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Technical Notes

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Aerodynamic Design of a Conventional Windmill Using Numerical Optimization

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Introduction

DESIGN of the blading for a windmill is an area of critical interest because moderate improvements in aerodynamic performance can provide a significant increase in overall efficiency and a major reduction in windmill size and cost. Stewart[1] presents an analysis and numerical investigation of aerodynamic performance using vortex theory to determine optimum blade twist and chord distribution. These results are valuable in two respects. First, they provide insight into the basic design problem; and second, they identify the benefits of design optimization. However, any increase in the number of design parameters or in the complexity of the analysis makes solution by direct analytical techniques cumbersome, if not impossible. Furthermore, realistic design must be based on structural as well as aerodynamic considerations. The overall objective must be to provide a design of the complete system for the most cost efficient production of electrical energy over the design life of the windmill. This suggests the need for an integrated design synthesis capability which includes aerodynamic, climatological, structural, mechanical, and economic considerations.

The purpose of this Note is to identify numerical optimization techniques as an effective tool for automated synthesis of windmill design. Several simple design examples are presented to show the efficiency and generality of these methods. The extension to more sophisticated design problems is discussed, and the advantages and limitations of these techniques are identified.

Performance Analysis

Consider a windmill operating at constant angular velocity Ω in an undisturbed wind of velocity V. The ratio of blade speed to wind speed at some radial location r is $x=\Omega r/V$. The power coefficient for the element of blade at radius r is given by Stewart[1] as

$$C_p = 4a(1-a)(x)(\tan\phi - \epsilon) \tag{1}$$

where ϕ is the blade approach angle, and ϵ is the ratio of profile drag to lift, i.e., $\epsilon = C_D/C_L$. The induction coefficient, a, is a function of the dimensionless blade loading

$$\lambda = BcC_L/8\pi r = (a\tan\phi\sin\phi)/(1-a) \tag{2}$$

where B is the number of blades, and c is the local chord length. For a specified velocity ratio, x, and approach angle, ϕ, the induction coefficient is

$$a = (1 - x\tan\phi)\cos^2\phi \tag{3}$$

For a precise definition of approach angle, see Fig. 1 in Stewart.[1] The design objective is now to maximize C_p in Eq. (1) for a specified velocity ratio, x. The only independent design variable is the angle ϕ.

Wind velocity varies at different times of the day and at different seasons. Also, because the windmill is operating in the Earth's boundary layer, the wind will vary as a function of elevation above the ground surface. Therefore, addition of cyclic blade pitch control improves performance; blade pitch control is necessary to allow the windmill to be feathered under extreme wind conditions. Equations (1-3) are valid for variable pitch blades simply by replacing ϕ in each equation by $\phi+\tau$ where τ is the pitch angle. Cyclic pitch control implies $\tau=\tau(\theta)$, where θ is angular location of a blade.

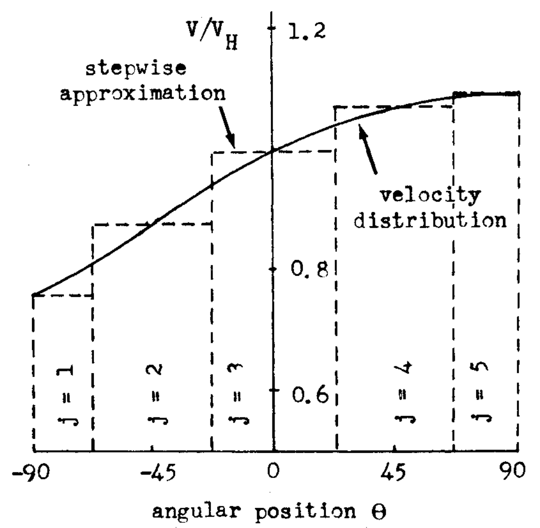
Design Automation

Numerical optimization techniques[2-4] can be effectively used to solve the general nonlinear problem

$$\text{minimize or maximize} \quad F(\bar{X}) \tag{4}$$

$$\text{subject to} \quad g_j(\bar{X}) \le 0 \quad j=1,m \tag{5}$$

where $\bar{X}$ is the vector of independent design variables, x_i, $i=1...n$. $F(\bar{X})$ is the design objective, in this case performance. The $g_j(\bar{X})$ are constraints which the design must satisfy. In the simple examples considered here, no constraints are imposed. In the general case, constraints will include such structural limits as allowable stress in blades and windmill



j	1	2	3	4	5
V/V_H	0.757	0.875	1.000	1.070	1.092
x	6.554	6.420	6.000	5.252	4.544
K_j	0.125	0.250	0.250	0.250	0.125

Fig. 1 Velocity profile due to boundary layer.

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Table 1 Comparison of CONMIN and Stewart optimization results[a]

x	6.75		13.83	
ϵ	0.01	0	0.01	0
ϕ	5.717° (5.71°)	5.617° (5.61°)	2.863° (2.86°)	2.757° (2.76°)
a	0.3211 (0.3208)	0.3329 (0.3328)	0.3076 (0.3075)	0.3333 (0.3332)
η_f	0.8949 (0.8948)	0.9952 (0.9952)	0.7955 (0.7960)	0.9988 (0.9988)
λ	0.00472 (0.00470)	0.00480 (0.00479)	0.00111 (0.00111)	0.00116 (0.00116)

[a] Numbers in parentheses are from Ref. 1.

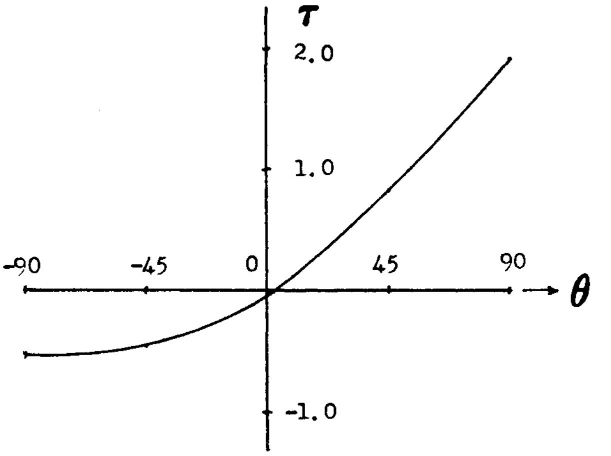

Fig. 2 Optimum pitch schedule.

tower and aeroelastic limits such as blade tip deflection and blade and/or tower flutter. The only mathematical assumptions of Eqs. (4) and (5) are that the functions be continuous and have continuous first derivatives. The optimization program, CONMIN,[5] was used to generate the results. This program uses the Fletcher-Reeves algorithm[6] for unconstrained problems and a modification of Zoutendijk's method of feasible directions[7] for constrained problems.

Design Examples

For purposes of comparison, Stewart's first case, where the optimum blade pitch is determined as a function of x and ϵ, is calculated using CONMIN. That is, for various combinations of x and ϵ,ϕ is determined to maximize C_p. The results are presented in Table 1. The numbers in parentheses are the theoretical results presented by Stewart.[1] The parameter η_f is the local Froude efficiency, $\eta_f=(27/16)C_p$. In this simple one-design-variable example, the CONMIN and Stewart results are seen to agree very well.

Now consider the design of a blade operating in a wind where the velocity varies according to the one-seventh power law, $V=V_0(y/H)^{1/7}$, where y is the height above the surface, H is the thickness of the boundary layer, and V_0 is the velocity of the top of the boundary layer. Assuming $H=300$ m, $R=30$ m, and the windmill axis is 35 m above the surface

$$V=V_H(0.11667+0.1\sin\theta)^{1/7} \tag{6}$$

where V_H is the wind velocity at the hub and θ is the blade rotation angle measured from the horizontal plane.

Such a velocity distribution is shown in Fig. 1 together with a piecewise approximation to the velocity profile. For constant angular velocity Ω, Eq. (6) also represents implicitly the variation of velocity with time. It is now desired to find the best blade pitch to maximize aerodynamic performance under this cyclic wind condition. Define the objective as the sum of efficiencies weighted by the fraction of time the blade is subjected to a given velocity. That is

$$C_{p,\mathrm{tot}}=\sum_{j=1}^{5}K_jC_p[(\phi+\tau_j),x_j] \tag{7}$$

Equation (7) is applicable at a given station, r, and is the numerical approximation to the integral of the power coefficient for one-half revolution of the blade. The design objective is now $C_{p,\mathrm{tot}}$, and the design variables are ϕ and τ_j. If all $\tau_j=0$, the design will be optimum for a blade of constant pitch; whereas if the τ_j are treated as design variables, the optimum pitch schedule will be determined.

Design results are presented in Table 2 for $\epsilon=0$ and $\epsilon=0.01$. For variable pitch, a reference ϕ is taken as optimum value for the constant pitch case, i.e., $\phi^*=6.359$ when $\epsilon=0$. The five design variables are τ_i for $i=1,2,\ldots,5$. As expected, efficiency is reduced in the presence of drag, and efficiency is increased by variable pitch. While the constant pitch solution varies slightly in the presence of drag, the pitch schedule for variable pitch is the same for each case since ϵ is constant. The optimum pitch schedule is shown graphically in Fig. 2.

Discussion

Numerical optimization techniques have been applied to aerodynamic design of windmill blades. The purpose here is to demonstrate the ease with which these methods can be applied to this design problem, rather than to present specific design results. These techniques are directly extendable to the more general design problem. For example, the optimum blade twist can be determined by treating ϕ as a design variable which is a function of r. For the case of variable ϕ, Eq. (7) would be integrated from the hub to R to obtain the total blade efficiency. Similarly, the velocity profile may be a function of both time and elevation above the Earth's surface. Additionally, structural, mechanical, climatological, and economic considerations can be directly incorporated into the design procedure. The number of design variables increases rapidly as the sophistication of the analysis is expanded.

Table 2 Design results

j	1	2	3	4	5	ϕ^*	$C_{p,\mathrm{tot}}$
			Initial design, $\tau_j=0, j=1,2,3,4,5$				
$\eta_f(\epsilon=0)$	0.9449	0.9318	0.8829	0.7712	0.6452	5.0	0.5009
$\eta_f(\epsilon=0.01)$	0.8369	0.8253	0.7820	0.6831	0.5715	5.0	0.4437
			Optimum design – constant pitch				
$\eta_f(\epsilon=0)$	0.9633	0.9759	0.9939	0.9554	0.8540	6.359	0.5679
$\eta_f(\epsilon=0.01)$	0.8667	0.8811	0.9045	0.8782	0.7900	6.458	0.5174
			Optimum design – variable pitch				
$\tau(\epsilon=0)$	−0.5730	−0.457	−0.055	0.826	1.917	6.359	0.5886
$\eta_f(\epsilon=0)$	0.9949	0.9947	0.9939	0.9921	0.9895		
$\tau(\epsilon=0.01)$	−0.572	−0.456	−0.054	0.826	1.917	6.458	0.5374
$\eta_f(\epsilon=0.01)$	0.8975	0.8993	0.9047	0.9140	0.9218		

However, at the preliminary design level, it is estimated that the problem can be defined in terms of 20-40 system variables, which is within the capacity of current optimization techniques to solve. With the availability of computer codes such as CONMIN to perform the design optimization, the need is now for development of analysis codes in each of the disciplines to provide overall system synthesis.

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Potential Application of Radial Splitter Diffuser to Shrouded Wind Turbines

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Nomenclature

AR	= diffuser exit area/turbine disk area
C_{PD}	= diffuser pressure-recovery coefficient (based on diffuser inlet dynamic pressure)
C_{PE}	= exit plane pressure coefficient (based on freestream dynamic pressure)
C_T	= turbine pressure-drop coefficient (based on local dynamic pressure)
D_E	= diffuser (or shroud) exit diameter
D_T	= diffuser inlet (or turbine) diameter
L	= total shroud length
R_E	= turbine power ratio (based on exit area)
R_T	= turbine power ratio (based on turbine disk area)
η_D	= diffuser pressure-recovery efficiency $= C_{PD}/(1-1/AR^2)$

Introduction

THE diffuser-augmented shrouded wing turbine has attracted renewed interest in recent years, as it offers several technical advantages over the conventional free rotors. Experimental studies[1-2] have indicated that a proper shroud design can produce energy concentration factors of as much as 3 to 4 at the turbine. To assess its economic viability, however, the shroud performance has to be considered in relation to its size and complexity.

The key aerodynamic characteristic of the shroud is the static pressure recovery downstream of the turbine, made up of the *internal* recovery in the shroud diffuser (indicated by the pressure rise coefficient C_{PD}) and the *external* recovery in the wake (characterized by the exit pressure coefficient C_{PE}). These together with the turbine pressure drop coefficient C_T determine the turbine power output ratio R_T relative to the unshrouded rotor of the same diameter and at the same wind speed, according to the following relation[1]

$$R_T = \frac{27}{16} \cdot C_T \cdot \left(\frac{1-C_{PE}}{1-C_{PD}+C_T} \right)^{1.5} \tag{1}$$

(based on one-dimensional analysis, neglecting inlet losses).

The shroud design problem is to combine an efficient diffuser for high C_{PD} with a suitable external shape to maximize $|C_{PE}|$, while maintaining compact overall dimensions (viz. minimum L/D_T and D_E/D_T) and simplicity for economical manufacture.

The efforts so far to produce compact as well as efficient shrouds have sought to utilize the external flow energy, either through ejector action with the aid of ring-airfoils placed at the exit of a low-angle diffuser[1] or to prevent separation in large-angle diffuser by slot injection.[2] In each case, a multielement shroud results, held together with slender connections for minimizing aerodynamic interference, a configuration that is relatively expensive to manufacture and maintain against storm damage. Also, a distinct advantage of the plain-diffuser shrouded rotor viz. the insensitivity of its output to fairly large angles of misalignment with the wind direction is lost when ring-airfoils are added,[1] as may also occur with slot-injection diffusers due to insufficient energization available in the leeward regions of the shroud.

This Note proposes the application of a new concept of efficient wide-angle conical diffuser utilizing radial splitters,[3] which overcomes the aforementioned drawbacks. With this diffuser concept a significant pressure recovery has been demonstrated with expansion angle as high as 40° together with good exit flow uniformity. Competitive aerodynamic performance will be shown possible with the proposed shroud, with potential for improved cost-effectiveness over the current shroud designs.

Radial Splitter Diffusers (RSD)

The conical diffuser is compartmented full length by means of equispaced, thin radial vanes or splitters. A small disk (about 2% area blockage), symmetrically located on the forward apex of the splitter junction, forces separation of the flow in the corners between the adjacent radial surfaces. The outward displacement due to this axisymmetric separation ensures attached flow on the diffuser wall; turbulent mixing in the compartments promotes diffusion and bubble closure, resulting in substantial pressure recovery and a well-filled exit section.

The disk size is critical to the success of the radial-splitter concept; if too small, a properly developed bubble is not realized and the flow separates off the diffuser wall, whereas too large a disk produces a bubble extending beyond the diffuser exit thereby reducing the effective area ratio and pressure recovery. Moving the disk some distance into the diffuser (by cutting away the splitter leading-edges) eliminates the inlet blockage and improves recovery. A typical RSD geometry is depicted in the upper portion of Fig. 1.

Unpublished data obtained recently at NASA Langley Research Center on a 40°, $AR = 4.5$ radial-splitter diffuser (for a possible wind tunnel application) provide a starting point for evaluating the potential of the RSD shroud. Typical wall static pressure distributions are shown in Fig. 1. In the empty diffuser, asymmetric separation occurs immediately after entry. The effect of an 8-vane assembly without disk is to make the flow separation symmetrical, with only a minor improvement in the pressure recovery. Addition of an optimum-size disk to the splitter produces a radical change in the flow pattern: a suction peak appears at the sharp inlet junction indicating a well-attached flow turning through the expansion angle, followed by an almost continuous pressure rise up to the exit.

In the limited RSD work so far, the emphasis has been on producing uniform flow at the exit of a given diffuser with a

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