = 0 \qquad (5)$$

let $x = ar$

then since

$$\frac{dR}{dr} = \frac{dR}{dx}\frac{dx}{dr} \quad \text{and} \quad \frac{dx}{dr} = a$$

equation (5) becomes

$$r^2R'' + rR' + (x^2 - p^2)R = 0 \qquad (6)$$

$$\text{where } R = f(x) = f(ar)$$

This equation is Bessel's differential equation (Hildebrands Advanced Calculus of Engineers, pp 165-167), and the general solution for "p" an integer can be written

$$R = C_1 J_p(x) + C_2 K_p(x) \qquad (7)$$

where $K_p(x)$ is given as follows (Woods, Advanced Calculus, pp 288)

$$K_p(x) = J_p(x)\ln(x) + P(x)$$

$$\text{where } P(x) = -\frac{1}{2}\sum_{n=0}^{\infty}\frac{(p-n-1)!\,x^{-p+2n}}{2^{-p+2n}\,n!} - \frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n x^{p+2n}}{2^{p+2n}\,n!\,(p+n)!}\left[1+\frac{1}{2}\cdots\frac{1}{n}\right]$$

Since equation (7) is applied to a solid cylinder, Bessel's function $K_p(x)$ becomes infinite at $x = 0$ ($\ln 0 = -\infty$), and it follows that $C_2 = 0$. Thus for the solid cylinder, the general solution is taken as:

$$R = CJ_p(x) = CJ_p(ar) \qquad (8)$$

Taking the remaining equation in our solution

$$Z'' - \frac{V_z}{\alpha} Z' - a^2 Z = 0 \tag{9}$$

Using the operator method of solution, the equation becomes

$$\left[D^2 - \frac{V_z}{\alpha} D - a^2\right] Z = 0 \tag{10}$$

The solution, therefore, of equation (9) becomes

$$Z = C_3 e^{m_1 z} + C_4 e^{m_2 z} \tag{11}$$

where

$$m_1 = \frac{V_z}{2\alpha} + \sqrt{\left(\frac{V_z}{2\alpha}\right)^2 + a^2}$$

$$m_2 = \frac{V_z}{2\alpha} - \sqrt{\left(\frac{V_z}{2\alpha}\right)^2 + a^2}$$

Hence for the cylinder the solution of the differential equation is given by substituting equations (4), (8), and (11) into equation

$$t = R(r)\,\Phi(\phi)\,Z(z)$$

then $$t = C J_p(ar)\left[C_1 \sin p\phi + C_2 \cos p\phi\right]\left[C_3 e^{m_1 z} + C_4 e^{m_2 z}\right]$$

Since the temperature distribution is a function of "r" and "z" only, then, p = 0, and the solution of the equation becomes

$$t = J_0(ar)\left[C_3 e^{m_1 z} + C_4 e^{m_2 z}\right]$$

Taking the remaining equation in the solution

$$Z'' - \frac{V}{\alpha} Z' - a^2 Z = 0 \qquad (9)$$

Using the operator method of solution, the equation becomes

$$\left[D^2 - \frac{V}{\alpha} D - a^2\right] Z = 0 \qquad (10)$$

The solution, therefore, of equation (9) becomes

$$Z = C_3 e^{m_1 z} + C_4 e^{m_2 z} \qquad (11)$$

where

$$m_1 = \frac{V}{2\alpha} + \sqrt{\left(\frac{V}{2\alpha}\right)^2 + a^2}$$

$$m_2 = \frac{V}{2\alpha} - \sqrt{\left(\frac{V}{2\alpha}\right)^2 + a^2}$$

Hence for the cylinder the solution of the differential equation is given by substituting equations (7), (8), and (11) into equation

$$t = R(r)\,\Phi(\phi)\,Z(z)$$

then $$t = C J_p(ar)\left[C_1 \sin p\phi + C_2 \cos p\phi\right]\left[C_3 e^{m_1 z} + C_4 e^{m_2 z}\right]$$

Since the temperature distribution is a function of r and z only, then, p = 0, and the solution of the equation becomes

$$t = J_0(ar)\left[C_3 e^{m_1 z} + C_4 e^{m_2 z}\right]$$