Analytical and experimental determination of the characteristics of a transonic axial turbine

Boatright, Billy Carrol
Monterey, California. Naval Postgraduate School

https://hdl.handle.net/10945/17673

Downloaded from NPS Archive: Calhoun
ANALYTICAL AND EXPERIMENTAL DETERMINATION OF THE CHARACTERISTICS OF A TRANSONIC AXIAL TURBINE

Billy Carrol Boatright
ANALYTICAL AND EXPERIMENTAL DETERMINATION OF THE CHARACTERISTICS OF A TRANSonic AXIAL TURBINE

by

Billy Carrol Boatright

December 1976

Thesis Advisor: R.P. Shreeve

Approved for public release; distribution unlimited.
**Title:** Analytical and Experimental Determination of the Characteristics of a Transonic Axial Turbine

**Authors:** Billy Carrol Boatright

**Performing Organization:**
Naval Postgraduate School
Monterey, California, 93940

**Report Date:** December 1976

**Number of Pages:** 1

**Abstract:** An analysis and test rig measurements of the performance of a transonic axial turbine are reported. The purpose was to confirm the accuracy of measurements made in a test rig which was designed to separate the losses occurring in the stator from the losses occurring in the rotor blade rows. The analysis was programmed for the Hewlett-Packard 21-MX computer. Reasonable agreement between predicted and measured characteristics was obtained using experimentally determined losses in the computer program. Lack of
agreement was noted using theoretical values. It was concluded that the rotor was not choked at the conditions in the tests, and that the test rig measurements were valid. A successful technique for smoothing the data obtained from the rig is also reported.
Analytical and Experimental Determination of the Characteristics of a Transonic Axial Turbine

by

Billy Carrol Boatright
Lieutenant Commander, United States Navy
B.S., University of Idaho, 1965

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1976
# TABLE OF CONTENTS

I. INTRODUCTION ................................................. 9

II. TURBINE TEST RIG INSTALLATION .............................. 12
   A. DESCRIPTION ........................................... 12
   B. TEST MEASUREMENTS AND ACCURACY .................... 13
   C. TESTING AND DATA REDUCTION ........................ 13

III. DATA SMOOTHING TECHNIQUE .................................. 14

IV. ANALYSIS OF TURBINE PERFORMANCE .......................... 17
   A. APPROACH ............................................. 17
   B. ANALYTICAL PREDICTIONS .............................. 17
   C. EXPERIMENTAL TURBINE TEST ......................... 18
   D. COMPARISON OF ACTUAL AND PREDICTED PERFORMANCE ..... 20
   E. DISCUSSION ........................................... 23

V. CONCLUSIONS AND RECOMMENDATIONS .......................... 25

APPENDIX A: TURBINE PERFORMANCE ANALYSIS .................... 65
   A-1 INTRODUCTION ....................................... 65
   A-2 ASSUMPTIONS ......................................... 65
   A-3 BASIC RELATIONS .................................... 68
   A-4 STATOR EXIT CONDITIONS ............................ 70
   A-5 INTERBLADE SPACE ................................... 72
   A-6 ROTOR CONDITIONS ................................... 75
   A-7 STAGE PERFORMANCE PARAMETERS ...................... 77
   A-8 THE COMPUTER PROGRAM ................................ 79
      A-8.1 OVERVIEW ....................................... 79
      A-8.2 DESCRIPTION OF THE PROGRAM ................... 82
   A-9 OPERATING PROCEDURE ............................... 87
   A-10 PROGRAM LISTING .................................... 90

APPENDIX B: TURBINE TEST RIG DATA REDUCTION AND PROCESSING ................................................. 112
   B-1 INTRODUCTION ....................................... 112
   B-2 DESCRIPTION OF PROGRAMS ............................ 113
   B-3 RAW DATA STORAGE ................................... 114
   B-4 PROCEDURE FOR DATA REDUCTION PROGRAM ............ 114

LIST OF REFERENCES ............................................. 120

INITIAL DISTRIBUTION LIST ..................................... 121
LIST OF FIGURES

1. Turbine Test Rig 34
2. Test Rig Piping Installation 35
3. The Floating Stator Assembly 36
4. Turbine Blading Arrangement 37
5. Turbine C 38
6. Pressure Tap Location for TTR Converging-Diverging Nozzles 39
7. Turbine Test Rig Geometry for Turbine Configuration C 40
8. Illustration of Smoothing Using First Degree Polynomial 41
9. Stator Loss vs. Isentropic Head Coefficient, before Smoothing 42
10. Stator loss vs. Isentropic Head Coefficient, after Smoothing 43
11. Rotor Loss vs. Isentropic Head Coefficient, before Smoothing 44
12. Rotor Loss vs. Isentropic Head Coefficient, after Smoothing 45
13. Predicted Performance Range, RPM=15,000, Input Rotor Loss=.2514 46
14. Predicted Performance Range, RPM=18,000, Input Rotor Loss=.2514 47
15. Predicted Performance Range, RPM=18,000, Input Rotor Loss=.1 48
16. Predicted Referred Horsepower vs. $k_{b1}$ 49
17. Predicted Efficiency vs. $k_{b1}$ 50
18. Predicted Stator Loss vs. $k_{b1}$ 51
19. Predicted Rotor Loss vs. $k_{b1}$ 52
20. Referred Horsepower vs. Pressure Ratio 53
21. Referred Horsepower vs. Pressure Ratio 54
22-31 Velocity Diagrams for Turbine Test 55
| A-1 | PRESSURE DISTRIBUTION THROUGH THE STATOR | 102 |
| A-2 | ILLUSTRATION OF SOLUTION AREA | 103 |
| A-3 | MAXIMUM ø AND SONIC LINE | 104 |
| A-4 | TYPICAL SOLUTION PROCESS | 105 |
| A-5 | PROGRAM SCHEMATIC | 106 |
| A-6 | FLOW CHART | 107 |
| A-7 | THERMODYNAMIC PROCESS OF FLUID IN AN AXIAL TURBINE STAGE | 111 |
| B-1 | DATA REDUCTION SCHEMATIC | 118 |
## LIST OF TABLES

I. TURBINE GEOMETRIES ............................. 27
II. PARAMETERS FOR THE TURBINE TEST .......... 28
III. TURBINE TEST RESULTS ......................... 29

A-1 DEFINITION OF VARIABLES ....................... 98

B-1 VARIABLES ADDED TO THE DATA REDUCTION PROGRAM .... 117
ABSTRACT

An analysis and test rig measurements of the performance of a transonic axial turbine are reported. The purpose was to confirm the accuracy of measurements made in a test rig which was designed to separate the losses occurring in the stator from the losses occurring in the rotor blade rows. The analysis was programmed for the Hewlett-Packard 21-MX computer. Reasonable agreement between predicted and measured characteristics was obtained using experimentally determined losses in the computer program. Lack of agreement was noted using theoretical values. It was concluded that the rotor was not choked at the conditions in the tests, and that the test rig measurements were valid. A successful technique for smoothing the data obtained from the rig is also reported.
I. INTRODUCTION

The transonic turbine test rig installation at the Naval Postgraduate School Turbopropulsion Laboratory was designed to study the effects on turbine performance of varying axial and tip clearances, to study the effects of blading design on turbine performance, and to allow the separate determination of stator and rotor losses in an operating machine.

Until the work of Solms (Ref. 1) the separation of rotor and stator losses through test rig measurements had not been attained satisfactorily. Through improvements in hardware and instrumentation and improvements in the data reduction process the stator and rotor losses were determined separately and were reported in Ref. 1.

Anomalies remained, however. Specifically, the turbine configuration designated as Turbine C in Ref. 1 gave different results when compared in terms of "referred" quantities depending on whether the discharge was to atmospheric pressure or to a region of reduced pressure. (Turbine C had converging-diverging stator passages in an axial entry, single impulse stage. The turbine was designed to operate in the transonic range.) In addition, Ref. 1 reported considerable scatter in the loss coefficients.

In the work of Robbins (Ref. 2) it was shown that discharge pressure affected the measurements of flow rate into the stage and also affected the labyrinth leak rate.
Accordingly, Robbins determined accurately the flow rate into the stage and the leak rate through the labyrinth seal (See Fig. 1) for all operating conditions. The results then obtained for turbine C were reported fully in Ref. 2. The continued presence of scatter in the measured loss coefficients was reported and a smoothing technique to eliminate the scatter was suggested.

Before the Turbine Test Rig could be used to measure the effects of varying parameters:

1. The scatter in the loss coefficients had to be eliminated.

2. The overall accuracy of the performance results evaluated from the rig measurements had to be verified in some way.

The resolution of these problems was the goal of the present work and is the subject of this report. First, a satisfactory method was found for smoothing the loss coefficients. The method is described in Section III.

The approach taken to verify the performance of the rig was to first devise an analysis which predicted the performance of the turbine in terms of unknown loss coefficients, and then to show that the measured loss coefficients were consistent with the predicted behaviour.

A description of the Turbine Test Rig is given in Section II. The analysis of the behaviour of the turbine, involving a comparison of an analytical prediction with the results of a short test program, is described in
Section IV. Details of the analysis and the computer program are given in Appendix A.
II. TURBINE TEST RIG INSTALLATION

A. DESCRIPTION

The test installation consists of three major components; an Allis-Chalmers twelve stage axial flow compressor, an exhauster assembly, and the turbine test rig (TTR) itself.

The compressor is the source of driving air for the TTR and for the exhauster assembly. Fig. 2 shows the piping arrangement. Turbine air passes through the first settling tank into an eight-inch pipe containing a flow nozzle, into the second settling tank and into the turbine.

Fig. 3 shows the plenum, the floating stator assembly, the rotor, and the dynamometer (Ref. 1). Pressure ratios of 6:1 can be achieved when the system is hooded. The hood was needed to achieve high pressure ratios in the tests reported here. Fig. 4 shows the turbine blading of the stator and rotor. Ref. 3 contains detailed descriptions of the test rig hardware.

The floating stator assembly shown in Fig. 3 permits measurements of the axial force and the torque on the assembly. Axial and rotational movements are constrained by calibrated force transducers that are heat insensitive. These measurements, together with wall static pressure measurements, allow the determination of the average axial and tangential velocity components at the stator exit.

In this report one configuration designated Turbine C
was tested, the geometry of which is shown in Fig. 5. Table I describes the geometry quantitatively. The stator blade profile is shown in Fig. 6. The blades of the stator generate a converging-diverging nozzle shape. Pressure measurements were taken at the locations shown in Fig. 6. The pressures necessary to the analysis of the stator axial force were taken at the locations shown in Fig. 7.

B. TEST MEASUREMENTS AND ACCURACY

1. Mass Flow Rates

Appendix A of Ref. 2 gives a detailed description of the method used to determine both the turbine flow rate and the labyrinth seal leak rate.

2. Forces, Torques, Temperatures, and Pressure

Ref. 3 and Ref. 5 give calibration procedures for the TTR. Identical procedures were employed here. Table II of Ref. 1 gives the expected accuracies of the measurements.

C. TESTING AND DATA REDUCTION

The TTR data collection system is described in Ref. 4. Appendix D of Ref. 1 gives a detailed explanation of the turbine test procedures. Those procedures were followed here with the exception that a constant RPM was held and the pressure ratio varied over the desired range. The data reduction method developed in Ref. 1 and Ref. 2 was revised as described in Appendix B.
III. DATA SMOOTHING TECHNIQUE

The sensitivity of the loss coefficients to variations in measured quantities was shown in Ref. 2. It was also stated that some variation is unavoidable since measurements are taken over a period of more than a minute. It was also pointed out in Ref. 2 that the parameter most important to the calculation of the loss coefficients was $P_1$ (the average pressure at the stator exit). $P_1$ is an average pressure that can not be measured directly. It is derived as described in Ref. 1 from many other measurements. Significant scatter was observed in the variation of $P_1$ as speed was varied at fixed pressure ratio. However, it was found that the hub and tip pressures ($P_h$ and $P_t$) measured just downstream of the stator varied smoothly over the same range. Since these two pressure were measured directly, and since they varied smoothly, it was assumed that the pressure behind the stator must vary smoothly also. Consequently, it was determined that a polynomial curve fit could be used to describe the variation of $P_1$ as speed was varied. The variation of $P_1$ was represented as a function of $P_h$, $P_t$, and the isentropic head coefficient ($K_{is}$) in the form:

$$
\sigma = \frac{P_1 - P_t}{P_t - P_h} = A_0 + A_1 K_{is} + A_2 K_{is}^2 \cdots \quad (1)
$$

Where $A_1$ is the polynomial coefficient.

Then, $P_1/P_{to}$ (where $P_{to}$ is total pressure upstream of the stator) was computed using the expression
\[
\frac{P_1}{P_{to}} = \sigma \left( \frac{P_t - P_h}{P_{to}} + \left( \frac{P_h}{P_{to}} \right) \right)
\]

Fig. 8 is an illustration of this procedure, which was added to the "bulk process" data reduction program. The data points in Fig. 8 were taken from Runs 6 and 7 in Ref. 2. The two Runs were for the same conditions but were made at different times. It can be seen that the trends are the same for both runs but the scatter is considerable. The scatter is the cause of the scatter in the calculated loss coefficients. The lines on Fig. 8 are the polynomial approximations according to Eq. (1) for the two runs. It can be seen that the polynomial approximation averages the data and maintains the original trend. The chosen smoothing function was a 1st degree polynomial.

Fig. 9 shows the stator loss coefficient prior to smoothing for the points in Run #7. Fig. 10 shows the same data after smoothing.

Similarly, Fig. 11 shows the rotor loss coefficients for Run #7 prior to smoothing and Fig. 12 shows the same data after smoothing.

It can be seen from these figures that the scatter has been removed. This is particularly true for the rotor loss coefficients. As pointed out above, it is believed that the observed scatter was due to the sensitivity of the loss coefficients to small changes in measured quantities during the data collection process. The smoothing technique removes the random variations recorded during
data collection and results in a much more realistic representation of the variation in the losses.

The smoothing technique was incorporated into the data reduction program as described in Appendix B.
IV. ANALYSIS OF TURBINE PERFORMANCE

A. APPROACH

In order to determine if the performance evaluated from the rig measurements was accurate an analysis to predict the behaviour of the test turbine was carried out and programmed in BASIC language.

A performance test was then conducted in a particular way in order to provide a comparison of the measured with the predicted behaviour.

B. ANALYTICAL PREDICTIONS

The flow through the turbine was analysed using a pseudo-1 Dimensional compressible approach. The analysis is described in detail in Appendix A, together with the computer program which was used to obtain predictions of the turbine performance. One of the inputs which the program requires is the rotor passage loss coefficient at zero incidence. Using the method given by Vavra in Ref. 6 it was determined that the rotor loss coefficient should have the value .2514. However, results of previous tests of the turbine indicated that the rotor loss coefficient was rarely as high as .2514 and could be as low as .1. Therefore, the performance of the turbine was analysed using rotor loss coefficients of .2514 and .1. The prediction program was run for both 15,000 and 18,000 RPM with an assumed rotor loss coefficient of .2514, and for 18,000 RPM with an assumed rotor loss coefficient of .1
The above parameters were chosen to obtain a prediction of the turbine performance in a range in which experimental data could be obtained. In particular, the analysis could be used to predict the pressure ratios at which choking would occur in the rotor as well as in the stator. The pressure ratio at which choking occurred in the rotor could then be established experimentally in a test conducted at fixed speed. An examination of the choking condition was considered to be a first test of the performance analysis.

The results of the analysis for the parameters given above are shown in Fig. 13, Fig. 14, and Fig. 15. What is shown in the figures is the map of the range of values which unknown parameters in the analysis can have, that leads to a solution.

Figures 16-19 show predicted performance parameters for the case of 18,000 RPM and assumed rotor loss of .1. These results will be discussed in conjunction with the results of the turbine test run.

C. EXPERIMENTAL TURBINE TEST

In order to examine the occurrence of rotor choking the turbine was run at constant RPM and the pressure ratio across the stage was increased in increments by lowering the back pressure. The point at which the horsepower ceased to increase for an increase in pressure ratio was examined to determine the choking point. At the condition where the flow reaches a Mach number of unity at the exit of the rotor, the power produced by the turbine can not be changed
by altering the downstream pressure.

Tests were conducted in this manner at 15,000 and 18,000 RPM. The controlled parameters for the tests are given in Table II. The reduced data is given in Table III. The referred horsepower is shown plotted versus the pressure ratio for the 15,000 RPM run in Fig. 20 and for the 18,000 RPM run in Fig. 21.

The results are discussed in the next section.
D. COMPARISON OF ACTUAL AND PREDICTED PERFORMANCE

Figures 13, 14, and 15 are maps of possible solutions for the flow through the turbine, when the parameters which are unknown are allowed to vary. It is noted first that the predicted range of solutions extends to pressure ratios \((P_2/P_{to})\) below the predicted choking line. The explanation for this apparent anomaly is that the program calculates all possible solutions, and for any point below the line a particular combination of losses and blockage factors existed which would allow a solution at that point without choking. It should be pointed out that any point on the plot can be brought to the choking line by reducing the stator loss by a very small amount. This is the procedure that was followed to get the range of values at choking shown in Figures 16-19.

Fig. 13 is the map produced by the program for RPM equal 15,000 and for an assumed rotor passage loss coefficient equal to .2514. As can be seen from the figure the pressure ratio, \((P_2/P_{to})\), at choking was about .28, corresponding to a stage pressure ratio \((P_{to}/P_2)\) of 3.57. Fig. 14 is the map for RPM=18,000. Note that the predicted stage pressure ratio for choking is again about 3.57.

Fig. 20 and Fig. 21 show the variation of the referred horsepower vs. the pressure ratio which were measured in turbine tests at 15,000 and 18,000 RPM respectively. It can be seen that the referred horsepower did not become independent of the back pressure at the pressure ratios
achieved in these tests. It was concluded that the stage did not choke at the predicted pressure ratio of 3.57. It was noted however that at the highest pressure ratios obtained at 18,000 RPM, the slope of the horsepower curve was becoming smaller.

The velocity diagrams in Figures 23-31 are also consistent with the argument that the rotor did not choke during the turbine tests. The velocity from the rotor, $V_2$, behaves smoothly and in a predictable manner throughout the pressure range at both test speeds. This might not be expected if, following choking, shock waves appeared downstream of the rotor exit plane.

Note the values of rotor loss coefficient given for the test results in Table III. With the exception of point #10, which was considered to be in error, they were all smaller than the value (.2514) assumed in the first performance calculations. As explained in Appendix A, the value of the rotor passage loss coefficient used in the program is the smallest value that can be calculated for the overall rotor loss coefficient. Therefore, if the experimental results were accurate, the computer program could not predict the correct performance since it could never calculate a rotor loss less than the input value of the passage loss, which was .2514. It was as a consequence of this observation that the performance was re-calculated using a rotor passage loss coefficient equal to .1.

Figures 16-19 show the choking behaviour of the
turbine predicted using the computer program for an assumed rotor passage loss of .1. In these figures, the data for test point #6 is also shown. In this case it can be seen that the predicted pressure ratio \((P_2/P_{t0})\) at choking is about .245, corresponding to a stage pressure ratio \((P_{t0}/P_2)\) of about 4.08. The latter is slightly higher than the maximum pressure ratio that was attained in the turbine tests. (At 18,000 RPM the highest attainable pressure ratio was 3.97, due to the characteristics of the dynamometer.)

Fig. 17 is the range of efficiencies predicted for the choked condition. It is noted that the efficiency measured at test point #6 was reasonably close to the predicted maximum efficiency.

Fig. 18 shows the range of stator losses for which solutions existed, in comparison with the value of stator loss measured at test point #6. It can be seen that the measured value intersects the predicted range of possible solutions. Fig. 19 shows the predicted range of the rotor coefficient. Again, it can be seen that the measured value at test point #6 overlaps the predicted range of possible solutions.

Fig. 16 shows the predicted range of horsepower compared with the horsepower measured at test point #6. The measured horsepower was slightly greater than the maximum value which was predicted.
E. DISCUSSION

The results shown in Figures 16-19 illustrate the uncertainty in the prediction of the performance of the turbine when only the rotor passage loss and blockage factor are known. It is recalled that the analysis satisfies only continuity through the stage, and values of an additional loss coefficient and an additional blockage factor must be established to obtain a unique solution. It is the aerodynamic shaping of the surfaces which determines these factors. Figures 16-19 show the possible range of performance that results simply from the areas of the passages and the blade angles.

The irregular shapes of the bounds on the possible solutions in Figures 16-19 are interesting. There is no obvious explanation for the reduced range of solutions near $k_{b1} = .81$.

The results obtained in the present work suggest that the performance of the turbine as measured in the turbine test rig is reliable. It had been thought previously that the rotor loss measurements were too low. Here, an analysis was carried out to predict the performance of the turbine, and the predicted results have shown very good agreement with the performance measurements in the test rig.

Whether or not the rotor was choking had also been a question in the past. In the present work both the analysis and experimental tests have shown that the rotor was not
choked in any test to date. The computer program with an assumed rotor loss of .1 has shown that the test rig should not be choked at any pressure ratio below about 4.08. Test data has shown that the rotor was not choked at a pressure ratio just slightly below 4.0.

It is also of interest to note that for a rotor passage loss coefficient of .2514 the predicted pressure ratio was shown to be independent of RPM. At both 15,000 and 18,000 RPM the choking pressure ratio \((P_{to}/P_2)\) was calculated to be about 3.57. More results are needed to confirm that the choking pressure ratio is indeed independent of speed.
V. CONCLUSIONS AND RECOMMENDATIONS

1. The rotor of Turbine C was not choked at any pressure ratio tested so far. This conclusion is based on the prediction of the computer program and also on the turbine test results.

2. The loss coefficients measured in the TTR and smoothed as described in this report can be accepted as being truly representative of the losses in the separate blade rows of the turbine.

3. In particular, the results from the computer program suggest that the magnitude of the measured rotor loss coefficients is probably correct. The method used by Vavra in Ref. 6 predicts much larger values of loss coefficients for the present rotor geometry than those which were measured. When the measured value of the rotor loss coefficient was entered into the computer program, there was good agreement between all the calculated and the measured turbine performance parameters. When the higher (calculated) value of the rotor loss was entered, there was a pronounced disagreement between the calculated and the measured performance.

4. The computer program should be used (and progressively developed) in conjunction with all tests carried out in the turbine test rig. Most importantly, a test
should be conducted to determine the rotor choking condition experimentally. This will require the purchase of a water-brake dynamometer to extend the power-speed range of the test rig.

5. Since the accuracy of the test rig measurements is no longer in doubt, experiments to determine the effect of parameter changes (e.g. axial and tip clearances) on turbine performance can go ahead.
### TABLE I

**TURBINE GEOMETRY**

<table>
<thead>
<tr>
<th>TURBINE</th>
<th>STATOR</th>
<th>ROTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DESCRIPTION</td>
<td>(A*) (in(^2))</td>
<td>NUMBER OF BLADES</td>
</tr>
<tr>
<td>STATOR 1</td>
<td>CONVERGING DIVERGING NOZZLES</td>
<td>2.9058</td>
</tr>
<tr>
<td>ROTOR 1</td>
<td>CIRCULAR ARC</td>
<td>7.119</td>
</tr>
</tbody>
</table>
### TABLE II

PARAMETERS FOR TURBINE TEST

<table>
<thead>
<tr>
<th>POINT #</th>
<th>RPM</th>
<th>$P_{to}/P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15,000</td>
<td>2.01</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>2.42</td>
</tr>
<tr>
<td>3</td>
<td>15,000</td>
<td>2.93</td>
</tr>
<tr>
<td>4</td>
<td>15,000</td>
<td>3.49</td>
</tr>
<tr>
<td>5</td>
<td>15,000</td>
<td>3.68</td>
</tr>
<tr>
<td>6</td>
<td>18,000</td>
<td>3.94</td>
</tr>
<tr>
<td>7</td>
<td>18,000</td>
<td>3.47</td>
</tr>
<tr>
<td>8</td>
<td>18,000</td>
<td>2.98</td>
</tr>
<tr>
<td>9</td>
<td>18,000</td>
<td>2.52</td>
</tr>
<tr>
<td>10</td>
<td>18,000</td>
<td>2.02</td>
</tr>
</tbody>
</table>
### Table III Turbine Test Results

#### Point #1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Ratio</td>
<td>2.01185938</td>
</tr>
<tr>
<td>Ref Flow Rate</td>
<td>1.020000000</td>
</tr>
<tr>
<td>Ref RPM</td>
<td>14262.15617</td>
</tr>
<tr>
<td>Ref Rotor Moment</td>
<td>102.5192326</td>
</tr>
<tr>
<td>Ref HP</td>
<td>23.19933152</td>
</tr>
<tr>
<td>Eff. T-S</td>
<td>0.712728663</td>
</tr>
<tr>
<td>Stator Loss Theor.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>Stator Loss Coeff.</td>
<td>0.220295023</td>
</tr>
<tr>
<td>Rotor Loss Coeff.</td>
<td>0.198187353</td>
</tr>
<tr>
<td>Rotor Loss Theor.</td>
<td>0.385181386</td>
</tr>
</tbody>
</table>

#### Volumetric Triangle Data

<table>
<thead>
<tr>
<th>V1</th>
<th>973.73327</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2</td>
<td>204.33594</td>
</tr>
<tr>
<td>VA1</td>
<td>226.65467</td>
</tr>
<tr>
<td>VA2</td>
<td>155.42281</td>
</tr>
<tr>
<td>VU1</td>
<td>946.99688</td>
</tr>
<tr>
<td>VU2</td>
<td>132.49932</td>
</tr>
</tbody>
</table>

| ALPHA 1    | 76.53987 |
| BETA 1     | 60.44410 |
| ALPHA 2    | 49.44789 |
| BETA 2     | -69.84574 |

#### Point #2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Ratio</td>
<td>2.420527383</td>
</tr>
<tr>
<td>Ref Flow Rate</td>
<td>1.020000000</td>
</tr>
<tr>
<td>Ref RPM</td>
<td>14309.05682</td>
</tr>
<tr>
<td>Ref Rotor Moment</td>
<td>132.334797</td>
</tr>
<tr>
<td>Ref HP</td>
<td>30.04483385</td>
</tr>
<tr>
<td>Eff. T-S</td>
<td>0.748970550</td>
</tr>
<tr>
<td>Stator Loss Theor.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>Stator Loss Coeff.</td>
<td>0.183632754</td>
</tr>
<tr>
<td>Rotor Loss Coeff.</td>
<td>0.183337254</td>
</tr>
<tr>
<td>Rotor Loss Theor.</td>
<td>0.455681440</td>
</tr>
</tbody>
</table>

#### Volumetric Triangle Data

<table>
<thead>
<tr>
<th>V1</th>
<th>1126.06784</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2</td>
<td>184.36934</td>
</tr>
<tr>
<td>VA1</td>
<td>265.46943</td>
</tr>
<tr>
<td>VA2</td>
<td>178.98521</td>
</tr>
<tr>
<td>VU1</td>
<td>1094.32844</td>
</tr>
<tr>
<td>VU2</td>
<td>44.23066</td>
</tr>
</tbody>
</table>

| ALPHA 1    | 76.36422 |
| BETA 1     | 64.02313 |
| ALPHA 2    | 13.88078 |
| BETA 2     | -70.79960 |
**TABLE III TURBINE TEST RESULTS (CONT.)**

**POINT #3**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO</td>
<td>2.93257874</td>
</tr>
<tr>
<td>REF FLOW RATE</td>
<td>1.020000000</td>
</tr>
<tr>
<td>REF RPM</td>
<td>14297.06424</td>
</tr>
<tr>
<td>REF ROTOR MOMENT</td>
<td>151.7256387</td>
</tr>
<tr>
<td>REF HP</td>
<td>36.6888547</td>
</tr>
<tr>
<td>EFF. T-S</td>
<td>0.769522864</td>
</tr>
<tr>
<td>STATOR LOSS THEOR.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF.</td>
<td>0.172065787</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF.</td>
<td>0.093520223</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR.</td>
<td>0.43179246</td>
</tr>
</tbody>
</table>

ISEN. HEAD COEFF. = 5.8829423922  
TH. DEG. OF REACTION = -0.031839189  
ACT. DEG. OF REACTION = -0.241795592

**VELOCITY TRIANGLE DATA**

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>VA1</th>
<th>VA2</th>
<th>VU1</th>
<th>VU2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1248.51552</td>
<td>218.93763</td>
<td>314.71895</td>
<td>206.88375</td>
<td>1208.19824</td>
<td>-71.64356</td>
</tr>
</tbody>
</table>

ALPHA 1 = 75.39969  
BETA 1 = 64.49981

**POINT #4**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO</td>
<td>3.490951547</td>
</tr>
<tr>
<td>REF FLOW RATE</td>
<td>1.020000000</td>
</tr>
<tr>
<td>REF RPM</td>
<td>14290.47554</td>
</tr>
<tr>
<td>REF ROTOR MOMENT</td>
<td>181.9713712</td>
</tr>
<tr>
<td>REF HP</td>
<td>41.16922187</td>
</tr>
<tr>
<td>EFF. T-S</td>
<td>0.758535626</td>
</tr>
<tr>
<td>STATOR LOSS THEOR.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF.</td>
<td>0.158068259</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF.</td>
<td>0.134491185</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR.</td>
<td>0.427987152</td>
</tr>
</tbody>
</table>

ISEN. HEAD COEFF. = 6.703669585  
TH. DEG. OF REACTION = -0.014684530  
ACT. DEG. OF REACTION = -0.213884738

**VELOCITY TRIANGLE DATA**

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>VA1</th>
<th>VA2</th>
<th>VU1</th>
<th>VU2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1333.15077</td>
<td>282.81378</td>
<td>355.00633</td>
<td>238.26855</td>
<td>1285.01420</td>
<td>-151.43638</td>
</tr>
</tbody>
</table>

ALPHA 1 = 74.55631  
BETA 1 = 64.26912
### TABLE III TURBINE TEST RESULTS (CONT.)

#### POINT #5

<table>
<thead>
<tr>
<th>POINT #5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO</td>
<td>3.677359655</td>
</tr>
<tr>
<td>REF FLOW RATE</td>
<td>1.020000000</td>
</tr>
<tr>
<td>REF RPM</td>
<td>14292.77432</td>
</tr>
<tr>
<td>REF ROTOR MOMENT</td>
<td>186.8682436</td>
</tr>
<tr>
<td>REF HP</td>
<td>42.3776356</td>
</tr>
<tr>
<td>EFF. T-S</td>
<td>0.752803647</td>
</tr>
<tr>
<td>STATOR LOSS THEOR.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF.</td>
<td>0.140979499</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF.</td>
<td>0.192867968</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR.</td>
<td>0.435423887</td>
</tr>
<tr>
<td>ISEN. HEAD COEFF.</td>
<td>6.950634955</td>
</tr>
<tr>
<td>TH. DEG. OF REACTION</td>
<td>3.98802E-03</td>
</tr>
<tr>
<td>ACT. DEG. OF REACTION</td>
<td>-0.223636759</td>
</tr>
<tr>
<td>VELOCITY TRIANGLE DATA</td>
<td></td>
</tr>
<tr>
<td>V1</td>
<td>1362.18537</td>
</tr>
<tr>
<td>VA1</td>
<td>361.19549</td>
</tr>
<tr>
<td>VU1</td>
<td>1313.45034</td>
</tr>
<tr>
<td>V2</td>
<td>301.42998</td>
</tr>
<tr>
<td>VA2</td>
<td>250.09963</td>
</tr>
<tr>
<td>VU2</td>
<td>-168.39007</td>
</tr>
<tr>
<td>ALPHA 1</td>
<td>74.62753</td>
</tr>
<tr>
<td>BETA 1</td>
<td>64.69083</td>
</tr>
<tr>
<td>ALPHA 2</td>
<td>-33.96162</td>
</tr>
<tr>
<td>BETA 2</td>
<td>-71.92123</td>
</tr>
</tbody>
</table>

#### POINT #6

<table>
<thead>
<tr>
<th>POINT #6</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO</td>
<td>3.938522427</td>
</tr>
<tr>
<td>REF FLOW RATE</td>
<td>1.020000000</td>
</tr>
<tr>
<td>REF RPM</td>
<td>17091.82469</td>
</tr>
<tr>
<td>REF ROTOR MOMENT</td>
<td>169.2232867</td>
</tr>
<tr>
<td>REF HP</td>
<td>45.89160403</td>
</tr>
<tr>
<td>EFF. T-S</td>
<td>0.784488861</td>
</tr>
<tr>
<td>STATOR LOSS THEOR.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF.</td>
<td>0.141347589</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF.</td>
<td>0.187722740</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR.</td>
<td>0.421689292</td>
</tr>
<tr>
<td>ISEN. HEAD COEFF.</td>
<td>5.950902877</td>
</tr>
<tr>
<td>TH. DEG. OF REACTION</td>
<td>0.104266156</td>
</tr>
<tr>
<td>ACT. DEG. OF REACTION</td>
<td>0.175212937</td>
</tr>
<tr>
<td>VELOCITY TRIANGLE DATA</td>
<td></td>
</tr>
<tr>
<td>V1</td>
<td>1318.93669</td>
</tr>
<tr>
<td>VA1</td>
<td>334.28819</td>
</tr>
<tr>
<td>VU1</td>
<td>1274.94005</td>
</tr>
<tr>
<td>V2</td>
<td>269.74423</td>
</tr>
<tr>
<td>VA2</td>
<td>266.54759</td>
</tr>
<tr>
<td>VU2</td>
<td>-63.83484</td>
</tr>
<tr>
<td>ALPHA 1</td>
<td>75.30782</td>
</tr>
<tr>
<td>BETA 1</td>
<td>61.53822</td>
</tr>
<tr>
<td>ALPHA 2</td>
<td>-15.96438</td>
</tr>
<tr>
<td>BETA 2</td>
<td>-70.56818</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>POINT #7</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO = 3.467558140</td>
</tr>
<tr>
<td>REF FLOW RATE = 1.0200000000</td>
</tr>
<tr>
<td>REF RPM = 17039.58315</td>
</tr>
<tr>
<td>REF ROTOR MOMENT = 155.9660642</td>
</tr>
<tr>
<td>REF HP = 42.16691707</td>
</tr>
<tr>
<td>EFF. T-S = 0.784673443</td>
</tr>
<tr>
<td>STATOR LOSS THEOR. = 0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF. = 0.165054929</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF. = 0.126352524</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR. = 0.418863835</td>
</tr>
<tr>
<td>ISEN. HEAD COEFF. = 4.668401576</td>
</tr>
<tr>
<td>TH. DEG. OF REACTION = 0.051715826</td>
</tr>
<tr>
<td>ACT. DEG. OF REACTION = 0.080815056</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VELOCITY TRIANGLE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 = 1280.67141 V2 = 234.50211</td>
</tr>
<tr>
<td>VA1 = 326.93646 VA2 = 234.79915</td>
</tr>
<tr>
<td>VU1 = 1238.22418 VU2 = -1.17934</td>
</tr>
<tr>
<td>ALPHA 1 = 75.29720 ALPHA 2 = -0.28778</td>
</tr>
<tr>
<td>BETA 1 = 60.69243 BETA 2 = -70.61495</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>POINT #8</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO = 2.974737893</td>
</tr>
<tr>
<td>REF FLOW RATE = 1.0200000000</td>
</tr>
<tr>
<td>REF RPM = 17058.70933</td>
</tr>
<tr>
<td>REF ROTOR MOMENT = 137.689044</td>
</tr>
<tr>
<td>REF HP = 37.26749796</td>
</tr>
<tr>
<td>EFF. T-S = 0.774777057</td>
</tr>
<tr>
<td>STATOR LOSS THEOR. = 0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF. = 0.169146155</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF. = 0.083159985</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR. = 0.414274730</td>
</tr>
<tr>
<td>ISEN. HEAD COEFF. = 4.169275243</td>
</tr>
<tr>
<td>TH. DEG. OF REACTION = 0.038828949</td>
</tr>
<tr>
<td>ACT. DEG. OF REACTION = 0.096859972</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VELOCITY TRIANGLE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 = 1200.68325 V2 = 219.93553</td>
</tr>
<tr>
<td>VA1 = 298.71298 VA2 = 208.69610</td>
</tr>
<tr>
<td>VU1 = 1162.84940 VU2 = 69.40873</td>
</tr>
<tr>
<td>ALPHA 1 = 75.59334 ALPHA 2 = 13.39624</td>
</tr>
<tr>
<td>BETA 1 = 59.51871 BETA 2 = -70.71206</td>
</tr>
</tbody>
</table>
### TABLE III TURBINE TEST RESULTS (CONT.)

#### POINT #9

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO</td>
<td>2.519408034</td>
</tr>
<tr>
<td>REF FLOW RATE</td>
<td>1.02090008000</td>
</tr>
<tr>
<td>REF RPM</td>
<td>17170.86109</td>
</tr>
<tr>
<td>REF ROTOR MOMENT</td>
<td>113.7867884</td>
</tr>
<tr>
<td>REF HP</td>
<td>31.00049275</td>
</tr>
<tr>
<td>EFF. T-S</td>
<td>0.743362597</td>
</tr>
<tr>
<td>STATOR LOSS THEOR.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF.</td>
<td>0.189633321</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF.</td>
<td>0.191292857</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR.</td>
<td>0.413088844</td>
</tr>
<tr>
<td>ISEN. HEAD COEFF.</td>
<td>3.567666122</td>
</tr>
<tr>
<td>TH. DEG. OF REACTION</td>
<td>0.058183678</td>
</tr>
<tr>
<td>ACT. DEG. OF REACTION</td>
<td>0.108307260</td>
</tr>
</tbody>
</table>

#### VELOCITY TRIANGLE DATA

<table>
<thead>
<tr>
<th>V1</th>
<th>1103.46913</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2</td>
<td>249.75975</td>
</tr>
<tr>
<td>V11</td>
<td>259.39382</td>
</tr>
<tr>
<td>V22</td>
<td>183.84024</td>
</tr>
<tr>
<td>VU1</td>
<td>1072.54789</td>
</tr>
<tr>
<td>VU2</td>
<td>169.95088</td>
</tr>
<tr>
<td>ALPHA 1</td>
<td>76.40416</td>
</tr>
<tr>
<td>ALPHA 2</td>
<td>42.50319</td>
</tr>
<tr>
<td>BETA 1</td>
<td>57.94743</td>
</tr>
<tr>
<td>BETA 2</td>
<td>-69.80025</td>
</tr>
</tbody>
</table>

#### POINT #10

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE RATIO</td>
<td>2.018415708</td>
</tr>
<tr>
<td>REF FLOW RATE</td>
<td>1.02090008000</td>
</tr>
<tr>
<td>REF RPM</td>
<td>17158.62871</td>
</tr>
<tr>
<td>REF ROTOR MOMENT</td>
<td>82.24683045</td>
</tr>
<tr>
<td>REF HP</td>
<td>22.39168077</td>
</tr>
<tr>
<td>EFF. T-S</td>
<td>0.688180960</td>
</tr>
<tr>
<td>STATOR LOSS THEOR.</td>
<td>0.107100795</td>
</tr>
<tr>
<td>STATOR LOSS COEFF.</td>
<td>0.114113589</td>
</tr>
<tr>
<td>ROTOR LOSS COEFF.</td>
<td>0.489236838</td>
</tr>
<tr>
<td>ROTOR LOSS THEOR.</td>
<td>0.351684730</td>
</tr>
<tr>
<td>ISEN. HEAD COEFF.</td>
<td>2.787522807</td>
</tr>
<tr>
<td>TH. DEG. OF REACTION</td>
<td>0.114732223</td>
</tr>
<tr>
<td>ACT. DEG. OF REACTION</td>
<td>-1.156972-03</td>
</tr>
</tbody>
</table>

#### VELOCITY TRIANGLE DATA

<table>
<thead>
<tr>
<th>V1</th>
<th>987.09080</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2</td>
<td>345.21705</td>
</tr>
<tr>
<td>V11</td>
<td>210.24041</td>
</tr>
<tr>
<td>V22</td>
<td>153.85815</td>
</tr>
<tr>
<td>VU1</td>
<td>964.44140</td>
</tr>
<tr>
<td>VU2</td>
<td>309.03475</td>
</tr>
<tr>
<td>ALPHA 1</td>
<td>77.70237</td>
</tr>
<tr>
<td>ALPHA 2</td>
<td>63.33281</td>
</tr>
<tr>
<td>BETA 1</td>
<td>55.61835</td>
</tr>
<tr>
<td>BETA 2</td>
<td>-66.77659</td>
</tr>
</tbody>
</table>
FIGURE 1  TRANSonic TURBINE TEST RIG
FIGURE 4
TURBINE BLADING ARRANGEMENT
FIGURE 5  TURBINE C
FIGURE 6

PRESSURE TAP LOCATIONS.

TURBINE TEST RIG.

CONV-DIV. NOZZLES.

NOTE:
1. HOLES IN BLADES ARE AT MID SPAN; USE #72 (0.020) DRILL.
2. DIMENSIONS FOR EACH BLADE ARE GIVEN WITH RESPECT TO "RADIAL PLANE" IN THAT BLADE.
   NO. OF BLADES = 31
   ANGLE BETWEEN RADIAL PLANES = 11.613° (11°36'46")
3. ACTUAL LOCATION OF TAPS TO BE RECORDED ON COMPLETION.
4. TAPS AND SURFACES TO BE FREE OF BURS.
5. DRILL RADIAL HOLES VIBRAX, CONNECTING TAPS TO INNER RING SURFACE.
   FOR TAP #5 USE #60 (0.030) DRILL.
   LOCATION HOLES AWAY FROM BLADE SURFACE AT LEAST 0.055.
6. FINISH UNNEEDED SURFACE PENETRATIONS TO BE FRESH-FREE AND FLUSH (94/8Y)
FIGURE 7 TURBINE TEST RIG GEOMETRY FOR TURBINE CONFIGURATION C
FIGURE 8 ILLUSTRATION OF SMOOTHING USING FIRST DEGREE POLYNOMIAL
FIG. 9
DATA FROM RUN #7
UNSMOOTHED

FIGURE 9
STATOR LOSS VS. ISENTROPIC HEAD COEFFICIENT, BEFORE SMOOTHING

STATOR LOSS COEFFICIENT

0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00

ISENTRIC HEAD COEFFICIENT

2.5
3.5
4.5
5.5
6.5
7.5
8.5
FIGURE 10 STATOR LOSS VS. ISENTROPIC HEAD COEFFICIENT, AFTER SMOOTHING

DATA FROM RUN #7
SMOOTHED

STATOR LOSS COEFFICIENT

0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05

8.5
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2

ISENTROPIC HEAD COEFFICIENT
FIGURE 11
DATA FROM RUN #7
UNSMOOTHED

ROTOR LOSS VS. ISENTROPIC HEAD COEFFICIENT, BEFORE SMOOTHING
FIGURE 12 Rotor Loss vs. Isentropic Head Coefficient, After Smoothing
FIGURE 13 PREDICTED PERFORMANCE RANGE,
RPM=15,000, INPUT ROTOR LOSS=.2514
FIGURE 14  PREDICTED PERFORMANCE RANGE,
RPM=18,000, INPUT ROTOR LOSS=.2514
FIGURE 15 PREDICTED PERFORMANCE RANGE, RPM=18,000, INPUT ROTOR LOSS=.1
FIGURE 16  PREDICTED REFERRED HORSEPOWER VS. $K_{b1}$
FIGURE 17  PREDICTED EFFICIENCY VS. $K_{b1}$
FIGURE 18  PREDICTED STATOR LOSS VS. $K_{b1}$
FIGURE 19 PREDICTED ROTOR LOSS VS. $K_{b1}$

![Graph showing predicted rotor loss versus $K_{b1}$ with data points plotted.](image-url)
FIGURE 20 REFERRED HORSEPOWER VS. PRESSURE RATIO
FIG. 21
REFERRED HORSEPOWER
VS. PRESSURE RATIO
FOR RPM=18,000

MEASURED
REFERRED
HORSEPOWER

PTO/P2
FIGURE 22 VELOCITY DIAGRAM FOR TURBINE TEST
FIGURE 23 VELOCITY DIAGRAM FOR TURBINE TEST

Stator Exit Plane

Point \( P \)

\( \frac{P_{1}}{P_{2}} = 2.4 \)

Referred \( \text{RPM} = 14389 \)

\( V_{1} = m_{1} \)

\( V_{2} = m_{2} \)

\( \alpha_{1} = \alpha_{2} \)

\( \beta_{1} = \beta_{2} \)

\( \text{N1 = 595} \)

\( \text{N1 = 1654} \)

\( \text{N1 = 656} \)

\( \text{N1 = 175} \)

\( \text{N2 = 514} \)

\( \text{N2 = 175} \)

\( \text{P1 = 595} \)

\( \text{P1 = 1654} \)

\( \text{P1 = 656} \)

\( \text{P1 = 175} \)

\( \text{P2 = 514} \)

\( \text{P2 = 175} \)

\( \text{P2 = 656} \)

\( \text{P2 = 175} \)
FIGURE 24 VELOCITY DIAGRAM FOR TURBINE TEST
FIGURE 25 VELOCITY DIAGRAM FOR TURBINE TEST
FIGURE 26 VELOCITY DIAGRAM FOR TURBINE TEST

STATOR EXIT PLANE

POINT 5
PT0/P2 = 3.68
REFERRED RPM = 14293

ROTOR EXIT PLANE

V1 = 1362
W1 = 645
VA1 = 361
VU1 = 1313
NU1 = 764
V2 = 301
W2 = 769
VA2 = 250
VU2 = -168
NU2 = -727
ALPHA1 = 75
ALPHA2 = -34
BETA1 = 65
BETA2 = -71
FIGURE 27 VELOCITY DIAGRAM FOR TURBINE TEST

POINT # 5
PTD/P2 = 3.94
REFERRED RPM = 17052

VI = 1318
HI = 781
VAI = 1334
U1 = 1275
VU1 = 277
W2 = 261
V2 = 276
W2 = 261
V2 = 276
H2 = -739
ALPHA1 = -15
BETA1 = -62
BETHA2 = -71
FIGURE 28 VELOCITY DIAGRAM FOR TURBINE TEST

POINT 4
PT0/P2 = 3.47
REF: RPM = 170000

V1 = 1281
H1 = 558
Y1 = 327
Y12 = 1238
K1 = 235
K2 = 235
V12 = -567
阿尔法 = 75
贝塔 = 51
贝塔2 = 71
FIGURE 29 VELOCITY DIAGRAM FOR TURBINE TEST
FIGURE 30 VELOCITY DIAGRAM FOR TURBINE TEST
FIGURE 31 VELOCITY DIAGRAM FOR TURBINE TEST
APPENDIX A
TURBINE PERFORMANCE ANALYSIS

A-1 INTRODUCTION

This appendix gives a detailed description of the theory and analytical technique used to predict the performance of a turbine in the Turbine Test Rig. Variables used in this appendix are defined as they are introduced. Also given are the details of the computer program used to implement the analytical method. The variables used in the computer program are listed and defined in Table A-I.

A-2 ASSUMPTIONS

The following assumptions were made:

1. The stage was choked in the stator.

2. In solving the continuity equation between the stator and rotor the solution corresponding to the lower static pressure was taken.

Assumption (1) was made on the basis of past experience with the test rig. Ref. 1 states that there is supersonic flow in the stator and an examination of the pressure distribution in the stator passages confirms this statement. Fig. A-1 is a plot of the pressure distribution through the stator by pressure tap number (See Fig. 6). It can be seen in this plot that there was a sharp pressure rise between tap #3 and tap #4, which indicates the presence of a shock in the divergent section of the passage. Also, the level of static pressure at the throat taps
in comparison to the supply pressure, was observed to be independent of downstream pressure for all operating conditions.

Assumption (2) limits the range of possible solutions as shown in Fig. A-2. This figure is a plot of flow coefficient ($\phi$) vs. pressure ratio ($P_1/P_{to}$) for various loss coefficients ($z$). (The flow coefficient, a non-dimensional mass flux, is introduced in Section A-2.) There are two possible pressures for a given flow coefficient and given loss coefficient. Solutions were limited to the left side of the line connecting the points of maximum flow function. The procedure was chosen because it resulted in the "supersonic root" for the flow from the stator. Also, conditions corresponding to choking were sought for the rotor, and the locus of locally sonic conditions is as shown in Fig. A-3. It should be noted that because of the definitions of the flow coefficient and loss coefficient, the "choking" condition at which the downstream pressure has no effect upstream of the plane in question, does not correspond to the maximum mass flux from given stagnation conditions.

The analysis is pseudo-1-dimensional. Blockage factors are introduced with the physical cross sectional area to account for non-uniform flow conditions. Loss coefficients are defined on the basis of kinetic energies. The analysis, which requires only mass flow continuity through the stage, is divided into three stages; the stator, the interblade
region and the rotor.

While the analysis itself is general, solutions can only be obtained by assigning values to chosen parameters. In the solutions reported here, the flow angle at the stator exit ($\alpha_j$), the blockage factors for the stator throat ($k_{bS}^*$) and rotor exit plane ($k_{bR}$), and the rotor passage loss coefficient ($z_R$) were given values. The value of $\alpha_j$ was fixed at $\alpha_j = 75^\circ$ based on the results of the turbine tests and from the blading geometry. The blockage factors were set at $k_{bS}^* = .965$ and $k_{bR} = .85$. The rotor passage loss coefficient ($z_R$) was an input variable which was set at .2514 (theoretical value) or .1 (lowest value obtained in test results). The "rotor loss coefficient" output from the program, $z_1-2$, includes all losses from the stator exit plane to the rotor exit plane. $z_1-2$ corresponds to the "rotor loss coefficient" evaluated from test rig measurements; consequently, $z_1-2 \geq z_R$. 
The sketch shows a general adiabatic process with entropy increase (losses) on a T-S diagram on which the temperature scale has been divided by the constant stagnation temperature of the process. A perfect gas is assumed. \((0)\) represents the initial stagnation state and \((\cdot)\) represents the final state.

The non-dimensional velocity, \(X\), is defined as
\[
X = \frac{V}{V_{to}} = \sqrt{\frac{C_p T_o}{\gamma}}
\]
where \(V_{to}\) is the "total" or "limiting" velocity, and the loss coefficient, \(z\), is defined as shown. Note that the stagnation velocity is constant throughout the process.

The "flow coefficient", \(\phi\), is defined here as
\[
\phi = \frac{\dot{m} V}{S_{to} \gamma A k_b}
\]
where \(\dot{m}\) is the flow rate, \(S_{to}\) is the density at the initial stagnation temperature and pressure, \(A\) is the flow area and \(k_b\) is the blockage factor. The flow coefficient defined in this way is a non-dimensional flow rate per unit area, or mass flux, referred to initial stagnation conditions.

The flow rate is given by
\[
\dot{m} = \frac{\rho A k_b V}{}}
so that Eq. A(1) can be written

$$\phi = \frac{pV}{\bar{p}_{to}V_{to}} = \left(\frac{p}{\bar{p}_{to}}\right)\left(\frac{T_{to}}{T}\right)\left(\frac{V}{V_{to}}\right)$$

which, in terms of the non-dimensional velocity defined above, becomes

$$\phi = \left(\frac{p}{\bar{p}_{to}}\right)\left(\frac{X}{1 - X^2}\right)$$  \hspace{1cm} A(2)

The loss coefficient is defined, as shown in the above sketch, as

$$z = \frac{T - T_{is}}{T_{to} - T_{is}} = 1 - \frac{X^2}{X_{is}^2}$$  \hspace{1cm} A(3)

Rearranging Eq. A(3),

$$\left(1 - z\right) = \frac{X^2}{X_{is}^2} = \frac{X^2}{1 - \frac{T_{is}}{T_{to}}}$$

and using the isentropic relationship between pressure and temperatures

$$X = \sqrt{(1 - z)\left[1 - \left(\frac{p}{\bar{p}_{to}}\right)\frac{T_{is}}{T}\right]}$$  \hspace{1cm} A(4)

Using Eq. A(4), Eq. A(2) becomes a single equation for \(\phi\) as a function of pressure ratio \(P/P_{to}\). In analysing the turbine, generally the flow rate is known so that the flow coefficient can be calculated using Eq. A(1). On specifying a value for the loss coefficient, \(\phi(z)\), the pressure ratio can be calculated from Eq. A(2) using Eq. A(4). An iterative technique is used (Newton's Method) starting with an initial estimate of the pressure ratio. Fig. A(2) shows \(\phi\) as a function of \(P/P_{to}\) for various values of \(z\).
in graphical form. The solution to the left of the maxima is obtained by beginning the iteration with a small value of $P_1/P_{to}$.

The analysis of the turbine begins by specifying that the flow rate is set by the choking of the stator nozzles. The pressures at successive stations are then calculated in turn from the flow coefficient as described above.

A-4 STATOR EXIT CONDITIONS

The flow coefficient at the throat of the stator is the value of the flow coefficient corresponding to locally sonic conditions at the minimum area. In the present work, the maximum flow coefficient at a loss coefficient equal to .05 (defined as $\phi^*$) was taken at the stator throat, throughout the calculations.

Since

$$\phi^* = \frac{\dot{W}}{S_{to} V_{to} A_s k_{bs}}$$

A(5)

where $A_s$ and $k_{bs}$ are the area and blockage factor at the throat station, the flow rate being constant through the machine requires that

$$\dot{W} = S_{to} V_{to} A_s k_{bs} \phi^* = \text{constant}$$

at stations ahead of the rotor, where $T_{to}$ is constant; this implies that

$$\rho_{to} A_s k_{bs} \phi^* = \text{constant}$$
The value of $\phi^*$ can be obtained analytically when the loss coefficient is known. The maximum occurs where $\frac{d\phi}{d(p)} = 0$, which leads to the solution (denoting values at the stator throat by an asterisk),

$$(X^*)^2 = B - \sqrt{B^2 - C}$$

where $B = \frac{y + (\frac{y-1}{2})z}{y + 1}$, $C = (1-z)(\frac{y-1}{y+1})$

and $P^*/P_{to} = \left(\frac{D}{1+D}\right)^{\frac{y}{y-1}}$

where $D = \left(\frac{2y}{y-1}\right)\left(1 - X^*\right)^2\left(1 + X^*\right)^2$

Then $\phi^*$ is given by Eq. A(2).

With $\phi^*$ known, continuity of flow rate requires that at the stator exit,

$$\phi_1 = \phi^* \frac{A_1 k_{\beta s}}{A_0 k_{\beta s} \cos \alpha_1}$$

where subscript 1 represents the stator exit plane (station 1) and $\alpha_1$ is the flow angle at the stator exit.

Knowing $\phi_1$, $P_1/P_{to}$ can be found by iteration as described in section A-3. Solutions on the left side of Fig. A-2 were selected by beginning the iteration with $P_1/P_{to}$ close to zero.

After obtaining the value for $P_1/P_{to}$ then $X_1$ can be
calculated using

\[ x_1 = \sqrt{\frac{1}{1-z_s} \left[ 1 - \left( \frac{p_i}{p_t} \right)^{\frac{1}{y-1}} \right]} \]  

A(9)

Then, \( P_1/P_{t1} \) can be found from

\[ P_1/P_{t1} = (1-x_1^2)^{\frac{x}{y-1}} \]  

A(10)

and

\[ P_{t1}/P_{to} = (P_1/P_{to}) (P_{t1}/P_1) \]  

A(11)

A-5 INTERBLADE SPACE

The interblade space, from Station (1) to Station (i), is shown in the sketch.

Conditions at (i) are calculated in a manner similar to those at Station (1).

First, by continuity,

\[ \phi_i = \phi^* \frac{A_5 k_b s}{A_i k_{bi} \cos \alpha_i} \frac{P_{to}}{P_{t1}} \]  

A(12)

where \( \alpha_i \) is the flow angle at the rotor entrance.
The value of $\alpha_i$ is obtained from the condition that angular momentum is conserved in the interblade space. Then

$$x_{ui} = \frac{R_i}{R_1} x_1 \sin \alpha_i$$  \hspace{1cm} A(13)

where $R_i$ = mean radius at Station 1, and $R_i$ = mean radius at Station $i$.

Then

$$\alpha_i = \sin^{-1} \left( \frac{x_{ui}}{x_1} \right)$$  \hspace{1cm} A(14)

and, also

$$\bar{U}_i = \left( \frac{\pi N}{360} \cdot R_i \right) \sqrt{\frac{N}{\omega}}$$  \hspace{1cm} A(15)

where $N$ is the RPM, and the tangential velocity ($V_{ui}$) has been non-dimensionalized as

$$x_{ui} = \frac{V_{ui}}{V_{to}}$$  \hspace{1cm} A(16)

Knowing $\phi_i$, a value for $P_i/P_{t1}$ can be found if a value of the interblade loss coefficient ($z_i$) is assumed. Then

$$P_i/P_{to} = \left( \frac{P_i}{P_{t1}} \right) \left( \frac{P_{t1}}{P_{to}} \right)$$  \hspace{1cm} A(17)
and

\[ X_i = \sqrt{(1-z_i) \left[ 1 - \left( \frac{p_i}{p_{ci}} \right)^{\gamma/\theta} \right]^2} \]  

With the absolute flow properties established at the rotor entrance, the relative flow conditions can be calculated. First, from the geometry of the velocity diagram, the relative flow angle \( (\theta_i) \) is given by

\[ \theta_i = \tan^{-1} \left[ \frac{X_i \sin \alpha_i - \bar{U}_i}{X_i \cos \alpha_i} \right] \]  

where

\[ \bar{U}_i = \frac{U_i}{V_{to}} \]  

is the non-dimensional rotor speed at radius \( R_i \). Then the non-dimensional relative velocity \( (X_{wi}) \) is, from the velocity diagram, given by

\[ X_{wi} = \frac{\cos \alpha_i}{\cos \theta_i} \]  

and the temperature \( T_i \), by

\[ \frac{T_i}{T_{to}} = 1 - X_i^2 \]
The equivalent temperature (Ref. 1), is given by

\[
\frac{T_{Ei}}{T_{to}} = 1 - x_i^2 + x_{wi}^2 + U_i^2 \left( \frac{R_2}{R_i} \right)^2 - 1
\]

A(23)

and therefore

\[
\frac{P_{Ei}}{P_{to}} = \frac{P_{Ei}}{P_i} \frac{P_i}{P_{to}} = \left( \frac{T_{Ei}}{T_{to}} \right)^\frac{\gamma}{\gamma-1} \frac{P_i}{P_{to}}
\]

A(24)

where

\[
T_{Ei} = \text{equivalent temperature into rotor (See Fig. A-7)}
\]

\[
P_{Ei} = \text{equivalent pressure into the rotor (See Fig. A-7)}
\]

A-6 ROTOR CONDITIONS

Using the values calculated thus far it is now possible to calculate the conditions in the rotor. Continuity requires that

\[
\phi_R = \phi^* \sqrt{\frac{T_{Ei}}{T_{to}}} \left( \frac{A_i}{A_R} k_{ls} \right)
\]

A(25)

where a subscript R denotes the exit plane in the rotor blading.

It should now be noted that the process in the rotor frame from the equivalent conditions at (i) to the exit of the rotor is entirely similar to the basic process described in Section A-3. However, the equivalent temperature takes the place of stagnation temperature and relative velocity replaces velocity in the given equations.
The pressure ratio which satisfies the flow coefficient given by Eq. A(25), for an assumed rotor passage loss coefficient \( z_R \) is \( P_2/P_{Ei} \). This is obtained as described in Section A-3. The corresponding non-dimensional velocity is now

\[
y_{w2} = \sqrt{(1-z_R) \left[ 1 - \left( \frac{P_2}{P_{Ei}} \right)^{\frac{Y-1}{\gamma}} \right]}
\]

A(26)

where

\[
y_{w2} = \frac{W_2}{\sqrt{2C_p T_{Ei}}}.
\]

A(27)

Hence

\[
x_{w2} = y_{w2} \sqrt{\frac{T_{Ei}}{T_{to}}}.
\]

A(28)

From the velocity diagram,

\[
x_2^2 = (U_2 - x_{w2} \sin \theta_1)^2 + (x_{w2} \cos \theta_1)^2
\]

A(29)

where \( \theta_1 = 71^\circ \) (from the geometry of the rotor), and

\[
U_2 = \frac{U_i R_2}{R_1}.
\]

The temperature at the rotor exit is given by

\[
\frac{T_2}{T_{to}} = \left[ (1-z_R) \left( \frac{P_2}{P_{Ei}} \right)^{\frac{Y-1}{\gamma}} + z_R \right] \left( \frac{T_{Ei}}{T_{to}} \right)
\]

A(30)

so that \( \frac{T_2}{T_{to}} = \frac{T_2}{T_{to}} + x_2^2 \)

A(31)
Next the loss coefficient that includes all of the
losses in the stage from Station 1 to Station 2 can be
calculated. This coefficient includes all of the losses in
the inter-blade region as well as those in the rotor
passage. It is called \( z_{1-2} \) here but, in fact corresponds
to the measured rotor loss coefficient in the TTR since all
losses aft of the stator in the TTR are included in the
"rotor loss coefficient."

\[
\begin{align*}
z_{1-2} &= 1 - \frac{T_e - T_2}{T_e - T_{2_{is}}} \\
&= 1 - \frac{T_2}{T_{is}} \\
&= 1 - \frac{T_{2_{is}}}{T_{is}} \\
&= 1 - \frac{T_2}{T_{is}} \left( \frac{P_2}{P_{to}} \right)^{\frac{z-1}{Y}} \\
&= A(32)
\end{align*}
\]

A-7 STAGE PERFORMANCE PARAMETERS

In the notation used for the turbine test rig in
Ref. 1 and Ref. 2, the following equations determine the
turbine performance parameters from quantities calculated
in the above steps:

\[
\begin{align*}
\Delta T_w &= T_{to} \left( 1 - \frac{T_{t2}}{T_{to}} \right) \\
\Delta T_{is} &= T_{to} \left[ 1 - \left( \frac{P_2}{P_{to}} \right)^{\frac{z}{Y}} \right] \\
\eta_{T-S} &= \frac{\Delta T_w}{\Delta T_{is}} \\
&= A(33) \\
&= A(34) \\
&= A(35)
\end{align*}
\]
\[
\hat{w} = \varphi^* \left( \int_{t_0}^{V_{to}} A_{o-k_{bo}} \right) \quad A(36)
\]

\[
H.P. = \hat{w} \left( 0.2402 \right) \frac{778}{550} \Delta T_w \quad A(37)
\]

\[
M = H.P. \left( 550 \right) \frac{360}{\pi N} \quad A(38)
\]

\[
\delta = \frac{p_{to}}{p_{ref}} \quad A(39)
\]

\[
\theta = \frac{T_{to}}{T_{ref}} \quad A(40)
\]

\[
\hat{w}^* = \frac{\hat{w} \sqrt{\theta}}{\delta} \quad A(41)
\]

\[
H.P.^* = \frac{H.P.}{\delta \sqrt{\theta}} \quad A(42)
\]

\[
M^* = \frac{M}{\delta} \quad A(43)
\]

\[
N^* = \frac{N}{\sqrt{\theta}} \quad A(44)
\]
A-8 THE COMPUTER PROGRAM

A-8.1 Overview

There are nine degrees of freedom in the computer program:

1. $\alpha$, the flow angle out of the stator
2. $k_{b1}$, the stator exit blockage factor
3. $k_{bi}$, the interstage blockage factor
4. $k_{bR}$, the rotor passage blockage factor
5. $z_s$, the stator loss coefficient
6. $z_i$, the interstage loss coefficient
7. $z_R$, the rotor passage loss coefficient
8. $N$, turbine RPM
9. $T_{to}$, total temperature upstream of the stator

$\alpha$ is input on the basis of turbine test results. At high pressure ratios the value of $\alpha$ was measured to be $75^\circ$-$76^\circ$. Therefore, the value of $\alpha$ was set and held at $75^\circ$. The values of RPM and $T_{to}$ are those conditions for which a solution is being sought.

All the other parameters, the rotor passage loss coefficient ($z_R$) and blockage factor ($k_{bR}$) were given chosen values. This choice was made because the incidence losses were included in the interblade calculation. It was thought that the parameters describing the flow inside the rotor passages could be held fixed as the speed was allowed to vary.

Consequently, there are four degrees of freedom with which the program works. The goal for the program is to
find those solutions which result in flow coefficients in the rotor that are less than, or equal to, the "choking" value. This is done in the program by iteration on the remaining four degrees of freedom. Fig. A-5 is a block diagram showing schematically the iteration process. The order of iteration is (from the most frequent to the least frequent), \( z_i \), \( z_s \), \( k_{bi} \), and \( k_{b1} \). The loss coefficients for both the stator and interblade always begin at zero and they are incremented as the program seeks solutions. A graphical representation of a typical iteration is shown in Fig. A-4. It can be seen that the program can find a solution for the condition where the loss coefficient is equal to zero (denoted as (1) on Fig. A-4). On the next iteration the program will set the loss coefficient equal to .1 and it can be seen that a solution still exists at point (2). On the next iteration, however, it can be seen that a solution does not exist for a loss coefficient equal to .2. In this case the program will go back and add a smaller increment to the previous loss coefficient (.1).

Each time an added increment results in a loss coefficient greater than the maximum allowable the program subtracts that increment and adds a smaller one. In this way the program eventually finds the maximum loss coefficient which will yield a solution. At this point, the given flow coefficient is at the maximum point for the calculated loss coefficient line. At this point "choking", as defined here, has occurred.
It should be pointed out that this "choking" condition (the point of maximum flow coefficient for any given loss coefficient) is not the point corresponding to locally sonic conditions. In Fig. A-3 the locus of points of maximum flow coefficient and the locus of points of local sonic flow are both shown. The point of locally sonic flow always falls to the left of the "choking" point (as defined in this work), and the flow coefficient for sonic flow is always less than the flow coefficient at the defined choking point. The reason is that the point of maximum mass flux (maximum flow coefficient) is for given upstream stagnation conditions (see next section). Because of the way the flow coefficient is defined, the maximum for a given loss coefficient will not correspond to local sonic conditions except where the loss coefficient is equal to zero.

The difference between the "choking" flow coefficient corresponding to the maximum and the flow coefficient at the local sonic condition is about 10% at a loss coefficient of .3 (the highest rotor loss coefficient predicted by the program). The difference is about 4% at a loss coefficient equal to .25 (the highest stator loss predicted), and insignificant (less than 1%) at a loss coefficient of about .15.
A-8.2 Description of the Program

The following discussion refers to Fig. A-6, which is a detailed flow chart of the program, to Table A-I, which is a list of program variables and their definitions, and to the program listing given in Section A-10.

The first step in the program is to input the variables. \( \alpha \), is input in line 280 (it is also converted to radians in that line). \( k_{b1} \) is input in line 286 and \( k_{b1} \) is input in line 284. All other variables are input as requested by the program (lines 20-110). Note that \( P_2 \) and \( P_{to} \) are input also. They are not involved in the calculations and were originally input as the conditions for which a particular solution was sought.

The next step is to go to a subroutine to calculate the value of \( \phi^* \) (line 170). \( \phi^* \) is defined as the value of the flow coefficient at the throat of the stator which sets the values of the flow coefficients at all downstream stations. \( \phi^* \) is generated in the subroutine by the method covered in Section A-4 assuming a loss coefficient of .05 to the throat of the stator (line 160).

Next the program calculates the conditions in the stator exit plane. This is done by first calculating the value for \( \phi_1 \) (line 320). As explained earlier, the value for \( \phi_1 \) determines the upper limit of the stator loss coefficient.

Using the method described in Section A-3, the program calculates the value for \( P_1/P_{to} \). Note that if the stator
loss coefficient is large enough, so that a solution does not exist, this will be detected in the decision, "$P_1/P_{to}<0 \text{ or } >1$" (lines 350 and 370). If "yes" then the stator loss ($z_s$) is too large and the following steps are taken:

1. The last increment added to the stator loss is subtracted (line 420). (Note, the initial stator loss increment is .1.)

2. The existing stator loss increment is multiplied by some factor less than 1 (line 430).

3. The new, smaller, increment is then added back to the stator loss coefficient (line 1570).

If stator loss increment is sufficiently small (line 1575) then the upper limit of the loss coefficient has been reached as discussed earlier. At this point no further solutions are attainable and $k_{bi}$ is incremented (line 2290). Note that the upper limit of stator loss will not normally be attained until several iterations have been made through the entire program.

Next the interblade calculations are made. The solution for $P_1/P_{to}$ found for the stator exit plane is used to make the calculations for $\phi$ (line 770). Again, $\phi$ determines the upper limit on the interblade loss. In the interblade region, however, there is another criterion that must be met. The value for $\theta_i$ generated within the program must be equal to $\theta_a$ ($\theta_a=69^\circ$ and is fixed by the geometry). This criterion is met by iterating the interblade loss coefficient as follows: On every pass through
this section the interblade loss is initially set to zero and the interblade loss increment is initially set to .1 (lines 720, 740). The value for $P_i/P_t$ is calculated using the same procedure as described earlier for $P_1/P_0$. Again, the value of $P_i/P_t$ is checked on each iteration to ensure that it falls between 0 and 1 (line 780). If it does not then the interblade loss is reduced in a method analogous to that covered for the stator loss.

After generating a value for $P_i/P_t$ the program calculates $\varphi_i$ (line 1208). When the value for $\varphi_i$ is generated it is compared to $\varphi_B$ and three results are possible. If $\varphi_i$ is equal to $\varphi_B \pm .05$ then convergence has occurred and the program will continue. If $\varphi_i$ is less than $\varphi_B$ (line 1250) then the value for interblade loss coefficient is increased by the current value of interblade increment and the program goes back to the start of the interblade calculations and begins anew. If $\varphi_i$ is greater than $\varphi_B$ (line 1240) the value of interblade loss is decreased in the same way as was described for the stator loss.

Two results are possible in the interblade region: Either the program finds the value of the interblade loss that gives flow angle convergence ($\varphi_i = \varphi_B$), or the interblade loss increment becomes sufficiently small ($10^{-6}$) to trigger termination.

Note that the interblade loss increment is allowed to become very small ($10^{-6}$) (line 850). This is because the above mentioned angle convergence requirement is very
sensitive. Note, also, that when the interblade loss increment falls to $10^{-6}$ and the iteration process in the interblade region is terminated, the stator loss coefficient is incremented (line 850). Stator loss is incremented because a higher value of stator loss may result in conditions that allow convergence in the interblade region. If not, the stator loss calculations eventually reach the termination criterion mentioned earlier.

Once angle convergence has been attained in the interblade region then the value of the rotor flow coefficient can be calculated (line 1470). If the program gets to this point for any given set of conditions then a solution exists and the only determination to be made is whether the resulting rotor flow coefficient is greater than the "choking" value (line 1530). If "yes" then it is disallowed, since a flow coefficient greater than the "choking" value cannot occur physically. If the calculated coefficient is less than, or equal to the "choking" value then the results are printed, the stator loss is incremented, and the calculations start again at the stator exit plane.

The choking value of the flow coefficient for the rotor ($\theta^*_R$) is determined in exactly the same way as the choking value for the stator ($\theta^*$), with the assumed value of rotor loss used as the loss coefficient. (At present the value for $\theta^*_R$ is calculated separately and inserted in line 121. It is suggested that a "GOSUB 2140" be inserted at this point. Then the input value for rotor loss
coefficient can be changed easily and the "choking" value for the rotor will be calculated in the program.) If the resulting flow coefficient is greater than "choking", the program increments stator loss coefficient and returns to the stator plane calculations to begin anew.

Therefore, in all cases the program will seek the highest value of stator loss coefficient for which solutions exist. At some point, the rotor flow coefficient normally attains a value less than the "choking" value and all solutions for higher stator loss coefficients will be acceptable solutions. Or, if the "choking" condition is never reached, the program will attain the highest possible stator loss in attempting to achieve solutions less than "choking".

When the highest value of the stator loss is reached, there are two possible avenues in the program. If the program was generating solutions less than "choking" at the time of termination in the stator calculations, then \( k_{b1} \) will be incremented, stator loss will be set back to zero, and the process begun anew (line 2280). If the solutions being generated at the time of termination were greater than "choking", then the upper value of \( k_{b1} \) has been reached and any increase in \( k_{b1} \) will yield no further solutions. In this case \( k_{b1} \) is incremented, \( k_{b1} \) is set to the pre-determined lower limit, and stator loss is reset to zero (line 2330).

The program continues in the above manner until the
pre-determined upper limit of \( k_{b1} \) is reached (line 2335), at which point the program is terminated.

Since the 21-MX computer has only 6 digits of accuracy, round-off error has been a problem. Where Newton's Method is used to solve for pressure ratio the computer lacks sufficient accuracy to converge to the desired accuracy under certain conditions. When this happens, the program will stay inside the iteration and never converge. This problem has been solved by putting a counter inside each iteration loop (lines 540 and 980). More than 60 iterations is treated as a non-convergent condition, and the appropriate loss coefficient is reduced in the manner covered previously.

A-9 OPERATING PROCEDURE

The procedures for operating the Hewlett-Packard 21-MX computer will not be covered in this paper since the applicable manuals are available at the laboratory. The computer must be on, with the RTE-B operating system "READY".

1. Load the paper tape program labeled "TTR 11".

2. Edit into the program the minimum values of \( k_{b1} \) (line 284) and \( k_{b1} \) line (286). Edit the desired value of \( \alpha \), into line 280. If it is desired to get a print-out of every solution then either remove lines 1724 and 1726 or place "REM" in front of them. If these two lines are left in the program then a solution will be printed out only if the calculated pressure ratio \( (P_2/P_{to}) \) is less
than, or greater than, the previously calculated lowest or highest pressure ratio respectively, for the value of $k_{b1}$ then being used.

3. Type "RUN", "RETURN" on the keyboard and the program will begin execution by asking for inputs:

   "INPUT STATOR LOSS COEFFICIENT" The normal input is zero. However, if some specific case is required then any desired loss coefficient can be input.

   "INPUT PT0" Input the upstream total pressure.
   "INPUT P2" Input the hood pressure.
   "INPUT RPM" Input the RPM desired.
   "INPUT TTO" Input the total temperature into the stage.

This is all that is required to operate the program. Depending on the range of $k_{b1}$ requested the program will take from 30 minutes to 36 hours to run on the 21-MX.

It is important to realize that the above procedure will produce solutions that give values of the rotor flow coefficient less than "choking". If the turbine performance at the "choking" condition is desired then after the computer run is complete a further step must be taken. It is necessary to take the values of $k_{b1}$ and $k_{bi}$ for which a solution is desired and force the choking condition. This is done as follows:

1. Scan the output results at the desired $k_{b1}$ and $k_{bi}$ and locate the point that has a value of rotor flow coefficient closest to the choking value and a non-zero
2. Re-start the program, this time putting in a value of the stator loss coefficient slightly less than the value printed on the output.

As stator loss is manually decreased, the rotor flow coefficient will increase towards the choking value. When the rotor flow coefficient is within the limits specified in line 2370 the words "CONVERGENT CONDITIONS" will appear on the teletype output preceding the printed results.

If the stator loss coefficient is decreased too much, the rotor flow coefficient will increase to a value above choking. In this case no output will be printed on the teletype. However, the flow coefficient will appear on the video display. In this case increase the stator loss slightly.

It has been found in this work that the extreme values of referred horsepower, stator loss, rotor loss, and efficiency occur at the lowest and highest values of k_{bi} for each value of k_{b1}. With this in mind it is possible to find the range of predicted values at choking by forcing to choking only the lowest value of k_{bi} (with non-zero stator loss) and the highest value of k_{bi} for each k_{b1}. 
REM**TRI(VI)**THIS PROGRAM IS TO PREDICT THE LOSSES THRU THE TIR

10 REM**TRI(VI)**THIS PROGRAM IS TO PREDICT THE LOSSES THRU THE TIR
11 LET Q8=1
12 LET Q9=1
13 PRINT "INPUT PTO"
14 PRINT "INPUT PTO"
15 PRINT "INPUT PTO"
16 PRINT "INPUT PTO"
17 PRINT "INPUT PTO"
18 PRINT "INPUT PTO"
19 PRINT "INPUT PTO"
20 PRINT "INPUT PTO"
21 PRINT "INPUT PTO"
22 PRINT "INPUT PTO"
23 PRINT "INPUT PTO"
24 PRINT "INPUT PTO"
25 PRINT "INPUT PTO"
26 PRINT "INPUT PTO"
27 PRINT "INPUT PTO"
28 PRINT "INPUT PTO"
29 PRINT "INPUT PTO"
30 PRINT "INPUT PTO"
31 PRINT "INPUT PTO"
32 PRINT "INPUT PTO"
33 PRINT "INPUT PTO"
34 PRINT "INPUT PTO"
35 PRINT "INPUT PTO"
36 PRINT "INPUT PTO"
37 PRINT "INPUT PTO"
38 PRINT "INPUT PTO"
39 PRINT "INPUT PTO"
40 PRINT "INPUT PTO"
41 PRINT "INPUT PTO"
42 PRINT "INPUT PTO"
43 PRINT "INPUT PTO"
44 PRINT "INPUT PTO"
45 PRINT "INPUT PTO"
46 PRINT "INPUT PTO"
47 PRINT "INPUT PTO"
48 PRINT "INPUT PTO"
49 PRINT "INPUT PTO"
50 PRINT "INPUT PTO"
51 PRINT "INPUT PTO"
52 PRINT "INPUT PTO"
53 PRINT "INPUT PTO"
54 PRINT "INPUT PTO"
55 PRINT "INPUT PTO"
56 PRINT "INPUT PTO"
57 PRINT "INPUT PTO"
58 PRINT "INPUT PTO"
59 PRINT "INPUT PTO"
60 PRINT "INPUT PTO"
61 PRINT "INPUT PTO"
62 PRINT "INPUT PTO"
63 PRINT "INPUT PTO"
64 PRINT "INPUT PTO"
65 PRINT "INPUT PTO"
66 PRINT "INPUT PTO"
67 PRINT "INPUT PTO"
68 PRINT "INPUT PTO"
69 PRINT "INPUT PTO"
70 PRINT "INPUT PTO"
71 PRINT "INPUT PTO"
72 PRINT "INPUT PTO"
73 PRINT "INPUT PTO"
74 PRINT "INPUT PTO"
75 PRINT "INPUT PTO"
76 PRINT "INPUT PTO"
77 PRINT "INPUT PTO"
78 PRINT "INPUT PTO"
79 PRINT "INPUT PTO"
80 PRINT "INPUT PTO"
81 PRINT "INPUT PTO"
82 PRINT "INPUT PTO"
83 PRINT "INPUT PTO"
84 PRINT "INPUT PTO"
85 PRINT "INPUT PTO"
86 PRINT "INPUT PTO"
87 PRINT "INPUT PTO"
88 PRINT "INPUT PTO"
89 PRINT "INPUT PTO"
90 PRINT "INPUT PTO"
275 REM INPUT INITIAL GUESS FOR ALPHA1
277 LET J=3.14159
280 LET Q1=75*J/180
281 REM INPUT BLOCKAGE FOR STATOR
282 LET K5=.851
283 REM INPUT INTERSTAGE BLOCKAGE
284 LET K4=.58
285 REM INPUT "1" BLOCKAGE
286 LET K6=.772
291 REM Z8=INITIAL STATOR LOSS INCREMENT
292 LET Z8=.1
300 LET P1=.001
310 REM CALCULATE PHI1
320 LET F1=FO*(AO*K1)/(K6*COS(01))
330 LET Y9=0
340 REM THE FOLLOWING CALCULATES P1/P10
350 IF (P1>0) THEN 370
360 GO TO 380
370 IF ((1-Z0)*(1-P1*G0)>0) THEN 450
370 PRINT # 1"LOWUP IN CALC. P1/P10, EXCESS STATOR LOSS"
380 PRINT # 1"AT LOWUP F1="F1
390 PRINT # 1"AT BLOWUP Z0="Z0
400 PRINT # 1"DECREASE INCREMENT SIZE AND CONTINUE"
410 LET Z0=Z0-Z8
420 IF Z0<0 THEN 2280
430 LET Z8=Z8*.8
440 GO TO 1570
450 LET X1=SQR((1-Z0)*(1-P1*G0))
460 LET A9=P1*(X1/(1-X1^2))-F1
470 LET D1=(P1*G0)*G0/2*(X1/(1-X1^2)/(1-P1*G0))
480 LET D1=(X1/(1-X1^2))*(1-(D1)/(1/(1-X1^2)+(2*X1^2)/(1-X1^2)^2)))
490 LET E=A9/D1
500 LET A8=P1-E
502 LET X1=SQR((1-Z0)*(1-P1*G0))
504 LET W9=P1*(X1/(1-X1^2))
510 IF F1=W9 AND (F1-W9<.000005 OR ABS(P1-A8)<.0005) THEN 635
520 LET Y9=Y9+1
530 LET P1=A8
540 IF (Y9>60) THEN 380
550 GOTO 350
635 LET P1=A8
640 LET X1=SQR((1-Z0)*(1-P1*T0))
650 LET P3=(1-X1+2)*(1/T0)
660 LET P4=P1/P3
667 PRINT
668 PRINT# 1"THE FOLLOWING ARE FOR STATOR LOSS="Z0"ALPHA1="01*180/J
669 PRINT# 1"hPM="N"k1="K4"K1="K6
670 PRINT# 1"P1/P10="P1
680 PRINT# 1"P1/P11="P3
690 PRINT# 1"P11/P10="P4
700 REM CALCULATE INTERSTAGE PHII AND ITERATE WITH INCREASING
710 REM INTERSTAGE LOSS UNTIL BETA(1)=BETA(WLAGE)
720 LET Z3=-.1
730 LET Z9=.1
740 LET Z3=Z3+.9
750 LET P5=.001
760 LET Y9=0
770 LET F3=F0*(1/P4)*((A0*k1)/(K4*.134902*COS(1.20443)))
780 IF (P5<0 OR P5>1) THEN 850
790 GOTO 890
850 IF (Z9>.000001) THEN 1261
860 PRINT# 1"NO CONVERGENCE FOR THIS STATOR LOSS(Z9<.000001),"
870 PRINT# 1"DECREASE STATOR LOSS AND CONTINUE"
880 GOTO 420
890 LET X3=SQR((1-Z3)*(1-P5*T0))
900 LET A9=P5*(X3/(1-X3+2))-F3
910 LET D1=(P5*T0)*G0/Z*(1-X3+2)/Z*(1-P5*T0))
920 LET D1=X3/(1-X3+2))*(1/(1/(1-X3+2)+(2*X3+2)/(1-X3+2)*2))
930 LET E=A9/D1
940 LET A8=P5-E
942 LET X3=SQR((1-Z3)*(1-P5*GO))
944 LET W9=P5*(X3/(1-X3*2))
950 IF F3>W9 AND (F3-W9<.000005 OR ABS(P5-A8)<.0005) THEN 1075
960 LET P5=A8
970 LET Y9=Y9+1
980 IF (Y9>60) THEN 1000
990 GOTO 780
1000 PRINT"1""CONVERGENCE NOT POSSIBLE FOR STATOR LOSS="Z0
1001 PRINT"1""DECREASE STATOR LOSS AND CONTINUE"
1010 GOTO 420
1075 LET P5=A8
1080 LET P6=P5*P4
1090 LET X3=SQR((1-Z3)*(1-P5*GO))
1100 LET Q1=(1-X3*2)*(1/GO)
1110 LET P=P6/Q1
1170 LET X4=(4.18375/4.195)*X1*SIN(Q1)
1180 IF (X4/X3>1) THEN 1281
1190 LET O3=MNA(X4/X3)
1200 LET U3=((J*X)/360)*4.195)/V0
1205 LET V3=(X3*SIN(O3)-U3)/(X3*COS(O3))
1210 LET V3=ATN(V3)
1215 LET V3=180*(V3/J)
1230 IF ((69-V3)<0 AND (69-V3)<.1) THEN 1318
1240 IF (V3>69) THEN 1290
1250 GOTO 740
1261 REM "BLOW-UP CAUSED BY EXCESSIVE INTERSTAGE LOSS"
1270 GOTO 1290
1280 PRINT
1281 REM "X4/X3>1, DECREASE INTERSTAGE LOSS AND CONTINUE"
1290 LET Z3=Z3-Z9
1292 IF (Z3<0 AND (C1=1)) THEN 2280
1293 LET C1=0
1300 LET Z9=Z9*.8
1305 IF Z3<0 THEN 2270
1308 IF Z9<.000001 THEN 860
1310 GO TO 740
1318 PRINT
1319 PRINT
1320 PRINT"AT ANGLE CONVERGENCE FOLLOWING CONDITIONS EXISTED:" Print
1330 PRINT"STATOR LOSS COEFFICIENT=".L0
1340 PRINT"INTERSTAGE LOSS COEFFICIENT=".L3
1350 PRINT".BETA(I)=".L3
1360 PRINT"P1/PT0=".P1
1370 PRINT"P(I)/PT1=".P5
1380 LET B3=B3*0180
1390 LET X5=X3*X0(CO3)/COS(B3)
1400 REM "XW1="X5
1410 LET T3=1-X5Y2
1420 REM "T1/TO="T3
1430 LET T4=1-X3T2+X5Y2+U3T2+((4,25275/4,195)Y2-1)
1440 LET T4=T4/T3*3,48756*P6
1450 LET P7=(T4/T3)*3,48756*P6
1460 REM "PE1/PT0="P7
1470 LET F2=F0*(SQR(T4)/P7)*((A0&1)/(.04871*X5))
1475 LET 01=01*180/J
1490 PRINT"ALPHA1=".01
1491 LET 01=01*0180
1500 PRINT"CALCULATED ROTOR FLOW COEFFICIENT=".F2
1510 PRINT"ISENTROPIC ROTOR FLOW COEFFICIENT (WITH LOSS=.2514)=".F4
1530 IF F2>F4 THEN 1550
1540 GO TO 2370
1550 PRINT"CALCULATED ROTOR FLOW COEFFICIENT>ISENTROPIC ROTOR FLOW"
1560 PRINT"COEFFICIENT (WITH ROTOR LOSS=.2514). THEREFORE, WILL"
1561 PRINT"RE-COMPUTE WITH STATOR LOSS COEFFICIENT INCREMENTED".28
1570 LET L0=L0+28
1572 IF ((L8<.0008) AND (F2>.22)) THEN 2330
1575 IF (L8<.0008) THEN 2280
1580 GO TO 300
1700 REM COMPUTE P1/PT0 FOR THESE CONDITIONS AND 2D COMPARE TO THEOR.
1705 PRINT"CONVERGENT CONDITIONS"
1706 LET S9=.2514
1707 GOSUB 7000
1710 LET R9=S8*P7
1720 PRINT# 1"P2/PTO(Actual)="P2", P2/PTO(Calculated)="R9
1721 IF (R9<69) THEN 1724
1722 IF (R9>68) THEN 1726
1723 GOTO 1570
1724 LET Q9=R9
1725 GOTO 2430
1726 LET Q8=R9
1729 GOTO 2430
2130 DEF FNA(X)=ATN(X/SQR(1-X*X))
2140 LET P8=.5
2150 LET X8=SQR((1-L1)*(1-P8*G0))
2160 LET D1=(P8*G0)*G0/2*((1-X8)+(1-X8*2)/(1-P8*G0))
2170 LET D1=(X8/(1-X8*2))*(1-(D1*(1/(1-X8*2)+(2*X8*2)/(1-X8*2))2))
2180 LET D2=(1.61604*(1-X8*2)+(1-P8*G0))
2190 LET D2=D2-((P8*G0*.161604*(1-X8*2))+(3.93121*(1-P8*G0)*2))
2200 LET D3=D3+(D1*(1/(1-X8*2)+(2*X8*2)/(1-X8*2))2)
2210 LET E=D1/D2
2220 LET A9=P8*E
2230 IF (AtS(P8-A9)<.000001) THEN 2255
2240 LET P8=A9
2250 GOTO 2150
2255 LET P8=A9
2260 RETURN
2270 PRINT# 1"INTERSTAGE LOSS LESS THAN ZERO"
2271 IF (Z0=0) THEN 1570
2272 GOTO 420
2280 LET C1=1
2290 LET K4=K4+.01
2300 LET Z0=0
2310 GOTO 291
2320 LET K4=1
2330 LET K6=K6+.01
2335 IF (K6>.82) THEN 2620
2336 LET Q8=0
2337 LET Q9=1
2340 LET K4=.4
2341 LET C1=1
2350 LET Z0=0
2360 GOTO 291
2370 IF (Abs(F2-F4)<.0005) THZ N 1705
2375 GOTO 1706
2430 PRINT # 6"K1="K1"K4="K4"PT1/PTO="h9"K6="K6
2440 PRINT # 6"ROTOR FLOW COEFFICIENT="F2
2450 PRINT # 6"STAIRK LOSS="L0
2460 PRINT # 6"INTERSTAGE LOSS="L3
2470 PRINT # 6"P1/PTO="P1
2480 PRINT # 6"P1/PTL="P5
2490 PRINT # 6"RPW="N
2602 GOTO 3000
2620 STOP
3000 LET Q0=R9/P7
3010 LET Q=1-Q0*90
3020 LET S=(1-O)\0
3030 LET L0=(Q0/F2)^2/0
3040 LET Z6=L0*(Suk((1+(2+5+1)/L0)-1)-S
3050 PRINT # 6"PREDICTED Rotor LOSS="L6
3060 LET Y2=Suk((1-.2514)*(1-Q0*90))
3070 LET X6=Y2*90\(T4)
3080 LET U2=U3*(4.13875/4.159)
3090 LET T2=((1-.2514)*90*0+.2514)*T4
4000 LET X2=(U2-X6*(IN(71*J/180))t2+X6t2*(COS(71*J/180))t2
4010 LET T1=T2+X2
4020 LET D3=T0*(1-T1)
4030 LET D4=T0*(1-K9t30)
4040 LET E0=D3/D4
4050 LET W0=F0*9.0*90*40*K1
4060 LET H0=W0*.4402*(778/550)*D3
4070 LET M1=H0*550*(360/(J*N))
4080 LET D=P0/2992
4090 LET 09=00/518.7
5000 LET W1=W0*SQR(09)/D
5010 LET H1=HO/(D*SQR(09))
5020 LET M2=M1/D
5030 LET N0=N/SQR(09)
5035 LET U1=(J*N**4.18375)/(360*V0)
5040 LET X0=(X1*COS(01*U/180))**2+(X1*SIN(01*U/180)-U1*4.18375/4.195)**2
5050 LET T5=1-X1**2
5070 LET T6=1-X1**2+X0**2+(U1**2*(((4.2503/4.18375)**2-1))
5080 PRINT #6 "CALCULATED EFFICIENCY="E0
5090 PRINT #6 "CALCULATED FLOW RATE="W0
5100 PRINT #6 "CALCULATED HORSEPOWER="H0
5110 PRINT #6 "CALCULATED ROTOR TORQUE="M1
5120 PRINT #6 "CALCULATED REFERRED FLOW RATE="W1
5130 PRINT #6 "CALCULATED REFERRED HORSEPOWER="H1
5140 PRINT #6 "CALCULATED REFERRED ROTOR TORQUE="M2
5150 PRINT #6 "CALCULATED REFERRED RPM="N0
5160 LET Z7=1-((T4-T2)/(T4-(K9/P4)*T30))
5170 PRINT #6 "CALCULATED ROTOR LOSS COEFFICIENT="Z7
5171 PRINT #6 ""
5172 GOTO 1570
5180 GOTO 1570
7000 LET SR=.001
7010 LET S7=SQR((1-S9)*(1-S8*G0))
7020 LET A9=SR*(S7/(1-S7**2))-1.2
7030 LET D1=(S8*G0)*G0/2*((1-S7**2)/(1-S8*G0))
7040 LET D1=(S7/(1-S7**2))*1-(D1*(1/(1-S7**2)+(2*S7**2)/(1-S7**2)**2))
7050 LET E=A9/D1
7060 LET A8=S8-E
7070 IF (A8*(SR-A8)<.0001) THEN 7100
7080 LET S8=A8
7090 GOTO 7010
7100 LET S8=A8
7110 LET S7=SQR((1-S9)*(1-S8*G0))
7120 RETURN
<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>(A*) Area at the Stator Throat</td>
</tr>
<tr>
<td>A1</td>
<td>(A1) Area at the Stator Exit Plane</td>
</tr>
<tr>
<td>A8</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>A9</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>C1</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>D</td>
<td>((\delta = P_{to}/P_{ref}))</td>
</tr>
<tr>
<td>D1</td>
<td>(\frac{d \phi}{d \left(\frac{P_{to}}{P_{ref}}\right)})</td>
</tr>
<tr>
<td>D2</td>
<td>(\frac{d^2 \phi}{d \left(\frac{P_{to}}{P_{ref}}\right)^2})</td>
</tr>
<tr>
<td>D3</td>
<td>(\Delta T_w)</td>
</tr>
<tr>
<td>D4</td>
<td>(\Delta T_{is})</td>
</tr>
<tr>
<td>E</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>E0</td>
<td>((\eta_{T-S})) Total-to-Static Efficiency</td>
</tr>
<tr>
<td>F0</td>
<td>((\phi^*)) Flow Coefficient at Stator Throat</td>
</tr>
<tr>
<td>F1</td>
<td>((\phi_1)) Flow Coefficient at Stator Exit Plane</td>
</tr>
<tr>
<td>F2</td>
<td>((\phi_R)) Flow Coefficient Through Rotor Passage</td>
</tr>
<tr>
<td>F3</td>
<td>((\phi_i)) Flow Coefficient Through Interblade Area</td>
</tr>
<tr>
<td>F4</td>
<td>Input Value of (\phi_R) Based on Assumed Rotor Loss</td>
</tr>
<tr>
<td>F8</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>F9</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>G0</td>
<td>((\text{Gamma} - 1)/\text{Gamma})</td>
</tr>
<tr>
<td>H0</td>
<td>Horsepower</td>
</tr>
<tr>
<td>H1</td>
<td>Referred Horsepower</td>
</tr>
<tr>
<td>H8</td>
<td>((\phi)) Rho</td>
</tr>
<tr>
<td>H9</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>J</td>
<td>(N) Pi</td>
</tr>
<tr>
<td>K1</td>
<td>( k_{bs} ) Blockage Factor at Stator Throat</td>
</tr>
<tr>
<td>K4</td>
<td>( k_{bi} ) Interblade Blockage Factor</td>
</tr>
<tr>
<td>K5</td>
<td>( k_{bR} ) Blockage Factor in the Rotor</td>
</tr>
<tr>
<td>K6</td>
<td>( k_{b1} ) Blockage Factor at the Stator Exit Plane</td>
</tr>
<tr>
<td>M1</td>
<td>Rotor Torque</td>
</tr>
<tr>
<td>M2</td>
<td>Referred Rotor Torque</td>
</tr>
<tr>
<td>N</td>
<td>RPM</td>
</tr>
<tr>
<td>N0</td>
<td>Referred RPM</td>
</tr>
<tr>
<td>O1</td>
<td>( (\alpha, ) ) Flow Angle at Stator Exit Plane</td>
</tr>
<tr>
<td>O3</td>
<td>( (\alpha, ) ) Flow Angle at Rotor Entrance Plane</td>
</tr>
<tr>
<td>O4</td>
<td>( (\Delta k_{bi}) ) Increment for ( k_{bi} )</td>
</tr>
<tr>
<td>P</td>
<td>( P_{ti}/P_{to} )</td>
</tr>
<tr>
<td>P0</td>
<td>( P_{to} )</td>
</tr>
<tr>
<td>P1</td>
<td>( P_{1}/P_{to} )</td>
</tr>
<tr>
<td>P2</td>
<td>( P_{2}/P_{to} )</td>
</tr>
<tr>
<td>P3</td>
<td>( P_{1}/P_{t1} )</td>
</tr>
<tr>
<td>P4</td>
<td>( P_{t1}/P_{to} )</td>
</tr>
<tr>
<td>P5</td>
<td>( P_{i}/P_{t1} )</td>
</tr>
<tr>
<td>P6</td>
<td>( P_{i}/P_{to} )</td>
</tr>
<tr>
<td>P7</td>
<td>( P_{Ei}/P_{to} )</td>
</tr>
<tr>
<td>P8</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>P9</td>
<td>( P_{2} )</td>
</tr>
<tr>
<td>R</td>
<td>( (\rho_{to}) ) Total Density at Stagnation Conditions</td>
</tr>
<tr>
<td>R7</td>
<td>Relaxation Parameter</td>
</tr>
<tr>
<td>R8</td>
<td>Relaxation Parameter</td>
</tr>
<tr>
<td>R9</td>
<td>Predicted ( P_{2}/P_{to} )</td>
</tr>
</tbody>
</table>
S7-S9 Used in GOSUB 7000 to Find $P_2/P_{Ei}$

- $T_0$ - $T_{to}$
- $T_1$ - $T_{t2}/T_{to}$
- $T_2$ - $T_2/T_{to}$
- $T_3$ - $T_i/T_{to}$
- $T_4$ - $T_{Ei}/T_{to}$
- $T_5$ - $T_1/T_{to}$
- $T_6$ - $T_{E1}/T_{to}$
- $U_1$ - Dimensionless Velocity (See Appendix A)
- $U_2$ - Dimensionless Velocity (See Appendix A)
- $U_3$ - Dimensionless Velocity (See Appendix A)
- $V_0$ - $V_{to}$ Total Velocity
- $W_0$ - $\dot{w}$ Flow Rate
- $W_1$ - $\dot{w}^*$ Referred Flow Rate
- $W_9$ - Decision Variable
- $X_0$ - $X_{w1}$ Non-Dimensional Relative Velocity
- $X_1$ - $X_1$ Non-Dimensional Velocity
- $X_2$ - $X_2$ Non-Dimensional Velocity
- $X_3$ - $X_i$ Non-Dimensional Velocity
- $X_4$ - $X_{ui}$ Non-Dimensional Velocity
- $X_5$ - $X_{wi}$ Non-Dimensional Relative Velocity
- $X_6$ - $X_{w2}$ Non-Dimensional Relative Velocity
- $X_8$ - Decision Variable
- $Y_2$ - Non-Dimensional Velocity
- $Y_6$ - Increment for $k_{b1}$
- $Y_7$ - Increment for $k_{bi}$
- $Y_8$-$Y_9$ Counters
20 $z_s$ Stator Loss Coefficient
23 $z_1$ Interblade Loss Coefficient
27 $z_{1-2}$ Predicted Rotor Loss Coefficient
28 Increment for Stator Loss Coefficient
29 Increment for Interblade Loss Coefficient
FIG. A-1
PRESSURE DISTRIBUTION THROUGH THE STATOR

SHOCK BETWEEN TAP #3 AND TAP #4

.1 INCHES BETWEEN TAPS
FIGURE A-3
LOCUS OF POINTS OF MAXIMUM $\phi$ AND LOCUS OF SONIC POINTS

FLOW COEFFICIENT

SONIC LINE

MAXIMUM $\phi$

$\frac{p_1}{p_T}$
FIGURE A-4 TYPICAL SOLUTION PROCESS

FLOW COEFFICIENT

PI/PTO

EVENTUAL SOLUTION POINT

---

FLOW COEFFICIENT

105
FIG. A-5 PROGRAM SCHEMATIC

106
FIG. A-6 FLOW CHART
107
1. Calculate $\phi_i$
   Input first guess for $\frac{P_i}{P_{to}}$

2. Calculate $X_i$
3. Calculate $f\left(\frac{P_i}{P_{t1}}\right)$
4. Calculate $f\left(\frac{P_i}{P_{t1}}\right)'$

$$\frac{P_i}{P_{t1}}_{n+1} = \frac{P_i}{P_{t1}}_n - \frac{f\left(\frac{P_i}{P_{t1}}\right)}{f\left(\frac{P_i}{P_{t1}}\right)'}$$

5. $\left| f\left(\frac{P_i}{P_{t1}}\right) - \phi_i \right| < \epsilon$
6. $\frac{P_i}{P_{t1}} < 0$ or $> 1$

7. Calculate $\frac{P_i}{P_{t1}}$
8. Calculate $X_i$
9. Calculate $X_{ui}$
10. Calculate $\alpha_i$
11. Calculate $|U_i|$
12. Calculate $Q_i$

$ii =$ interblade loss increment
$si =$ stator loss increment
Calculate $X_{wi}$
Calculate $\frac{T_i}{T_{to}}$
Calculate $\frac{TE_i}{T_{to}}$
Calculate $\frac{PE_i}{P_{to}}$

If $\beta_i < \beta_B$

$z_i = z_i + ii$

$z_i = z_i - ii$

$ii < 10^{-6}$

$z_s = z_s + si$

Increment $k_{b1}$ and return to 1

$z_s = z_s + si$

GOSUB to Calculate $\frac{P_2}{PE_i}$

$\phi_R < \phi_R^*$

$si < 0.0008$

$z_s = z_s + si$

$\bar{\phi}_r < \phi_R^*$

yes

no

yes

no

no

yes

no

no

yes

no

no
Calculate $\frac{P_2}{F_{EI}}$
Calculate $Y_{w2}$
Calculate $X_{w2}$
Calculate $x^2_2$
Calculate $T_2$
Calculate $T_{t_{20}}$
Calculate $T_{t_{20}}$
Calculate $z_{1-2}$

PRINT

Increment $k_{b_1}$ or $k_{b_1}$
and return to start
FIGURE A-7 THERMODYNAMIC PROCESS OF FLUID IN AN AXIAL TURBINE STAGE
APPENDIX B
TURBINE TEST RIG (TTR) DATA REDUCTION AND PROCESSING

B-1 INTRODUCTION

This appendix describes only those changes that have been made to the data reduction procedure given in Ref. 2. The program numbers and the functions of those programs given in Ref. 2 have not changed. The channel and port assignments for data collection and storage have not changed. Variables also have not changed, although additional variables were defined in the course of modifying the data reduction process. Those variables that now exist in addition to those given in Ref. 2 are given in Table B-I.

Modifications made during the course of this work were:

1. Raw data can now be read directly from mass memory. Previously it was necessary to read the paper tape for a given point in order to reduce it.

2. Provisions have been written into the programs for smoothing the reduced data.

3. All of the parameters used in calculating the theoretical loss coefficients that were previously entered from charts are now in polynomial form in the program.

4. Storage of raw data is now a separate program. Previously it was necessary to run the reduction sequence to store data.
B-2 DESCRIPTION OF PROGRAMS

The data reduction is divided into ten separate programs with an additional program to store the raw data. Fig. B-1 shows the contents of each program and also shows the sequence in which the programs are chained. Following is a description of the reduction sequence referred to Fig. B-1 which will be followed by a description of how to run the reduction program.

TTR in Fig. B-1 is used only to store the raw data. TTR does not chain to any other program.

The data reduction process begins in TTR1. In order to smooth the reduced data as discussed in Section III it is necessary to have n values for \((P_1 - P_{hub})/(P_{tip} - P_{hub})\). Therefore, the first step in the data reduction process is to go from TTR1 to the point in TTR2 where the values of \(P_1\), \(P_{tip}\), and \(P_{hub}\) have been calculated. On completion, the program returns to TTR1 and repeats the process n times until the n values of \((P_1 - P_{hub})/(P_{tip} - P_{hub})\) have been calculated. The program then proceeds to TTR9 where a polynomial curve fit for the n values of \((P_1 - P_{hub})/(P_{tip} - P_{hub})\) is generated. Then the program returns the polynomial coefficients to TTR1B where the reduction process begins at the first point in the run. However, when the point in TTR2 is reached where \(P_1/P_{to}\) is calculated, it is calculated using the polynomial as described in Section III.

The rest of the reduction process in Fig. B-1 is the
same as described in Ref. 2 with the exception that no further keyboard inputs are needed until the raw and reduced data is tabulated.

B-3 RAW DATA STORAGE

The procedure for storing raw data on the mass memory is outlined in Ref. 2, Appendix B. Sec. B-3, lines 10-19. Note that line 10 should read, "GET 'TTR'". Although it is now a separate program the procedures for storing data have not changed.

B-4 PROCEDURE FOR DATA REDUCTION PROGRAM

1. Key in GET "TTR1B"
2. Press "RUN EXECUTE".
3. The following check list will be printed on the HP 9830 printer:

"PRIOR TO RUNNING THIS SEQUENCE ENSURE FOLLOWING:
"THAT TTR2 LINE 590 HAS PROPER FACTOR IN IT;
   1.02 FOR HOODED
   1.01 FOR UNHOODED
"IF IT IS NECESSARY TO INPUT ALPHA 1 THEN CHANGE TTR2
"LINE 960 TO 'INPUT A3'
"IF BLOCKAGE FACTOR OTHER THAN 1, CHANGE TTR2-1060
"TO 'INPUT X7', AND PUT SEMICOLON AFTER TTR2-1050
"ENSURE THAT TTR1, TTR1B HAVE THE PROPER FILENAME IN 441
"IF IT ISN'T DESIRED TO STORE REDUCED DATA CHANGE
TTR6-580 TO 'G1=0'
"ENSURE TTR6-610, 630 HAVE PROPER FILENAME FOR REDUCED
DATA
"ENSURE TTR7-220 HAS PROPER FILENAME FOR RAW DATA
"ENSURE TTR8-220 HAS PROPER FILENAME FOR REDUCED DATA
"IF IT IS DESIRED TO RUN A SINGLE POINT, WITHOUT
SMOOTHING, INSERT TTR2-1127 'GOTO 1200'. OTHERWISE
OMIT TTR2-1127

The above checklist will prepare all ten programs. The
last item is a provision for elimination of smoothing if
it is desired to run a single point. If smoothing is not
desired then each point must be run separately with
TTR2-1127 inserted.

4. Press CONTINUE EXECUTE

5. The display will read LOWEST, HIGHEST RECORD #
   THIS RUN. Input the lowest record number and the
   highest record number from which it is desired to read
   raw data. Note, the record number on which the reduced
data will be stored will be the same number as the raw
data record from which the raw data was read. Make sure
the filename designated for the storage of the reduced
data can accept data on those record numbers without
writing over previous information.

6. Next the display will read PRINT OUT RESULTS?
   YES=1, NO=0. If a 1 is selected here then at the conclusion
   of data reduction, for all the points in the run, the
   program will begin to print all results. If 0 is selected
   then all reduced data will be stored on the mass memory.

7. It will take about 15 minutes for the program to
   read the raw data and make the calculations necessary for
smoothing the data. When this is completed the program will start reducing each point and printing the results.

8. At the completion of data reduction for all points the option to tabulate the data is available. ENTER RECORD #'S: LOWEST, HIGHEST will appear on the display. If it is desired to tabulate the raw data then enter the same record numbers that were entered to initiate the program. The raw data will then be tabulated.

9. Next the display will read ENTER LOWEST, HIGHEST RECORD NUMBER. If it is desired to tabulate the reduced data then enter the appropriate record numbers. The reduced data will be tabulated.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>Lowest Record Number</td>
</tr>
<tr>
<td>A5</td>
<td>Counter</td>
</tr>
<tr>
<td>A6</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>A7</td>
<td>Highest Record Number</td>
</tr>
<tr>
<td>B</td>
<td>(Array) Polynomial Coefficients From TTR9</td>
</tr>
<tr>
<td>F8</td>
<td>Decision Variable</td>
</tr>
<tr>
<td>Q</td>
<td>(Array) Values of Isentropic Head Coefficient</td>
</tr>
<tr>
<td>T4</td>
<td>Polynomial Approximation of Reynolds Number</td>
</tr>
<tr>
<td>Z</td>
<td>Decision Variable</td>
</tr>
</tbody>
</table>
TTR
1.) RAW DATA STORAGE
2.) RAW DATA PRINTED

TTR 9
1.) GENERATE POLYNOMIAL CURVE FIT FOR n
\[ \frac{P_1-P_h}{P_t-P_h} \]
VALUES OF \( V \)

TTR 1B
1.) READ RAW DATA
2.) CHANNEL ASSIGNMENT
3.) TEMPERATURE SUBROUTINE

TTR 2
1.) CALCULATE MASS FLOW RATE
2.) EVALUATION OF CONTROL VOLUME A

TTR 3
1.) EVALUATION OF CONTROL VOLUME B AND TEMPERATURES

FIGURE B-1 DATA REDUCTION SCHEMATIC
TTR 4
1.) CALCULATE STATOR EXIT VELOCITIES
2.) CALCULATE THEORETICAL LOSSES

TTR 5
1.) PRINT REDUCED DATA

TTR 6
1.) STORE REDUCED DATA

TTR 7
1.) TABULATE RAW DATA

TTR 8
1.) TABULATE REDUCED DATA
LIST OF REFERENCES


3. Commons, P.M., Instrumentation of the Transonic Turbine Test Rig to Determine the Performance of Turbine Inlet Guide Vanes by the Application of the Momentum and Moment of Momentum Equations, M.S.A.E. Thesis, Naval Postgraduate School, Monterey, California, 1967


### INITIAL DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>No.</th>
<th>Copies</th>
<th>Name and Address</th>
</tr>
</thead>
</table>
| 1.  | 2      | Defense Documentation Center  
|     |        | Cameron Station  
|     |        | Alexandria, Virginia 22314 |
| 2.  | 2      | Library, Code 0212  
|     |        | Naval Postgraduate School  
|     |        | Monterey, California 93940 |
| 3.  | 1      | Chairman, Department of Aeronautics  
|     |        | Naval Postgraduate School  
|     |        | Monterey, California 93940 |
| 4.  | 1      | Associate Professor R.P. Shreeve, Code 57Sf  
|     |        | Department of Aeronautics  
|     |        | Naval Postgraduate School  
|     |        | Monterey, California 93940 |
| 5.  | 1      | Mr. J.E. Hammer, Code 57  
|     |        | Department of Aeronautics  
|     |        | Naval Postgraduate School  
|     |        | Monterey, California 93940 |
| 6.  | 8      | Turbopropulsion Laboratory  
|     |        | Department of Aeronautics  
|     |        | Naval Postgraduate School  
|     |        | Monterey, California 93940 |
| 7.  | 1      | Lt. Cdr. B.C. Boatright  
|     |        | 969 Edwin Drive  
|     |        | Virginia Beach, Virginia 23462 |
Analytical and experimental determination of the characteristics of a transonic axial turbine.