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CONTRIBUTIONS TO HYDRODYNAMICS :

Vortex Motion in Homogeneous and Stratified Media

By T. Sarpkaya, S.K. Johnson, W.E. Gray, and J.J. Daly, Department of Mechanical Engineering

Introduction

The research activities at the Naval Postgraduate School (NPS) in the general area of hydrodynamics concern vortex breakdown, harmonically-oscillating flow about smooth and rough bluff bodies, wave forces on offshore structures, hydroelastic oscillation of cables in steady and oscillating flow, impulsively-started flow about various types of bodies, discrete vortex analysis of separated time-dependent flows, and the rise and demise of trailing vortices in homogeneous and density-stratified media. This article focuses on the effect of ambient turbulence and density stratification on the migration of trailing vortices.

Vortices and vortex wakes have become a major theme of aerodynamics research since the advent of the large aircraft, and their evolution required an examination of many fundamental problems in fluid mechanics. Much of the progress made during the past two decades has been extremely useful in understanding the behavior and interpreting the consequences of vortices generated by the control surfaces of bodies submerged in homogeneous and stratified ocean environment.

The flow over a wing of finite span is three-dimensional having both chordwise and spanwise velocity components. The difference in spanwise velocity components causes the fluid to roll up into a number of streamwise vortices distributed along the span. These small vortices roll up into two, counter-rotating, large vortices (trailing vortices) just inboard of the wing tips (see Figure 1). They descend (downwash) gradually because of mutual induction. In the oceans, the motion of a submerged body produces numerous vortices from various control surfaces (e.g., sail planes and stern) which rise (upwash) due to mutual induction. These vortices decay partly due to large-scale instabilities (sinusoidal instability[1]) and linking (Figure 2), leading to vortex rings, and vortex breakdown[2,3] (Figure 3), leading to the bulging and bursting of the vortex core and partly due to small-scale effects (viscous and turbulent diffusion). These, in turn, depend on the intensity and scale of the ambient turbulence. With a goal of minimizing the adverse consequences of trailing vortices (internal waves, surface signatures, vortex-aircraft interactions, etc.), the relationship between the lifespan of the vortices and the various demise mechanisms are being explored through analysis and experiments with the support of the Office of Naval Research.

Figure 1

Roll-up of vortex sheets over a delta-wing model.

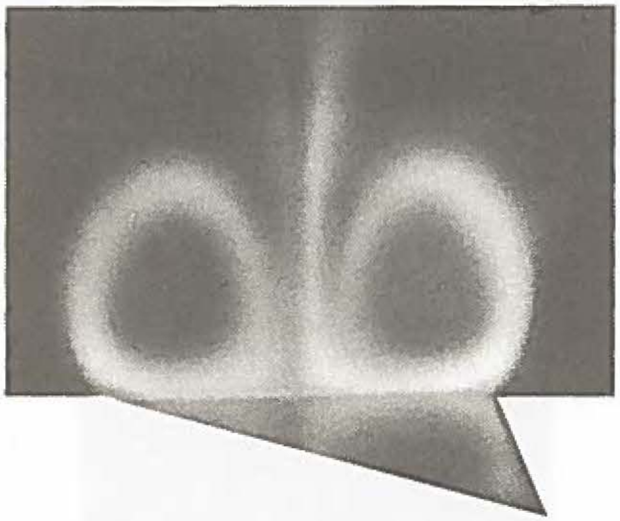


Figure 2

Rise of trailing vortices and the inception of sinusoidal instability (time increases upwards).

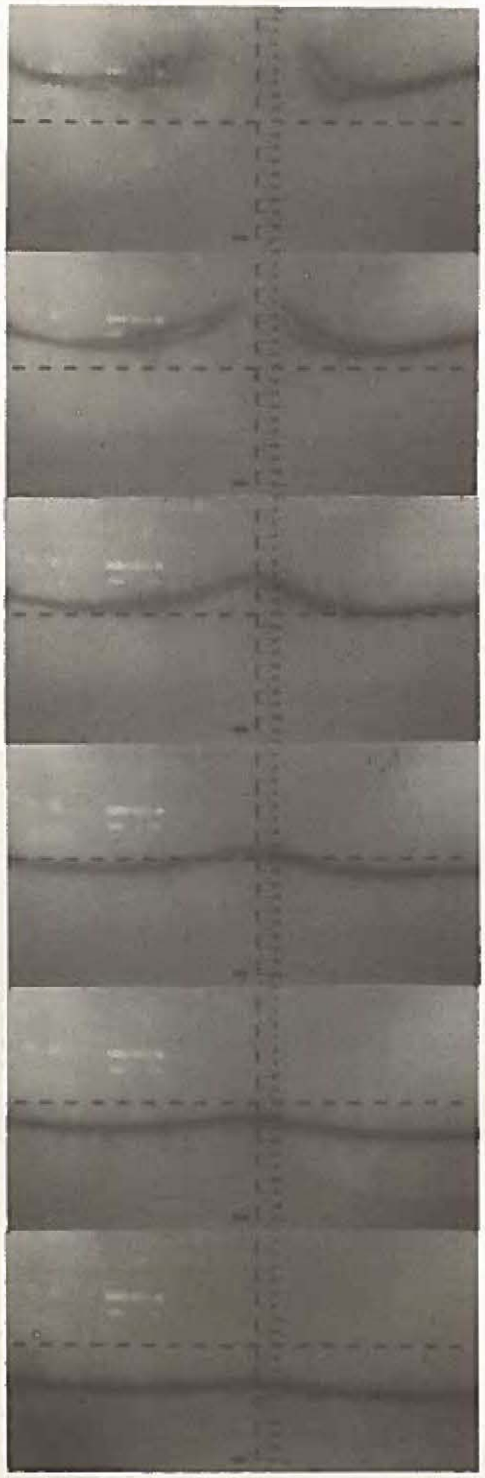
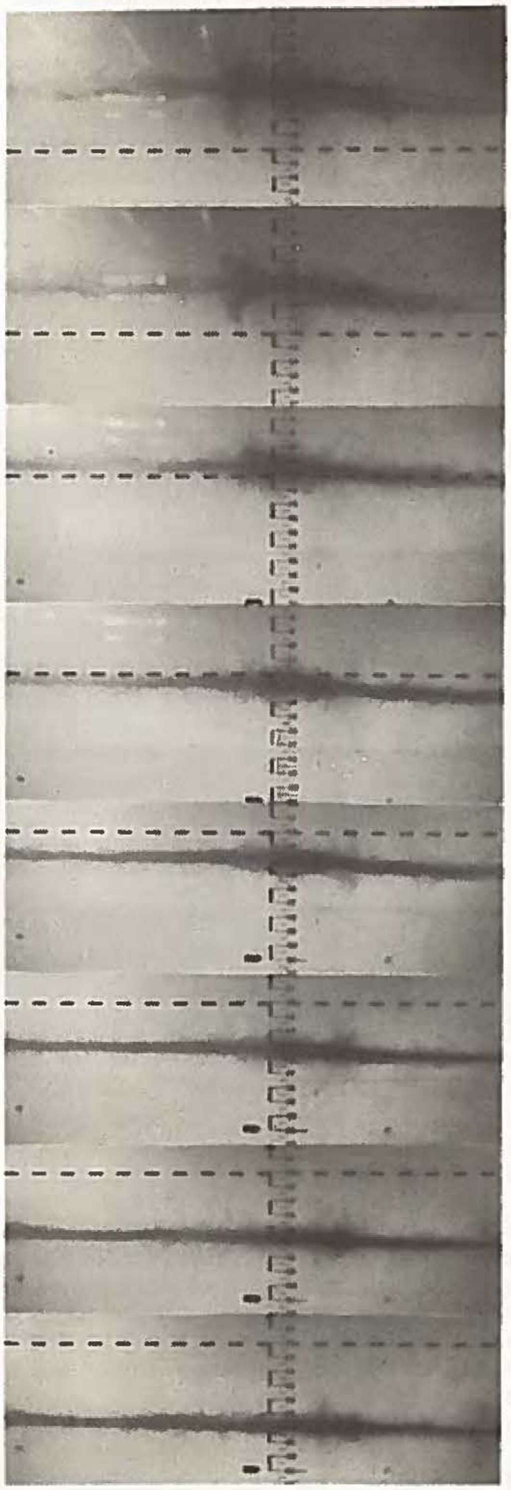


Figure 3

Evolution of vortex breakdown (core bulging and bursting) in trailing vortices (time increases upwards).



Vortex Motion in Homogeneous Media

In the absence of stratification, the rise of a vortex pair is governed primarily by the relation,[4]

$$H^* = f(t^*, D^*, \epsilon, r^*) \qquad (1)$$

where $H^* = H/b_0$, $t^* = V_0t/b_0$, $D^* = D/b_0$, $\epsilon^* = (\epsilon b_0)^{1/3}/V_0$, $r^* = r_c/b_0$, and H, the vertical displacement of the vortex pair; b_0, the initial separation of the vortices; V_0, the initial mutual induction velocity; t, the time; D, the initial depth of the vortex pair; ϵ, the dissipation rate of the grid-generated turbulence per unit mass, and r_c, the initial vortex core radius. The effect of the aspect ratio and the angle of attack of the lifting surface enter into the Equation (1) indirectly through V_0 and b_0 [note that $b_0 = (\pi/4)B$ where B is the base width of the foil]. Whereas the parameters t^*, D^*, and ϵ^* may be changed independently, r^* is taken as nature provides it ($r^* \approx 0.1$). The primary reason for this is that a century of theoretical and experimental aerodynamics research has been incapable of describing the details of the structure of the tip vortex to be used as initial conditions in a viscous solution. It is surprising, but true, that up until recently, the importance of the wing-tip shape and its influence upon both the initial tangential velocity profile and initial turbulence in the vortex has not been fully appreciated. Here the said influences are characterized in terms of an effective core radius, with full awareness of its shortcomings.

The experiments were performed in a long towing tank (with a cross section of 3 ft by 6 ft).[5,6] Two parallel rails are mounted at the bottom of the tank. A carriage rides smoothly on these rails and provides the test body with a constant velocity through the use of an endless cable and a variable speed motor. The two rails, the carriage and the filling pipes are located under a turbulence management system (polyurethane foam, sandwiched between two perforated aluminum plates).

The ambient turbulence (nearly isotropic) is generated by means of a bi-planar grid. The grid is attached to another carriage, mounted on two parallel rails along the top of the towing tank. The test model is placed in the basin and the tank is filled with water (or stratified as desired through a computer-controlled stratification system). The hollow interior of the model is filled with neutrally buoyant fluorescent dye to seed the vortex core. After sufficient time for equilibrium, the grid is set in motion at the desired speed. When the grid has moved a prescribed distance of x/M (M = mesh size of the grid), the model is set in motion at the same speed. Thus, the horizontal distance between the model and the grid is kept constant.

A large number of grid-turbulence measurements have shown that the turbulence parameter ϵ^* may be written as[4]

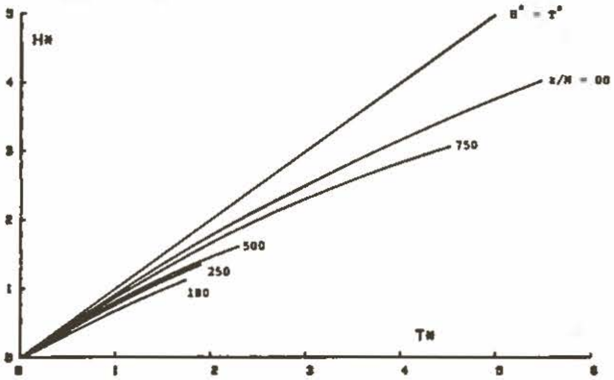
$$\epsilon^* = C(V_0/U)^{-1}(M/b_0)^{1/3}(x/M)^{-0.767} \qquad (2)$$

where C is a constant (≈ 0.44). Thus, for a given model and angle of attack, i.e., for a given V_0/U and M/b_0, the turbulence parameter varies with x/M as given by Equation (2).

The motion of the trailing vortices was recorded on high-speed film at the test section, together with two digital timers, at regular intervals. It is from these photographs that the data similar to that shown in Figure 4 are deduced. Clearly, the path of the vortices in the turbulent field does not significantly deviate from that of the vortices in the non-turbulent medium until the vortex pair rises at least one initial separation distance. Subsequently, the vortices slow down considerably before the onset of large scale instabilities (mostly vortex breakdown) and eventual dissipation. Figure 4 shows that the smaller the x/M (i.e., the stronger the intensity of turbulence), the smaller is the ultimate rise and the lifespan of the vortex pair. This is primarily due to the fact that turbulence enhances the instabilities leading to the vortex breakdown and, at the same time, diffuses vorticity rapidly, eventually leading to the total destruction of the vortices. This mechanism is far more powerful than the sinusoidal instability leading to the linking of the vortices and to the formation of irregular vortex rings. It remains to be determined what the individual effects of the turbulence intensity and scale are in bringing about the destruction of the vortices due to the vortex breakdown.

Figure 4

Effect of ambient turbulence on the rise of trailing vortices.



The maximum height H*(max) attained by the vortices is of special importance. Figure 5 shows the maximum height data as a function of ϵ^*. Also shown in this figure is the flight data obtained by Tombach. ⅛ There is reasonable agreement between the laboratory and flight data in spite of the considerable scatter due to the difficulty of determining the maximum height attained by the vortices. The uncertainties stem primarily from the randomness in the inception of the instabilities leading to the destruction of the vortices. Figure 6 shows the lifespan of the vortex pair as a function of ϵ^*. The data points on the T*(max) axis correspond to the non-turbulence case. Clearly, the effect of turbulence is to reduce both the lifespan and the maximum height attained by the vortices. The data have also shown that the vortices in the non-turbulent medium break up primarily due to linking and in the turbulent medium, mostly due to vortex breakdown.

Figure 5

Maximum height attained by the vortices as a function of the turbulence parameter.

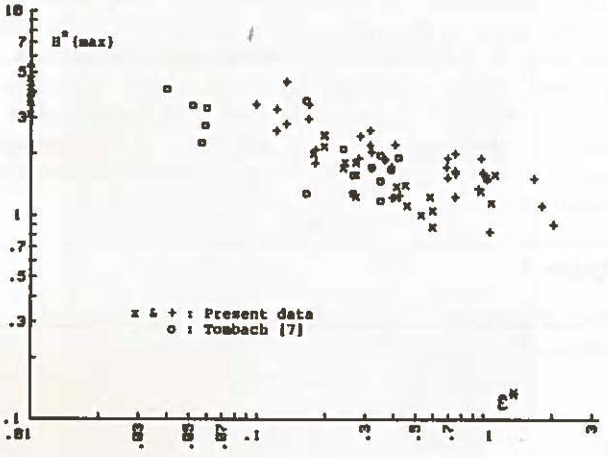
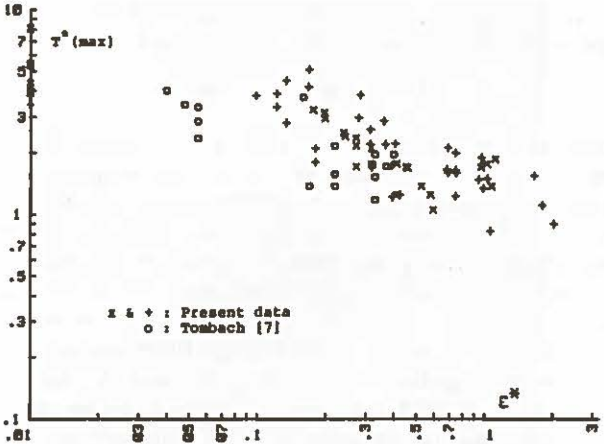


Figure 6

Lifespan of the vortices as a function of the turbulence parameter.



Vortex Motion in Stratified Media

The generation of the internal waves and the rise and demise of the vortices in a stratified medium may be analyzed through the use of the equations of motion for an incompressible fluid for both laminar and turbulent motions provided that a suitable turbulence closure model is adopted and the usual Boussinesq approximation (gravitational acceleration is much larger than the fluid acceleration) is made. For the type of motions considered herein, the Boussinesq approximation is quite valid and has been used in the investigation of all types of internal waves in stratified fluids.

The governing equations of motion in normalized form are

$$\frac{\partial \zeta_m}{\partial t_m} + F_v\left(\frac{\partial u_m \xi_m}{\partial x_m} + \frac{\partial v_m \xi_m}{\partial y_m}\right) = \frac{F_v}{Re}\,\nabla^2 \xi_m + \frac{\partial \varrho'_m}{\partial x_m} \tag{3}$$

and

$$\frac{\partial \varrho'_m}{\partial t_m} + F_v\left(\frac{\partial u_m \varrho_m}{\partial x_m} + \frac{\partial v_m \varrho_m}{\partial y_m}\right) = F_v n^2 v_m \tag{4}$$

in which the appropriately normalized variables are: ζ the vorticity, t the time, u and v the velocity components, x and y the coordinates of a point, ϱ the density of fluid, ϱ' the fluctuating component of the density, F_v a Froude number, Re a Reynolds number, and n the Vaisala-Brunt frequency (for additional details see Reference 4).

Equations (3) and (4) are valid when $F_v \ll 1$ and buoyancy dominates the flow. When F_v approaches zero, the equations that result from Equations (3) and (4) describe the propagation of linear internal waves. The $F_v \ll 1$ regime is of interest in the present investigation because for submerged bodies of naval interest the Froude number is about 0.01. The results presented below are obtained by considering the full nonlinear equations (3) and (4).

A sufficiently large grid is chosen, the appropriate boundary conditions are imposed, the initial vorticity distribution in the vortex pair is taken to be Gaussian, and the velocities are calculated at all points through the use of the well-known Biot-Savart law. Equations (3) and (4) are integrated through the use of an efficient upwind differencing scheme[4] at each time step. Among the several calculations carried out only one utilizing the initial parameters corresponding to those of an experiment will be reported here, (i.e., $D^* = 8$, $r_c = 0.09$, $n = 0.0135$, and $F_v = 0.018$).

Figures 7 and 8 show the velocity and constant density contours at time $T^* = 3.64$. Clearly, the motion of the vortices gives rise to two regions of circulation with countersigned vorticity (in the upper right and lower left regions of the vortex in Figure 7). Similar regions of circulation develop around the left-half of the vortex pair. These counter vortices not only reduce the rise velocity of the original vortex pair, but also push the pair against each other. Consequently, the vorticity is lost in the overlapping regions of the vortex pair, and the rise velocity is further reduced. As time increases, the countersigned vorticity begins to dominate the flow and the vortex migration stops. With further increases in time, the vortex begins to migrate downward, provided that it is not yet subjected to large scale instabilities (sinusoidal instability and/or vortex breakdown).

The density contours reveal the same phenomena in a different context. As the vortices rise, fluid of greater density is pushed upwards (Figure 8) into regions of lesser density. Since such a migration cannot go on indefinitely, the vortices rise to a maximum height and then begin to sink downwards. The calculations do not take into account sinusoidal instability, vortex breakdown, and ambient turbulence. In reality, of course, the vortex pair begins to break up as it nears the end of its maximum migration and eventually disappears.

The experimental and calculated values are compared in Figure 9. The correspondence between the measured and calculated values is surprisingly good up to the time of maximum rise. This is partly because of the experimental fact that the migration of vortices in a highly stratified medium is inhibited primarily due to the reduction of vorticity of the initial vortices and the creation of the countersigned vorticity.

Figure 7

Velocity field and the evolution of counter-sign vorticity at $T^* = 3.64$.

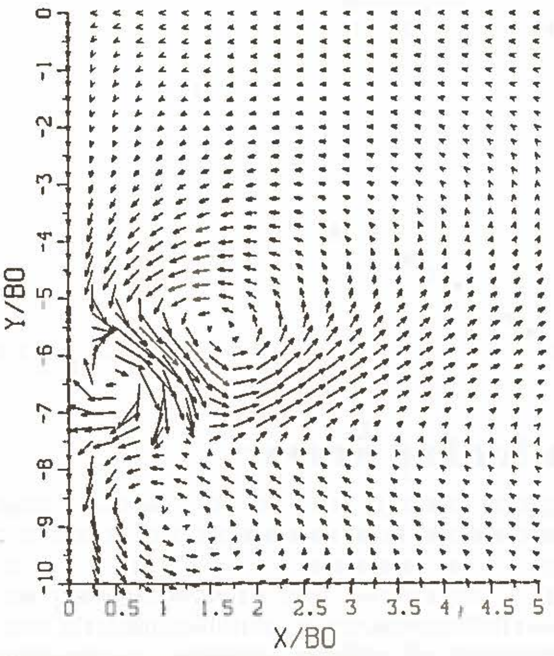


Figure 8

Constant density countours at $T^* = 3.64$.

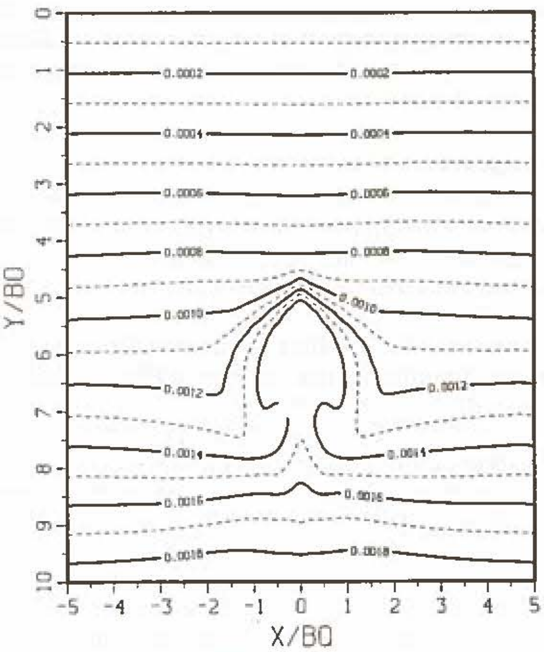
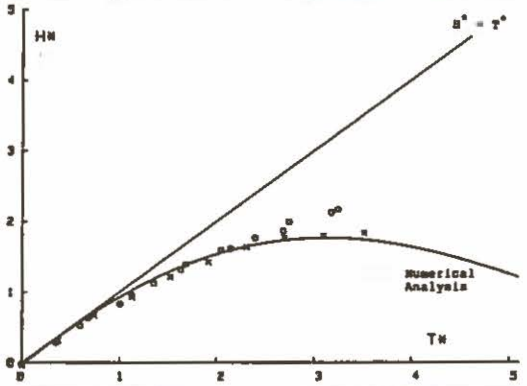


Figure 9

Comparison of numerical and experimental results for the example cited in the text (SP = 0.75, F_v = 0.018).



Concluding Remarks

The migration of trailing vortices in the ocean environment is of vital importance to the Navy because these vortices give rise to surface and temperature scars (if they reach the free surface) and to internal waves. The experimental and analytical efforts face numerous difficulties due to the complex interactions of ambient turbulence, stratification, sinusoidal instability, vortex breakdown, and laminar and turbulent diffusion. This article attempted to summarize briefly the efforts undertaken in the hydrodynamics laboratory of the Naval Postgraduate School. The problems investigated concern the effect of ambient turbulence in stratified medium, surface signatures in homogeneous[8] and arbitrarily-stratified ocean, scale effects, numerical investigation of the dependence of the calculated internal wave characteristics on the turbulence model used, etc.

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Biographies

Dr. T. Sarpkaya, a Distinguished Professor in the Department of Mechanical Engineering, joined the Naval Postgraduate School in 1967. He has been teaching and doing research in the general area of hydrodynamics for the past 30 years. He is primarily interested with time-dependent flows, vortex breakdown, discrete vortex models, and marine hydrodynamics. In 1981, his book, *Mechanics of Wave Forces on Offshore Structures* was published.

LCDR Steven K. Johnson graduated from the U.S. Naval Academy with a M.S. degree in Mechanical Engineering in June 1974. In 1982 he graduated with distinction from the Naval Postgraduate School. His thesis "Trailing Vortices in Stratified Fluids" was the basis of his receiving the Newborn Student Research Award, an annual award sponsored by the Sigma Xi Society. LCDR Johnson also was the recipient of the Naval Sea Systems Command Award for excellence in Naval Engineering, and the Navy League Award for highest school-wide academic achievement. He is now serving aboard the USS BUCHANAN (DDG-14) as the Executive Officer.

LT William E. Gray, USN, received his Bachelor of Science degree in Micro-Biology in 1976 from the University of Ohio. He then attended Surface Warfare Officers School, San Diego, California. In 1982 he reported to the Naval Postgraduate School and graduated in 1985 with the degrees Master of Science in Mechanical Engineering and Mechanical Engineer. His thesis entitled "Surface Disturbances Due to Trailing Vortices." He was awarded the Naval Sea Systems Command Award in Naval Engineering recognizing his achievements at the Naval Postgraduate School. Following his graduation he reported to the Naval Reactors Representative Office at Portsmouth Naval Shipyard, Portsmouth, NH where he is currently stationed.

LT John J. Daly, USN, received his Bachelors Degree in Mechanical Engineering from the University of Illinois in 1978. He then attended the Naval Nuclear Power School and after other tours of duty was assigned to the Naval Postgraduate School graduating in 1985 with the degrees Master of Science in Mechanical Engineering and Mechanical Engineer. His Master's Thesis was entitled "Effects of Ambient Turbulence and Stratification on the Demise of Trailing Vortices." Following graduation LT Daly attended Surface Warfare Officers School – Department Head Course in Newport, RI. He reported to USS PETERSON (DD-969) in December 1986 as the Chief Engineer.

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About the Cover

Growth and motion of vortices generated by a plate at an angle of attack of 60 degrees. The picture is taken in a recirculating water table and the vortices are visualized by means of aluminum dust. The alternate shedding of vortices takes place practically about all bluff bodies (cylinders, cables, missiles, etc.) and gives rise to large drag, oscillating lift force, and hydro- or aero-elastic oscillations. The flow field may be simulated numerically through the use of the fundamental equations of motion. The visualization of flow helps to our physical understanding of the phenomenon and provides data for comparison with those obtained in numerical experiments. (See article beginning on page 3.)

Photograph is the courtesy of Professor Turgut Sarpkaya (NPS).

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