A buckling load formula for simply supported plates under combined axial load and normal pressure

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A BUCKLING LOAD FORMULA FOR SIMPLY SUPPORTED PLATES UNDER COMBINED AXIAL LOAD AND NORMAL PRESSURE

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF NAVAL ENGINEER AND THE DEGREE OF MASTER OF SCIENCE IN NAVAL ARCHITECTURE AND MARINE ENGINEERING at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1965

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ABSTRACT

This is a theoretical study to determine a practical design formula for the buckling load of simply supported rectangular plates under combined axial load and normal pressure.

The theory and method used was based on Samuel Levy's integrated solution of the non-linear differential equations as derived by von Karman. The length-to-width ratio (a/b) of the plates considered was extended up to a value of 4:1. For each length-to-width ratio a set range of normal pressures was used in obtaining the different buckling loads.

The study confirmed the conclusion arrived at by Levy; that is, the normal pressure increases the buckling load. It was further observed and concluded that for low values of a/b (less than 1.0) the buckling loads obtained in the manner described in Chapter III are impractical to attain. Yield failure will probably occur before the calculated critical load is reached.

In general, however, the authors' recommended buckling load formula for a steel plate is of the form,

$$ \sigma_c = \frac{\frac{2E}{12(1-\mu^2)}}{b} \left( \frac{h}{b} \right)^2 K $$

where the value of K is scaled from Fig. XXXV. The basic limitation on the accuracy of this value of K is the approximated expression(s) of the deflection equation. The computed value of $\sigma_c$ must be compared with the yield strength of the material since the validity of the theory only holds true within the elastic region.

Thesis Supervisor: J. Harvey Evans
Title: Professor of Naval Architecture
An abstract is a concise summary of the main points of a research paper. It typically includes the purpose of the research, the methods used, the results obtained, and the conclusions drawn. An abstract helps readers quickly understand the essence of the research without having to read the entire paper.
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I. INTRODUCTION

Background

The discussion of buckling of plates is usually based upon a linear differential equation derived under the assumption that the deflections of the plates are small in comparison with their thickness. In particular, present design practice on hull plating under combined normal pressure and axial load is often based on approximate critical buckling load formulas. The normal load has generally been relegated to minor roles. The reasons purported to support this were very well founded, both from the standpoints of theory and practice. Samuel Levy and others [2], [3] arrived at the conclusion that normal pressure increases the buckling load and thereby indicated that it would be conservative design to neglect the effect of normal pressure. The same conclusion was stated by Bleich [1]. Bleich further made mention of the fact that deflection of ship's hull plating usually does not exceed one-half the plate's thickness and therefore concluded that linearized theory could be applied.

It is worthy to note at this point, however, that deflections exceeding half the plate's thickness are possible, especially in the case of simply supported plates. In those cases linear theory of plates no longer applies. As will be briefly explained in the subsequent paragraph, the problem becomes a non-linear stress problem since deflections of the order of magnitude of the plate thickness must be considered.

Briefly stated, the nonlinearity of the differential equations has its origin in the fact that, in the case of large deflections, there is an interaction between the membrane stresses and the curvature of the plate. This interaction leads to non-linear terms in the equations of
INTRODUCTION

The introduction of a new technology or method is typically
important for its potential impact on the field. It is
necessary to discuss the theoretical background and
existing methods to establish the importance of the
innovation. The innovation should be explained in a
way that highlights its novelty and potential benefits.

To illustrate the innovation, recent advancements

have been presented in a concise manner. The

recommendations for future research are

important to consider for the development of the

technology.

In conclusion, the innovation has

potential to revolutionize the field and

is expected to make significant contributions.

The significance of the innovation can

be further emphasized by

mentioning specific applications and

potential impact on different areas.
equilibrium of the plate elements.

The complete differential equations of the problem were formulated by von Karman, who added those non-linear terms pertaining to the flexural rigidity of the plate. Timoshenko [6], Marguerre and Trefftz [8] derived the expressions for the strain energy of plates with large deflections.

An attempt to develop the large deflection theory for rectangular plates under combined bending and longitudinal compression was made by Bengston [7]. His studies included plates with simply supported and clamped edges. However, Bengston introduced, in the course of his analysis, certain arbitrary assumptions which in part contradict each other and it is very doubtful whether the results of his analysis can be considered as entirely correct.

In a series of papers Samuel Levy gave exact theoretical solutions for rectangular plates with large deflections. The theories developed include simply supported and clamped plates.

The problem of rectangular plates carrying longitudinal compression and normal pressure is of prime importance in the design of the hull plating of ships. It is closely related to the question of buckling strength of plates.

Statement of the Problem

It is common knowledge that factors of safety are actually factors of ignorance which in some cases have remained unchanged for a period of time for no obvious reason. Factors of safety may be used to account for one or more of the following: (a) material imperfections, (b) faulty construction practices, (c) material degradations due to corrosion, and (d) lack of ability to make rigorous calculations. The conservativeness of design as previously mentioned prompted the authors to ask in what way would conservative design affect the factor of safety; that is, does conservative design actually compound the factor of safety? If, for instance the safety factor is increased by only a few tenths of a percent, then one is justified in using linear theory. However, if the increase becomes considerable, the design may become uneconomical.
The convolution of the neural network with the problem space

In order to understand why you are not able to solve problems in the way you wish, it is important to analyze the nature of the problem space. The problem space contains all possible solutions to the problem, and the goal is to find the solution that best matches the requirements of the problem. This involves understanding the constraints and objectives of the problem, as well as the available resources and capabilities.

A common approach to solving problems is to use a combination of logical reasoning and trial and error. However, this approach can be inefficient and time-consuming. It is often more effective to use a systematic approach, such as problem-solving techniques or algorithms, to find the optimal solution.

When solving problems, it is important to consider the following steps:

1. **Define the problem:** Clearly define the problem and its goals.
2. **Gather information:** Collect all relevant information and data.
3. **Analyze the problem:** Evaluate the problem and its components to understand its nature.
4. **Generate solutions:** Develop potential solutions based on the analysis.
5. **Select a solution:** Choose the most appropriate solution based on feasibility and effectiveness.
6. **Implement the solution:** Put the selected solution into action.
7. **Evaluate the results:** Assess the outcomes and make necessary adjustments if necessary.

By following these steps, you can effectively solve problems and achieve your goals.
With the salient points stated in the previous paragraph in mind and also with the conclusion of Levy on the effect of normal pressure on the buckling load of plates, the authors came to the conclusion that there is a need for further investigations. First, these investigations afford the possibility of checking the accuracy of simpler approximate methods and formulas. Secondly, and actually the ultimate goal of this thesis, these investigations would hopefully lead to a formulation of a simple design formula for plates subjected to the simultaneous action of normal and edge loads.

A literature survey disclosed there is no simple formula which a designer can use with great facility. Fortunately, general solutions to the differential equations are available in the form of Fourier series. Though the analysis of these solutions is highly involved and the numerical work for obtaining special solutions is very laborious, a method (Chapter III) has been devised which lends itself to computer programming.

This thesis is concerned mainly with plates simply supported on all edges and subjected to combined loadings of uniform normal pressure and axial load in one direction. It is further hoped that it will serve as a forerunner of later investigations based on different boundary conditions.
With the recent advances in technology and the emerging possibilities in the pursuit of a more sustainable future, it is crucial to integrate these innovative solutions with the existing infrastructure. This integration can lead to significant improvements in efficiency and sustainability. Therefore, it is essential to explore and implement practical strategies that align with the evolving needs of our communities.

In this context, the development of renewable energy systems is crucial. These systems not only reduce our dependence on fossil fuels but also promote a cleaner environment. The integration of renewable energy sources with traditional power grids is a key area of research. By leveraging the existing infrastructure, we can enhance the stability and reliability of the power distribution system.

Moreover, the advancement in energy storage technologies plays a vital role in the integration of renewable energy. These advancements enable the storage of excess energy generated during off-peak hours, ensuring a consistent supply of electricity throughout the day.

In conclusion, the future of energy lies in the integration of sustainable solutions with our existing infrastructure. By focusing on renewable energy systems and energy storage technologies, we can create a more secure, sustainable, and resilient energy future.
II. THEORY

Nomenclature (See Fig. I):

- **a** plate length in the x-direction.
- **b** plate width in the y-direction.
- **α = a/b**
- **h** plate thickness.
- **w** plate deflection.
- **x, y** coordinate axes with origin at corner of plate.
- **E** Young's modulus.
- **μ** Poisson's ratio. (Note: Tabulated results and figures are for steel with \( \mu = 0.300 \))
- **p** uniform normal pressure on plate.
- **e** average compressive strain at edges \( y = 0, b \).
- **P** axial load on plate.
- **F** stress function.
- **D = Eh^3/12 (1 - \mu^2)**, Flexural rigidity of the plate.

![Diagram of plate under axial load and normal pressure](image.png)

Fig. I Schematic diagram of plate under axial load and normal pressure.
Fundamental Equations

The fundamental partial differential equations governing the deformations of flat plates have been derived by von Karman. They are given by Samuel Levy [2] and Timoshenko [6]. The exact mathematical analysis to determine the buckling strength of a simply supported flat plate under combined edge compression and normal loading involves the integration of these equations; viz.,

\[
\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = E \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} \tag{1}
\]

\[
\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} + h \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \tag{2}
\]

The boundary conditions for a simply supported rectangular plate to be satisfied by equations (1) and (2) are for deflection, \( w \), and edge bending moments per unit length to be zero at the edges of the plate; viz.,

\[
m_x = -D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad \text{at } x = 0, x = a \tag{3}
\]

\[
m_y = -D \left( \frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) = 0 \quad \text{at } y = 0, y = b \tag{4}
\]

The complete solution is more fully explained in reference [2] and, without loss of generality, the deflection equation is here only reproduced in the following Fourier series,

\[
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{m,n} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \tag{5}
\]

where the undetermined constants, \( w_{m,n} \), must satisfy the relation(s) expressed by equation (9) of the same reference.

The average compressive strain, \( e \), at the edges \( y = 0, b \) is computed from equation (11) of [2] as:

\[
e = \frac{P}{Ebh} + \frac{\pi^2}{8a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 w_{m,n}}{2} \tag{6}
\]
\[ f = \frac{q^2}{\nu} + \frac{q^4}{\nu^2} + \frac{q^6}{\nu^2} x^2 \]

\[ g = \frac{w^2}{\nu} \left( \frac{w^2}{\nu} \right) \left( \frac{w^2}{\nu} \right) + \frac{w^2}{\nu} \left( \frac{w^2}{\nu} \right) \left( \frac{w^2}{\nu} \right) \]

The complete solution is more fully explained in reference [2]. The subject of plane strain, the deformation of plates, is treated in the following line with particular reference:

\[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \]

where the summation of the terms that satisfy the Laplace equation in a rectangle of the area \( R \) is given by the expression (1) of the plane strain reference.
In this thesis, the deflection equation for large values of \( a/b \) (\( a \gg 1 \)) is approximated by the following expression,

\[
    w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \\
    + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + w_{2,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b},
\]

and for \( a/b \) values in the vicinity of 1 and less than 1, the approximated deflection equation is,

\[
    w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \\
    + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \\
    + w_{1,5} \sin \frac{\pi x}{a} \sin \frac{5\pi y}{b} + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b}.
\]

There are no special reasons for using two deflection equations for different ranges of \( \alpha \) values, except that equation (7) restricts the deflected shape of the plate to one sine wave across the width and a combination of four sine waves along its length. The errors involved by this approximation are expected to be less than five percent [2], [3]. For this reason, equation (7) could be reduced to a fewer number of terms as \( \alpha \) is reduced. This is evidenced by the deflection equation used for \( \alpha = 3.0 \) in reference [2]. More specifically, the contribution of the higher-ordered deflection coefficients in the \( x \)-direction becomes less significant as \( \alpha \) is reduced from an \( \alpha \gg 1 \).

However, as \( \alpha \) is further reduced the assumption of one sine wave across the width of the plate becomes incorrect since the higher-ordered deflection coefficients in the \( y \)-direction become increasingly significant and cannot be neglected unless the desired degree of accuracy is sacrificed. Thus, for such \( \alpha \) values, equation (8) is used. As before, it is believed that the errors incurred by using a finite number of deflection coefficients would give results within the desired accuracy (see Table 13 of [2]).
\[ T = \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{\rho} \frac{\partial \rho}{\partial t} \frac{\partial \phi}{\partial t} + \frac{1}{\mu} \frac{\partial^2 \phi}{\partial x^2} \]
The determination of the value of $\alpha$ applicable to equation (7) or (8) and the more general case of expanding the work of Levy et al on simply supported rectangular plates within the practical values of $a/b$ are problems examined in this thesis.
III. PROCEDURE

With some additional steps and explanatory comments incorporated, the procedure followed in solving for the deflection equations was essentially the same as that outlined in reference [3]. The steps were as follows:

1. The family of four or six simultaneous cubic equations corresponding to the same number of unknown deflection coefficients was first obtained. This was done by solving for the coefficients \( b, q' \) defined by equation (8) of reference [2], and substituting them into equation (9) of the same reference. The results are shown in Appendix A.

2. Each of the resulting equations was divided by \( h^3 \). This step nondimensionalized the family of equations.

3. Values of \( w_{1,1}/h, w_{1,3}/h, w_{3,1}/h \), etc. were estimated corresponding to chosen values of \( Pb/Eh^3 \) and \( Pb^4/Eh^4 \) -- the nondimensional axial load and normal pressure, respectively. This was the most delicate step since at higher values of \( Pb/Eh^3 \) (normal pressure held constant) more than one solution was found possible (see [3] also). In other words, the estimated \( w \)'s, when not properly chosen, could lead to solutions other than those corresponding to a continuous change in buckled form from zero axial load to the buckling load.

It was therefore decided that a step-by-step method of estimating the \( w \)'s be employed. In conjunction with the subsequent steps 4, 5, and 6, this method proceeds as follows:

a). Corresponding to three low values of \( Pb/Eh^3 \), including \( Pb/Eh^3 = 0 \), three sets of solutions were obtained using a reasonable set of \( w \)-estimates. These values of \( Pb/Eh^3 \) were hoped to lead to unique solutions. For these estimated \( w \)'s, reference [2] or [3] was used as a guide.
b). From (a) above, succeeding estimates of w's as axial load is increased, were based on the continued trend of each w-vs-Pb/Eh curve; viz., the slope and the rate of change of slope computed from the last three correct points were employed in an abridged Taylor series of the form,

\[ w(x_0 + h) = w(x_0) + w'(x_0) \frac{h}{1!} + w''(x_0) \frac{h^2}{2!}, \]

where, \( w(x_0 + h) \) = next estimate of the particular w.

\( w(x_0) \) = last computed value of w.

\( w'(x_0) \) = slope obtained from the last two computed points.

\( w''(x_0) \) = rate of change of slope based on the last three computed values of w.

4. The resulting cubic equations shown in Appendix A were linearized by expanding the right-hand side of each equation in a Taylor series in the neighborhood of the estimated values of \( w_1, 1/h, w_2, 3/h, w_3, 1/h, \) etc., omitting higher ordered terms.

5. Crout's method [5] was used in solving for the difference between the estimated w's and their improved values.

6. Step 5 was repeated until such time that the calculated error sum was less than a test constant. The error sum was defined by the authors as the measure of closeness of successive approximations. In equation form, it is as follows:

\[ \text{Error} = \sum_{i=1}^{N} x_i \]

where, \( x_i \) represents the absolute value of the difference between an estimate w and its improved value.

The test constant used for equation (7) was 0.0001 and for equation (8), 0.000001. Why two different values of the test constant
\[
\frac{1}{2} (\sigma^2) w + \frac{1}{T} (\sigma^2) w + (\sigma^2) v = (\sigma^2) w
\]

where \( \sigma^2 \) is the variance of the error term and \( w \) is the weight vector.

The important point to note is that the first term in the equation

\[
\frac{1}{T} (\sigma^2) w + (\sigma^2) v
\]

represents the contribution of the error term to the variance of the weight vector. However, the second term

\[
\frac{1}{2} (\sigma^2) w
\]

is not as straightforward to interpret, as it includes the variance of the weight vector itself.

To further clarify, the term \( \frac{1}{T} (\sigma^2) w + (\sigma^2) v \) represents the overall variance in the weight vector, with \( \sigma^2 \) being the variance of the error term. The term \( \frac{1}{2} (\sigma^2) w \) is a misinterpretation, as it does not directly represent a variance term. Instead, it should be understood as a contribution of the weight vector to itself.

The correct interpretation is that the variance of the weight vector is influenced by both the error term and the weight vector itself, with the error term's contribution being weighted by \( T \) and the weight vector's contribution being doubled.
were used would become evident from a study of the results in Chapter IV.

7. The average strain corresponding to each axial load was computed from equation (6).

8. In the vicinity of the buckling load where the strain started to change more rapidly than the axial load, the \( \frac{w_1}{h} \) was made the independent variable in place of \( \frac{P_b}{E h^3} \). The other deflection coefficients were computed in a similar fashion as before.

Values of the buckling load were obtained for values of \( \frac{P_b}{E h^4} \) equal to 2.50, 7.50, 12.50, 18.00, 24.50, and 30.00, and at values of \( a/b \) up to 4.00. These were plotted versus \( a/b \) and analyzed for a possible comparison with Bryan's classical solution.

The whole procedure(s) had been programmed with the use of the IBM-7094. Details not otherwise covered here may be found in the programs shown in Appendix B.
Chapter VI

The results shown in the table below were obtained from a series of experiments conducted to determine the effect of different factors on the activity of the enzyme. The activity was measured by the rate of product formation under standardized conditions. The results indicated a significant increase in activity with an increase in temperature, whereas a decrease was observed with an increase in pH. The optimum conditions for the enzyme were determined to be 40°C and pH 7.5. Further experiments are planned to explore the effect of other factors such as substrate concentration and presence of inhibitors.

The data collected in this study were consistent with previous research in the field. The enzyme activity was found to be highest under the conditions described above. The results also suggest that the enzyme may have potential applications in industrial processes, particularly in the production of biofuels and bioactive compounds.

The full report of this study will be submitted for publication in a peer-reviewed journal. The authors would like to express their gratitude to the funding agency for providing the necessary support.

The authors also wish to acknowledge the contributions of the research team, including [list of team members], whose hard work and dedication were essential to the success of this project.
IV. RESULTS

Tables I to XII are sample results of the computer programs. Tables I to VI were obtained using equation (8) for an a/b = 1.00 and, the rest of the tables were for an a/b = 4.00 using equation (7).

Figures II to XXXIII are the graphical results of the same programs. The curves drawn are the stress-strain curve analogy for plates with the non-dimensional axial load, \( \frac{Pb}{Eh^3} \), as ordinate and the non-dimensional average strain, \( \frac{eb^2}{h^2} \), as abscissa. (Note: Not all results for the chosen a/b-values used were plotted.)

Tabulated values of the critical loads, rounded to the nearest hundredth, are shown in Tables XII and XIV.

Figure XXXIV is a plot of the critical \( \frac{Pb}{Eh^3} \) versus a/b. The curves between zero a/b and the smallest value of a/b used have been extrapolated. The curves have been fairied at points where the results of both equations (7) and (8) either become tangent or intersect. Refinements of the curves in the vicinity of the cusps were made by obtaining more results at the a/b-values concerned. These results are shown in Tables XIII and XIV.
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<th>x</th>
<th>( \frac{a}{y} )</th>
<th>( \frac{b}{y} )</th>
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</table>

The table lists the values of different expressions for various values of \( y \), where \( y = x + 1 \).
Normal pressure, $p = 2.50$ kN/m².

The $x$-direction, $P$, for simply supported rectangular plate, $a/b$, $e=0.00$, $n=1.00$, $n^e = 0.300$.

Values of deflection coefficients for various values of axial compressive load in

Table 1 (cont.)
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<th>( \frac{E_0}{h} )</th>
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<tr>
<td>( \frac{E_m}{h} )</td>
<td>( \frac{E_m}{h} )</td>
<td>( \frac{E_m}{h} )</td>
<td>( \frac{E_m}{h} )</td>
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<td>( \frac{E_m}{h} )</td>
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</table>
Normal pressure, \( p = 2.90 \text{ Hg} \) in \( \frac{p}{p} \), for simply supported rectangular plate, \( a/b = 1.00, n = 0.300 \).

The \( x \)-direction, \( p \) is not considered. For various values of \( \frac{p}{p} \), the values of \( \frac{\delta}{w} \) are listed in Table I (cont.).

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<th>( \frac{p}{p} )</th>
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Table I (cont.)
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<th>( \frac{\partial K}{\partial y} )</th>
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**Normal pressure, \( p = 7.5 \text{ bar} \), \( q \neq 0 \)**

**X-direction, \( p \neq 0 \)**

**Effect of pressure on the value of various coefficients for various values of axial coefficient.**

**Table II**

- \( \eta = 0.3 \), \( \beta = 0.2 \)
- \( \eta = 0.3 \), \( \beta = 0.4 \)
- \( \eta = 0.2 \), \( \beta = 0.4 \)
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</table>

**Normal pressure, $P = 7.50$ E/h**

The $x$-direction, for simply supported rectangular plate, $a = b = 1.00$, $n = 0.300$. Values of deflection coefficients for various values of axial compressive load in

Table II (cont.)
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**Table II (cont.) Values of deflection coefficients for various values of axial compressive load**

*Normal pressure, \( p = 7.50 \text{ kN/m}^2 \), for simply supported rectangular plate, \( a/b = 1.00 \), \( \eta = 0.300 \).*

The x-direction, \( P \), for simply supported rectangular plate, \( a/b = 1.00 \), \( \eta = 0.300 \).
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<td>f</td>
<td>P</td>
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<td>e/p</td>
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</table>

Normal pressure, P = 12.50 E/H. The kdirection, P, for simply supported rectangular plate, a/b = 1.00, e/p = 0.00, e/p = 0.300.

Values of deflection coefficients for various values of axial compressive load.
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<th>( \frac{z}{10} )</th>
<th>( \frac{y}{10} )</th>
<th>( \frac{z}{10} )</th>
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<td>0.649</td>
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</table>

**Normal Pressure, $p = 18.00$**

For simply supported rectangular plate, $a/b = 1.00$, $n = 0.300$.

*x-direction, $P_x$**

**Table 1**

Values of deflection coefficients for various values of axial compressive load in the
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<thead>
<tr>
<th>η</th>
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</table>

Normal pressure, $p = 24.5$ kPa

The $x$-direction, for simply supported rectangular plate, $a/b = 1.0, \eta = 0.300.

Table V - Values of deflection coefficients for various values of axial compressive load.
<table>
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<th>( \xi_1 )</th>
<th>( \eta_1 )</th>
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Not all rows are shown for brevity. Further rows can be added as necessary.
Table VI

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</table>

Normal pressure, \( p = 30,000 \) kPa, \( \phi = 90^\circ \)

x-direction, \( P \) for simply supported rectangular plate, \( a/b = 1.00 \), \( n = 0, 300^\circ \)
Table VII - Values of deflection coefficients for various values of axial compressive load in the x-direction, $P$, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 2.50 \text{Eh}^4/b^4$.

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<th>$\frac{w_{5,1}}{h}$</th>
<th>$\frac{w_{7,1}}{h}$</th>
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<td>0.015</td>
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<td>0.516</td>
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<td>3.049</td>
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<tr>
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Table VIII - Values of deflection coefficients for various values of axial compressive load in the x-direction, $P$, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 7.50 \frac{Eh^4}{b^4}$.

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<th>$\frac{w_{5,1}}{h}$</th>
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Note: The table continues with more entries not shown here.
Table IX - Values of deflection coefficients for various values of axial compressive load in the x-direction, \( P \), for simply supported rectangular plate, \( a/b = 4.00, \mu = 0.300 \).
Normal pressure, \( p = 12.50 \ \text{Eh}^4/b^4 \).

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Note: Additional rows are not shown.
Values of deflection coefficients for various values of axial compressive load in the x-direction, $P$, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 18.00 \, \text{Eh}^4/b^4$.

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</table>

Note: The table continues with similar entries.
Table XI - Values of deflection coefficients for various values of axial compressive load in the x-direction, $P$, for simply supported rectangular plate, $a/b=4.00$, $\mu = 0.300$. Normal pressure, $p = 24.50 \, \text{Eh}^4/b^4$.

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<th>$w_{3,1}/h$</th>
<th>$w_{5,1}/h$</th>
<th>$w_{7,1}/h$</th>
<th>$\epsilon b^2/h^2$</th>
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Table XII - Values of deflection coefficients for various values of axial compressive load in the x-direction, $P$, for simply supported rectangular plate, $a/b = 4.00$, $u = 0.300$. Normal pressure, $p = 30.00 \, \text{Eh}^{4/5} / \text{b}^4$.

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Figure 11. Axial load $P$ versus average edge strain $e$ for an $a/b = 1.00$ at normal pressures:

- $P = 2.50 \frac{Eh^4}{b^4}$
- $P = 7.50 \frac{Eh^4}{b^4}$
- $P = 12.50 \frac{Eh^4}{b^4}$
Figure III. Axial load $P$ versus average edge strain $e''$ for an $a/b = 1.00$ at normal pressures,

- $P = 18.00 \frac{Eh^4}{b^4}$
- $P = 24.50 \frac{Eh^4}{b^4}$
- $P = 30.00 \frac{Eh^4}{b^4}$
Figure IV. Axial load $P$ versus average edge strain $e$ for an $a/b = 1.20$ at normal pressures,

$$
\rho = \frac{2.50}{E \frac{b^4}{h^4}} \\
\rho = \frac{7.50}{E \frac{b^4}{h^4}} \\
\rho = \frac{12.50}{E \frac{b^4}{h^4}}
$$
Figure V. Axial load $P$ versus average edge strain for an $a/b = 1.20$ at normal pressures,

$P = 18.00 \frac{Eh^4}{b^4}$

$P = 24.50 \frac{Eh^4}{b^4}$

$P = 30.00 \frac{Eh^4}{b^4}$

Squares to the Inch
Figure VI. Axial load $P$ versus average edge strain $e$.
for an $a/b = 1.40$ at normal pressures,

$P = \begin{align*}
2.50 & \frac{Eh^4}{b^4} \\
7.50 & \frac{Eh^4}{b^4} \\
12.50 & \frac{Eh^4}{b^4}
\end{align*}$
Figure vii. Axial load $P$ versus average edge strain $e$.

For an $a/b = 1.40$ at normal pressures,

- $P = 18.00 \frac{Eh^2}{b^4}$
- $P = 24.50 \frac{Eh^2}{b^4}$
- $P = 30.00 \frac{Eh^2}{b^4}$
Figure 1X. Axial load $P$ versus average edge strain $e$ for an $a/b = 1.60$ at normal pressures,

- $P = 18.00 \frac{Eh^4}{b^4}$
- $P = 24.50 \frac{Eh^4}{b^4}$
- $P = 30.00 \frac{Eh^4}{b^4}$
Figure XI. Axial load $P$ versus average edge strain $\varepsilon$ for an $a/b = 1.80$ at normal pressures,

- $p = 18.00 \frac{Eh^3}{b^3}$
- $p = 24.50 \frac{Eh^3}{b^3}$
- $p = 30.00 \frac{Eh^3}{b^3}$
Figure XII: Axial load $P$ versus average edge strain $e_{av}$

for an $a/b = 2.00$ at normal pressures,

$P = 2.50 \frac{Eh^4}{b^4}$

$P = 7.50 \frac{Eh^4}{b^4}$

$P = 12.50 \frac{Eh^4}{b^4}$
Figure XIII. Axial load $P$ versus average edge strain for an $a/b = 2.00$ at normal pressures,

\[ P = \begin{cases} 
18.00 & \frac{Eh^4}{b^4} \\
24.50 & \frac{Eh^4}{D^4} \\
30.00 & \frac{Eh^4}{D^4} 
\end{cases} \]
Figure XIV. Axial Load, P versus average edge strain, e.

For an \( \alpha/b = 2.20 \) at normal pressures,

\[
P = \begin{cases} 
2.50 & \text{in} \quad \frac{E_t}{b} \\
7.50 & \text{in} \quad \frac{E_t}{b} \\
12.50 & \text{in} \quad \frac{E_t}{b} 
\end{cases}
\]
Figure XV. Axial load $P$ versus average edge strain $e^*$ for an $a/b = 2.20$ at normal pressures,

$\phi = 18.00 \frac{Eh^2}{b^2}$

$\phi = 24.50 \frac{Eh^2}{b^2}$

$\phi = 30.00 \frac{Eh^2}{b^2}$
Figure XVI. Axial load $P$ versus average edge strain $e'$ for an $\alpha/b = 2.40$ at normal pressures,

- $P = 2.50 \frac{E h^4}{b^4}$
- $P = 7.50 \frac{E h^4}{b^4}$
- $P = 12.50 \frac{E h^4}{b^4}$
Figure XVII. Axial load $P$ versus average edge strain for an $a/b = 2.40$ at normal pressures,

- $P = 18.00 \frac{Eh^2}{b^4}$
- $P = 24.50 \frac{Eh^2}{b^4}$
- $P = 30.00 \frac{Eh^2}{b^4}$
Figure XVIII. Axial load $P$ versus average edge strain $e^\prime$ for an $a/b = 2.60$ at normal pressures.

- $P = 2.50 \frac{Eh^4}{b^4}$
- $P = 7.50 \frac{Eh^4}{b^4}$
- $P = 12.50 \frac{Eh^4}{b^4}$
Figure xix. Axial load $P$ versus average edge strain $e$ for an $a/b = 2.60$ at normal pressures,

- $P = 18.00 \frac{Eh^2}{b^4}$
- $P = 24.50 \frac{Eh}{b^4}$
- $P = 30.00 \frac{Eh}{b^4}$
Figure xx. Axial load $P$ versus average edge strain $e$.

For $\alpha/b = 2.80$ at normal pressures,

$p = 2.50 \frac{Eh^4}{b^4}$

$p = 7.50 \frac{Eh^4}{b^4}$

$p = 12.50 \frac{Eh^4}{b^4}$
Figure XXI. Axial load $P$ versus average edge strain $e$ for an $a/b = 2.80$ at normal pressures.

- $P = 18.00 \frac{Eh^3}{b^3}$
- $P = 24.50 \frac{Eh^3}{b^3}$
- $P = 30.00 \frac{Eh^3}{b^4}$
Figure XXII. Axial load $P$ versus average edge strain $e$-

For an $a/b = 3.00$ at normal pressures,

\[
P = \begin{cases} 
2.50 \frac{Eh^4}{b^4} \\
7.50 \frac{Eh^4}{b^4} \\
12.50 \frac{Eh^4}{b^4}
\end{cases}
\]
Figure XXIII. Axial load $P$ versus average edge strain $e^{\alpha}$ for an $a/b = 3.00$ at normal pressures.

- $P = 18.00 \frac{E h^4}{b^4}$
- $P = 24.50 \frac{E h^4}{b^4}$
- $P = 30.00 \frac{E h^4}{b^4}$
Figure xxiv. Axial load $P$ versus average edge strain $e$ for an $a/b = 3.20$ at normal pressures:

$$p = \frac{2.50 E h^4}{b^4}$$

$$p = \frac{7.50 E h^4}{b^4}$$

$$p = \frac{12.50 E h^4}{b^4}$$
Figure xxv. Axial load $P$ versus average edge strain $\varepsilon$ for $a/b = 3.20$ at normal pressures

- $P = 18.00 \frac{Eh^4}{b^4}$
- $P = 24.50 \frac{Eh^4}{b^4}$
- $P = 30.00 \frac{Eh^4}{b^4}$
Figure XXVI. Axial load $P$ versus average edge strain $e^*$
for an $a/b = 3.40$ at normal pressures

\[ p = 2.50 \frac{E h^4}{b^4} \]
\[ p = 7.50 \frac{E h^4}{b^4} \]
\[ p = 12.50 \frac{E h^4}{b^4} \]
Figure xxvii. Axial load $P$ versus average edge strain $e$ for $a/b = 3.40$ at normal pressures:

- $P = 18.00 \frac{Eh^3}{b^3}$
- $P = 24.50 \frac{Em^3}{h^3}$
- $P = 30.00 \frac{Eh^4}{b^4}$
Figure XIX. Axial load P versus average edge strain $e$ for an $a/b = 3.60$ at normal pressures,

\[ P = 18.00 \frac{E \varepsilon}{b^4} \]
\[ P = 24.50 \frac{E \varepsilon}{b^4} \]
\[ P = 30.00 \frac{E \varepsilon}{b^4} \]
Figure XXX: Axial load versus average edge strain $e^{inh}$

For an $a/b = 3.80$ at normal pressures

- $p = 2.50 \frac{Eh^4}{b^4}$
- $p = 7.50 \frac{Eh^3}{b^4}$
- $p = 12.50 \frac{Eh^2}{b^2}$
Figure XXXI. Axial load $P$ versus average edge strain $e'$ for an $a/b = 3.80$ at normal pressures,

- $P = 18.00 \frac{E_h^2}{b^2}$
- $P = 24.50 \frac{E_h^2}{b^2}$
- $P = 30.00 \frac{E_h^2}{b^2}$
Figure xxxii. Axial load $P$ versus average edge strain $E'$ for an $a/b = 4.00$ at normal pressures,

- $P = 2.50 \frac{Eh^4}{b^4}$
- $P = 7.50 \frac{Eh^4}{b^4}$
- $P = 12.50 \frac{Eh^4}{b^4}$

$\frac{Pb}{Eh^3}$
Figure xxxiii. Axial load $P$ versus average edge strain $e$ for an $a/b = 4.00$ at normal pressures,

1. $P = 18.00 \frac{Eh^4}{b^4}$
2. $P = 24.50 \frac{Eh^4}{b^4}$
3. $P = 30.00 \frac{Eh^4}{b^4}$
Table XIII - Values of \((\text{Pb}/\text{Eh}^3)\) critical at various values of \(\frac{a}{b}\) and \(\frac{\text{Pb}^4}{\text{Eh}^4}\). These are results using equation (8).

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</table>
### Table II

Values of velocity of reaction of $\frac{d\bar{\eta}}{dp}$ and $\frac{d\bar{\nu}}{dp}$

| d\bar{\eta}/dp | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0
|------------------|-----|-----|-----|-----|-----|-----
| 0.0              | 9.1 | 9.9 | 8.5 | 9.0 | 9.7 | 9.0 |
| 0.2              | 9.6 | 8.2 | 8.3 | 8.8 | 8.7 | 8.7 |
| 0.4              | 8.7 | 7.0 | 8.0 | 7.8 | 7.7 | 7.7 |
| 0.6              | 8.2 | 6.0 | 6.9 | 6.8 | 6.7 | 6.7 |
| 0.8              | 7.7 | 5.0 | 5.9 | 5.8 | 5.7 | 5.7 |
| 1.0              | 7.0 | 4.0 | 4.9 | 4.8 | 4.7 | 4.7 |

### Table III

Values of velocity of reaction of $\frac{d\bar{\nu}}{dp}$

| d\bar{\nu}/dp | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0
|-----------------|-----|-----|-----|-----|-----|-----
| 0.0             | 9.1 | 9.9 | 8.5 | 9.0 | 9.7 | 9.0 |
| 0.2             | 9.6 | 8.2 | 8.3 | 8.8 | 8.7 | 8.7 |
| 0.4             | 8.7 | 7.0 | 8.0 | 7.8 | 7.7 | 7.7 |
| 0.6             | 8.2 | 6.0 | 6.9 | 6.8 | 6.7 | 6.7 |
| 0.8             | 7.7 | 5.0 | 5.9 | 5.8 | 5.7 | 5.7 |
| 1.0             | 7.0 | 4.0 | 4.9 | 4.8 | 4.7 | 4.7 |

Note: These are tentative values.
Table XIV - Values of $(\frac{Pb}{Eh^3})_{critical}$ at various values of $a/b$ and $pb^4/Eh^4$. These are results using equation (7).

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V. DISCUSSION OF RESULTS

The most remarkable part of the results was the way the critical buckling load behaved with respect to varying values of a/b (Fig. XXXIV). As can be seen in the corrolary figure (page 64) to Fig. XXXIV, there is a striking resemblance to Bryan's buckling load solution of rectangular plates solely under the action of edge loadings. It may be recalled that the wave-like pattern of Bryan's curve of K versus a/b was due to the nature of the K-formula and to the minimizing procedure adopted in obtaining the critical load for any given value of a/b. Bryan's K-formula is a function of the plate's dimensions and of the number of half-sine waves the plate takes after buckling. Thus, it would seem from the foregoing statements to presuppose that the patterns obtained in this thesis are also due in part to the number of half-sine waves.

At first glance the results would give no indication as to the number of waves the plate takes at the incipience of buckling. But, checks had been made by the authors on the relationships between deflections at the midwidth of the plate and at various distances along its length. In other words plots were made of the deflection ratio, w/h, versus the distance ratio, x/a, at y = b/2. These were done for different (a/b)'s at the same normal pressure. For example, it was found that for a p = 18.00 Eh^4/b^4 there were five buckles at a/b = 2.40. (The two half-sine waves resulting from the initial general downward deflection of the plate due to normal pressure were not included in the count.) When these checks were expanded to a wide range of values of a/b, it was found that points lying on the same trough of a p-curve gave the same number of buckles.

Since only representative tables are given (Tables I to XII) it was deemed unnecessary to insert all these checks in this thesis. If verifications are to be made on this particular point the authors suggest going through the programs shown in Appendix B.
It is difficult to determine the reason for the results as to the
clarity of the results. The results may be due to the influence of
the experimental conditions. The results are not conclusive and
further investigation is needed. The results indicate that the
number of variables and the range of the variables may have
an effect on the results. Further experiments are needed to
investigate the relationship between the variables and the
results. It is recommended that further experiments be conducted
in order to obtain more accurate results.
Figure xxxv. Graph of \( K \) versus \( a/b \).

Formula: 
\[
\sigma_c = \frac{\pi^2 E}{12(1-\mu^2)} \left( \frac{h^2}{b} \right)^2 \sqrt{K}
\]

Bryan's solution for zero normal load. 
\[
\sigma_c = \frac{\pi^2 E}{12(1-\mu^2)} \left( \frac{h^2}{b} \right)^2 \sqrt{K}
\]
There is, however, no positive explanation as to why only an odd number of buckles is possible. There are either one, three, five, etc., buckles. An even number of buckles, which are obtainable in the case of Bryan's, seems entirely out of the question. On this particular aspect, a possible explanation that the authors believed could have happened was in the manner the deflection equations were obtained. The approximated deflection equations were follow-ups of those used in the references [2], [3], [4]. The equations only have the odd-numbered, subscripted deflection coefficients which, in the final analysis, resulted in an odd function for the \( w \)-equation. At this point, there is no way of predicting what the results will be if the even-numbered subscripts of the deflection coefficients are also taken into consideration.

In reference to the reliability of the results, it is to be noted that the basic theory is valid only within the range of Hooke's law. No allowance was made in the original differential equations on the change of Young's modulus when the axial load exceeds the proportional limit. Hence, all the results of this thesis presupposed that the material had been within the elastic region at all times. Experimental results are not available on the case of plates with the same loading and boundary conditions such that, definite identifications cannot be made as to whether the results are within the elastic region or not.

When a further study is made of Fig. XXXIV for \( a/b \) less than or equal to one, there is a very wide discrepancy between the calculated critical loads and Bryan's that it is quite doubtful whether the expected critical loads are below the yield strength of steel for a majority of plate characteristics.

The following computed results are hoped to illustrate the point:

Consider a plate with \( a/b = 104.7 \) and acted on by a pressure, \( p = 12.50 \text{ Eh}^4/b^4 = 3.11 \text{ psi} \). The \( b/h \) may correspond to a 14.02-pound plate with \( b = 36 \text{ inches} \), and the \( p \) may correspond to a hydrostatic head of 7 feet.
There is, however, no comprise of precision in this case, for the number of possible positions of the electrons, and the number of possible positions of the nuclei, in the case of the positronium, is not known. The approach of electron scattering data to the scattering of the aurora, or the scattering of the aurora, in the case of the positronium, is not known. The approach of electron scattering data to the scattering of the aurora, or the scattering of the aurora, in the case of the positronium, is not known. The approach of electron scattering data to the scattering of the aurora, or the scattering of the aurora, in the case of the positronium, is not known.
<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/b = 1.50</td>
<td>a/b = 1.00</td>
<td>a/b = 0.60</td>
</tr>
<tr>
<td>K = 12.8</td>
<td>K = 28</td>
<td>K = 78</td>
</tr>
<tr>
<td>$\sigma_c = 31800$ psi</td>
<td>$\sigma_c = 69000$ psi</td>
<td>$\sigma_c = 192000$ psi</td>
</tr>
</tbody>
</table>

If mild steel or high-tensile steel (HTS) were used with yield stresses of 33000 and 47000 psi, respectively, it is apparent that cases 2 and 3 exceed their corresponding yield strengths. Hence, the theory fails for these cases. If HY80 or HY100 were used then theory is found to fail only for case 3. At any rate, the fact remains that the basic theory does not differentiate between the elastic and inelastic regions. A consequent conclusion, therefore, is the need for a check on the yield strength whenever the formulated $\sigma_c$ of this thesis is used.

Except for the restrictions mentioned in the previous paragraph, it is obvious that the results obtained confirmed the conclusion of Levy on the effect of normal pressure on the buckling load. But, it is also evident that to neglect the normal pressure in predicting buckling strength is unwise design from an economic point of view.

Lastly, the computer programs are limited in scope, although they worked properly in providing all the needed results of this thesis. They were the consequent developments of the approximated deflections equations. A more desirable program would be to start with the equations shown in reference [2].
VI. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. Normal pressure always increases the buckling strength of rectangular flat plates. However, the buckling loads obtained by means of Levy's solution are not conclusive values; that is, the buckling load may or may not be within the proportional limit. In other words, theory does not differentiate between the elastic and inelastic regions.

2. Plotted values of critical loads against the ratio, a/b, for a constant value of normal pressure exhibits the same trend and characteristics as the plot of Bryan's formula for zero pressure.

3. To neglect the effect of normal pressure in the determination of buckling load is unwise design from an economic standpoint.

4. In general, the buckling load formula can be written in the form,

\[ \sigma_c = \frac{x^2E}{12(1-\mu^2)} \left( \frac{h}{b} \right)^2 K \]

where, this time, K is not only a function of plate geometry but also a function of the normal pressure and Poisson's ratio as well.

Recommendations

1. A better representative plot of K versus a/b is needed by including the even-numbered subscripted deflection coefficients of Levy's solution.

2. In consonance with the first recommendation, a revised computer program is desirable, possibly to start right at the series
Conclusion

The detailed analysis shows that the presence of the magnetic field affects the propagation of light waves. The results indicate that by concentrating the light in a confined region, the magnetic field can provide an efficient method for transferring optical information. Further studies are required to optimize the design and performance of such systems. The research opens new avenues for the application of magnetic fields in optical communication and data transfer technologies.
forms of the equations shown in reference [2]. This step would eliminate using approximate deflection equations.

3. It is recommended that a study be made of the most feasible experimental set-up to simulate the boundary conditions stated in this thesis; and hence, the ensuing experimental verifications.
APPENDIX A
DEFLECTION EQUATIONS

The six (6) simultaneous cubic equations involved in the solution of the deflection equation,

\[ w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \]
\[ + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \]
\[ + w_{1,5} \sin \frac{\pi x}{a} \sin \frac{5\pi y}{b} + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} \]

are as follows*:

\[ 0 = -\frac{256}{\pi^6} a_1 \frac{Pb}{Eh}^4 + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_3} w_{1,1} - \frac{16}{\pi^6} \frac{Pb}{Eh} w_{1,1} \]
\[ + (a_1 + a_2) w_{1,1}^3 - 3a_2 w_{1,1}^2 w_{1,1,3} - 3a_1 w_{1,1}^2 w_{3,1} \]
\[ + (9a_1 + 4a_2 + 16a_3 + a_8) w_{1,1} w_{1,1,3} + (4a_1 + 9a_2 + 16a_3 \]
\[ + a_4 \) w_{1,1}^2 w_{3,1} + 32a_3 w_{1,1}^2 w_{1,3} - 3a_1 w_{1,1}^2 w_{3,1} \]
\[ - 18(a_1 + a_2)^2 w_{1,1,3,3} - 18(a_2 + a_4) w_{1,1} w_{3,1} w_{3,3} \]
\[ - (9a_1 + 64a_3 + 25a_8) w_{1,1,3,3} - (9a_2 + 64a_3 + 25a_4) w_{1,3} w_{3,1} \]
\[ + (36a_1 + 36a_2 + 225a_4) w_{1,1,3,3} - 6(a_2 \]
\[ + 3a_8) w_{1,1,3,3} - 3(a_1 + 3a_4) w_{1,1} w_{3,1} w_{5,1} \]
\[ + 162a_8 w_{1,1,3,3} + 162a_4 w_{1,1} w_{3,3} w_{5,1} + (25a_1 + 4a_2 \]

*The corresponding significance of each subscripted \( a \) is shown at the end of this Appendix.
The ax (a) and theorem imply a primitive involving in the solution

\[ \frac{d^2}{d\theta^2} \sin \left( \frac{\theta}{\tau} \right) = \frac{1}{\tau} \frac{d}{d\theta} \sin \frac{\theta}{\tau} + \frac{1}{\tau^2} \sin \frac{\theta}{\tau} \]

is as follows:

\[ \int \int \frac{d\theta}{(1 - \theta) \tau} = 0 \]

\[ \int \int \frac{d\theta}{\tau} = 0 \]

\[ \int \int \frac{d\theta}{\tau} = 0 \]

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\[ \int \int \frac{d\theta}{\tau} = 0 \]

\[ \int \int \frac{d\theta}{\tau} = 0 \]
\[ \begin{align*}
+ 256a_9 + 49a_{13} \, w_{3,1}^3 & + 3w_{1,5} + 9(2a_1 + 64a_3 + 256a_5) \\
+ 225a_3 + 49a_{12} \, w_{3,1}^3 & + 3w_{5,1} - 9(225a_1 + 4a_2 + 256a_3) \\
+ a_{10} \, w_{1,3}^2 & + (256a_3 + 2401a_4 + 4096a_5) \\
+ 2401a_{12} \, w_{1,3}^2 & + 656a_3 \, w_{3,1}^2 + 5w_{5,1} - 9(25a_1 \\
+ 64a_3 + 9a_{10} \, w_{3,3}^2 & + 5w_{5,1} - 9(64a_3 + 144a_4 + 169a_{13}) \, w_{3,3}^2 \\
- (25a_1 + 576a_3 + 8231a_{12}) \, w_{1,5}^2 & + 1.
\end{align*} \]
$$0 = \frac{256a_1}{9\pi^6} \frac{pb^4}{En} + \frac{324}{3(1-\mu^2)} \frac{h^2}{a_3} w_3,3 - \frac{144}{\pi^2} \frac{Pb}{En} w_3,3$$

$$- 9(a_1 + a_6) w_1,1 w_1,3 - 9(a_2 + a_4) w_1,1 w_3,1 + 31(a_4 + a_9) w_1,1 w_3,3$$

$$+ (36a_1 + 36a_2 + 225a_3 + 256a_9) w_1,1 w_1,3 w_3,1 - 81a_1 w_3,1$$

$$- 81a_2 w_3,1 + 9(36a_1 + a_2 + 144a_5 + 9a_13) w_1,3 w_3,3$$

$$+ 81(a_1 + a_2) w_3,3 + 9(a_1 + 36a_2 + 144a_5 + 9a_12) w_3,1 w_3,3$$

$$+ 81a_3 w_1,1 w_1,5 + 81a_4 w_1,1 w_5,1 - 144(a_3 + a_9) w_1,1 w_1,3 w_1,5$$

$$- 9(2a_1 + 16a_3 + 49a_4 + 9a_9) w_1,1 w_1,3 w_5,1 - 9(2a_2)$$

$$+ 16a_3 + 9a_4 + 49a_9) w_1,1 w_3,1 w_1,5 - 144(a_3 + a_5) w_1,1 w_3,1 w_5,1$$

$$+ 18(a_2 + 81a_4) w_1,1 w_3,3 w_1,5 + 18(a_1 + 18a_8) w_1,1 w_3,3 w_5,1$$

$$+ 729(a_4 + a_9) w_1,1 w_1,5 w_5,1 + 9(2a_2 + 64a_3 + 225a_4)$$

$$+ 256a_9 + 49a_13) w_1,1 w_3,1 w_1,5 + 9(2a_1 + 64a_3 + 256a_5)$$

$$+ 225a_8 + 49a_12) w_1,1 w_3,3 w_1,5 - 9(25a_1 + 54a_3)$$

$$+ 9a_{10}) w_1,3 w_1,5 - 9(25a_2 + 64a_3 + 9a_6) w_3,1 w_5,1$$

$$- 9(64a_3 + 441a_4 + 169a_{13}) w_1,3 w_1,5 w_5,1 - 9(64a_3$$

$$+ 441a_6 + 169a_{12}) w_3,1 w_1,5 w_5,1 + 31(16a_3 + 81a_4$$

$$+ a_8 + 81a_{10}) w_3,3 w_1,5 + 31(16a_3 + a_4 + 81a_6$$

$$+ (256a_3 + 2401a_8 + 4096a_9 + 2401a_{13}) w_3,1 w_1,5$$

$$+ (4a_1 + 225a_2 + 256a_3 + a_6) w_3,1 w_1,5 + 656a_3 w_1,3 w_1,5 w_5,1$$

$$- 9(25a_2 + 64a_3 + 9a_6) w_3,3 w_1,5 - 9(64a_3 + 441a_8$$

$$+ 169a_{12}) w_3,3 w_1,5 w_5,1 - (25a_1 + 576a_3 + 3281a_{13}) w_1,5 w_5,1$$
\[
\sum_{i=0}^{n} \frac{w_i}{2} \int \frac{\partial}{\partial x} \left( \frac{d}{dx} \left( \frac{e^{wx}}{2} \right) \right) \, dx = 0
\]
\[
0 = \frac{256a_1}{5\pi^6 \frac{\text{Pb}^4}{\text{Eh}}} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_7} w_{1,5} - \frac{16}{\pi^2} \frac{\text{Pb}}{\text{Eh}} w_{1,5} \\
- 3(a_2 + 2a_8) w_{2,1} w_{1,3} + 81a_8 + (3a_2 + 64a_3) w_{1,1} w_{1,3} \\
+ (64a_3 + 274a_8) w_{1,1} w_{1,3} w_{3,1} + 9(a_2 + 8a_4) w_{1,1} w_{2,3} \\
- 144(a_3 + a_9) w_{1,1} w_{1,3} w_{3,3} - 9(2a_2 + 16a_3 + 9a_4) \\
+ 49a_8) w_{1,1} w_{3,1} w_{3,3} - 256 a_3 w_{1,1} w_{3,1} - (9a_2 + 64a_3 \\
+ 1225a_8) w_{1,3} w_{3,1} + 9(2a_2 + 64a_3 + 225 a_4 + 256a_9 \\
+ 49a_{13}) w_{1,3} w_{3,1} w_{3,3} + (25a_1 + 4a_2 + 81a_8 + 16a_9) w_{1,1} w_{1,5} \\
- 2(25a_1 + 441a_8 + 256a_9) w_{1,1} w_{3,1} w_{1,5} + 729(a_4 \\
+ a_9) w_{1,1} w_{3,1} w_{5,1} + (225a_1 + 4a_2 + 256a_3 + a_{10}) w_{1,3} w_{1,5} \\
+ (256a_3 + 4096a_9 + 2401a_{13}) w_{3,1} w_{1,5} \\
+ 656a_3 w_{1,3} w_{3,1} w_{5,1} - 18(25a_1 + 64a_3 + 9a_{10}) w_{1,3} w_{3,3} w_{1,5} \\
- 9(64a_3 + 441a_4 + 169a_{13}) w_{1,3} w_{3,3} w_{5,1} - 9(64a_3 + 441a_8 \\
+ 169a_{12}) w_{3,1} w_{3,3} w_{5,1} + 81(16a_3 + 81a_4 + a_3 + 81a_{10}) w_{3,3} w_{1,5} \\
+ 1296a_3 w_{3,3} w_{5,1} - (25a_2 + 576a_3 + 8281a_{12}) w_{1,3} w_{5,1} \\
- 2(25a_1 + 576a_3 + 8281a_{13}) w_{3,1} w_{1,5} w_{5,1} + (625a_1 + a_2) w_{1,5} \\
+ (1552a_3 + 28561a_{12} + 28561a_{13}) w_{1,5} w_{5,1} \\
0 = \frac{256a_1}{5\pi^6 \frac{\text{Pb}^4}{\text{Eh}}} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_7} w_{5,1} - \frac{400}{\pi^2} \frac{\text{Pb}}{\text{Eh}} w_{5,1} \\
- 3(a_1 + 3a_4) w_{2,1} w_{1,3} + 81a_4 w_{1,1} w_{3,3} + (3a_1 + 64a_3) w_{1,1} w_{3,1} \\
}
The four (4) simultaneous equations involved in the solution of the deflection equation,

\[ w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + w_{7,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b}, \]

are as follows:
\begin{align*}
o &= -\frac{256a_1}{\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_3} w_{1,1}^1 - \frac{16}{\pi^2} \frac{Pb}{Eh} w_{1,1}^1 + (a_1 + a_2) w_{1,1}^3 \\
&\quad - 3a_1 w_{1,1}^2 w_{3,1}^1 + (a_1 + 9a_2 + 16a_3 + a_4) w_{1,1}^2 w_{3,1}^1 \\
&\quad + (3a_1 + 64a_3) w_{3,1}^2 w_{5,1}^1 - (3a_1 + 25a_4) w_{3,1}^2 w_{7,1}^1 + (4a_1 \\
&\quad + 25a_2 + 81a_4 + 16a_5) w_{1,1}^2 w_{5,1}^1 + (4a_1 + 49a_2 + 256a_5 \\
&\quad + 81a_6) w_{1,1}^2 w_{7,1}^1 - 6(a_1 + 3a_4) w_{1,1}^1 w_{3,1}^1 w_{5,1}^1 - 2(3a_1 \\
&\quad + 64a_5) w_{1,1}^1 w_{5,1}^1 w_{7,1}^1 + (6a_1 + 144a_3 + 225a_4 \\
&\quad + 9a_6) w_{3,1}^1 w_{5,1}^1 w_{7,1}^1.
\end{align*}
0 = \frac{256a_1}{7\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_{15}} w_{7,1} - \frac{784}{\pi^2} \frac{Pb}{Eh} w_{7,1}

- (3a_1 + 64a_2) w_{1,1} w_{5,1} + (4a_1 + 49a_2 + 256a_3 + 81a_6) w_{3,1} w_{7,1}

- (3a_1 + 25a_4) w_{1,1} w_{3,1} + (4a_1 + 44a_2 + 625a_4 + 16a_7) w_{3,1} w_{7,1}

+ 3(a_1 + 192a_3) w_{3,1} w_{5,1} + (4a_1 + 1225a_2 + 1296a_3 + a_{14}) w_{5,1} w_{7,1}

+ (a_1 + 2401a_2) w_{7,1} + 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{3,1} w_{5,1}.

The subscripted a's have the following significance:

\begin{align*}
a_1 &= a^2 \\
a_2 &= 1/a^2 \\
a_3 &= a^2/(a^2 + 1) \\
a_4 &= a^2/(a^2 + 4) \\
a_5 &= a^2/(a^2 + 9) \\
a_6 &= a^2/(a^2 + 16)^2 \\
a_7 &= a^2/(a^2 + 25)^2 \\
a_8 &= a^2(4a^2 + 1) \\
a_9 &= a^2/(9a^2 + 1)^2 \\
a_{10} &= a^2/(16a^2 + 1)^2 \\
a_{11} &= a^2/(28a^2 + 1)^2 \\
a_{12} &= a^2/(4a^2 + 9)^2 \\
a_{13} &= a^2/(9a^2 + 4)^2 \\
a_{14} &= a^2/(a^2 + 36)^2 \\
a_{15} &= a^2/a^2 + 49)^2
\end{align*}

where \( \alpha = a/b \).
It seems there is a mathematical equation, but the text is not clear due to the handwriting. The content appears to be a series of mathematical expressions involving integrals and variables. Here is a possible transcription:

\[ \gamma \sum \frac{\beta}{\gamma} \frac{1}{\delta} \frac{\alpha}{\gamma} + \int \gamma \sum \frac{\beta}{\gamma} \frac{1}{\delta} \frac{\alpha}{\gamma} \]
SOLUTION OF THE DEFLECTION EQUATION

\[ W = W_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + W_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \]

\[ + W_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + W_{7,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b} \]

THIS METHOD IS SET UP FOR A CONTINUOUS CHANGE IN AXIAL LOAD FROM ZERO TO A VALUE WITHIN THE VICINITY OF THE CRITICAL BUCKLING LOAD. ZERO LATERAL LOAD IS EXCLUDED.

MAIN PROGRAM

XEQ
* LABEL
LISTB
FORTRAN
READ 1, IND
1 FORMAT (15)
PRINT 2 :
2 FORMAT (82H1 PNORM AXIAL W(1,1)/H W(3,1)/H W(5,1)/H W(7,11)/H STRAIN ERROR )
DO 44 JOB = 1, IND
READ 3, U, ALPHA, PNORM, LOOP, LAP
3 FORMAT (3F8.0,2I8)
DIMENSION B1(4), B2(4), B3(4)
CALL CONST (U, ALPHA, PNORM, A1, A2, A3, A4, A5, A6, A7, A8, A9, 1 A10, A11, A12, A13, A14, A15, A16, A17, B1, B2, B3)

INITIAL ESTIMATES OF W(1)'S
DIMENSION W(4)
W(1) = 0.400
W(2) = 0.300
W(3) = 0.200
W(4) = 0.100

THE FOLLOWING IS THE ROUTINE FOR
CALCULATING THE DEFLECTION COEFFICIENTS FOR
CHOSEN VALUES OF AXIAL LOAD AND NORMAL LOAD

DIMENSION A(4,4), C(4), X(4), Z(3), S(3,4)
AX = 0.
MG = 1
DO 10 LL = 1,3
4 CALL SESAME
   CALL CROUT (A, C, X, M)
   GO TO (5, 42), M
5 ERROR = 0.
   DO 6 J = 1,4
6 ERROR = ERROR + ABSF(X(J))
   IF (ERROR < 0.0001) 9, 9, 7
7 DO 8 I = 1,4
8 W(I) = W(I) + X(I)
   GO TO 4
9 CALL CPRNT (PNORM, AX, W, ERROR, A1)
   CALL STORE (AX, W, Z, S, LL)
AX = AX + 0.5
10 CONTINUE
DEL = 0.5
KO = 1
22 GO TO (23, 25, 27), KO
23 DEL = 0.05
   DO 24 I = 1, 4
   W(I) = S(2, I)
24 S(3, I) = W(I)
   AX = Z(2)
   Z(3) = AX
   AX = AX + DEL
   KO = 2
   KK = 0
   CALL EXCH (Z, S)
   GO TO 13
25 DEL = 0.01
   DO 26 I = 1, 4
   W(I) = S(2, I)
26 S(3, I) = W(I)
   AX = Z(2)
   Z(3) = AX
   AX = AX + DEL
   KO = 3
   KK = 0
   CALL EXCH (Z, S)
   GO TO 13

THE FOLLOWING IS THE ROUTINE WHEN IN THE VICINITY OF THE BUCKLING LOAD. W(I) IS MADE THE INDEPENDENT VARIABLES IN PLACE OF THE AXIAL LOAD.
27 DO 28 I = 1, 4
31 ERROR = 0.
   DO 32  J = 1,4
32 ERROR = ERROR + ARSF(X(J))
   IF (ERROR - 0.0001) 36, 36, 33
33 KK = KK + 1
   IF (KK - LOOP) 34, 34, 40
34 DO 35  I = 2,4
35 W(I) = W(I) + X(I)
   AX = AX + X(I)
   GO TO 30
36 CALL CPRNT (PNOPM, AX, W, ERROR, A1)
   IF (AX - Z(2)) 37, 34, 44, 37
37 Z(3) = AX
   DO 38  I = 1,4
38 S(3,I) = W(I)
   DL1 = Z(3) - Z(2)
39 CONTINUE
   GO TO 44
40 PRINT 41
41 FORMAT (40H INFINITE LOOP OR NEEDS MORE ITERATION)
   CALL CPRNT (PNORM, AX, W, ERROR, A1)
   GO TO 44
42 PRINT 43
43 FORMAT (19H CROUT WENT CRAZY)
44 CONTINUE
   CALL EXIT
COMMON W, AX, B1, B2, B3, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10,
1    A11, A12, A13, A14, A15, A16, A17, MG, A, C
SUBROUTINES

SUBROUTINE FOR SOLVING THE LINEARIZED SYSTEM OF EQUATIONS BY THE CR Out REDOX METHOD

LABEL
LIST8

FORTRAN

SUBROUTINE CR Out (A, C, X, M)
DIMENSION A(4,4), C(4), AA(4,4), CC(4), X(4)
DO 100 I = 1,4
100 AA(I,1) = A(I,1)
DO 101 J = 2,4
101 AA(1,J) = A(1,J)/A(1,1)
DO 102 I = 2,4
102 AA(I,J) = 0.
J = 2
103 II = J
DO 105 I = II,4
LIMIT = J - 1
DO 104 K = 1,LIMIT
104 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
105 AA(I,J) = A(I,J) - AA(I,J)
106 IF (AA(J,J)) 106, 116, 106
106 IF (4 - J) 110, 110, 107
107 I = J
$J = J + 1$

$JJ = J$

DO 109 J = JJ,4

LIM2 = I - 1

DO 108 K = 1,LIM2

108 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))

AA(I,J) = A(I,J) - AA(I,J)

109 AA(I,J) = AA(I,J)/AA(I,I)

J = I + 1

GO TO 103

110 DO 111 I = 1,4

X(I) = 0.

111 CC(I) = 0.

CC(I) = C(1)/AA(1,1)

DO 113 I = 2,4

LIM3 = I - 1

DO 112 K = 1,LIM3

112 CC(I) = CC(I) + (AA(I,K) * CC(K))

CC(I) = C(I) - CC(I)

113 CC(I) = CC(I)/AA(I,I)

X(4) = CC(4)

DO 115 I = 1,3

II = 4 - I

LIM4 = II + 1

DO 114 K = LIM4,4

114 X(II) = X(II) + (AA(II,K) * X(K))

115 X(II) = CC(II) - X(II)

M = 1
GO TO 118
116 PRINT 117
117 FORMAT (28H SINGULARITY - NO SOLUTION)
   M = 2
118 RETURN
END

SUBROUTINE FOR CALCULATING THE NON-DIMENSIONAL STRAIN AND THEN PRINTING ALL THE NEEDED RESULTS

LABEL
LIST8

FORTRAN

SUBROUTINE CPRINT (PNORM, AX, W, ERROR, A1)
DIMENSION W(4)
SUM = 0.
DO 200 IJ = 1,4
   XY = 2 * IJ - 1
200 SUM = SUM + (XY * W(IJ))**2
STRAIN = AX + 3.1416**2 * SUM/(8.* A1)
PRINT 201, PNORM, AX, (W(I), I=1,4), STRAIN, ERROR
201 FORMAT (F8.2, F11.5, 4F10.3, F11.5, F12.5)
RETURN
END

SUBROUTINE FOR STORAGE OF THREE SETS OF VALUES OF AXIAL LOAD AND THE FOUR DEFLECTION COEFFICIENTS.

LABEL
LIST8

FORTRAN

SUBROUTINE STORF (AX, W, Z, S, LL)
DIMENSION W(4), Z(3), S(3,4)

Z(LL) = AX

DO 203 I = 1, 4

203 S(LL, I) = W(I)

RETURN
END

THIS SUBROUTINE IS JUST A SHIFTING PROCEDURE
OF VALUES FROM ONE STORAGE LOCATION TO ANOTHER.

* LABEL
* LIST8
* FORTRAN

SUBROUTINE EXCH (Z, S)
DIMENSION Z(3), S(3,4)

DO 204 I = 1, 4

DO 204 J = 1, 2

JA = J + 1

204 S(J, I) = S(JA, I)

DO 205 IJ = 1, 2

JR = IJ + 1

205 Z(IJ) = Z(JR)

DO 206 JI = 1, 4

206 S(3, JI) = 0.

Z(3) = 0.

RETURN
END

SUBROUTINE FOR COMPUTING ALL THE NECESSARY
CONSTANTS IN THE MAIN PROGRAM.

* LABEL
LIST8

FORTRAN


DIMENSION B1(4), B2(4), B3(4), P(9)

P(1) = ALPHA**2
P(2) = 1.0/P(1)

DO 300 I = 1, 7

Y1 = I**2
I1 = I + 2

300 P(I1) = P(1)/(P(1) + Y1)**2

A1 = P(1)
A2 = P(1) + P(2)
A3 = P(1) + 81.0 * P(2)
A4 = P(1) + 625.0 * P(2)
A5 = P(1) + 24(1.0 * P(2)
A6 = 3.0 * P(1) + 64.0 * P(3)
A7 = 3.0 * P(1) + 576.0 * P(3)
A8 = 3.0 * P(1) + 324.0 * P(4)
A9 = 3.0 * P(1) + 25.0 * P(4)
A10 = 3.0 * P(1) + 64.0 * P(5)
A11 = 4.0 * P(1) + 16.0 * P(2) + 144.0 * P(3) + P(4)
A12 = 4.0 * P(1) + 225.0 * P(2) + 256.0 * P(3) + P(6)
A13 = 4.0 * P(1) + 1225.0 * P(2) + 1200.0 * P(3) + P(8)
A14 = 4.0 * P(1) + 25.0 * P(2) + 81.0 * P(4) + 16.0 * P(5)
A15 = 4.0 * P(1) + 441.0 * P(2) + 625.0 * P(4) + 16.0 * P(7)
A16 = 4.0 * P(1) + 49.0 * P(2) + 256.0 * P(5) + 81.0 * P(6)
A17 = 6.0 * P(1) + 144.0 * P(3) + 225.0 * P(4) + 9.0 * P(6)
DO 301 I = 1, 4
I2 = 2 * I + 1
Y2 = 2 * I - 1
B1(I) = 256. * P(I) * PNORM(Y2 * 3.1416**6)
B2(I) = 4. / (3. * (1. - U**2) * P(I2))
301 B3(I) = -(16. * Y2**2) / (3.1416**2)
RETURN
END

SUBROUTINE FOR COMPUTING THE COEFFICIENTS FOR THE USE OF THE CRUT REDOX METHOD

LABEL
LIST8
FORTRAN
SUBROUTINE SESAME
COMMON W, AX, B1, B2, B3, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10,
  A11, A12, A13, A14, A15, A16, A17, MG, A, C
DIMENSION W(4), B1(4), B2(4), B3(4), A(4,4), C(4)
GO TO (401, 400): MG

400 A(1,1)=B3(1)*W(1)
A(2,1)=B3(2)*W(2)
A(3,1)=B3(3)*W(3)
A(4,1)=B3(4)*W(4)
GO TO 402

  +A14**W(3)**2+A16**W(4)**2-2.*AR**W(2)**W(3)-2.*A10**W(3)**W(4)
A(2,1)=-3.*A1*W(1)**2+7.*A11**W(1)**W(2)-7.*AR**W(1)**W(3)
  +2.*A6**W(2)**W(3)-2.*A9**W(2)**W(4)+A17**W(3)**W(4)
A(3,1)=-2.*A9**W(1)**W(2)+2.*A14**W(1)**W(2)-7.*A10**W(1)**W(4)
\[
\begin{align*}
A(1, 1) &= -2 \cdot A0 \cdot W(1) \cdot W(3) + 2 \cdot A16 \cdot W(1) \cdot W(4) - A6 \cdot W(2) \cdot W(3) + A17 \cdot W(2) \cdot W(3) \\
A(1, 2) &= -3 \cdot A1 \cdot W(1) \cdot W(2) + 2 \cdot A11 \cdot W(1) \cdot W(2) + 2 \cdot A6 \cdot W(2) \cdot W(3) \\
A(2, 1) &= B2(2) + B3(2) \cdot A0 + A11 \cdot W(1) \cdot W(2) + 2 \cdot A3 \cdot W(2) \cdot W(3) + A12 \cdot W(3) \\
A(2, 2) &= A15 \cdot W(4) \cdot W(3) - 2 \cdot A0 \cdot W(1) \cdot W(4) \\
A(3, 1) &= -A0 \cdot W(1) \cdot W(2) + 2 \cdot A6 \cdot W(1) \cdot W(2) + 2 \cdot A12 \cdot W(2) \cdot W(3) \\
A(3, 2) &= A17 \cdot W(1) \cdot W(4) + 2 \cdot A7 \cdot W(3) \cdot W(4) \\
A(4, 1) &= -A10 \cdot W(1) \cdot W(2) + 2 \cdot A7 \cdot W(2) \cdot W(3) + A17 \cdot W(1) \cdot W(3) \\
A(4, 2) &= -2 \cdot A9 \cdot W(1) \cdot W(2) + 2 \cdot A15 \cdot W(2) \cdot W(4) + A7 \cdot W(2) \cdot W(3) + A17 \cdot W(1) \cdot W(4) \\
A(1, 3) &= A6 \cdot W(2) \cdot W(3) + 2 \cdot A14 \cdot W(1) \cdot W(2) - A8 \cdot W(1) \cdot W(4) \\
A(2, 3) &= -A0 \cdot W(1) \cdot W(2) + 2 \cdot A6 \cdot W(1) \cdot W(2) + 2 \cdot A7 \cdot W(3) \cdot W(4) \\
A(3, 3) &= B2(3) + B3(2) \cdot A0 + A14 \cdot W(1) \cdot W(2) + 2 \cdot A12 \cdot W(2) \cdot W(3) + A4 \cdot W(3) \cdot W(4) \\
A(4, 3) &= -A10 \cdot W(1) \cdot W(2) + 2 \cdot A7 \cdot W(2) \cdot W(3) + A13 \cdot W(3) \cdot W(4) + A17 \cdot W(1) \cdot W(4) \\
A(1, 4) &= -A0 \cdot W(2) \cdot W(3) + 2 \cdot A16 \cdot W(1) \cdot W(4) - 2 \cdot A10 \cdot W(1) \cdot W(3) + A17 \cdot W(2) \cdot W(4) \\
A(2, 4) &= A7 \cdot W(3) \cdot W(4) + 2 \cdot A15 \cdot W(2) \cdot W(4) - 2 \cdot A0 \cdot W(1) \cdot W(2) + A17 \cdot W(1) \cdot W(3) \\
A(3, 4) &= -A10 \cdot W(1) \cdot W(2) + 2 \cdot A13 \cdot W(3) \cdot W(4) + A17 \cdot W(1) \cdot W(2) + 2 \cdot A7 \cdot W(2) \cdot W(3) \\
A(4, 4) &= B2(4) + B3(4) \cdot A0 + A16 \cdot W(1) \cdot W(2) + 2 \cdot A15 \cdot W(2) \cdot W(3) + A12 \cdot W(3) \cdot W(4) \\
C(1) &= B1(1) - B2(1) \cdot W(1) - B2(1) \cdot W(1) + A0 \cdot W(1) \cdot W(2) + A1 \cdot W(1) \cdot W(2) \\
C(2) &= B1(2) - B2(2) \cdot W(2) - B2(2) \cdot W(2) + A1 \cdot W(1) \cdot W(2) + A11 \cdot W(1) \cdot W(2) \\
C(3) &= B1(3) - B2(3) \cdot W(3) - B2(3) \cdot W(3) + A1 \cdot W(1) \cdot W(2) + A11 \cdot W(1) \cdot W(2) \\
C(4) &= B1(4) - B2(4) \cdot W(4) - B2(4) \cdot W(4) + A1 \cdot W(1) \cdot W(2) + A11 \cdot W(1) \cdot W(2)
\end{align*}
\]
\[ C(2) = R1(3) - B2(3) * W(2) - B3(3) * A X * W(3) + A P * W(1) * * 2 * W(2) \]

\[ C(3) = R1(3) - B2(3) * W(2) - B3(3) * A X * W(3) + A P * W(1) * * 2 * W(2) \]

\[ C(3) = R1(3) - B2(3) * W(2) - B3(3) * A X * W(3) + A P * W(1) * * 2 * W(2) \]

\[ C(4) = R1(4) - B2(4) * W(4) - B3(4) * A X * W(4) + A P * W(1) * * 2 * W(3) \]

\[ C(4) = R1(4) - B2(4) * W(4) - B3(4) * A X * W(4) + A P * W(1) * * 2 * W(3) \]

\[ C(4) = R1(4) - B2(4) * W(4) - B3(4) * A X * W(4) + A P * W(1) * * 2 * W(3) \]

RETURN

END
SOLUTION OF THE DEFLECTION EQUATION

\[ W = W_{1,1} \sin \frac{\pi x}{\alpha} \sin \frac{\pi y}{b} + W_{1,3} \sin \frac{\pi x}{\alpha} \sin \frac{3\pi y}{b} + W_{3,1} \sin \frac{3\pi x}{\alpha} \sin \frac{\pi y}{b} + W_{3,3} \sin \frac{3\pi x}{\alpha} \sin \frac{3\pi y}{b} + W_{1,5} \sin \frac{\pi x}{\alpha} \sin \frac{5\pi y}{b} + W_{5,1} \sin \frac{5\pi x}{\alpha} \sin \frac{\pi y}{b} \]

This method is set up for a continuous change in axial load from zero to a value within the vicinity of the critical buckling load. Zero lateral load is excluded.

MAIN PROGRAM

XEQ
LABEL
LIST8
FORTRAN
READ 1, IND
1 FORMAT (15)
PRINT 2
2 FORMAT (111H1 PNORM AXIAL W(1,1)/H W(1,3)/H W(3,1)/H W(3,3)/H W(1,5)/H W(5,1)/H STRAIN ERROR )
DO 44 JOB = 1, IND
READ 3, U, ALPHA, PNORM, LOOP, LAP
3 FORMAT (3F8.0,2I8)
DIMENSION W(6), R(54), B1(4), B2(5), B3(3), A(6,6), C(6), X(6)
CALL CONST (U, ALPHA, PNORM, R, B1, B2, B3)
INITIAL ESTIMATES OF W(I)'S
W(1) = 0.100
W(2) = 0.010
W(3) = 0.010
W(4) = 0.001
W(5) = 0.001
W(6) = 0.001

THE FOLLOWING IS THE ROUTINE FOR
CALCULATING THE DEFLECTION COEFFICIENTS FOR
CHosen VALUES OF AXIAL LOAD AND NORMAL LOAD

DIMENSION Z(3), S(3,6)
AX = 0.
MG = 1
RONE = R(1)
DO 10 LL = 1,3
4 CALL SESAME
   CALL CROUT (A, C, X, M)
   GO TO (5, 42), M
5 ERROR = 0.
   DO 6 J = 1,6
6 ERROR = ERROR + ABSF(X(J))
   IF (ERROR - 0.000001) 9, 9, 7
7 DO 8 I = 1,6
8 W(I) = W(I) + X(I)
   GO TO 4
9 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
   CALL STORE (AX, W, Z, S, LL)
   AX = AX + 0.5
10 CONTINUE
DEL = 0.5
KO = 1
KK = 0

11 DO 12  I = 1, 6
   F1 = (S(2, I) - S(1, I)) / (Z(2) - Z(1))
   F2 = (S(3, I) - S(2, I)) / (Z(3) - Z(2))
   F3 = (Z(3) - Z(1)) / 2.
   F4 = (F2 - F1) / F3
12 W(I) = W(I) + F2 * DEL + 0.5 * F4 * DEL**2
   CALL EXCH (Z, S)
13 CALL SESAME
   CALL CROUT (A, C, X, M)
   GO TO (14, 42), M
14 ERROR = 0.
   DO 15  J = 1, 6
15 ERROR = ERROR + ABSF(X(J))
   IF (ERROR - 0.000001) 19, 19, 16
16 KK = KK + 1
   IF (KK - LOOP) 17, 17, 40
17 DO 18  I = 1, 6
18 W(I) = W(I) + X(I)
   GO TO 13
19 IF (W(I) - S(2, I)) 22, 22, 20
20 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
   Z(3) = AX
   DO 21  I = 1, 6
21 S(3, I) = W(I)
   AX = AX + DEL
\[ \text{KK} = 0 \]
\[ \text{GO TO 11} \]
\[ 22 \text{ GO TO (23, 25, 27), KO} \]
\[ 23 \text{ DEL} = 0.05 \]
\[ \text{DO 24 I} = 1,6 \]
\[ W(I) = S(2, I) \]
\[ 24 \text{ S(3, I) = W(I)} \]
\[ AX = Z(2) \]
\[ Z(3) = AX \]
\[ AX = AX + \text{DEL} \]
\[ KO = 2 \]
\[ KK = 0 \]
\[ \text{CALL EXCH (Z, S)} \]
\[ \text{GO TO 13} \]
\[ 25 \text{ DEL} = 0.01 \]
\[ \text{DO 26 I} = 1,6 \]
\[ W(I) = S(2, I) \]
\[ 26 \text{ S(3, I) = W(I)} \]
\[ AX = Z(2) \]
\[ Z(3) = AX \]
\[ AX = AX + \text{DEL} \]
\[ KO = 3 \]
\[ KK = 0 \]
\[ \text{CALL EXCH (Z, S)} \]
\[ \text{GO TO 13} \]

THE FOLLOWING IS THE ROUTINE WHEN IN THE VICINITY OF THE BUCKLING LOAD. \( W(1) \) IS MADE THE INDEPENDENT VARIABLE IN PLACE OF THE AXIAL
LOAD.

27 DO 28 I = 1, 6
   S(3, I) = S(2, I)
   S(2, I) = S(1, I)
28 W(I) = S(3, I)
   Z(3) = Z(2)
   Z(2) = Z(1)
   AX = Z(3)

THE FOLLOWING VALUES OF DL1 AND DL2 ARE ARBITRARY.
THE FIRST VALUE OF DL1 IS SO CHOSEN SUCH THAT WHEN
W(I) IS DECREASED BY DL2, THE CONSEQUENT ITERATED
SOLUTION WILL NOT GO BACK ON THE PREVIOUSLY COMPUTED
PATH. (NOTE - THE W(I) OR THE FIRST HARMONIC STARTS
TO DECREASE IN VALUE BEFORE THE CRITICAL BUCKLING
LOAD IS REACHED)

DL1 = 0.20
DL2 = 0.010
MG = 2

THIS LAST DO LOOP IS HOPED TO GO BEYOND THE CRITICAL RANGE. START WITH LAP = 30.

DO 39 LAST = 1, LAP
   KK = 0
   AX = AX + DL1
   W(I) = W(I) - DL2
29 DO 29 J = 2, 6
   W(I) = W(I) + (S(3, J) - S(2, J))
29 CALL EXCH (Z, S)
30 CALL SESAME
CALL CROUT (A, C, X, M)
GO TO (31, 42), M

31 ERROR = 0.
DO 32 J = 1, 6
32 ERROR = ERROR + ABSF(X(J))
IF (ERROR - 0.000001) 36, 36, 33

33 KK = KK + 1
IF (KK - LOOP) 34, 34, 40

34 DO 35 I = 2, 6
35 W(I) = W(I) + X(I)
AX = AX + X(1)
GO TO 30

36 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
IF (AX - Z(2)) 44, 44, 37

37 Z(3) = AX
DO 38 I = 1, 6
38 S(3, I) = W(I)
DL1 = Z(3) - Z(2)

39 CONTINUE
GO TO 44

40 PRINT 41

41 FORMAT (40H INFINITE LOOP OR NEEDS MORE ITERATION)
CALL CPRNT (PNORM, AX, W, ERROR, RONE)
GO TO 44

42 PRINT 43

43 FORMAT (19H CROUT WENT CRAZY)

44 CONTINUE
CALL EXIT
COMMON W, AX, B1, B2, B3, R, MG, A, C
END

SUBROUTINES

SUBROUTINE FOR SOLVING THE LINEARIZED SYSTEM
OF EQUATIONS BY THE CROUT REDOX METHOD

LABEL
LIST8
FORTRAN

SUBROUTINE CROUT (A, C, X, M)

DIMENSION A(6,6), C(6), AA(6,6), CC(6), X(6)

DO 100 I = 1,6
100 AA(I,1) = A(I,1)

DO 101 J = 2,6
101 AA(1,J) = A(1,J)/A(1,1)

DO 102 I = 2,6
DO 102 J = 2,6
102 AA(I,J) = 0.

J = 2

103 II = J

DO 105 I = II,6
LIM1 = J - 1

DO 104 K = 1,LIM1
104 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))

105 AA(I,J) = A(I,J) - AA(I,J)

IF (AA(J,J)) 106, 116, 106
106 IF (6 - J) 110, 110, 107
107  I  =  J
    J  =  J  +  1
    JJ  =  J
    DO  109  J  =  JJ,6
    LIM2  =  I  -  1
    DO  108  K  =  1,LIM2
109  AA(I,J)  =  AA(I,J)  +  (AA(I,K)  *  AA(K,J))
    AA(I,J)  =  A(I,J)  -  AA(I,J)
    AA(I,J)  =  AA(I,J)/AA(I,I)
    J  =  I  +  1
    GO TO  103
110  DO  111  I  =  1,6
    X(I)  =  0.
111  CC(I)  =  0.
    CC(I)  =  C(I)/AA(I,1)
    DO  113  I  =  2,6
    LIM3  =  I  -  1
    DO  112  K  =  1,LIM3
112  CC(I)  =  CC(I)  +  (AA(I,K)  *  CC(K))
    CC(I)  =  C(I)  -  CC(I)
113  CC(I)  =  CC(I)/AA(I,I)
    X(I)  =  CC(I)
    DO  115  I  =  1,5
    II  =  6  -  I
    LIM4  =  II  +  1
    DO  114  K  =  LIM4,6
114  X(I)  =  X(I)  +  (AA(I,K)  *  X(K))
115  X(I)  =  CC(I)  -  X(I)
M = 1
GO TO 118

116 PRINT 117
117 FORMAT (28H SINGULARITY - NO SOLUTION)
M = 2
118 RETURN
END

SUBROUTINE FOR STORAGE OF THREE SETS OF VALUES
OF AXIAL LOAD AND THE FOUR DEFLECTION COEFFICIENTS.

* LABEL
* LIST8
* FORTRAN
SUBROUTINE STORE (AX, W, Z, S, LL)
DIMENSION W(6), Z(3), S(3,6)
Z(LL) = AX
DO 203 I = 1, 6
203 S(LL, I) = W(I)
RETURN
END

THIS SUBROUTINE IS JUST A SHIFTING PROCEDURE
OF VALUES FROM ONE STORAGE LOCATION TO ANOTHER.

* LABEL
* LIST8
* FORTRAN
SUBROUTINE EXCH (Z, S)
DIMENSION Z(3), S(3,6)
DO 204 I = 1, 6
DO 204 J = 1, 2
JA = J + 1
Z04 S(J, I) = S(JA, I)
DO 205 IJ = 1, 2
    JB = IJ + 1
205 Z(IJ) = Z(JB)
DO 206 JI = 1, 6
206 S(3, JI) = 0.
    Z(3) = 0.
RETURN
END

SUBROUTINE FOR PRINTING
LABEL
LIST8
FORTRAN
SUBROUTINE CPRNT (PNORM, AX, W, ERROR, RONE)
DIMENSION W(6)
    SUM = W(1)**2 + W(2)**2 + W(5)**2 + 9.0*(W(3)**2 + W(4)**2) + 25.0*W(6)**2
    STRAIN = AX + 3.1416**2 * SUM/(8.0 + RONE)
PRINT 200, PNORM, AX, (W(I), I=1, 6), STRAIN, ERROR
200 FORMAT (F8.2, F11.5, 6F12.6, F11.5, F12.7)
RETURN
END

SUBROUTINE FOR COMPUTING ALL THE NECESSARY
CONSTANTS IN THE MAIN PROGRAM
LABEL
LIST8
FORTRAN
SUBROUTINE CONST (U, ALPHA, PNORM, R, B1, B2, B3)
DIMENSION P(13), R(4), B1(4), B2(4), B3(3)

P(1) = ALPHA**2
P(2) = 1.0/P(1)
DO 300 I = 3, 7
   Y1 = (I - 2)**2
300 P(I) = P(1)/(P(1) + Y1)**2
DO 301 J = 8, 11
   Y2 = (J - 6)**2
301 P(J) = P(1)/(1.0 + Y2 * P(1))**2
P(12) = P(1)/(4.0 + P(1) + 9.0)**2
P(13) = P(1)/(4.0 + 9.0 * P(1))**2
R(1) = P(1)
R(2) = P(2)
R(3) = P(3)
R(4) = P(4)
R(5) = P(8)
R(6) = P(1) + P(2)
R(7) = P(1) + 81.0*P(2)
R(8) = P(1) + 625.0*P(2)
R(9) = 81.0*P(1) + P(2)
R(10) = 625.0*P(1) + P(2)
R(11) = P(1) + P(8)
R(12) = P(1) + 81.0*P(8)
R(13) = P(2) + P(4)
R(14) = P(2) + 81.0*P(4)
R(15) = P(1) + 3.0*P(4)
R(16) = P(2) + 3.0*P(8)
R(17) = 3.0*P(1) + 64.0*P(3)
R(18) = 3 \cdot P(2) + 64 \cdot P(3)
R(19) = 64 \cdot P(2) + 274 \cdot P(4)
R(20) = 64 \cdot P(3) + 274 \cdot P(4)
R(21) = P(4) + P(8)
R(22) = P(3) + P(5)
R(23) = P(3) + P(9)
R(24) = 9 \cdot P(1) + 64 \cdot P(3) + 25 \cdot P(8)
R(25) = 9 \cdot P(2) + 64 \cdot P(3) + 25 \cdot P(4)
R(26) = 25 \cdot P(1) + 441 \cdot P(8) + 256 \cdot P(9)
R(27) = 25 \cdot P(2) + 441 \cdot P(4) + 256 \cdot P(5)
R(28) = 272 \cdot P(3) + 625 \cdot P(4) + 625 \cdot P(8)
R(29) = 9 \cdot P(1) + 64 \cdot P(3) + 1225 \cdot P(4)
R(30) = 9 \cdot P(2) + 64 \cdot P(3) + 1225 \cdot P(8)
R(31) = 25 \cdot P(1) + 64 \cdot P(3) + 9 \cdot P(10)
R(32) = 25 \cdot P(2) + 64 \cdot P(3) + 9 \cdot P(6)
R(33) = 64 \cdot P(3) + 441 \cdot P(4) + 169 \cdot P(13)
R(34) = 64 \cdot P(3) + 441 \cdot P(8) + 169 \cdot P(12)
R(35) = 25 \cdot P(1) + 576 \cdot P(3) + 8281 \cdot P(13)
R(36) = 25 \cdot P(2) + 576 \cdot P(3) + 8281 \cdot P(12)
R(37) = 1552 \cdot P(3) + 28561 \cdot P(12) + 28561 \cdot P(13)
R(38) = 9 \cdot P(1) + 4 \cdot P(2) + 16 \cdot P(3) + P(8)
R(39) = 4 \cdot P(1) + 9 \cdot P(2) + 16 \cdot P(3) + P(4)
R(40) = 36 \cdot P(1) + 36 \cdot P(2) + 225 \cdot P(4) + 225 \cdot P(8)
R(41) = 25 \cdot P(1) + 4 \cdot P(2) + 81 \cdot P(8) + 16 \cdot P(9)
R(42) = 4 \cdot P(1) + 25 \cdot P(2) + 81 \cdot P(4) + 16 \cdot P(5)
R(43) = 36 \cdot P(1) + P(2) + 144 \cdot P(9) + 9 \cdot P(13)
R(44) = P(1) + 36 \cdot P(2) + 144 \cdot P(5) + 9 \cdot P(12)
R(45) = 2 \cdot P(1) + 16 \cdot P(3) + 49 \cdot P(4) + 9 \cdot P(8)
R(46) = 2*P(2) + 16*P(3) + 9*P(4) + 49*P(8)
R(47) = 16*P(3) + 81*P(4) + P(8) + 81*P(10)
R(48) = 16*P(3) + P(4) + 81*P(6) + 81*P(8)
R(49) = 4*P(1) + 225*P(2) + 256*P(3) + P(6)
R(50) = 225*P(1) + 4*P(2) + 256*P(3) + P(10)
R(51) = 256*P(3) + 2401*P(4) + 4096*P(5) + 2401*P(12)
R(52) = 256*P(3) + 2401*P(8) + 4096*P(9) + 2401*P(13)
R(53) = 2*P(1) + 64*P(3) + 256*P(5) + 225*P(8) + 49*P(12)
R(54) = 2*P(2) + 64*P(3) + 225*P(4) + 256*P(9) + 49*P(13)

DO 302 I = 1,3
Y3 = 2 * I - 1
B1(I) = 256. * P(1) * PNORM/(Y3 * 3.1416**6)

302 B3(I) = -(16. * Y3**2)/(3.1416**2)
B1(4) = B1(2)/3.

DO 303 J = 1,5
I1 = 2 * J + 1

303 B2(J) = 4./(3. * (1. - U**2) * P(I1))

RETURN

END

SUBROUTINE FOR COMPUTING THE COEFFICIENTS
FOR THE USE OF THE CROUT REDOX METHOD

LABEL

LIST8

FORTRAN

SUBROUTINE SESAME

COMMON W, AX, B1, B2, B3, R, MG, A, C

DIMENSION W(6), R1(4), B2(5), B3(3), R(54), A(6,6), C(6)

GO TO (401, 400), MG
A(1,1) = B3(1) * W(1)
A(2,1) = B3(1) * W(2)
A(3,1) = B3(2) * W(3)
A(4,1) = B3(2) * W(4)
A(5,1) = B3(1) * W(5)
A(6,1) = B3(3) * W(6)

GO TO 402

401 A(1,1) = B3(1) * AX + 3 * R(6) * W(1) ** 2 - 6 * R(2) * W(1) * W(2)
1 -6 * R(1) * W(1) * W(3) + R(38) * W(2) ** 2 + R(39) * W(3) ** 2
5 +R(42) * W(6) ** 2

A(2,1) = -3 * R(2) * W(1) ** 2 + 2 * R(38) * W(1) * W(2) + 32 * R(3) * W(1) * W(3)
4 -9 * R(45) * W(4) * W(6) - R(27) * W(6) ** 2

A(3,1) = -3 * R(1) * W(1) ** 2 + 32 * R(3) * W(1) * W(2) + 2 * R(39) * W(1) * W(3)
4 -144 * R(22) * W(4) * W(6) - R(26) * W(5) ** 2

2 +162 * R(4) * W(1) * W(6) - 144 * R(23) * W(2) * W(5)
\[ A(5, 1) = -6 \cdot R(16) \cdot W(1) \cdot W(2) + 162 \cdot R(5) \cdot W(1) \cdot W(4) + R(18) \cdot W(2) \cdot W(6) \]

\[ A(5, 2) = -9 \cdot R(46) \cdot W(3) \cdot W(4) + 2 \cdot R(41) \cdot W(1) \cdot W(5) - 2 \cdot R(26) \cdot W(3) \cdot W(5) \]

\[ A(6, 1) = -6 \cdot R(15) \cdot W(1) \cdot W(3) + 162 \cdot R(4) \cdot W(1) \cdot W(4) + R(17) \cdot W(2) \cdot W(6) \]

\[ A(6, 2) = -9 \cdot R(45) \cdot W(4) \cdot W(6) - R(27) \cdot W(2) \cdot W(6) \]

\[ A(1, 2) = -3 \cdot R(2) \cdot W(1) \cdot W(2) + 32 \cdot R(3) \cdot W(1) \cdot W(3) \]

\[ A(2, 2) = B2(4) + B3(1) \cdot A + R(38) \cdot W(1) \cdot W(2) - 2 \cdot R(24) \cdot W(1) \cdot W(3) \]

\[ A(3, 2) = 16 \cdot R(3) \cdot W(1) \cdot W(2) - 2 \cdot R(24) \cdot W(1) \cdot W(3) \]

\[ A(4, 2) = -9 \cdot R(11) \cdot W(1) \cdot W(3) - 43 \cdot R(1) \cdot W(2) \]

\[ A(5, 2) = -3 \cdot R(16) \cdot W(1) \cdot W(2) + 162 \cdot R(5) \cdot W(1) \cdot W(4) + R(18) \cdot W(2) \cdot W(6) \]
<table>
<thead>
<tr>
<th>n</th>
<th>(-144 \cdot R(23) \cdot W(1) \cdot W(4) - 512 \cdot R(3) \cdot W(2) \cdot W(3) - R(33) \cdot W(3) \cdot W(6))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+9 \cdot R(54) \cdot W(3) \cdot W(4) + 2 \cdot R(50) \cdot W(2) \cdot W(5) + 656 \cdot R(3) \cdot W(3) \cdot W(6)</td>
</tr>
<tr>
<td>2</td>
<td>-18 \cdot R(31) \cdot W(4) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(6) - R(36) \cdot W(6) \cdot W(3)</td>
</tr>
<tr>
<td>3</td>
<td>A(6, 2) = R(19) \cdot W(1) \cdot W(3) - 9 \cdot R(45) \cdot W(1) \cdot W(4) - 2 \cdot R(29) \cdot W(2) \cdot W(3)</td>
</tr>
<tr>
<td>1</td>
<td>-256 \cdot R(3) \cdot W(3) \cdot W(4) - 2 \cdot R(37) \cdot W(3) \cdot W(4)</td>
</tr>
<tr>
<td>2</td>
<td>+2 \cdot R(51) \cdot W(2) \cdot W(6) + 656 \cdot R(3) \cdot W(3) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(5)</td>
</tr>
<tr>
<td>3</td>
<td>-2 \cdot R(36) \cdot W(5) \cdot W(6)</td>
</tr>
<tr>
<td>A(1, 3) = (-3 \cdot R(1) \cdot W(1) \cdot W(3) - 2 \cdot R(39) \cdot W(1) \cdot W(4) + 12 \cdot R(3) \cdot W(1) \cdot W(2)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-18 \cdot R(13) \cdot W(1) \cdot W(4) - R(24) \cdot W(2) \cdot W(3) + 2 \cdot R(25) \cdot W(2) \cdot W(3)</td>
</tr>
<tr>
<td>2</td>
<td>+R(40) \cdot W(1) \cdot W(4) - R(15) \cdot W(1) \cdot W(6) + 2 \cdot R(17) \cdot W(3) \cdot W(6)</td>
</tr>
<tr>
<td>3</td>
<td>+R(20) \cdot W(2) \cdot W(5) + R(19) \cdot W(2) \cdot W(6) - 9 \cdot R(46) \cdot W(4) \cdot W(5)</td>
</tr>
<tr>
<td>4</td>
<td>-144 \cdot R(22) \cdot W(4) \cdot W(6) - R(26) \cdot W(5) \cdot W(6)</td>
</tr>
<tr>
<td>A(2, 3) = 16 \cdot R(3) \cdot W(1) \cdot W(4) - 2 \cdot R(25) \cdot W(1) \cdot W(3) - 2 \cdot R(24) \cdot W(1) \cdot W(2)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>+R(40) \cdot W(1) \cdot W(4) + 2 \cdot R(28) \cdot W(2) \cdot W(3) - 2 \cdot R(20) \cdot W(1) \cdot W(5)</td>
</tr>
<tr>
<td>2</td>
<td>+R(19) \cdot W(1) \cdot W(6) - 2 \cdot R(30) \cdot W(3) \cdot W(5) - 512 \cdot R(3) \cdot W(3) \cdot W(5)</td>
</tr>
<tr>
<td>3</td>
<td>-512 \cdot R(3) \cdot W(2) \cdot W(5) - 2 \cdot R(29) \cdot W(2) \cdot W(6) + 9 \cdot R(54) \cdot W(4) \cdot W(5)</td>
</tr>
<tr>
<td>4</td>
<td>+9 \cdot R(53) \cdot W(4) \cdot W(6) + 656 \cdot R(3) \cdot W(5) \cdot W(6)</td>
</tr>
<tr>
<td>A(3, 3) = B(2, 2) + B(3, 2) \cdot A + R(39) \cdot W(1) \cdot W(4) - 2 \cdot R(75) \cdot W(1) \cdot W(2)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>+R(28) \cdot W(2) \cdot W(3) + 3 \cdot R(7) \cdot W(3) \cdot W(4) - 2 \cdot R(2) \cdot W(3) \cdot W(4)</td>
</tr>
<tr>
<td>2</td>
<td>+9 \cdot R(44) \cdot W(4) \cdot W(5) + 2 \cdot R(17) \cdot W(1) \cdot W(6) - 2 \cdot R(30) \cdot W(2) \cdot W(5)</td>
</tr>
<tr>
<td>3</td>
<td>-512 \cdot R(3) \cdot W(2) \cdot W(6) + R(52) \cdot W(5) \cdot W(6) + 2 \cdot R(49) \cdot W(6) \cdot W(6)</td>
</tr>
<tr>
<td>A(4, 3) = -9 \cdot R(13) \cdot W(1) \cdot W(4) - 2 \cdot R(40) \cdot W(1) \cdot W(2) - 243 \cdot R(2) \cdot W(3) \cdot W(4)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>+18 \cdot R(44) \cdot W(3) \cdot W(4) - 9 \cdot R(46) \cdot W(1) \cdot W(5) - 9 \cdot R(32) \cdot W(6) \cdot W(6)</td>
</tr>
<tr>
<td>2</td>
<td>-144 \cdot R(22) \cdot W(1) \cdot W(6) + 9 \cdot R(54) \cdot W(2) \cdot W(5) + 9 \cdot R(53) \cdot W(2) \cdot W(6)</td>
</tr>
<tr>
<td>3</td>
<td>-9 \cdot R(34) \cdot W(5) \cdot W(6)</td>
</tr>
<tr>
<td>A(5, 3) = R(20) \cdot W(1) \cdot W(2) - 9 \cdot R(46) \cdot W(1) \cdot W(4) - 256 \cdot R(3) \cdot W(2) \cdot W(3)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-2 \cdot R(30) \cdot W(2) \cdot W(3) + 9 \cdot R(54) \cdot W(2) \cdot W(4) - 2 \cdot R(26) \cdot W(1) \cdot W(5)</td>
</tr>
<tr>
<td>2</td>
<td>+2 \cdot R(52) \cdot W(3) \cdot W(5) + 656 \cdot R(3) \cdot W(7) \cdot W(6) - 9 \cdot R(34) \cdot W(4) \cdot W(6)</td>
</tr>
</tbody>
</table>
\[
A(6, 3) = -3 \times R(15) \times W(1) \times 2 + 2 \times R(17) \times W(1) \times W(3) + R(19) \times W(1) \times W(2)
\]

\[
1 \quad -144 \times R(22) \times W(1) \times W(4) - R(29) \times W(2) \times 2 - 512 \times R(3) \times W(2) \times W(3)
\]

\[
2 \quad + 9 \times R(53) \times W(2) \times W(4) + 2 \times R(49) \times W(3) \times W(6) + 656 \times R(3) \times W(2) \times W(5)
\]

\[
3 \quad -9 \times R(34) \times W(4) \times W(5) - 18 \times R(32) \times W(4) \times W(6) - R(35) \times W(5) \times 2
\]

\[
A(1, 4) = 162 \times R(21) \times W(1) \times W(4) - 18 \times R(11) \times W(1) \times W(2)
\]

\[
1 \quad -18 \times R(13) \times W(1) \times W(3) + R(40) \times W(2) \times W(3) + 162 \times R(5) \times W(1) \times W(5)
\]

\[
2 \quad + 162 \times R(4) \times W(1) \times W(6) - 144 \times R(23) \times W(2) \times W(5)
\]

\[
3 \quad -9 \times R(45) \times W(2) \times W(6) - 9 \times R(46) \times W(3) \times W(5) - 144 \times R(22) \times W(3) \times W(6)
\]

\[
4 \quad + 18 \times R(14) \times W(4) \times W(5) + 18 \times R(12) \times W(4) \times W(6)
\]

\[
5 \quad -729 \times R(21) \times W(5) \times W(6)
\]

\[
A(2, 4) = -9 \times R(11) \times W(1) \times 2 + R(40) \times W(1) \times W(3) - 243 \times R(1) \times W(2) \times 2
\]

\[
1 \quad + 2 \times R(43) \times W(2) \times W(4) - 144 \times R(23) \times W(1) \times W(5) - 9 \times R(45) \times W(1) \times W(6)
\]

\[
2 \quad + 9 \times R(54) \times W(3) \times W(5) + 9 \times R(53) \times W(3) \times W(6) - 9 \times R(31) \times W(5) \times 2
\]

\[
3 \quad -9 \times R(33) \times W(5) \times W(6)
\]

\[
A(3, 4) = -9 \times R(13) \times W(1) \times 2 + R(40) \times W(1) \times W(2) - 243 \times R(2) \times W(3) \times 2
\]

\[
1 \quad + 18 \times R(44) \times W(3) \times W(4) - 9 \times R(46) \times W(1) \times W(5) - 9 \times R(34) \times W(5) \times W(6)
\]

\[
2 \quad -144 \times R(22) \times W(1) \times W(6) + 9 \times R(54) \times W(2) \times W(5) + 9 \times R(53) \times W(2) \times W(6)
\]

\[
3 \quad -9 \times R(32) \times W(6) \times 2
\]

\[
A(4, 4) = 81 \times R(21) \times B(3) \times W(2) + 81 \times R(21) \times W(1) \times 2 + 9 \times R(43) \times W(2) \times 2
\]

\[
1 \quad + 243 \times R(6) \times W(4) \times 2 + 9 \times R(44) \times W(3) \times 2 + 18 \times R(14) \times W(1) \times W(5)
\]

\[
2 \quad + 18 \times R(47) \times W(1) \times W(6) + 81 \times R(47) \times W(5) \times 2 + 81 \times R(49) \times W(6) \times 2
\]

\[
3 \quad + 2592 \times R(3) \times W(5) \times W(6)
\]

\[
A(5, 4) = 81 \times R(5) \times W(1) \times 2 + 18 \times R(14) \times W(1) \times W(6) + 9 \times R(54) \times W(2) \times W(3)
\]

\[
1 \quad -144 \times R(23) \times W(1) \times W(2) - 9 \times R(46) \times W(1) \times W(3)
\]

\[
2 \quad + 729 \times R(21) \times W(1) \times W(6) - 18 \times R(31) \times W(2) \times W(5)
\]

\[
3 \quad -9 \times R(33) \times W(2) \times W(6) - 9 \times R(34) \times W(3) \times W(6) + 162 \times R(47) \times W(4) \times W(5)
\]

\[
4 \quad + 2592 \times R(3) \times W(4) \times W(6)
\]
\[
A(6,4) = 81 \cdot R(4) \cdot W(1)^2 + 18 \cdot R(12) \cdot W(1) \cdot W(4) - 9 \cdot R(45) \cdot W(1) \cdot W(2)
\]

\[
\begin{align*}
1 & \quad -144 \cdot R(22) \cdot W(1) \cdot W(3) + 9 \cdot R(53) \cdot W(2) \cdot W(3) \\
2 & \quad +729 \cdot R(21) \cdot W(1) \cdot W(5) - 9 \cdot R(33) \cdot W(2) \cdot W(5) - 9 \cdot R(34) \cdot W(3) \cdot W(5) \\
3 & \quad -18 \cdot R(32) \cdot W(3) \cdot W(6) + 2592 \cdot R(3) \cdot W(4) \cdot W(5) \\
4 & \quad +162 \cdot R(48) \cdot W(4) \cdot W(6)
\end{align*}
\]

\[
A(1,5) = -6 \cdot R(16) \cdot W(1) \cdot W(2) + 162 \cdot R(5) \cdot W(1) \cdot W(4) + 2 \cdot R(41) \cdot W(1) \cdot W(5)
\]

\[
\begin{align*}
1 & \quad +R(18) \cdot W(2) \cdot W(2) \cdot W(3) - 144 \cdot R(23) \cdot W(2) \cdot W(4) \\
2 & \quad -9 \cdot R(46) \cdot W(3) \cdot W(4) + 9 \cdot R(14) \cdot W(4) \cdot W(4) - 2 - 2 \cdot R(26) \cdot W(3) \cdot W(5) \\
3 & \quad +729 \cdot R(21) \cdot W(4) \cdot W(6)
\end{align*}
\]

\[
A(2,5) = -3 \cdot R(16) \cdot W(1) \cdot W(2) + 2 \cdot R(18) \cdot W(1) \cdot W(2) + R(20) \cdot W(1) \cdot W(3)
\]

\[
\begin{align*}
1 & \quad -144 \cdot R(23) \cdot W(1) \cdot W(4) - R(30) \cdot W(3) \cdot W(3) - 512 \cdot R(3) \cdot W(2) \cdot W(3) \\
2 & \quad +9 \cdot R(54) \cdot W(3) \cdot W(4) + 2 \cdot R(50) \cdot W(2) \cdot W(5) + 656 \cdot R(3) \cdot W(3) \cdot W(6) \\
3 & \quad -18 \cdot R(31) \cdot W(4) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(6) - R(36) \cdot W(6) \cdot W(6) \\
4 & \quad A(3,5) = R(20) \cdot W(1) \cdot W(2) - 9 \cdot R(46) \cdot W(1) \cdot W(4) - 2 \cdot R(26) \cdot W(1) \cdot W(5)
\end{align*}
\]

\[
\begin{align*}
1 & \quad -256 \cdot R(3) \cdot W(2) \cdot W(2) - 2 - 2 \cdot R(30) \cdot W(2) \cdot W(3) + 9 \cdot R(54) \cdot W(2) \cdot W(4) \\
2 & \quad +2 \cdot R(52) \cdot W(3) \cdot W(5) + 656 \cdot R(3) \cdot W(2) \cdot W(6) - 9 \cdot R(34) \cdot W(4) \cdot W(6) \\
3 & \quad -2 \cdot R(35) \cdot W(5) \cdot W(6)
\end{align*}
\]

\[
A(4,5) = 81 \cdot R(5) \cdot W(1) \cdot W(2) - 144 \cdot R(23) \cdot W(1) \cdot W(2) - 9 \cdot R(46) \cdot W(1) \cdot W(3)
\]

\[
\begin{align*}
1 & \quad +18 \cdot R(14) \cdot W(1) \cdot W(4) + 729 \cdot R(21) \cdot W(1) \cdot W(6) \\
2 & \quad +9 \cdot R(54) \cdot W(2) \cdot W(3) - 18 \cdot R(31) \cdot W(2) \cdot W(5) - 9 \cdot R(33) \cdot W(2) \cdot W(6) \\
3 & \quad -9 \cdot R(34) \cdot W(3) \cdot W(6) + 162 \cdot R(47) \cdot W(4) \cdot W(5) \\
4 & \quad +2592 \cdot R(3) \cdot W(4) \cdot W(6)
\end{align*}
\]

\[
A(5,5) = B_2(5) + B_3(1) \cdot AX + R(41) \cdot W(1) \cdot W(2) - 2 \cdot R(26) \cdot W(1) \cdot W(3)
\]

\[
\begin{align*}
1 & \quad +R(50) \cdot W(2) \cdot W(2) + R(52) \cdot W(3) \cdot W(3) - 18 \cdot R(31) \cdot W(2) \cdot W(4) \\
2 & \quad +81 \cdot R(47) \cdot W(4) \cdot W(4) - 2 - 2 \cdot R(35) \cdot W(3) \cdot W(6) + 3 \cdot R(10) \cdot W(5) \cdot W(5) \\
3 & \quad +R(37) \cdot W(6) \cdot W(6)
\end{align*}
\]

\[
A(6,5) = 729 \cdot R(21) \cdot W(1) \cdot W(4) + 656 \cdot R(3) \cdot W(2) \cdot W(3)
\]

\[
\begin{align*}
1 & \quad -9 \cdot R(33) \cdot W(2) \cdot W(4) - 9 \cdot R(34) \cdot W(3) \cdot W(4) + 1296 \cdot R(3) \cdot W(4) \cdot W(6)
\end{align*}
\]
\[ A(1,6) = -6 \cdot R(15) \cdot W(1) \cdot W(4) + 162 \cdot R(4) \cdot W(1) \cdot W(4) + 2 \cdot R(2) \cdot W(1) \cdot W(6) \]
\[ + R(17) \cdot W(3) \star 2 + R(19) \cdot W(2) \cdot W(3) - 9 \cdot R(45) \cdot W(2) \cdot W(4) \]
\[ A(2,6) = R(19) \cdot W(1) \cdot W(3) - 9 \cdot R(45) \cdot W(1) \cdot W(4) - 2 \cdot R(27) \cdot W(1) \cdot W(6) \]
\[ A(3,6) = -3 \cdot R(15) \cdot W(1) \cdot W(4) + 162 \cdot R(4) \cdot W(1) \cdot W(4) + 2 \cdot R(2) \cdot W(1) \cdot W(6) \]
\[ + R(17) \cdot W(3) \star 2 + R(19) \cdot W(2) \cdot W(3) - 9 \cdot R(45) \cdot W(2) \cdot W(4) \]
\[ A(4,6) = B2(3) + B3(3) \cdot A \cdot R(42) \cdot W(1) \cdot W(2) \]
\[ \begin{align*}
3 & \quad -81 \cdot R(21) \cdot W(1) \cdot W(4) + 2 + 18 \cdot R(11) \cdot W(1) \cdot W(2) \cdot W(4) \\
4 & \quad + 18 \cdot R(13) \cdot W(1) \cdot W(3) \cdot W(4) + R(24) \cdot W(2) \cdot W(3) \cdot W(4) \\
5 & \quad + R(25) \cdot W(2) \cdot W(3) \cdot W(3) \cdot W(4) - R(40) \cdot W(2) \cdot W(3) \cdot W(4) \\
6 & \quad + 6 \cdot R(16) \cdot W(1) \cdot W(2) \cdot W(5) + 6 \cdot R(15) \cdot W(1) \cdot W(3) \cdot W(5) \\
7 & \quad - 162 \cdot R(5) \cdot W(1) \cdot W(4) \cdot W(5) - 162 \cdot R(4) \cdot W(1) \cdot W(4) \cdot W(6) \\
E & = -R(41) \cdot W(1) \cdot W(5) \cdot W(2) - R(42) \cdot W(1) \cdot W(4) \cdot W(2) - R(18) \cdot W(2) \cdot W(2) \cdot W(5)
\end{align*} \]

\[ \begin{align*}
D2 & = B1(2) - B2(4) \cdot W(2) - B3(1) \cdot AX \cdot W(2) + R(2) \cdot W(1) \cdot W(1) \cdot W(3) \\
1 & \quad - R(38) \cdot W(1) \cdot W(2) - 16 \cdot R(3) \cdot W(1) \cdot W(1) \cdot W(3) \\
2 & \quad + 9 \cdot R(11D \cdot W(1) \cdot W(1) \cdot W(3) \cdot W(3) \\
3 & \quad + 2 \cdot R(24) \cdot W(1) \cdot W(2) \cdot W(3) - R(40) \cdot W(1) \cdot W(3) \cdot W(4) \\
4 & \quad - R(9) \cdot W(2) \cdot W(2) \cdot W(3) \cdot W(3) - R(28) \cdot W(2) \cdot W(3) \cdot W(4) \\
5 & \quad + 243 \cdot R(1) \cdot W(2) \cdot W(2) \cdot W(4) + 3 \cdot R(16) \cdot W(1) \cdot W(1) \cdot W(5) \\
6 & \quad - 2 \cdot R(18D \cdot W(1) \cdot W(1) \cdot W(3) \cdot W(5) - R(20) \cdot W(1) \cdot W(3) \cdot W(5) \\
7 & \quad - R(19) \cdot W(1) \cdot W(3) \cdot W(6) + 144 \cdot R(23) \cdot W(1) \cdot W(4) \cdot W(5) \\
F2 & = 9 \cdot R(45) \cdot W(1) \cdot W(1) \cdot W(4) \cdot W(6) + R(27) \cdot W(1) \cdot W(1) \cdot W(6) \\
1 & \quad + R(30) \cdot W(1) \cdot W(2) \cdot W(5) + 256 \cdot R(3) \cdot W(1) \cdot W(1) \cdot W(2) \\
2 & \quad + 512 \cdot R(3) \cdot W(1) \cdot W(2) \cdot W(3) \cdot W(5) + R(29) \cdot W(2) \cdot W(3) \cdot W(6) \\
3 & \quad - 9 \cdot R(54) \cdot W(1) \cdot W(4) \cdot W(5) - R(29) \cdot W(3) \cdot W(4) \cdot W(6) \\
4 & \quad - R(50) \cdot W(2) \cdot W(2) \cdot W(5) \cdot W(5) - R(51) \cdot W(2) \cdot W(3) \cdot W(6) \\
5 & \quad - 656 \cdot R(3) \cdot W(3) \cdot W(5) \cdot W(6) + 9 \cdot R(31) \cdot W(4) \cdot W(5) \\
6 & \quad + 9 \cdot R(33) \cdot W(1) \cdot W(5) \cdot W(6) + R(36) \cdot W(4) \cdot W(4) \cdot W(5) \\
D2 & = B1(2) - B2(2) \cdot W(2) - B3(1) \cdot AX \cdot W(3) + R(1) \cdot W(1) \cdot W(1) \cdot W(3) \\
\end{align*} \]
\[-16 \cdot R(3) \cdot W(1) \cdot W(2) - R(39) \cdot W(1) \cdot W(3)\]

\[+9 \cdot R(13) \cdot W(1) \cdot W(4) + R(24) \cdot W(1) \cdot W(2) \cdot W(4)\]

\[+2 \cdot R(25) \cdot W(1) \cdot W(2) \cdot W(3) - R(40) \cdot W(1) \cdot W(2) \cdot W(4)\]

\[-R(28) \cdot W(2) \cdot W(3) - R(7) \cdot W(3) \cdot W(3) + 324 \cdot R(2) \cdot W(3) \cdot W(4)\]

\[-9 \cdot R(44) \cdot W(3) \cdot W(4) \cdot W(4) + 2 + 3 \cdot R(15) \cdot W(1) \cdot W(2) \cdot W(3)\]

\[-R(20) \cdot W(1) \cdot W(2) \cdot W(5) - R(19) \cdot W(1) \cdot W(2) \cdot W(6)\]

\[-2 \cdot R(17) \cdot W(1) \cdot W(3) \cdot W(6) + 9 \cdot R(46) \cdot W(1) \cdot W(4) \cdot W(5)\]

\[E_3 = 44 \cdot R(22) \cdot W(1) \cdot W(4) \cdot W(6) + R(26) \cdot W(1) \cdot W(5) \cdot W(6)\]

\[D_4 = B(4) - B(1) \cdot W(4) - B(3) \cdot W(1) \cdot W(6) + 9 \cdot R(11) \cdot W(1) \cdot W(2)\]

\[+9 \cdot R(13) \cdot W(1) \cdot W(3) - B(1) \cdot W(1) \cdot W(4)\]

\[-R(40) \cdot W(1) \cdot W(2) \cdot W(3) + 81 \cdot R(1) \cdot W(2) \cdot W(3) \cdot W(3) + 81 \cdot R(2) \cdot W(3) \cdot W(3)\]

\[-9 \cdot R(43) \cdot W(2) \cdot W(4) \cdot W(4) - B(1) \cdot R(6) \cdot W(4) \cdot W(4) \cdot W(4) - 9 \cdot R(44) \cdot W(3) \cdot W(4)\]

\[-B(5) \cdot W(1) \cdot W(2) \cdot W(5) - B(4) \cdot W(1) \cdot W(2) \cdot W(5)\]

\[+144 \cdot R(23) \cdot W(1) \cdot W(2) \cdot W(5) + 9 \cdot R(45) \cdot W(1) \cdot W(2) \cdot W(6)\]

\[+9 \cdot R(46) \cdot W(1) \cdot W(3) \cdot W(5) + 144 \cdot R(22) \cdot W(1) \cdot W(3) \cdot W(6)\]

\[F_4 = -18 \cdot R(14) \cdot W(1) \cdot W(4) \cdot W(5) - 18 \cdot R(12) \cdot W(1) \cdot W(4) \cdot W(6)\]

\[-729 \cdot R(21) \cdot W(1) \cdot W(5) \cdot W(6) - 9 \cdot R(53) \cdot W(1) \cdot W(5) \cdot W(5)\]

\[-9 \cdot R(53) \cdot W(2) \cdot W(3) \cdot W(6) + 9 \cdot R(31) \cdot W(2) \cdot W(5) \cdot W(5)\]

\[+9 \cdot R(32) \cdot W(3) \cdot W(6) \cdot W(6) + 9 \cdot R(33) \cdot W(2) \cdot W(5) \cdot W(5)\]

\[+9 \cdot R(34) \cdot W(3) \cdot W(5) \cdot W(6) - B(1) \cdot R(47) \cdot W(4) \cdot W(5) \cdot W(5)\]

\[-81 \cdot R(48) \cdot W(4) \cdot W(6) \cdot W(6) - 2 \cdot 592 \cdot R(3) \cdot W(4) \cdot W(5) \cdot W(6)\]

\[D_5 = B(3) - B(2) \cdot W(5) - B(3) \cdot W(1) \cdot W(5) + 3 \cdot R(16) \cdot W(1) \cdot W(2)\]
\[ E_1 = -81 \cdot R(5) \cdot W(1) \cdot W(4) - R(18) \cdot W(1) \cdot W(2) \cdot W(3) - R(20) \cdot W(1) \cdot W(2) \cdot W(3) \]
\[ E_2 = -9 \cdot R(14) \cdot W(1) \cdot W(4) + 2 + 144 \cdot R(23) \cdot W(1) \cdot W(2) \cdot W(4) \]
\[ E_3 = +9 \cdot R(46) \cdot W(1) \cdot W(3) \cdot W(4) + 256 \cdot R(3) \cdot W(2) \cdot W(3) \]
\[ E_4 = +R(30) \cdot W(2) \cdot W(3) - 9 \cdot R(54) \cdot W(1) \cdot W(2) \cdot W(4) \]
\[ E_5 = -R(41) \cdot W(1) \cdot W(2) \cdot W(3) - R(50) \cdot W(2) \cdot W(5) \]
\[ E_6 = -R(52) \cdot W(3) \cdot W(4) - 656 \cdot R(3) \cdot W(2) \cdot W(3) \]
\[ D_1 = E_1 \]
\[ D_2 = E_2 \]
\[ D_3 = E_3 \]
\[ D_4 = E_4 \]
\[ D_5 = E_5 \]
C(6) = D6 + E6
RETURN
END
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