A microwave frequency standard employing optically pumped sodium vapor.

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A MICROWAVE FREQUENCY STANDARD EMPLOYING OPTICALLY PUMPED SODIUM VAPOR

ROBERT G. WILLIAMS
THESIS

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by
Robert G. Williams
Captain, United States Marine Corps

Submitted in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
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1958
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EMPLOYING OPTICALLY PUMPED SODIUM VAPOR

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from the
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ABSTRACT

Sodium atoms have a natural resonance at about 1772 megacycles which is independent of magnetic field. This resonance can be made as narrow as 100 cycles. By transmitting light from a sodium lamp through sodium vapor in argon, a population difference is produced between the two quantum levels responsible for this resonance. The effect of changing the populations of the levels, for example by resonating with an applied radio frequency field, can be detected by changes in the light absorption. An instrument will be described in which this effect is used to stabilize a simple oscillator to the order of 1 part in $10^8$.

The investigation and construction of this standard was done at Varian Associates, Palo Alto, California, during the period January to March, 1958, while the author was a student at the United States Naval Postgraduate School, Monterey, California.

I would like to thank William E. Bell and Dr. Arnold L. Bloom of Varian Associates for their assistance and suggestions on this project. I would especially like to express my gratitude and thanks to Professor Carl E. Menneken of the United States Naval Postgraduate School for the encouragement and aid that he gave so freely.
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1. INTRODUCTION

1-1. Problem of Accurate Frequency Standard

New navigation procedures, certain secure forms of radio transmission, and new systems for radio communications in general require for their operation the generation and control of very precise frequencies. In the latter case of radio communications, the general adoption of the single sideband technique of radio communication is imposing a severe requirement on the frequency standards that are used to adjust the reinserted carriers. An error of 10 cycles in the frequency of the reinserted carrier will introduce appreciable distortion, while an error of 50 cycles will destroy nearly all intelligence in the signal. Considering that the carrier frequency will be in the hundreds of megacycles region, the accuracy of present frequency standards (of the order 1 part in $10^5 \rightarrow 10^6$), with several exceptions, will not satisfy the requirements needed. There are certain standards, such as the cesium beam clock, the atomichron, and other atomic beam instruments that have the accuracy necessary for use with systems mentioned above, but without exception, these standards are either extremely complex, need laboratory conditions for proper operation, or require a condition which is impractical for general use (e.g., liquid nitrogen is needed for proper operation of a maser).

1-2. A New Type of Frequency Standard

With the discovery of an optical method of detecting the hyperfine transition between the field independent lines ($m_F = 0 \rightarrow m_F = 0 \Delta F = 1$) of the ground state in sodium metal vapor, it was seen that this method might be utilized to form a new type of frequency standard that has the accuracy necessary to meet the requirements of the previously mentioned systems.
A frequency standard, of accuracy of 1 part in $10^8$, using the method of optical detection of this transition in sodium metal vapor, is described in this paper. The complete instrument is fairly simple, quite compact, and requires no peculiar conditions for its proper operation. It may be used in applications that require accurate and stable frequency standards and where facilities are such that special operating requirements cannot be maintained. With further engineering, it is believed that an instrument capable of accuracy of 1 part in $10^{10}$ or better can be achieved and that with this extended engineering, the equipment can be made more portable with a reduction in both size and weight.
2. THEORY

2-1. Energy Levels and Frequencies

Quantum theory explains quite well the discrete wavelengths of light that are characteristic in the line spectra of certain atoms. Bohr, in his postulates, stated that an electron could be in only one or another of a set of discrete energy levels for a particular atom. He further postulated that only one photon of energy was radiated or absorbed at a time. These postulates lead to the following formulation:

\[ \Delta E = \text{Separation of energy levels} \]

\[ \Delta E = h \nu \]

\[ h = \text{Planck's constant} \]

\[ \nu = \text{Frequency of emitted light} \]

From this formula it can be seen that only discrete frequencies may occur in a line spectra, for only a discrete set of energy levels exist for atoms of a certain element.

If the spectra of certain atoms are examined using a spectrometer of high resolving power, the major lines will sometimes seem to be formed by more than one separate line. This multiplicity of lines is due to principal quantum energy states within an atom being further divided into sub-states. All of the states of usual interest are related to the free electrons in the atom and are defined by their orbit and spin.

A further breakdown of energy levels in an atom is the so-called hyperfine splitting of energy levels. These hyperfine splitting are due to the interaction of the nuclear spin and the electron spin (Appendix I). The energy level separations in the hyperfine splittings are usually quite small in comparison with the energy levels that produce the line.

\(^1\)Harley Howe, Introduction to Physics, McGraw Hill Book Company, 1948, p. 545.
spectra. The frequencies resulting from a transition between the hyperfine levels are still defined by the formula resulting from Bohr's postulate ($\Delta E = h\nu$) and are in the radio and microwave range of frequencies.

2-2. Optical Pumping

An atom can be raised to a higher energy state, called an excited state, by several methods. This can be done by bombardment with electrons, subjection to high temperatures as from an electric arc or high flame, or by absorption of radiant energy from an external source. In the above methods, the population of atoms in a particular sample will in general be distributed uniformly among all energy levels the atom might occupy. This includes the ground state (the unexcited state) and all excited states. A method of achieving population differences between energy levels of an atom in vapor is optical pumping. In this method an absorption cell containing a gas is illuminated by a circularly polarized beam of light of a wavelength that corresponds to a transition of energy levels in the atom. Illumination by this polarized light will produce transitions from the sublevels of the ground state to an excited state. Circular polarization of the light causes the probabilities of these transitions to be different for the various sublevels of the ground state. The probabilities of transition are dependent on the sense of polarization. The relaxation time to disorient and return to the ground state from an excited state is quite short, but is lengthened by including a relatively high pressure (of the order of several centimeters of Hg) buffer gas in the absorption cell. This buffer gas is usually argon, or one of the other noble gases. Disorientation and loss of phase memory prior to re-emitting the absorbed optical energy and returning to the ground state gives an equal probability for the atom to return to any of
the sublevels of the ground state. Due to the difference in probabilities of being excited out of the various sublevels of the ground state, and the equal probability of returning to any sublevel, the pumping process will have a tendency to increase the population of the atoms in one particular energy sublevel of the ground state at the expense of depopulation of the other sublevels.

Because of the short life in the excited states, the pumping process can be treated macroscopically as an equivalent transition and relaxation process between sublevels of the ground state.

In the pumping process, let the pumping rate out of the \( i^{th} \) sublevel be designated by \( P_i \) and let \( a_i \) be the population of the \( i^{th} \) sublevel. The net pumping process can be described by the following equations:

\[
\frac{da_i}{dt} = -a_i P_i + \sum_{j=0}^{n} \frac{a_j P_j}{n}
\]

Where \( A \) is the total population of the sample and \( n \) is the number of sublevels in the \( S_1 \) ground state. The summation term in equation 1 says that the probability is the same for returning to any sublevel in the ground state upon optical re-emission from an excited state. From equation 1 it can be seen, provided no thermal relaxation is involved, that the energy state with the least probability of excitation from it will have an increase in population.

2-3. Choice of Alkali Metal Vapors

The alkali metals have been found to be the most suitable materials for use in connection with experiments involving optical pumping.
The main reason for the selection of the alkali metals is that there is only one free or valence electron in the outer shell. It is easy to raise an atom of the alkali metals to an excited state by the methods mentioned previously.

Another reason for the selection of the alkali metals is the fact that vapor of proper pressure for absorption of energy can be formed in the absorption cell at relatively low temperatures (30° C to 140° C). There are other metals that could be used, but the temperatures necessary to achieve the proper vapor pressure in the absorption cell would be impractical for most experiments and uses.

Another favorable factor for the alkali metals is the fact the lamps necessary to give the spectral frequencies peculiar to the certain metals are easily fabricated, in fact many are available commercially. Filters to remove the unwanted wavelengths of light are also quite readily available.

2-4. Optical Monitoring

The intensity of the beam of light passing through the absorption cell is the source of information concerning the events taking place within the cell. (1-3, 5) If the intensity of the light leaving the absorption cell is monitored in relation to a constant input, the removal of energy from the beam will be observed by the monitoring device as a decrease in the intensity of the light. The intensity of the light through the cell will be the greatest when the vapor in the cell has been "pumped", that is, there is an overpopulation in a level that has little probability of excitation from it.

If some characteristic, e.g., field strength, frequency of an applied field, etc., of the environment of the absorption cell is varied,
there will be a point where the frequency or field strength is such that the atoms in the energy state that have been increased in population by optical pumping will have a transition to some other energy state. When this occurs, the system will again be receptive to the pumping process.

The pumping process will then resume, causing a resultant decrease in passed light intensity. The state that is filled by optical pumping will be repopulated after the condition that caused it to be depopulated has passed or ceased. The conditions that cause a transition between energy levels after pumping, and a decrease of intensity of the transmitted light will be the phenomena of interest. The change in intensity of the transmitted light when atoms in the "pumped" or overpopulated state have transition to some other state will be called the signal.

The detection of the intensity of the light passing through the absorption cell is done by using an ordinary photocell. A photocell is sensitive to a particular band of wavelengths of light, depending on the type of material chosen for the photosurface. The photocell chosen for the monitoring device should have a region of high sensitivity and response corresponding to the wavelengths of light used for the pumping process.

2-5. Zeeman Splitting of Energy Levels

Zeeman found that the emission spectrum of an atom when placed in a magnetic field will have several lines with nearly the same wavelength, where only one existed in the absence of the magnetic field. This dependence on the magnetic field is called the Zeeman effect. The magnitude of separation of the multiple lines depends on the strength of the magnetic field. The separation of the hyperfine levels is also

affected by the magnetic field and is referred to as the Zeeman effect. Because of this, it was felt that optical pumping with optical monitoring might be utilized to provide a new way to measure the earth's field. Experiments were performed by Bell and Bloom\(^5\) and the results indicated that this was a feasible method of measuring the earth's field.

Figure 2-1 is a block diagram of the experimental setup and Figure 2-2 is a picture of the apparatus used. The inclination on the equipment is the dip of the magnetic field at the location where the experiment was performed. The beam of light used to pump the system lies parallel to the magnetic field in order to produce the maximum effect. The simplicity of the experimental setup may be seen from the Figures. The lamp and photocell to the sides of the glass oven were for another experiment and are to be disregarded.

The metal sodium is taken as an example throughout, but identical experiments have been performed with potassium and rubidium, and presumably could be performed with cesium. The absorption cell was a one liter pyrex flask with an equilibrium sodium vapor pressure of approximately \(2 \times 10^{-5}\) mm of Hg. The flask also contains the buffer gas, a spectroscopic grade argon at a pressure of 3 cm of Hg. The lamp was a sodium lamp that provided the wavelength of light necessary for excitation of the atoms in the absorption cell. The monitoring device was a common photocell with a photosurface of Si material, which is sensitive to sodium light.

The frequency corresponding to the magnitude of the Zeeman splitting caused by the earth's field at the location the experiment was performed (.5 gauss of 50,000 gamma), was of the order of magnitude corresponding to an RF frequency of 350 kilocycles. The RF generator
shown schematically in Figure 2-1 was a crystal controlled oscillator operating at an approximate frequency of 350 kilocycles. To prove feasibility of this method, it was decided to vary the magnetic field and to keep the frequency used to excite the absorption cell constant. This was done because of the ease in varying the field strength and the complexity of a swept frequency signal generator.

To vary the strength of the magnetic field, large sweep coils were placed around the small building where the experiment was performed. The coils were excited by using the 60 cycle line current which swept the magnitude of the field about the earth's field.

The polarizer caused circular polarization of the light of such a sense that it caused a transition of $\Delta m = +1$. This transition has a tendency to increase the population in the $F = 2, m = 2$ energy state. Atoms in this state have little probability of excitation out of it by the circularly polarized light of the sense of polarization that put them in that state.

A filter was used to remove unwanted wavelengths of light. In sodium the D lines correspond to the transitions from the $3\text{S}_1\frac{1}{2}$ ground state to the $3\text{P}_1\frac{1}{2}$ and $2\text{P}_2\frac{1}{2}$ excited states. These transitions result in light of wavelength $5896\text{Å}$ and $5890\text{Å}$ respectively. The $2\text{P}_2\frac{1}{2}$ has an $m_f = 3$ sublevel. If light of wavelength corresponding to the transition from the ground level to this excited state is passed through the absorption cell, it will be able to excite atoms from the $F = 2, m = 2$ level of ground state and no net pumping action will result. A filter was used to remove the unwanted $5890\text{Å}$. It was found later that a filter was not

$1$ By reversing the sense of polarization, transitions of $\Delta m = -1$ will result.
needed because almost all sodium vapor spectral lamps appear to have a difference of intensity of the two D lines.

As the magnetic field is swept, the changing field strength will cause the difference in the separation of the energy levels to vary. At some instant of time the separation between two levels will correspond to the frequency of the RF generator. When this situation occurs, there will be a transition due to excitation by the RF field of atoms from the energy state whose population has been increased by optical pumping to some other state. With a field of .5 gauss (50,000 gamma) and an applied RF field of 350 kilocycles transitions from the F = 2, m = 2 level to the F = 2, m = 1 level resulted. When this transition occurs, the atoms are again in a state of high probability of absorption. This will result in a decrease of light intensity which will be detected by the monitoring device.

If the sweep of the oscilloscope is also driven by the same source that drives the sweep coils, a trace will be given as shown in Figure 2-3. The double signal arises from the fact that the conditions which give a decrease in light intensity will occur twice per cycle of the sweep of the field.

This experiment did not try to measure the strength of the earth's field, instead was merely to show that the techniques involved in optical pumping and monitoring could be applied to this problem. If the frequency had been swept over a range, with only the earth's field present, the frequency at which a signal would be given will be determined by the strength of the field and would be a means of measuring this strength.

2-6. Field Independent Transitions

In the hyperfine splitting of energy levels in the ground state of alkali metals, the transition $m_f = 0 \rightarrow m_f = 0 \Delta F = 1$ is independent of
the strength of weak magnetic fields. Carver and Dicke observed this transition in $^{87}\text{Rb}$ using optical methods for polarization and microwave methods for detection.\(^1\) These transitions have also been observed in atomic beam experiments.\(^2\)

Following their experiments concerning the optical polarization and detection of the Zeeman splitting of energy levels, Bell and Bloom developed a method whereby the detection of this field independent transition in sodium metal vapor was performed using optical means for polarization and detection.\(^6\)

Figure 2-4 shows a schematic diagram of the experimental apparatus. The apparatus was similar to that used in the experiment involving the Zeeman splittings as shown in Figure 2-2 with the glass oven replaced by a larger oven containing a cavity and absorption cell. Unpolarized light from the sodium lamp was transmitted along the earth's magnetic field through the absorption cell, which was placed in a resonant cavity, and then observed by the photocell.

The absorption cell, made of fused quartz, was mounted in a resonant cavity operating in the $\text{TE}_{011}$ mode and tunable around the frequency producing the transitions. This mode was chosen for two reasons. In the resonating state of the cavity, there is a concentration of magnetic field line along the axis of the cavity. If the frequency associated with this field is the frequency corresponding to the transition between the energy state of interest of the sodium vapor, the magnetic field will give up energy to the vapor. The fact the only currents in the cavity

\(^1\)Private communication between Carver and Dicke and Bell and Bloom.

walls are circular currents, concentric about the axis, makes it possible to cut apertures in the ends of the cavity to pass the pumping light through the absorption cell.

The output of the microwave signal generator exciting the cavity was frequency modulated. This produced results similar to those obtained in the Zeeman splitting experiment in which the frequency was held constant and the magnetic field was swept. That is, when the frequency of the modulated signal corresponded to the transition \( m_f = 0 \rightarrow m' = 0 \) \( \Delta F=1 \), transition took place, and the system was again receptive to pumping, with a resultant decrease of light intensity coming from the absorption cell. Figure 2-5 shows the signal when transitions occur.

The two \( m_{f} = 0 \) levels always have the same probability of excitation out of them, no matter what the sense of polarization is. From the theory of optical pumping applicable to experiments investigating the Zeeman splitting of the sublevels, there would never be an increase in the population of either \( m_{f} = 0 \) sublevel. By this theory, with no difference in populations, no signal will be observed. The postulation was then advanced that a difference in the intensity of light that caused excitation out of the two levels under question produced the necessary differential of population.

Bell and Bloom explained the phenomena of producing an intensity difference in the following manner. The light causing transitions from the \( F = 1 \) and \( F = 2 \) states will be scattered proportional to the number of sublevels within each \( F \) state. For sodium in the ground state there are 3 and 5 sublevels in the \( F = 1 \) and \( F = 2 \) levels respectively. Reduction of components of the transmitted light will be in the ratio \( e^{-3x} \) to \( e^{-5x} \). The factor \( x \) is a function of the path length of the
light and the vapor density in the cell. From this it can be seen that there will be more light acting on the lower state than on the upper at distances farther along the cell. Therefore, the necessary intensity difference is produced at the rear of the cell, with the lower state having more light act on it, resulting in a greater population in the upper states.

2-7. Field Independent Transitions to Control a Frequency Standard

The frequencies resulting from the hyperfine transition between the $m_f = 0$ levels of the ground state of sodium and the other alkali metals, like the energy separation of these levels, are virtually independent of field strength in weak magnetic fields. Because of this independence and the fact the level separation is a constant, these frequencies are seen as possible sources of a stable frequency for a frequency standard application.¹

As was mentioned previously, detection of this transition by microwave methods has been performed by Carver and Dicke in rubidium vapor. The technique developed by Bell and Bloom, of detection of the transition by optical means, resulted in a much simpler system and produced a higher signal-to-noise ratio. Because of the simplicity of these techniques, it was felt that a frequency standard using these optical methods could be developed that would be of small physical size and simple instrumentation. Using these optical methods in a frequency standard, the factors that affect the frequency as well as the accuracy of any servo control system must be considered.

¹An instrument using this transition and employing a cesium beam and a beam focusing technique to achieve population differences is manufactured by the National Company, Maiden, Massachusetts.
The frequencies produced by the hyperfine transition \( m_f = 0 \rightarrow m_f = 0 \Delta F = 1 \) in alkali metal vapor will have a slight dependence on field strength in fields of the order of one gauss or larger. For an accuracy of 1 part in \( 10^8 \) in the frequency, the transition can be considered independent of field strength in weak magnetic fields. However, for greater accuracies the effect of the magnetic field must be taken into account.

Interactions among atoms produce a finite band of frequencies around the main transition frequency as is shown in Figure 2-5. The width of this line is due to Doppler broadening and collisions of the alkali metal atoms with each other. The addition of the buffer gas reduces the line width which is normally of the order of one kilocycle to the order of 100 cycles. If the entire line is used for control, i.e., if the servo system allows the frequency of the source exciting the cell to vary over the entire line width, an accuracy of slightly better than one part in \( 10^7 \) will result. If the servo control is such that the frequency varies only over the peak portion of the signal, a greater accuracy will result. It was felt that a servo system with a response such that the source frequency will vary only over this peak portion could be produced.

Other factors affecting the accuracy of the system will be the components of the system that tend to pull the frequency of the source when they drift, pressure of the metal vapor and buffer gas in the cell, and fluctuations in the lamp supplying the pumping light. These conditions appear to have effect only on accuracies greater than one part in \( 10^8 \).

To prove the feasibility of the method of optical detection of the hyperfine transition as a source of stable frequency for a frequency
standard, a goal of accuracy of one part in $10^8$ was chosen. This would alleviate many of the problems of control that are necessary to achieve greater accuracy. Methods of control necessary to produce higher accuracies will be discussed in a later section.
3. FREQUENCY STANDARD INSTRUMENTATION CONSIDERATIONS

3-1. Choice of a System

The most straightforward method to utilize optical detection of the field independent hyperfine transition as the source of a stable frequency in a frequency standard is to utilize it in some form of a closed loop servo type of system to maintain, somewhere in the system, a constant frequency. In this method an oscillator is either operated at the transition frequency or this frequency is produced by frequency synthesis methods. This will result, after exciting the absorption cell, in a signal from a photosensitive detector monitoring the light passed through the cell. Because optical detection gives only an indication of the transition and does not actually produce a frequency that can be detected by the photosensitive device, a method to control the oscillator so that it produces this transition frequency is described.

The signal produced by optical detection will be introduced into some form of servo control system. If the signal indicates a drift from the frequency causing transitions, the servo control will sense this and will determine the direction the frequency is shifting. The servo control will then generate the necessary correction signal to shift the oscillator back to the transition frequency.

The oscillator should have the feature that its frequency may be either raised or lowered by changing one of the parameters affecting the frequency of the oscillator. It is almost irrelevant how the frequency is changed as long as it can be done in a manner that produces the necessary frequency correction.

The oscillator must, in some manner, radiate enough energy to excite the absorption cell and cause the transitions. There are any
number of ways that this might be accomplished. Methods that might be used to excite the cell are the utilization of a cavity as was done in the original method of detecting the hyperfine transition, or a radiating horn as was done in a later method of observing the same transition. Other methods that might be used are a simple Lecher wire arrangement, various antenna configurations, or any other method that will radiate enough energy into the absorption cell to excite the sodium vapor and produce transitions. The method chosen to excite the absorption cell must permit the pumping light from the lamp to pass through the absorption cell and then to the photosensitive detector.

A sodium lamp will produce the wavelength of light necessary to accomplish the optical pumping in the cell. No polarizer or filter will be necessary in this application because the pumping action comes from the difference of intensities in the light exciting atoms out of the various levels. As mentioned previously, this is attributed to the scattering of the light exciting the various levels by different amounts at the front of the cell so the pumping process is accomplished near the rear of the cell.

The absorption cell should be made of a material that has few losses at the frequency corresponding to the transition. The cell should be quite carefully made because sodium will react with the air, and just a very minute leak will completely destroy all of the sodium metal in the cell within a very short time.

The light will be transmitted through the absorption cell and will excite a photosensitive surface that will detect any change of intensity of the pumping light. There are many photosensitive surfaces available that will cover every wavelength in the visible spectrum. The surface
used in the detecting device must have a photosurface of material that is sensitive to a band of wavelength that includes the particular wavelength of light used.

3-2. Description of System

Having chosen a system in an attempt to form a frequency standard, the various problems and methods of solution relative to each component of the system should be investigated to see what the major problems are and what the most feasible solutions of them are. Figure 3-1 gives a block diagram of the proposed system.

Oscillator

As stated above, there are two methods that could be used to generate the desired transition frequency. The first of these would be to generate some lower frequency and then multiply this frequency to correspond to the transition frequency. The second method would be to generate the frequency of transition directly. Frequency synthesis methods to raise a frequency to the microwave region are usually quite complicated and as this standard is merely to prove the theory, it was decided they were too involved to include in the system at this time. A microwave oscillator capable of operating at 1772 megacycles was chosen as the source to excite the absorption cell.

A method must be provided to locate the frequency produced by the oscillator in relationship to the transition frequency at any particular instant. It is impossible to use a certain intensity of light to show this relationship due to fluctuations in the lamp supplying the pumping light. Therefore, a dip in the intensity as shown by the signal of Figure 2-4 must be used to indicate transitions and the oscillator will have to be frequency modulated to produce this signal.
Another point to be considered is the method of shifting the average value of the modulated frequency. What type of signal will the servo control be required to produce in order to maintain this average frequency at the transition frequency must also be considered.

A microwave oscillator employing a lighthouse tube was chosen to be the frequency element of the system. An oscillator of this type using plate and grid cavities to control the frequency of oscillation has good inherent stability. Providing there are no extreme variations of temperatures to greatly change the physical dimensions of the cavities the resonant frequencies of these cavities will remain fairly constant. The frequency of an oscillator using a vacuum tube, such as a lighthouse tube is dependent to a small degree on the plate voltage of the tube. Therefore, the method to shift the average frequency will be to shift the magnitude of this voltage. This also provides a convenient method of performing the frequency modulation by superimposing a small sweep voltage on the plate voltage.

Radiation of Absorption Cell

Two methods have been used successfully to excite the absorption cell in the optical detection of the hyperfine transition in alkali metal vapor. In their original experiments Bell and Bloom used a resonant cavity with the absorption cell to be excited mounted in the cavity. More recently Arditi and Carver excited an absorption cell by use of a radiating horn. (10) The requirements placed on the method of excitation are not severe. The system used must be capable of exciting the cell at the frequency causing transition, it must have the magnetic field lines exciting the cell parallel to the pumping light, it need deliver less than a milliwatt of power to the absorption cell, and the system chosen should
permit the pumping light to pass from the lamp through the cell to the photosensitive detector.

As the cavity used by Bell and Bloom in their experiment was available, it was decided to form the system using this cavity as the means of radiating the absorption cell. Later, time permitting, an attempt would be made to radiate the cell by other means mentioned previously.

Using a cavity as the means of exciting the absorption cell, which is mounted inside the cavity, requires that the cavity be located in an oven, for the cell must be kept at a temperature sufficient to vaporize the sodium metal.

Unlike microwave detection of the hyperfine transition which requires an extremely high Q cavity, optical detection of this transition can be performed using a cavity of low Q. In microwave detection, a high Q is necessary because the energy released by the atoms in the transition excites the cavity. As this energy is quite small, a low Q cavity would have too much loss for the excitations to be maintained. In optical detection, the cavity supplies the energy to the vapor in the cell to cause transition. Since the power required to excite the vapor is small, an adequate amount will still be present despite the losses of a low Q circuit when a relatively small signal is applied. This is a convenient condition since the radiation losses from the cavity through the openings necessary for the light passage lowers the Q of the cavity.

The acceptability of a low Q cavity is convenient in another way. It removes the necessity for a careful temperature control of the cavity. This would be required if a high Q were necessary since the change in resonant frequency due to the temperature induced changes in physical dimensions would place the transition frequency outside the narrow band.
width associated with the high Q. The broad band width of the low Q cavity will permit significant changes in the center frequency and still have sufficient excitation to cause transitions.

The mode that the cavity resonates in must be such that the absorption cell will be located in a concentration of magnetic field lines that lie parallel to the light causing polarization in the cell. The cavity and mode originally selected was a cylindrical resonant cavity resonating in the TE_{011} mode. This mode has a concentration of magnetic field lines parallel to and along the axis of the circular cylinder. The absorption cell can be mounted in this field with comparative ease. A cavity resonant in this mode has only cylindrical currents flowing in the cavity walls and end plates. The cylindrical currents make it easy to cut apertures in the end plates without cutting across any current paths. The light passing through the opening will travel along and parallel to the concentration of the magnetic field lines in the cavity. See Figure 3-2.

The Sodium Lamp

Two methods are used to form sodium lamps. In both, a properly evacuated glass shell contains sodium metal and a slight atmosphere of argon or other noble gas. The evacuation must remove all oxygen in order to prevent loss of sodium due to oxidation. The argon or other noble gas is used merely to aid in starting the sodium discharge. In one lamp, electrodes are sealed into the tube and the vapor produced by a heating element in the tube is excited by a D.C. discharge between the electrodes. In the second method an electrodeless lamp is used. The vapor, produced by inductive heating of the sodium metal, is excited by a RF field that causes it to give off the characteristic sodium light.

The lamp will be the source of most of the noise in the system due to the light intensity being very sensitive to any ripple or fluctuation.
in the power supply driving the lamp and to any changes in the temperature of the bulb or of the plasma in it. Lamps are also subject to oscillations in the plasma which appears as a modulation at audio frequencies of the light intensity.

An electrodeless lamp was chosen for this application. The walls of a D.C. lamp become blackened after repeated operation. It is believed this blackening comes from a sputtering of the electrodes supplying the heat and excitation energy to the sodium in the lamp.

The Absorption Cell.

The absorption cell that contains the sodium vapor has to be made of a material that is transparent to the light causing polarization. The material should have as little loss at 1772 megacycles as possible in order that the additional power required from the source be insignificant. Fused quartz appears to be the best material for this application.

The cell with the sodium metal in it should be kept at a temperature that will cause approximately $2 \times 10^{-5}$ mm of Hg of sodium vapor to be in the cell. This occurs at approximately 125-135°C. Also present in the cell should be about 5 cm of argon, the buffer gas, to lengthen the relaxation time for disorientation in the Zeeman sublevels of the ground state.

Photosensitive Surface

About the only requirements placed on the signal detecting device are that the photosurface chosen should, as was stated before, have a sensitivity band that includes the particular wavelength of the pumping light and should be sensitive to change in intensity of this light. There are several forms of photosensitive material that can be used for this particular problem. The most common is the photocell with a photosurface of Si material which has been used in all previous experiments. There
are several new forms of photosensitive surfaces available that would be suitable. One of these is a wafer form of a device, making use of the photoconductivity properties of certain of the semiconductors. This particular device appears to be well suited to this application.

The Servo Control System

The signal from the photosensitive surface does not contain the frequency of transition but merely shows a dip in the intensity of light when the transition occurs. A servo control system must be formed to use this signal to control the frequency of the oscillator that supplies energy to the absorption cell. The servo control system will change the frequency of the oscillator by controlling the output of the regulated power supply of the oscillator. There are several methods of accomplishing this of which two will be described.

One form of a control system that might be used to do this is to frequency modulate the oscillator in a sawtooth manner. The resulting signal is then passed through a differentiating circuit and the output will be the slope of the signal. At the frequency of transition the slope of the signal is zero and the output of the differentiator will be zero. Positive and negative output signals will denote whether the center frequency is above or below the transition frequency. This error signal then could be made to change the plate voltage applied to the oscillator to again produce the transition frequency.

Another method of forming the servo control system is to vary the frequency from the oscillator in a sinusoidal manner. The resulting signal from the detecting device is then passed to a synchronous amplifier, a form of phase detector, that uses a chopper synchronized with the modulation frequency in deriving an error signal. In this method,
if the average frequency of the oscillator corresponds to the transition frequency a certain charge will be placed upon a condensor during the positive half of the frequency variation. During the negative half cycle the chopper switches and the charge on the condensor is neutralized. As the time constants are long relative to the period of the modulating frequency, zero error voltage appears at the output. If, however, the average frequency is shifted during a cycle of the modulating frequency, a net charge is left on the condensor. This charge has a polarity such that the resulting voltage will be an indication of the direction and amount of drift of the average oscillator frequency from the desired transition frequency. This voltage, as in the previous method, can be used to control the plate voltage of the oscillator in such a manner as to bring it back to the correct average frequency.

As a synchronous amplifier was available, it was decided to use this method of generating the error signal. This also eliminated the need for devising circuitry to cause the oscillator to be frequency modulated in a sawtooth manner.

3-3. Final Prototype Design

Oscillator

An Amerac Incorporated Model 192B5 microwave oscillator was chosen as the frequency source of the system. This oscillator was of the vacuum tube type using a 2C32 lighthouse tube. The frequency determining elements in the circuit were cavities for the plate and grid circuits. The frequency range of this oscillator was from 1700 to 1850 megacycles. A regulated power supply was used to supply the plate voltage to the oscillator and a D.C. filament supply was used in order to reduce any 60 cycle modulation the filaments might introduce. Small shifts in the operating
frequency of the oscillator could be controlled by adjusting the level of bias applied to the control grid of the regulator control tube of the power supply. Frequency modulation was accomplished by superimposing a small 60 cycle voltage on the D.C. level of the voltage on the screen grid of the regulator control tube.

Cavity

The cavity was a right circular resonant cavity operating in the $TE_{011}$ mode. See Figure 3-2. Apertures were cut in both end plates of the cavity to permit the "pumping" light to pass through the absorption cell and to facilitate the mounting of the cell in the cavity. Energy was introduced into the cavity by means of a small coupling loop exciting the magnetic field. A second loop was used to monitor the power in the field in order that the cavity might be tuned easily. Tuning was accomplished by adjusting the separation of the end plates.

The cavity with the absorption cell mounted in it was contained in an oven that kept the temperature of the cell at approximately $130^\circ$ C.

Absorption Cell

The absorption cell was made of a fused quartz tube two inches in diameter and about four inches long. The end facing the lamp source was sealed with a plain clear piece of fused quartz. The other end was tapered sharply down to one-half inch diameter where a graded seal of vycor to pyrex was attached. This graded seal was used for ease in sealing the cell when it was being removed from the vacuum system.

The cell was thoroughly cleaned in preparation for putting in the sodium metal. The inside of the graded seal was treated with potassium fluborate ($KBF_4$) so that the sodium would not react with and turn the glass a dark brown. The cell was then mounted on a vacuum system and a
vacuum in the region of $10^{-6}$ mm of Hg was drawn. Sodium metal was then driven by use of heat into the cell.

After the cell had a sufficient quantity of sodium metal in it, spectral grade argon at a pressure of five cm of Hg was introduced into the cell. The cell was then sealed off.

Sodium Lamps

A sodium lamp was also made on the same vacuum system used for the absorption cell. The lamp was a one-half inch diameter tube of pyrex about one and one-half inches long in its finished form. The tube was treated with potassium fluoborate for the same reason as the graded seal of the absorption cell. This lamp was prepared in the same manner as the cell with the exception that a pressure of argon of only 1 cm of Hg was used.

The lamp was excited by the RF field from the tank coil of an RF oscillator operating at around 50 megacycles. This field not only supplied the energy to excite the vapor, but also supplied the heat to vaporize the metallic sodium.

Photocell

The photocell used for detection of the transition was a standard photocell using a photosensitive surface of Si material. This surface is sensitive to a band of light that includes the wavelengths of sodium light responsible for the optical pumping.

The signal from the photocell was amplified by using a differential amplifier that used a second similar photocell masked to prevent any light from exciting it. This amplifier cancelled out noise resulting from ripple in the power supply that supplies voltage to the photocell circuit and amplifier.
Synchronous Amplifier

A synchronous amplifier was used to detect the frequency drift of the oscillator and generate the necessary signal to bring it back to the transition frequency. A schematic of the basic circuit of the synchronous amplifier is shown in Figure 3-3. A general description of the operation of a synchronous amplifier was given in a previous section. However, attention should be given to the fact that a phase shifting circuit to control the relative phase of the chopper is included. This is necessary to bring the phase of the chopper in synchronism with the phase of the modulating voltage on the oscillator.

The output of the synchronous amplifier was coupled to the control grid of the regulator control tube of the microwave oscillator power supply.

Packaging

The system was first assembled on a laboratory bench. After the system was operating and several tests had been conducted, it was then mounted on a small portable rolling frame. The completed standard is shown in Figure 3-4.

The optical path was such that extraneous light was not permitted to enter the photocell. External light, such as that produced by a fluorescent light, will cause noise that will exceed the signal.
4. EVALUATION AND CONCLUSIONS

4-1. Effect of Instabilities of Components

The component that introduces the major instabilities in the system is the sodium lamp that supplies the "pumping" or polarizing light.

The lamp is quite dependent on the amount of power that the RF field supplies to the lamp. As was mentioned, the RF field not only supplies energy to excite the vapor, but also heats the metal to produce the vapor.

It was found that only one amount of applied power would result in a stable operating condition. This amount of power varied for different lamps. If more power was supplied, the lamp would give off brilliant light, but would not polarize the sodium vapor. If less power was supplied, the intensity of the light was insufficient to cause a usable amount of light to pass through the absorption cell.

An excess amount of power appears to produce a runaway effect in the lamp. This effect might possibly be explained in the following manner. A portion of this excess power will heat the sodium metal by induction heating and produce more vapor. The remainder of the power will excite the increased vapor producing the characteristic sodium light. Atoms in the vapor will migrate at a faster velocity due to the increase in excitation and temperature and will collide with the metallic sodium at this increased velocity. The metal will be heated further by the energy released in these collisions. This will cause more vapor and more collisions and so on until the light given off is very intense. The increase in the collisions of the atoms with one another, with the walls of the lamp and with the sodium metal will cause the spectral lines to become quite broad. The energy available will be spread over a wider band of wavelengths and there will be insufficient energy at the proper wavelengths to cause "pumping" of the vapor in the absorption cell.
A similar process occurs when the energy is insufficient to establish equilibrium and the lamp will cool until the signal disappears. When insufficient power is supplied to the lamp, there will be a certain point where the plasma in the lamp exhibits a negative resistance characteristic, and the lamp will break into oscillations which are invisible to the eye but which the photocell will detect. The signal resulting from these oscillations is many times greater than that produced by the transition and will completely mask the transition signal.

As long as enough energy is supplied by the cavity to the absorption cell to cause transitions, the cavity introduces no apparent instabilities in the system. However, the absorption cell is a possible source of system instabilities. The maximum signal is produced when the cell is operated around $135^\circ$ C. Signals sufficient to control the system will be produced when the temperature of the cell is in the range of $120^\circ$ C to $155^\circ$ C. Below $120^\circ$ C the temperature produces very little vapor and insufficient energy will be taken from the light beam when transition occurs to be detected. Above $155^\circ$ C the vapor becomes so dense that insufficient light is transmitted through the cell. Strong magnetic field will introduce an error into the system, due to the separation of the hyperfine levels being dependent on strong magnetic fields, but steel chassis and other ferrous substances have no apparent effect when placed near the absorption cell.

If the oscillator is such that it will have sudden large shifts of frequency, the servo system is not capable of tracking the shift and will not be able to keep the oscillator on frequency. However, by using a fairly stable oscillator, such as the Amerac oscillator, after a sufficient warm-up period, the synchronous amplifier will have no trouble keeping the oscillator on frequency.
The rest of the system, as long as voltages are not fluctuating greatly or power is not interrupted, will introduce no appreciable instabilities into the system.

4-2. Testing the Standard

As the time available for this project was quite limited, the instrument could not be tested thoroughly nor a complete evaluation made on it.

Tests conducted on the instruments in the limited time available for testing are as follows.

An oscilloscope was used to view the signal from the photocell to the synchronous amplifier. The signal appeared on the oscilloscope when the average frequency of the oscillator was the frequency at which transitions occurred. The stability of the display of this signal, Figure 4-1, indicated that the servo system kept the oscillator locked on the transition frequency.

The accuracy of the standard was determined in a rough manner from the oscilloscope display. The width of the line, as shown in Figure 4-1, was of the order of 3-4 hundred cycles. The ratio of the maximum deviation of the peak of the signal, representing the point where transition occurred, to the line width gave an estimate of the maximum drift of the average frequency of the oscillator. This method gave an accuracy of the order of a few parts in $10^8$. Tests necessary to obtain more precise evaluation of the stability required more time than was available.

The plate voltage of the microwave oscillator was slowly varied when the signal from the microwave oscillator was locked on the transition frequency to see if the system would remain locked on. If the voltage was varied slowly so as not to exceed the time constant of the
synchronous amplifier circuit and thus cause the oscillator to drop out of lockin, the plate voltage could be varied approximately five percent before the oscillator would pull out.

Various other tests were conducted. Among these were observing the effect of temperature of the oven on the operation of the system, checking the effect of the amount of light passing through the absorption cell, and varying the voltage on the photocell circuit. These tests all resulted in changing the amplitude of the signal coming from the photocell. It was found that the signal could be reduced to half of its maximum value before the oscillator would not remain locked in. In general, it was noted that unless a change took place so rapidly that the synchronous amplifier could not follow the change, the system would remain locked in and by observing the oscilloscope no large excursions from the average frequency were noticed.

4-3. Refinements for Greater Accuracy and Reduction of Instabilities

If greater accuracies are desired, steps must be taken to reduce errors resulting from strong magnetic fields as might be experienced in certain environments or in large localized magnetic disturbances. Several methods of eliminating the effect of strong magnetic fields will be discussed. The first method would be to control the field in such a manner that any variation in the field could be compensated. A method whereby this might be done is shown in Figure 4-2. In this method the hyperfine and Zeeman resonance would be observed in the same absorption cell. The Zeeman resonance which is directly proportional to field (700 kcs/gauss) could be used to control two field coils which would balance out any

1 Bell and Bloom have conducted experiments which verify this.
change. Another method that might be used to produce a constant field is the use of magnetic shields. However, a shield will not completely eliminate any large magnetic disturbances. A combination of shielding and field control may represent a practical solution.

The absorption cell will introduce errors in the frequency as the pressure of the buffer gas in the cell varies. Dicke found in his experiments with rubidium that there is an absolute frequency shift of 20 cps per mm of Hg pressure. However, Carver in recent experiments could find no such shifts in frequency with sodium using argon for the buffer gas. If rubidium is used, a method of handling this shift due to pressure would be to calibrate each absorption cell after manufacture, which calibration could be expected to remain permanent for the life of the cell. The temperature around the cell will have to be controlled so that there will not be a change in pressure due to temperature. Temperature control will have to be to a tolerance inversely proportional to the pressure in the absorption cell at calibration, e.g., if the pressure in the cell at calibration is 1 mm of Hg, the tolerance on the temperature is +1.5°C.

The lamp that furnishes the polarizing light is the main source of noise and instabilities in the system. A method whereby this could be corrected to some degree is to use two photocells in a balanced system. One of these photocells is used in the conventional manner of monitoring the light passing through the absorption cell. The other would look directly at the lamp. The response of this photocell could be used to cancel out fluctuations in the lamp intensity which would be picked up as noise by the other photocell which produces the signal. The lamp is very sensitive to bulb temperature and power supply ripple. These can be compensated by straightforward methods of control. The fact that the RF field exciting the vapor to produce the light also must heat the sodium
in the lamp to provide the vapor appears to be a source of considerable instability. A method whereby this might be corrected is to have a small tube containing the sodium metal attached to the bulb, but not in the main RF field. This tube will be heated in some manner to produce the vapor. In this method, the RF field need only excite the vapor. This will eliminate the runaway effect observed when the RF field both excited the vapor and vaporized the sodium.

4-4. Future Considerations

The use of an oscillator to generate the transition frequency directly makes the present instrument quite restricted in its application as a frequency standard. If a crystal is used to generate a lower frequency and then by frequency multiplication and frequency synthesis this frequency is raised to the transition frequency, a very versatile instrument can be formed. With further engineering, the use of transistorized circuits and refinement of the method used to excite the absorption cell, it is felt that a standard can be built and packaged in one or two feet of a standard relay rack. It is believed, because of the simplicity of the system and lack of complex operating and aligning procedures, no special attention will have to be given to the standard once it has been installed.

If rubidium is used as the alkali metal in the absorption cell, the amount of power needed to heat the cell is reduced for rubidium produces sufficient vapor for operation at approximately room temperature. This reduction in the power requirement and the fact that the remainder of the system can be made quite compact suggests the possibility of a portable instrument for field use.

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4-5. Summary

In summary, the advantages of a frequency standard employing optically pumped alkali metal vapor are:

1. Greater accuracies than are available from present standards other than those of great complexity or those that require special operating conditions are possible.

2. No special operating requirements such as liquid nitrogen are needed for the proper operation of the standard.

3. The simplicity and compactness of the instrument will make it ideal for general usage particularly in military applications.

4. The instrument will be able to operate continuously over extremely long periods of time without requiring replenishment or replacement of any of the alkali metal in the absorption cell or lamp.
FIGURE 2-1. SCHEMATIC DIAGRAM OF APPARATUS FOR OPTICAL DETECTION OF MAGNETIC RESONANCE
Figure 2-2. Experimental Apparatus for Optical Detection of Magnetic Resonance
Figure 2-3. Signal Produced by Optical Detection of Transition Between Energy States
FIGURE 2-4. SCHEMATIC DIAGRAM OF APPARATUS FOR OPTICAL DETECTION OF FIELD INDEPENDENT TRANSITION
Figure 2-5. Signal Produced by Optical Detection of Field Independent Transition
Figure 3-1. Basic Schematic of Frequency Standard Using Optical Polarization and Detection.
Figure 3-3. Schematic of Synchronous Amplifier
Figure 3-4. Frequency Standard Utilizing Optical Polarization and Detection
Figure 4-1. Signal from Photocell to Synchronous Amplifier
Figure 4-2. Modification of Frequency Standard using Optical Polarization and Detection to Achieve Greater Accuracy.
Hyperfine Splitting

The theory which says that discrete energy levels in an atom are related to the orbit and spin of the free electrons must be extended to include the effect of the magnetic moment of the nucleus on the energy levels.

In hyperfine splitting, the nucleus of an atom also possesses angular momentum and a magnetic moment. This magnetic moment of a nucleus is small in comparison with that of the electrons. However, account must be taken of the energies resulting from the interaction of the magnetic moment of the nucleus and the magnetic moment of the associated electrons.

As magnetic moments are represented by vectors, all mathematical calculations must be performed by vector means.

The total energy of the atom is the energy resulting from the sum of the magnetic moments of the nucleus and electrons forming the atom. Two $F$ levels result depending on whether the vectors of the nucleus and electrons are aiding or opposing each other, e.g.,

\[
\vec{F} = \vec{I} + \vec{J} \quad \vec{F} = \text{Magnetic moment of the atom in question.}
\]

\[
\vec{F} = \vec{I} - \vec{J} \quad \vec{I} = \text{Magnetic moment of its nucleus.}
\]

\[
\vec{J} = \text{Magnetic moment of its electrons.}
\]

The resulting moment will represent a certain quantized amount of energy, with only certain measurable components, represented by magnetic quantum numbers. In quantum theory, the components $I$ and $J$ can have values of only whole or half integers. Because of these and other restrictions, $F$ can be of only whole or half integers. $m_F$, the magnetic quantum number which represents the measurable components of energy, may have any of the $(2F + 1)$ values, $F$, $(F - 1) \ldots$, $-(F - 1)$, $-F$. 

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Because this paper is concerned with atoms in the ground state, the electrons of concern will have an intrinsic angular momentum of 1/2, and will have no magnetic orbital quantum number or energy dependent upon strength of the applied magnetic field, while the nucleus will have an intrinsic angular momentum of 3/2.

With \( I = 3/2 \) and \( J = 1/2 \), \( F \) will have the values of 2 and 1. The \( F = 2 \) level can have \( m_f \) values of +2, +1, 0, -1, -2, and the \( F = 1 \) level values of \( m_f \) of +1, 0, -1.

Zeeman showed that the strength of the magnetic field surrounding the atom in question would further separate the magnetic quantum energy levels, \( m_f \), and the amount of separation of the levels would be proportional to the strength of the applied field. This is represented in Figure I-1 by the curve of the \( m_f \) sublevels. From the Figure it can be seen that the separation of the \( m_f = 0 \) levels are independent of field strength for weak magnetic fields.
FIGURE 1-1. HYPERFINE SPLITTING OF 2S_\frac{1}{2} GROUND STATE OF SODIUM ATOM