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Optimization models for allocation of air strike assets with persistence

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OPTIMIZATION MODELS FOR ALLOCATION OF AIR STRIKE ASSETS WITH PERSISTENCE

by

Davi Rogério da Silva Castro

December 2002

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This thesis addresses the critical process of assigning strike aircraft to targets once the targets have been identified: How do we optimally employ available aircraft and weapons on the current set of targets, and how can we modify a previously optimized assignment list to face changes in the tactical situation? Our contribution to the strike-planning problem includes (1) a static allocation model in which each aircraft makes at most one sortie during the planning time horizon, (2) a dynamic model in which each aircraft may make more than one sortie during that horizon, and (3) extensions of these models with "persistence incentives," which discourage major plan changes in the results when partial but important changes in the tactical situation necessitate reoptimization. These optimization models are mixed-integer programs that solve in seconds on a personal computer for realistic scenarios with three weapons types, 156 aircraft at seven bases, and 100 potential targets. In a scenario in which two new high-priority targets arise and must be added to an air tasking order with eight original targets, persistence incentives reduce the number of major plan changes from five to two.

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ACRONYMS

AAA	Anti-Aircraft Artillery
AGM	Air-launched Surface-attack Guided Missile
AIM	Air-launched Intercept-aerial Guided Missile
ATO	Air Tasking Order
BDA	Battle Damage Assessment
CBU	Cluster Bomb Unit
COP	Common Operational Picture
EGBU	Enhanced Guided Bomb Unit
EKS	Expected Kills per Sortie
ETD	Estimated Time of Departure
GAMS	General Algebraic Modeling System
GBU	Guided Bomb Unit
MAAP	Master Air Attack Plan
MIP	Mixed-Integer Program
REDS	Real-Time Execution Decision Support
ROE	Rules of Engagement
SAM	Surface-to-Air Missile
SCL	Standard Conventional Load
SEAD	Suppression of Enemy Air Defense
SPAWAR	Space and Naval Warfare Center
SPINS	Special Instructions
TACC	Tactical Air Control Center

TAM	Theater Attack Model (USAF)
TCS	Tactical Control System
TOT	Time on Target
TVD	Target Value Destroyed

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I owe a final, heartfelt, and enduring word of thanks to my lovely wife, Rita, for her patience and encouragement, and to Matheus who learned to accept "daddy's got to go study" with never-ending smiles and hugs.

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EXECUTIVE SUMMARY

The explosion of information available to decision-makers at all levels on the battlefield has led to tremendous strides in the ability to process, decide and act. At the same time, there is increasing demand for faster and better decisions. This thesis addresses one type of critical decision that must be made quickly and effectively, namely how to assign strike aircraft to targets once the targets have been identified. More specifically, the primary questions that this research attempts to answer are:

"How do we optimally employ available aircraft and weapons on the current set of targets?"

"How can we modify a previously optimized assignment list to face changes in the tactical situation, in a manner that balances the desire to obtain a new optimal solution with the desire to minimize disruption of existing plans based on the previous solution?"

The general air strike planning problem is divided into five areas: target selection, weapon allocation, mission formation and assignment, mission routing and scheduling, and contingency planning. This thesis addresses weapon allocation, and it addresses mission allocation and assignment, for the aircraft of an air strike package (the "package" will typically contain other aircraft with non-strike roles such as suppression of enemy air defense; we do not directly consider these aircraft in this thesis).

Our contributions to the strike-planning problem include (1) a static allocation model in which each aircraft makes at most one sortie during the planning time horizon, (2) a dynamic model in which each aircraft may make more than one sortie during that horizon, and (3) extensions of these models with "persistence incentives," which discourage major changes in the results when partial but important changes in the tactical situation necessitate reoptimization. All of these models are mixed-integer programs.

We demonstrate these models in realistic scenarios. For instance, our models can allocate 156 air assets from seven bases to 100 targets in just few seconds on a personal computer. This particular problem has three types of aircraft and two possible weapons configurations for each aircraft. There are three types of weapons and 20 different packages can be formed from the aircraft available.

The persistence paradigm plays an important role in this thesis. We show that a new optimal plan can differ drastically from a previous plan after only small changes in the tactical situation. To handle these unexpected changes more conveniently, the persistent model adds a term to the original objective function that penalizes deviations from the original plan. The new plan is nearly optimal in the standard sense, and unit-level planners and pilots have fewer plan changes to handle.

This research is performed under the sponsorship and guidance of SPAWAR (Space and Naval Warfare Center). The result of this work is to be used inside REDS (Real-Time Execution Decision Support), a decision-support tool currently being developed at SPAWAR under sponsorship of the Office of Naval Research. The optimization modeling developed in this thesis will support SPAWAR's goal of having REDS provide the best solution for allocation of strike assets in a dynamic tactical environment.

I. INTRODUCTION

A. BACKGROUND AND PURPOSE

The Space and Naval Warfare System Command (SPAWAR) in San Diego is developing tools for improving the planning and execution of (air) strike warfare. More precisely, they are working on a project called REDS (Real-Time Execution Decision Support), which contains two modules: the Element Level Planner (ELP) and the Mission Monitor (MM). ELP automates the administrative work involved for planning at the combat unit level and MM will monitor all phases of a mission, making a wealth of real-time data available for planning, e.g., updates of target information, aircraft load-outs, aircraft positions and status, etc.

The Naval Postgraduate School’s Department of Operations Research has been invited to provide theoretical support in the development of the next stages of REDS. The effort will look for the use of the information coordinated by the Mission Monitor to partially automate the decision-making processes involved in mission planning. REDS will (i) help determine the composition of strike packages and the assignment of packages to targets, (ii) identify "efficient" (low risk, low fuel consumption, high success probability) routes for packages to take to targets, and (iii) provide probabilistic information on mobile target locations, potential actions by the adversary, etc. The mobile targets may be surface-to-air missiles, theater ballistic missiles, troops, etc.

This thesis will focus on item (i), investigating different models and possible objective functions for optimization of strike planning.

B. RESEARCH PROBLEM

Since its origins in World War II, operations research methods, such as optimization, have dealt with the problem of resource allocation. Optimization has been used in many military and civilian situations to determine the best assignment of tasks to agents. In these situations, optimization models ensure that the tasks are accomplished to the highest possible degree of performance, subject to constraints that must be enforced on the availability of the agents and the limitations of their resources.

In our specific problem the "agents" are air strike force assets and the "tasks" are military targets to be neutralized. The constraints refer to targeting priorities, weapons effects, aircraft availability, speed, location, and weapon capability. The main goal is to best support the overall campaign objectives. Examples of this type of modeling are discussed in the literature review.

We plan to go one step further than the solution of an optimization model for assigning agents to tasks, by developing a dynamic allocation model. This model allocates resources over a substantial time horizon in a changeable tactical environment, recommending good allocations even as real and contemplated modifications in the initial conditions occur. This means that the problems must be solved many times in rapid sequence, with the possibility of prior decisions constraining future ones.

This thesis supports the strike planning effort, by partially automating the decision-making processes involved in mission planning. Further, it addresses the situation when assets are already assigned to targets and a new tactical scenario emerges with new targets, new priorities, aircraft maintenance updates, and modified weather conditions affecting weapon performance.

The problem will be represented mathematically by a mixed-integer program (MIP). In a MIP, we have both continuous and integer variables to describe, quantify, and qualify the states and controls of the system. We can view the situation as a multiperiod problem: plans are made for multiple time periods in the future, and as one period elapses and better data and forecasts become available, the model is slid forward one period. Using ideas of persistence [Brown, Dell and Wood 1997], we are able to model and address the very changeable tactical scenarios that arise in such real-world situations. And note that, even if we are planning only one period in advance, a similar situation can arise: A plan is developed and promulgated, but the tactical situation changes before that plan can be executed. Persistence is important in this case, too.

C. AIR TASKING ORDERS AND AIR STRIKE PACKAGES

An Air Tasking Order (ATO) is the administrative vehicle used to disseminate daily plans to units and to command and control agencies. The ATO normally provides specific instructions to include radio call signs, times on targets, and other detailed information required for the execution of a plan. An ATO is the result of a complex process of target selection and allocation of assets covering a myriad of missions. It usually takes two days of planning and strict control of execution to develop an ATO. ATOs require careful coordination of tasks and consideration of weaponeering data, force structure, sortie and weapons availability, intelligence aspects of the enemy, weather, and numerous other pieces of information.

There are usually three ATOs in existence at any one time: (1) the ATO in execution (today’s plan), (2) the ATO in production (tomorrow’s plan), and (3) the ATO in planning (following day’s plan). The idea of reducing that cycle is represented in Figure 1 according to the concept of a real-time C^2 infrastructure.

Griggs [1994] precisely defines an air strike package: "An strike package is a group of fighter and bomber aircraft that have combined to provide mutual support against enemy threats while they achieve a common goal of destroying a set of targets. Strike packages are normally constructed in several steps. First, the mission planner must select the right type and number of aircraft and munitions to efficiently destroy each target. Next, all flights attacking targets in the same vicinity are grouped into packages if aircraft speed restrictions and tactics are compatible. Last, the mission planner must add suppression of enemy air defense (SEAD) aircraft and air-to-air fighter escort, or sweep, aircraft to protect the groups of attackers."

The addition of SEAD and escort aircraft depends on their availability, the enroute threats such as surface-to-air missiles, the mission, and the type of aircraft in the package. A group of flights attacking targets in the same vicinity together with SEAD and escort aircraft comprise a typical air strike package. Figure 2 shows an example of a strike package used during the Gulf War.

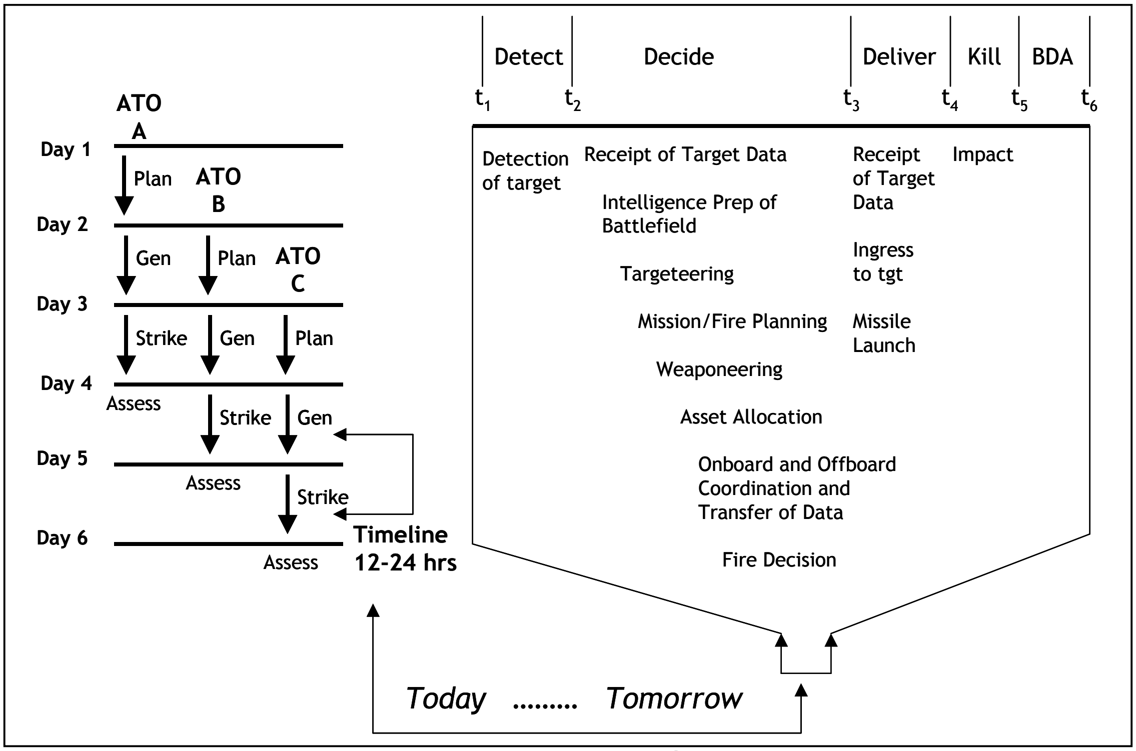

Figure 1. ATO timeline[1]

The actual production cycle of an ATO (Air Tasking Order) comprises three days of preparation and execution, but the increasing flow of information from the battlefield is driving the need for a more compressed timeline.

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[1] The source of this figure is the Naval Aviation Interoperability Assurance Office, (http://www.dtic.mil/ndia/systems/Clark.pdf) "Streamlining Acquisition Through Collaborative Engineering," 10/06/2000.

Strike force planning can quickly become very difficult to model and solve if we consider the huge number of possible combinations of aircraft, tactics, and weapons against each type of target, threat and environmental condition. In order to derive the best methodology for our specific objectives, we first review the current status of research on strike force allocation.

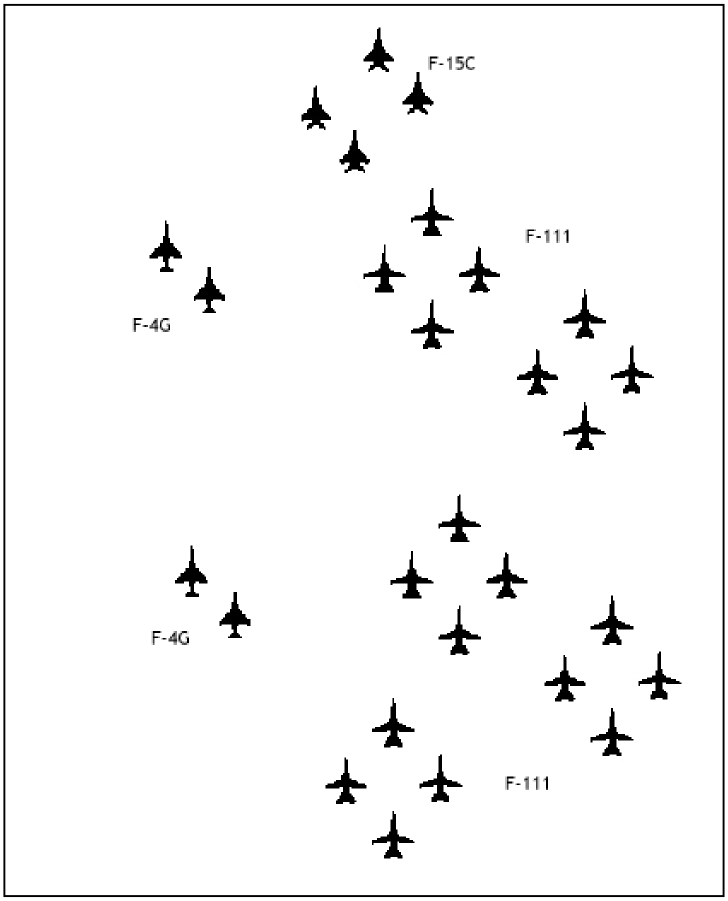

Figure 2. Gorilla Package[2]

A "gorilla package" was a type of strike package that placed a large number of aircraft over a target in a short period of time during the Gulf War. Planning time and effort increased dramatically with the size of the package.

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[2] Cohen 1993, Vol IV, pp. 164-165.

There are different problem characteristics that may be modeled depending on the model's purpose. The allocation of strike forces can be either part of a real-time planning tool or a long-term budget planning system. Each problem focuses on different aspects of mission formation and assignment. Acceptable solution time depends on the model's purpose.

D. LITERATURE REVIEW

Our research sponsor at SPAWAR, John McDonnell, introduced us to a model for composing strike packages from available assets and allocating those packages to targets. He and colleagues Gizzi and Louis [McDonnell et al. 2001] develop a nonlinear optimization formulation of the allocation problem, and solve it approximately using a genetic algorithm (heuristic). Their approach encompasses both strike and suppression responsibilities as well as multi-target and multi-threat allocations. The model assumes that weaponry load-outs have been predetermined and the strike force can be reassigned to new targeting objectives.

Abrahams [1998] and Balart [1996] are the most important reference for the work of McDonnell et al. Abrahams and Balart's model is based on a non-linear objective function for a static allocation, where crucial parameters such as target value and weapon effectiveness are independent of time. A genetic algorithm provides approximate solutions.

Li, Curry and Boyd [2002] are currently working on an integer programming model for the strike force asset allocation problem, sponsored by the Office of Naval Research. Although this report focuses on determining if a solver can handle a large integer model in real time, it contains interesting insights on how to build suppression packages to act against threats.

Criteria for building strike packages are found in the USAF models described by Griggs [1994], Jackson [1989], and Yost [1995]; although each author uses a different objective and different interpretations for "sortie," "target," "weapon," etc. Our research incorporates some of the main ideas from these Air Force models.

According to Yost [1996], the Time Strike munitions optimization model was introduced in 1995 for use by various USAF agencies to develop requirements for conventional munitions, to refine operational plans based on the availability of different mixes of munitions, and to assess the effects of procuring different types and quantities of munitions. Time Strike's objective pursues the phases (sets of goals for each target class) in a hierarchical order defined by the user. The model's notion of target classes is a major difference from previous models, and supports the fact that campaign objectives involve killing collections of related targets rather than individual targets. It is a multi-period linear programming model.

Koewler [1999] creates a prototype scheduling and allocation tool that strikes a balance between ease of use, accurately defining and solving the allocation problem, and generating solutions in an operationally acceptable amount of time. A combination of concepts from project scheduling, object-oriented programming and heuristics form the basis for the methodology developed.

Saling [1999] examines the Joint Air Operations Center structure, the relationship of information to the Master Air Attack Plan and methods of distributing that information to the warfighter through the Air Tasking Order and alternately through dynamic re-tasking.

Dolan [1993] offers a solution to the problem of producing a timely and flyable ATO (Air Tasking Order) that effectively uses assigned aircraft. His model decides which strike package should be assigned against each target and which available launch sites should provide the assets required in the selected strike packages. The output is an ATO used in theater level war games conducted at the Naval War College.

Crawford [1994] enhances a version of Dolan's optimization model to explicitly incorporate the dimension of time, thereby allowing multiple sorties per aircraft per day, something not allowed in Dolan's model. Crawford's model takes packages as inputs, which are provided by a team in a war game exercise.

The doctrine currently used by American military forces completely guides the process of mission planning. Joint Publication 3-01.2 [1986] provides us some insights into planning operations, discussing the Intelligence activity of targeting, as well procedures and techniques of Command and Control. This document describes some fundamental principles from which we define our objectives. Two of them are especially important when conducting counterair operations: the *concentration of force* and the *economy of effort*.

Concentration of force is the effective application of combat power, which requires that sufficient force be concentrated at the appropriate time and place to ensure achievement of the objective.

Economy of effort is the correct selection and use of weapon systems, sound distribution of forces and careful balance in the allocation of tasks. When applying this principle, the commander intends to achieve effective concentration of power at the decisive time and place while conserving weapons for countering enemy reattacks.

The same document also instructs on how to establish target priorities. Five criteria are of vital importance: threat, feasible effect (degree of positive effect, in terms of degrading every capability or enhancing friendly operations), delay in effect (time between the initial engagement and the desired effect; concentration of effort may compress that time), risk calculation, and forces available.

Part of our research is directed to answer the question of how to modify a previously optimized plan. We are particularly interested in solving a multiperiod model in a rolling-horizon format, where the model is solved at the beginning of some period or by suggestion of the decision maker. The recommendations of the solution for the previous period are remembered and deviations from them are charged a penalty in the objective function. This is called a "persistence incentive."

The method we use for handling persistence derives from Brown, Dell and Wood [1997]. These authors state that the lack of persistence is one of the most common sources of complaints about optimization. They address the issue using a series of case studies that demonstrates how persistence can mediate the differences in focus between managers and modelers, and show how to develop models from the start with persistence in mind.

E. PHASES OF AIR STRIKE PLANNING

The general air strike planning problem can be structured to identify and classify objectives, constraints, decisions, and influencing factors. Using this structure, decisions are divided into five problem areas: (i) target selection, (ii) weapon allocation, (iii) mission formation and assignment, (iv) mission routing and scheduling, and (v) contingency planning [Glenn 1980].

The *target selection activity* examines potential targets to determine military importance, priority of attack, and weapon feasibility to obtain a desired effect [USAF 1998]. The selected target systems are then further analyzed to determine their components and critical elements. This phase distills the commander's objectives into a list of targets. The product of this phase is a suggested target list with recommended priorities assigned and the extent of desired damage specified.

Weapon allocation (also called "weaponeering") estimates the quantity of a specific weapon type required to achieve the desired level of damage to a given target, considering target vulnerability, weapon effects, munitions delivery errors, damage criteria, weapon reliability, etc. [USAF 1998]. Weapon effectiveness varies according to the weapon, target, damage criteria, delivery conditions, and target environment. There are different ways of stating weapon effectiveness according to the target/weapon combination. We will derive and use only the probability of kill, based upon information about the target, the weapon and the aircraft available.

Mission formation and assignment designs the actual strike package. Usually, planners start with the allocation of strike assets and then assign the non-attack mission platforms (i.e., aircraft to escort, suppression of enemy air defenses, jamming, airborne control, tactical reconnaissance, air refueling, and search and rescue), which support the package's ingress and egress.

During the *mission routing and scheduling process*, planners ensure that the mission packages are "deconflicted" with other mission packages, tankers are available at the refueling points, times of launch and landing are synchronized, etc. The deconfliction process prevents conflicts such as aircraft on different missions using the same altitude or too many aircraft requiring refueling or landing at the same time. This coordination must still conform to precise arrivals at the required time on target. The decision-maker also considers the enemy's air defense and weather conditions enroute in this phase.

Finally, *contingency plans* specify secondary targets and conditions for diverting the strike to them, and variations in strike tactics if the weather conditions at the target differ from those anticipated.

This thesis addresses two areas of strike mission planning: (ii) weapon allocation and (iii) mission allocation and assignment. Only the strike components of the packages are considered. Additional work is required to include specialized escort and SEAD aircraft to increase the package's probability of success against the enemy's defenses.

1. Weapon Allocation

In the weapon allocation problem, we must consider the effectiveness of a specified number of weapons delivered by a given number of aircraft against a target that has an associated minimum damage criterion or minimum acceptable probability of kill. To accurately model this situation requires nonlinear functions for evaluation of the probability of killing the target. We would prefer, however, a linear optimization model for assigning strike packages to targets (linear in the optimization model's decision variables and constraints). So, how do we incorporate a nonlinear measure of effectiveness in a linear optimization model?

The solution is to enumerate the "strike options," i.e., possible strike packages, and calculate their effectiveness against each target as part of the mixed-integer programming (MIP) model's input, rather than as part of the model's decision process. Then the MIP can be defined with binary variables representing the possible pairings of targets to strike packages. Dolan's [1993] and Crawford's [1994] models also incorporate pre-enumerated strike packages, with a subjective preference rating for each given target type. Ratings from one to five are specified, with one being the most preferred and five the least.

Instead of a preference rating, we calculate the probability of kill for each package against each target type. This calculation is simplified and only depends on the type and quantity of weapons the aircraft in the package can launch. We also consider that some weapons don't work against certain types of targets or under certain weather conditions.

Each possible combination of weapons load for a given aircraft is called a *configuration*. An example of a strike package could be the assignment of two aircraft of type a in configuration c, where configuration c contains four weapons of type w_1 and two weapons of type w_2.

2. Mission Allocation and Assignment

The phase of strike planning that receives the most emphasis in this thesis is mission allocation and assignment. It is extremely difficult to address the problem with full generality. Therefore, choices must be made concerning alternate ways of handling various aspects of the problem. As a result, there could be many reasonable modeling approaches.

The first choice to be considered is whether the model is *static* or *dynamic*. A static model treats all strikes as taking place during a single time period. This does not mean the strikes are literally simultaneous, but the resolution of modeling ignores time effects. Thus, the static model does not explicitly consider the possibility of aircraft completing a strike, returning to its launch site, refueling, and launching for a second strike. The second launch would have to be considered in a second run of the static model.

A dynamic model, on the other hand, explicitly treats the passage of time, allowing aircraft to perform multiple strikes. The dynamic model can also include the feature of having the model decide when targets should be struck within a given time window. This thesis develops both static and dynamic models.

A second modeling choice is whether sorties must strike only one target or are allowed multiple targets. The single-target restriction may not be entirely realistic, but is much easier to model than the multiple target case. Throughout the thesis, the single-target restriction is enforced.

Table 1. Summary of modeling options

Time	Static vs. Dynamic
Targets per sortie	Single target vs. Multiple target
Strike package composition	Homogeneous vs. Heterogeneous

Finally, strike packages can be homogeneous or heterogeneous with respect to the types of strike aircraft in the package and with respect to the types of weapons carried in each aircraft. The static model developed in this thesis allows for heterogeneous strike packages. For computational reasons, the dynamic model is restricted to strike packages consisting of only one type of strike aircraft, each with the same weapons loadout. Table 1 presents a summary of modeling options for the air strike asset allocation problem.

3. Contingency

Although the principal emphasis of a strike plan must be on events that are judged most likely to occur, it is also important for the plan to offer responses to the less likely possibilities [Glenn 1980]. A strike plan should include contingency plans that provide secondary targets, backup assignments and variations in strike tactics if weather conditions, aircraft availability or target characteristics change.

Our models do not directly provide contingency plans, in contrast to Kuykendall’s [1998] Tomahawk missile assignment problem, which assigns backups for every mission. However, our models can be helpful for dealing with contingencies indirectly. As conditions change, the models can be re-run. The persistence feature will encourage the new solution to be similar to the previous one, making it easier to adapt to the necessary changes.

The models described in this thesis incorporate important aspects of the strike planning process to produce air tasking orders. Chapter II describes a static model. Chapter III treats a dynamic case. Chapter IV develops extensions of the static and dynamic models with persistence incentives. Finally, Chapter V summarizes our results and suggests directions for future research.

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II. STATIC MODEL WITH HETEROGENEOUS PACKAGES

This chapter develops a static optimization model for creating air tasking orders. The key decisions are: which combination of aircraft, with which weapon loadouts, coming from which launch site, should strike which target?

The mixed-integer program makes the best overall assignment of strike packages, taking into account the most important characteristics of the assets and targets. Key features of this model are given in Table 2.

The model in the next chapter is dynamic. Although some parameters and variables are similar for both models, the models are defined completely and independently in separate chapters.

Table 2. Static model summary

General	A MIP (mixed-integer program) with a static time approach allocates heterogeneous strike packages, allowing only one target struck per sortie
Objective Function	A linear combination of three objectives: minimize value of targets not assigned, minimize the effects of imperfect matching of targets to packages (incomplete damage, long-distance flight, etc.), and maximize value of unused aircraft
Campaign Objectives	Input as a required time on target (TOT)
Target Prioritization	Based on target values
Aircraft Packaging	Different types of aircraft carrying different mixes of weapons are allowed (not restricted to homogeneous packages)

A. INDICES

The indices used to define this model are:

a	aircraft type	{‘FA18’, ‘A-6’, ‘A10’,...}
c	configuration	{‘C1’, ‘C2’, ‘C3’,...}
i	launch site	{‘Airbase-01’, ‘CV71’,...}
j	target	{‘Safwan’, ‘SCUD’,...}
k	target type	{‘Airfield’, ‘Bridge’,...}
n	strike package	{‘N1’, ‘N2’,...}
r	threat type	{‘AAA’, ‘SAM’, ‘Air-to-Air’,...}
w	weapon	{‘MK84’, ‘GBU31’, ‘AGM65’,...}

Launch sites can be either airbases or aircraft carriers. With this understanding, the terms “site” and “base” are used interchangeably. An aircraft configuration is an instance of a possible loading of guns, ammunition, missiles, bombs, and external fuel tanks on a single aircraft, although this thesis ignores fuel tanks for simplicity. Potential configurations are specified in a descriptive document called standard conventional loads (SCLs). An example for the F/A-18 aircraft is given in Table 3:

Table 3. Example of configuration

Config ID	MG25	
Type A/C	F/A-18	
Gun	20mm	500 rounds
Missile	AIM7	2
Missile	AIM9	2
Bomb	MK82	5
External fuel	TANK	2

The strike package is a mixture of different aircraft and configurations. Table 4 gives an example:

Table 4. Examples of heterogeneous strike packages for the static model

package	aircraft	configuration	number
n1	A-10	c1	2
	A-10	c2	1
n2	F-16	c1	2
	F/A-18	c1	2

The strike packages are assumed to be given information for the MIP. In practice, they are obtained from past experience and/or software that enumerates feasible combinations and evaluates them with weapons effectiveness models such as in appendix A.

In attacking a target, strike aircraft will typically be subject to threats from surface-to-air missiles (SAM), anti-aircraft artillery (AAA), etc. Mission success will depend on surviving these threats so they are a part of our models. However, we do not explicitly model the auxiliary aircraft in a strike package that might be responsible for dealing with these threats.

B. PARAMETERS

The data parameters used to define this model are:

1. Asset Data

acval(a)	value of aircraft a if preserved for later use
acavail(i,a)	number of aircraft a available at site i
range(a,c)	range in nautical miles of aircraft a in configuration c
psv(a,c,r)	probability of survival against threat type r for aircraft a in configuration c. This will be increased

	when considering the use of support missions (escort and SEAD) in the same package

2. Target Data

tthreat(*j*,*r*)	equals one if threat type *r* is present at target *j*
mindamage(*j*)	minimum required damage level on target *j*
tgtval(*j*)	target value

The minimum required damage on a target is sometimes specified with subjective terms like "light", "moderate" and heavy." Throughout the thesis, the minimum required damage on a target is interpreted numerically as a specified probability of kill (pk). The mapping between the subjective terminology and probabilities is given in Table 5.

Table 5. Levels of damage and probability of kill[3]

Damage Level	Description	pk
Light	Minor damage, some functions lost, but still capable of operation	0.3
Moderate	Extensive damage, many functions lost. Operation still possible but at reduced effectiveness	0.7
Heavy	Unable to operate	0.9

3. Strike Package Data

numweap(*a*,*c*,*w*)	number of weapon *w* carried by aircraft *a* in configuration *c*
numac(*n*,*a*,*c*)	number of aircraft *a* in configuration *c* flown in

---

[3] Class notes from Introduction to Naval Weapons Engineering (ES 310), Damage Prediction (http://www.fas.org/man/dod-101/navy/docs/es310/dam_crit/dam_crit.htm)

	strike package *n*
pk(*n,k*)	probability of kill for package *n* against target type *k*. The method for computing pk prior to the optimization is given in Appendix A
refuelmax	maximum number of air refuelings per strike
minpk	minimum probability of kill. Used to ensure that a package without sufficient damage capability will not be assigned to any target

4. Geographic Data

dist(*i,j*)	distance in nautical miles between site *i* and target *j*

5. Derived Data

nac(*n,a*)	number of aircraft *a* in package *n*
nw(*n,w*)	number of weapon *w* in package *n*
refuels(*n,i,j*)	number of refuelings needed by package *n* from site *i* when attacking target *j*. The aircraft in the package with the lowest range will dictate the value of this parameter

$$\mathrm{nac}(n,a) \equiv \sum_{c} \mathrm{numac}(a,c)$$

$$\mathrm{nw}(n,w) \equiv \sum_{a,c} \mathrm{numac}(n,a,c) \cdot \mathrm{numweap}(a,c,w)$$

$$\mathrm{refuels}(n,i,j) \equiv \max_{(a,c)\text{ s.t. numac}(n,a,c)>0} \left\{ \left\lfloor \frac{\mathrm{dist}(i,j)}{\mathrm{range}(a,c)} \right\rfloor \right\}$$

dpen(*n,i,j*)	distance penalty when package *n* from site *i* strikes target *j*. Following Crawford [1994], the distance

	penalty increases as the length of the mission approaches the range of the aircraft. When a combat radius is exceeded, the aircraft must refuel which causes a jump in the distance penalty
m1	proportionality constant of dpen for distance
m2	proportionality constant of dpen for refueling

$$\begin{aligned}\mathrm{dpen}(n,i,j) \equiv \mathrm{m1} \cdot & \max_{(a,c) \text{ s.t. } \mathrm{numac}(n,a,c)>0} \left(\frac{\mathrm{dist}(i,j)}{\mathrm{range}(a,c)} \right) \\ & + \mathrm{m2} \cdot \mathrm{refuels}(n,i,j)\end{aligned}$$

pkpen(*n*,*j*)	penalty imposed if package *n*’s probability of kill is different from the required damage for target *j*
m3	proportionality constant of pkpen when the probability of kill is not enough
m4	proportionality constant of pkpen when the probability of kill is greater than that required

$$\mathrm{pkpen}(n,j) \equiv \mathrm{m3} \cdot \frac{\mathrm{mindamage}(j) - \mathrm{pk}(n,j)}{\mathrm{mindamage}(j)} \quad \text{if } \mathrm{mindamage}(j) > \mathrm{pk}(n,j)$$

$$\mathrm{pkpen}(n,j) \equiv \mathrm{m4} \cdot \frac{\mathrm{pk}(n,j) - \mathrm{mindamage}(j)}{\mathrm{mindamage}(j)} \quad \text{if } \mathrm{pk}(n,j) > \mathrm{mindamage}(j)$$

attrition(*n*,*j*)	expected attrition if package *n* strikes target *j*
m5	proportionality constant of attrition

$$\mathrm{attrition}(n,j) \equiv \mathrm{m5} \cdot \sum_{(a,c) \text{ s.t. } \mathrm{numac}(n,a,c)>0} \mathrm{numac}(n,a,c) \cdot \left[1 - \prod_{r \in \mathrm{tthreat}(j,r)} \mathrm{psv}(a,c,r) \right]$$

6. Objective Function Data

stress(n,i,j)	penalty for imperfect damage, long distance (refueling) and low probability of survival against enemy defenses if package n is flown from site i against target j

$$\text{stress}(n,i,j) \equiv \text{dpen}(n,i,j) + \text{pkpen}(n,j) + \text{attrition}(n,j)$$

C. DECISION VARIABLES

The primary decision variables are binary. They allow for selection of which package is assigned to each target and which site provides the assets required.

$$\text{STRIKE}(n,i,j) = \begin{cases} 1 & \text{if strike package } n \text{ is assigned from} \\ & \text{site } i \text{ to target } j \\ 0 & \text{otherwise} \end{cases}$$

This variable has its domain restricted to implicitly enforce constraints and to make solution easier. In particular, STRIKE(n,i,j) exists only if all the following conditions hold for (n,i,j):

$$\begin{aligned} &\text{refuels}(n,i,j) < \text{refuelmax} \\ &\text{nac}(n,a) \le \text{acavail}(i,a) \qquad \forall a \ \text{ s.t. } \ \text{nac}(n,a)>0 \\ &\text{pk}(n,i,j) > \text{minpk} \end{aligned}$$

There are two additional sets of variables:

$\text{NOGO}(j) = 1$ if target j not attacked, zero otherwise

PRESERVE(i,a) number of aircraft from site i not assigned to any target

These variables have integer interpretations but can be treated as continuous variables: by virtue of the constraints' structure, they cannot fractionate in any feasible solution.

D. OBJECTIVE FUNCTION

The objective function has three components which may be weighted or solved for lexicographically:

$$\begin{aligned}
&\text{Minimize} \quad \sum_{j} \text{tgtval}(j) \cdot \text{NOGO}(j) \\
&\text{Minimize} \quad \sum_{n,i,j} \text{stress}(n,i,j) \cdot \text{STRIKE}(n,i,j) \\
&\text{Maximize} \quad \sum_{i,a} \text{acval}(a) \cdot \text{PRESERVE}(i,a)
\end{aligned}$$

The first component term charges a penalty when a target is not struck. The second component accounts for imperfect damage, long distance (refueling), and lower probability of survival against enemy defenses. The last one rewards savings of aircraft for use in future missions and unforeseen contingencies.

Throughout this thesis, multiple objective-function components are combined into a weighted sum, with positive weights on the terms to be minimized and negative weight on the terms to be maximized. It is also possible to do a lexicographic optimization in which the highest priority objective is optimized first; then it is constrained to its optimal value while the second priority objective is optimized; and the process continues as long as there are alternative optima at each stage [Rosenthal 1985].

E. CONSTRAINTS

The static model needs only two sets of constraints. The first set ensures that the targets are struck or penalties are incurred through the variables NOGO(j):

$$\sum_{n,i} \text{STRIKE}(n,i,j) + \text{NOGO}(j) = 1 \quad \forall j$$

The second set of constraints ensures that we only use aircraft that are available:

$$\sum_{n,j} \text{nac}(n,a) \cdot \text{STRIKE}(n,i,j) + \text{PRESERVE}(i,a) = \text{acavail}(i,a) \quad \forall i,a$$

F. RESULTS

We have implemented the static model using GAMS with CPLEX and XA as solvers. The results for a 100-target problem are given in Table 6.

Table 6. Static model results for 100 targets
Running on a Dell Workstation Precision 340 (Pentium IV 2 GHZ 1 GB RAM). GAMS IDE environment version 2.08.3, revision module 117, October 16, 2000.

	CPLEX (version 6.6.1)	XA
presolver	1 row and 9 columns eliminated	-
problem size	108 rows, 5900 columns, 12300 nonzeros	109 rows, 5909 columns, 18209 nonzeros
optcr = 0.05	< 1 sec	< 1 sec
optcr = 0.0	< 2 sec	< 3 sec

This problem also has seven bases with three types of aircraft and two possible configurations for each aircraft. There are three types of weapons and 20 different packages are allowed to be formed from the total of 156 aircraft available.

Observe that CPLEX applies a "presolver" phase, which reduces the size of the MIP. The parameter "optcr" is a relative measure of optimality, a bound on how far from the best possible answer we are; "optcr = 0.05" means we are no more than 5% off. The smaller the optcr, the more time is needed for the solver to find a solution.

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III. DYNAMIC MODEL WITH HOMOGENEOUS PACKAGES

This chapter presents the dynamic extension of the static optimization model in the previous chapter. Adding the time dimension allows explicit consideration of assigning aircraft to multiple sorties. In exchange for this added realism, the dynamic model introduces the restriction of homogeneous strike packages. Each package contains only one type of strike aircraft, and each aircraft in a package carries the same type of weapon. The model is summarized as follows (Table 7):

Table 7. Dynamic model summary

General	A MIP allocates homogeneous strike packages to targets over a multi-period time horizon
Objective Function	Linear combination or lexicographic optimization of four objectives: minimize target value of targets not assigned, minimize attrition, minimize distance penalty, and maximize value of unused aircraft
Campaign Objectives	Input as a required time on target and minimum damage for each target. Target prioritization based on target values
Aircraft Packaging	Strike packages are created for valid combinations of aircraft and configurations, but are restricted to be homogeneous. Packages are formed by aircraft of the same type, from the same site, with the same configuration. We also require that each aircraft deliver the same amount of only one type of weapon
Planning Horizon	The user specifies the present time, time of the first period, time between periods, and total number of periods

A. INDICES

a	aircraft	{‘FA18’, ‘A-6’, ‘A10’,...}
c	configuration	{‘C1’, ‘C2’, ‘C3’,...}
i	site	{‘Airbase-01’,...}
j	target	{‘Safwan’, ‘SCUD’,...}
k	target type	{‘Airfield’, ‘Bridge’,...}
n	strike package	{‘N1’, ‘N2’,...}
nac	number of aircraft	{‘NAC1’, ‘NAC2’,...}
nw	number of weapons	{‘NW1’, ‘NW2’,...}
r	threat type	{‘AAA’, ‘SAM’, ‘Air-to-Air’,...}
t	time period	{‘T1’, ‘T2’, ‘T3’,...}
w	weapon	{‘MK84’, ‘GBU31’, ‘AGM65’,...}

The index *nac* is a device for representing general integer variables as binary variables. For example, if a binary variable for a package with *nac* = “nac4” is positive, then there are four aircraft in the selected package. Although contemporary integer programming solvers handle general integer variables, we use this device to allow for the calculation of the nonlinear probability of kill prior to optimization. The index *nw* is similar to *nac* but counts weapons per aircraft, rather than aircraft per package.

B. PARAMETERS

1. Asset Data

numweap(*a*,*c*,*w*)	number of weapons of type *w* carried by aircraft *a* in its configuration *c*
cep(*a*,*w*)	circular error probable (in feet) of weapon *w*

	delivered by aircraft a. This is interpreted as the radius of a circle within which half of the delivered weapons are expected to fall
lethal(w,k)	lethal radius (in feet) of weapon w against target type k. Beyond the lethal radius, we consider the weapon to have no effect, whereas within the lethal radius it has constant effect. This "cookie-cutter" approach is the conceptually simplest kind of weapon effect model
wstate(w)	minimum weather state in which weapon type w can operate. Equals one for "poor," two for "marginal" and three for "good"
psv(a,c,r)	probability of survival against threat type r for aircraft a in configuration c. This will be increased when considering the use of support missions (escort and SEAD) in the same package
speed(a,c)	speed (in knots) of aircraft a in configuration c
acval(a)	aircraft value
recover(a,c)	recovery time (in minutes) of aircraft a to configuration c, includes all services (reloading, refueling, etc.) needed to prepare the aircraft for the next sortie upon return from another
range(a,c)	range (in nautical miles) of aircraft a in configuration c
acavail(i,a)	number of aircraft of type a available at site i
dueback(i,a,t)	number of aircraft of type a from site i to be available in time period t. For the first time period, this is the number of planes initially available. For

	later time periods, it refers to aircraft from a previous plan that have not yet returned from earlier sorties
dist(i,j)	distance (in nautical miles) from site i to target j (default is "great circle" if other data are not available)
pack(n,a,c,nac)	number of aircraft a with configuration c in package n. The packages are generated considering the realistic combinations of a, c and number of aircraft
refuelmax	maximum number of air refuelings for all packages
minpk	minimum probability of kill. Used to ensure that a package without sufficient damage capability will not be assigned

2. Target Data

tthreat(j,r)	equals one if threat type r is present at target j
mindamage(j)	desired damage level on target j, i.e., the minimum acceptable probability of kill for the strike package to be assigned to target j
tgtval(j)	target value: this information will be important to derive objective function coefficients
reqtot(j)	required time on target (TOT)
forecast(t)	weather state forecast for period t. Equals one for "poor," two for "marginal" and three for "good"

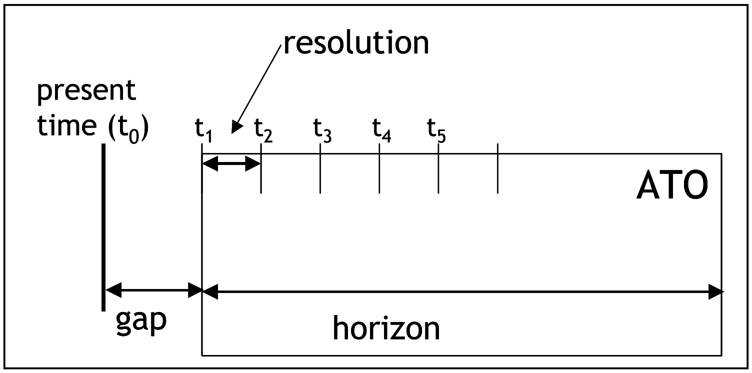

Figure 3. Time related parameters

Time in the dynamic model is discrete, which means we divide the planning cycle into a finite number of "periods." The planning of an ATO occurs at present time "t_0." Considering that some time is needed to process and disseminate the orders, we define a "gap" before the first time "t_1" when a launch can occur. We also define "resolution" as the interval between successive periods and "horizon" as the total cycle length, from t_1 to the end of the last period.

3. Time Control Parameters

The time related parameters described in Figure 3 are the following:

pt	present time (t_0)
gap	gap interval ($t_1 - t_0$), the minimum time required for an order to be completely processed
hz	horizon (t_{last}), the number of time periods
rs	resolution ($t_1 - t_2$), length of each time period

4. Derived Data

pk(*n*,*k*,*w*,*nw*)	probability of kill for package *n* with *nw* weapons *w* from each aircraft against target type *k*. Refer to appendix A for the calculations of this parameter
trvtime(*i*,*a*,*c*,*j*)	number of periods that aircraft *a* in configuration *c*

	will take to go from site *i* to target *j*
flt(*i*,*a*,*c*,*j*)	time (in minutes) required for a round-trip mission including recovery of aircraft *a* in configuration *c* from site *i* when attacking target *j*
flp(*i*,*a*,*c*,*j*)	number of periods required for a round-trip mission including recovery of aircraft *a* in configuration *c* from site *i* when attacking target *j*
refuels(*i*,*a*,*c*,*j*)	number of refuelings needed by aircraft type *a* with configuration *c* from site *i* when attacking target *j*

$$\text{trvtime}(i,a,c,j) \equiv \left\lceil \frac{60 \cdot \text{dist}(i,j)}{\text{rs} \cdot \text{speed}(a,c)} \right\rceil$$

$$\text{flt}(i,a,c,j) \equiv 2 \cdot \frac{60 \cdot \text{dist}(i,j)}{\text{speed}(a,c)} + \text{recover}(a,c)$$

$$\text{flp}(i,a,c,j) \equiv \left\lceil \frac{\text{flt}(i,a,c,j)}{\text{rs}} \right\rceil$$

$$\text{refuels}(i,a,c,j) \equiv \left\lfloor \frac{\text{dist}(i,j)}{\text{range}(a,c)} \right\rfloor$$

dpen(*i*,*a*,*c*,*j*)	distance penalty when aircraft *a* in configuration *c* from site *i* strikes target *j*
m1	proportionality constant of dpen for distance
m2	proportionality constant of dpen for refueling

$$\text{dpen}(i,a,c,j) \equiv \text{m1} \cdot \frac{\text{dist}(i,j)}{\text{range}(a,c)} + \text{m2} \cdot \text{refuels}(i,a,c,j)$$

pkpen(*n*,*j*,*w*,*nw*)	penalty imposed if package *n*'s probability of kill is different from the required pk for target *j*
m3	proportionality constant for pkpen when the

	probability of kill is less than required
m4	proportionality constant for pkpen when the probability of kill is greater than that required

$$\text{if } \ \text{mindamage}(j) > \text{pk}(n,j,w,nw):$$

$$\text{pkpen}(n,j,w,nw) \equiv \text{m3} \cdot \frac{\text{mindamage}(j) - \text{pk}(n,j,w,nw)}{\text{mindamage}(j)}$$

$$\text{if } \ \text{pk}(n,j,w,nw) > \text{mindamage}(j)$$

$$\text{pkpen}(n,j,w,nw) \equiv \text{m4} \cdot \frac{\text{pk}(n,j,w,nw) - \text{mindamage}(j)}{\text{mindamage}(j)}$$

dnacc(*n,a,c*)	number of aircraft *a* in configuration *c* flying with strike package *n*
dnac(*n,a*)	number of aircraft *a* in package *n*

$$\text{dnacc}(n,a,c) \equiv \sum_{nac} nac \cdot \text{pack}(n,a,c,nac)$$

$$\text{dnac}(n,a) \equiv \sum_{c} \text{dnacc}(n,a,c)$$

attrition(*n,j*)	expected attrition if package *n* strikes target *j*

$$\text{attrition}(n,j) \equiv nac \cdot \left[1 - \prod_{r \in \text{tthreat}(j,r)} \text{psv}(a,c,r) \right] \quad \forall a,c,j,nac$$

pot(*j,t*)	equals one if required time on target for target *j* is during period *t*, zero otherwise. Derive pot according to:

$$\text{pot}(j,t) = 1 \quad \text{if } \ \text{rs} \cdot (t-1) \le \text{reqtot}(j) - \text{pt} - \text{gap} < \text{rs} \cdot t$$

C. DECISION VARIABLES

$$\text{STRIKE}(n,i,j,w,nw,t) = \begin{cases} 1 & \text{if package } n \text{ coming from site } i \text{ is assigned} \\ & \text{to strike target } j \text{ with } nw \text{ weapons of type } w \\ & \text{from each airplane with take-off during period } t \\ 0 & \text{otherwise} \end{cases}$$

The binary variable STRIKE(n,i,j,w,nw,t) exists only if the following conditions hold:

$$\begin{array}{ll} \text{pot}(j, t+\text{trvtime}(i,a,c,j)) = 1 & \forall a,c \;\; \text{s.t. } \text{dnacc}(n,a,c) > 0 \\ \text{refuels}(i,a,c,j) < \text{refuelmax} & \forall a,c \;\; \text{s.t. } \text{dnacc}(n,a,c) > 0 \\ \text{assign}(i,a) > 0 & \forall a \;\; \text{s.t. } \text{dnac}(n,a) > 0 \\ \text{wstate}(w) \le \text{forecast}(t) & \\ \text{pk}(n,j,w,nw) > \text{minpk} & \end{array}$$

NOGO(j)	equals one if target j is not attacked, zero otherwise (continuous variable)
PRESERVE(i,a,t)	number of aircraft a from site i not assigned to any target in period t

The last two variables have integer interpretations but can be treated as continuous. By virtue of the constraints' structure, the variables cannot be fractional in any feasible solution.

D. OBJECTIVE FUNCTIONS

$$\min \quad \sum_j \text{tval}(j) \cdot \text{NOGO}(j)$$

$$\max \quad \sum_{i,a,t} \text{acval}(a) \cdot \text{PRESERVE}(i,a,t)$$

$$\min \quad \sum_{n,i,a,j,w,nw,t \in \text{dnac}(n,a)} [\text{dpen}(i,a,j) + \text{pkpen}(n,j,w,nw) + \text{attrition}(n,j)] \cdot \text{STRIKE}(n,i,j,w,nw,t)$$

E. CONSTRAINTS

Strike all targets or don't

$$\sum_{n,i,w,nw,t} \text{STRIKE}(n,i,j,w,nw,t) + \text{NOGO}(j) = 1 \quad \forall j$$

Balance of aircraft

$$\begin{aligned}\sum_{n,j,w,nw} \text{dnac}(n,a) \cdot \text{STRIKE}(n,i,j,w,nw,t) + \text{PRESERVE}(i,a,t) = \\ \text{dueback}(i,a,t) + \text{PRESERVE}(i,a,t-1) + \\ \sum_{n,c,j,w,nw} \text{dnacc}(n,a,c) \cdot \text{STRIKE}(n,i,j,w,nw,t-\text{flp}(i,a,c,j)) \quad \forall\ i,a,t\end{aligned}$$

The left-hand side of a balance constraint is the number of aircraft of type *a* that launch from site *i* during period *t* or remain on the ground for future use. The right-hand side of the balance constraint is the sum of previously preserved aircraft and those returning from earlier mission during period *t*. Earlier missions represented by the dueback parameter were launched prior to t_1, whereas the longer term on the right-hand side represents returning missions launched after t_1.

F. RESULTS

We have implemented the dynamic model using GAMS with CPLEX and XA as solvers. The results for 100 targets and 16 periods of 15 minutes each are given in Table 8.

This problem has seven bases with three types of aircraft and two possible configurations for each aircraft. There are three types of weapons and 24 different packages are allowed to be formed from a total of 36 available aircraft.

Table 8. Dynamic model results for 100 targets

Running on a Dell Workstation Precision 340 (Pentium IV 2 GHZ 1 GB RAM). GAMS IDE environment version 2.08.3, revision module 117, October 16, 2000.

	CPLEX (version 6.6.1)	XA
presolver	1213 rows and 1219 columns eliminated	-
problem size	98 rows, 10093 columns, 20094 nonzeros	1361 rows, 11362 columns, 32661 non-zeros
optcr = 0.05	< 1 sec	< 1 sec
optcr = 0.0	201 sec	> 1200 sec

IV. ALLOCATION WITH PERSISTENCE

Mathematical programming models can be stated and solved so that they exhibit varying degrees of persistence with respect to previous values of variables, constraints, or even exogenous considerations [Brown, Dell and Wood 1997]. The importance of the persistence emerges when small changes to input data lead to drastically different solutions.

A. PERSISTENCE FOR THE STATIC MODEL

The following scenario provides an example of how to achieve better results by considering persistence. Initially, we have a list of ten targets to be struck and a limited number of aircraft available. Table 9 shows the possible combinations of packages:

Table 9. Packages for the static model's scenario

package	aircraft	configuration	number of a/c
n01	A-10	c1	2
n02	A-10	c1	4
n03	A-10	c1	2
	A-10	c2	2
n04	F-16	c1	1
	F/A-18	c1	1
n05	F16	c2	1
	F/A-18	c2	1
n06	F16	c1	2
	F/A-18	c1	2

The configurations differ with respect to weapon loading, as described in Chapter II. These aircraft are located (refer to the map in Figure 4) at seven bases as follows (Table 10):

Table 10. Distribution of aircraft for example

base	A-10	F-16	F/A-18
OEAH	8		
KHARJ		4	
OEJB			4
OEDF		4	
OERY	4		
OERK		2	2
CV71			8

In this demonstration, we first run the static model with this information but consider only eight targets. Then we change the scenario and run the model again with the complete list of ten targets. This procedure simulates a situation in which new information becomes available moments after the first plan is disseminated. The added targets have more importance and must be attacked. Changes between the two solutions are highlighted in Table 11.

Table 11. Results for two consecutives runs of the static model
The first column shows the list of targets to be struck, but the first run only eight targets are available when the plan is generated. The second run simulates the situation where new information is suddenly available. Two targets with higher priority appear. The model generates a completely new plan.

target	1st run (8 targets)				2nd run (10 targets without persistence)			
	pckg	base	distance (NM)	pk	pckg	base	distance (NM)	pk
Safwan	n01	OEAH	353	.95	n01	OEAH	353	.95
Al Asad	n01	OEAH	724	.89	n01	OERY	678	.89
H2	n01	OEAH	773	.95	n01	OERY	703	.95
H3 Airbase	n01	OERY	707	.95	NOT STRUCK			
H3 Highway	n01	OERY	719	.95	n05	OERK	705	.91
Wadi Al Khirr	n04	OERK	495	.90	n04	OERK	495	.90
Tallil Air Base	n01	OEAH	442	.95	n01	OEAH	442	.95
Rasheed	n05	OERK	590	.79	NOT STRUCK			
Safwan2	-	-	-	-	n01	OEAH	353	.89
Safwan3	-	-	-	-	n01	OEAH	353	.84

Figure 4. Location of targets and bases used in the examples

The result is that we have the important targets struck but five out of eight original allocations show major changes. We would prefer a new plan for which the addition of two targets has a lower impact. This is why we need a persistent model.

The key outputs of the static model are optimal values of STRIKE(n,i,j) which equals one if strike package n is assigned from site i to target j. The persistent model will incorporate these results through the following parameters:

$$\text{prev}(n,i,j) = \text{ previous optimal value of STRIKE}(n,i,j)$$
$$\text{previj}(i,j) = \sum_n \text{prev}(n,i,j)$$
$$\text{prev\_nogo}(j) = \text{ previous optimal value of NOGO}(j)$$

A "persistence penalty" is applied to the new optimal value of STRIKE(n,i,j) as follows

route_pen	persistence penalty for changing the base from which a target is attacked
pkg_pen(n,n')	persistence penalty for keeping the same base from which a target is attacked but changing the package used from n' to n

$$\text{pers\_pen}(n,i,j) = \begin{cases} \text{route\_pen} \text{ if } \text{previj}(i,j) = 0 \text{ and } \text{prev\_nogo}(j) = 0 \\ \text{pkg\_pen}(n,n') \text{ if } \text{prev\_nogo}(j) = 0 \text{ and } \text{prev}(n',i,j) = 1 \\ 0 \quad \text{otherwise} \end{cases}$$

The value of the pkg_pen parameter should be small when there is just a change in configuration, it should be larger if the number of aircraft of the same type increases, and should be yet larger if there is a change in aircraft type.

Finally, the objective function will incorporate:

$$\text{Minimize} \quad \sum_{n,i,j} \text{pers\_pen}(n,i,j) \cdot \text{STRIKE}(n,i,j)$$

For the example, the objective function contains the following terms:

$$\begin{aligned}\text{Minimize} \quad & \sum_{j} \text{tgtval}(j) \cdot \text{NOGO}(j) \\ & + \sum_{n,i,j} \text{stress}(n,i,j) \cdot \text{STRIKE}(n,i,j) \\ & + \sum_{n,i,j} \text{pers\_pen}(n,i,j) \cdot \text{STRIKE}(n,i,j)\end{aligned}$$

Table 12. Results for runs of the static model with persistence
The same situation described in Table 11 is analyzed. But the model with persistence generates a plan with a reduced number of modifications of the original plan.

target	1st run (8 targets)				2nd run (10 targets with persistence)			
	pckg	base	distance (NM)	pk	pckg	base	distance (NM)	pk
Safwan	n01	OEAH	353	.95	n01	OEAH	353	.95
Al Asad	n01	OEAH	724	.89	n01	OEAH	724	.89
H2	n01	OEAH	773	.95	n01	OEAH	773	.95
H3 Airbase	n01	OERY	707	.95	NOT STRUCK			
H3 Highway	n01	OERY	719	.95	n01	OERY	719	.95
Wadi Al Khirr	n04	OERK	495	.90	n04	OERK	495	.90
Tallil Air Base	n01	OEAH	442	.95	n01	OEAH	442	.95
Rasheed	n05	OERK	590	.79	NOT STRUCK			
Safwan2	-	-	-	-	n01	OERY	380	.89
Safwan3	-	-	-	-	n01	OERK	363	.84

The solution based on the persistent model retains as much as possible from the original plan (Table 12). Only two modifications are necessary to accommodate the new targets. Table 13 shows that the persistent solution is sub-optimal by only 1%; comparing Tables 11 and 12 shows that the persistent solution would be easier to implement.

Table 13. Summary of the results for the static model example

	original 8 targets run	10 targets without persistence	10 targets with persistence
Major changes			
Targets not struck	-	2	2
Change of base	-	3	0
Total	-	5	2
Objective function value			
Value of targets not struck	-	180.00	180.00
Stress	206.67	151.75	155.20
Total	206.67	331.75	335.20

B. SUCCESSIVE EXECUTION OF THE DYNAMIC MODEL

Before developing the persistent version of the dynamic model, we need to address re-execution of the model in successive time periods.

Figure 5 represents the results of two successive executions. Time t_0 is the current time when the second problem is run. Time t_1 is the beginning of the first time period of the second run. If the first run plans a sortie for launch before t_1 and returns after t_1, then it must be regarded as a fixed decision in the second run. On the other hand, if the first execution plans a sortie that launches after t_1, then this part of the plan can possibly be changed. The persistent model allows, but discourages, these changes. To account for previous plans, we define:

prev_etd(n,i,j)	previous estimated time of departure of package n from site i to attack target j, derived from the previous optimal values of STRIKE(n,i,j,w,nw,t)
prev_flt(i,a,c,j)	flight time of aircraft a in configuration c from site

	i to target *j* in previous solution
prev_nogo(*j*)	previous optimal value of the variable NOGO(*j*)
fixed(*j*)	equals one if it is too late to change plans for target *j*, zero otherwise

This information and the updated time control parameters are used to specify fixed decisions and to re-derive the parameters dueback(*i*,*a*,*t*).

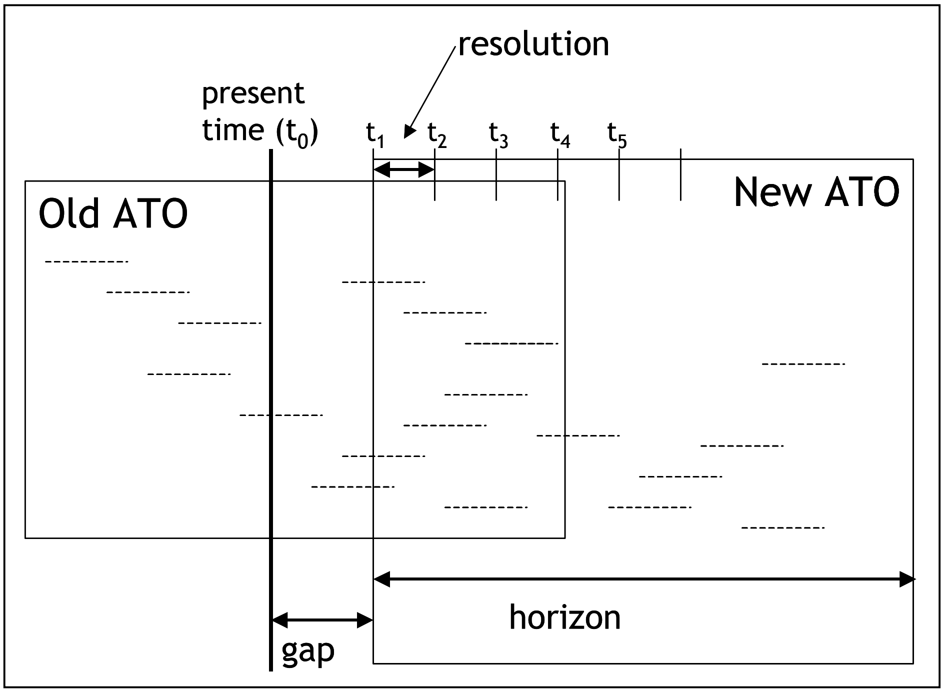

Figure 5. Super-imposition of a new plan onto an old plan

This figure represents the results of two successive executions of the dynamic model. Horizontal lines represent the selected sorties. Sorties launched between t_1 and t_4 will be considered by both plans.

C. PERSISTENCE FOR THE DYNAMIC MODEL

The persistent dynamic model can be re-applied many times, re-allocating strike packages considering new targets, threats and changes in priorities, weather or asset availability.

The scenario for illustrating persistence in the dynamic model is the same as for the static model except that the packages are formed under the conditions described in Chapter III (refer to Table 7). The targets are given in Table 14, including the required time on target.

Table 14. Target data example for the dynamic persistent model

target	latitude		longitude		target value	time on target	minimum damage
	deg	min	deg	min			
Safwan	30	08	47	39	100	1000	.80
Al Asad	33	47	42	26	150	1015	.75
H2	33	21	40	35	160	1140	.90
H3 Airbase	32	55	39	44	100	1055	.50
H3 Highway	32	50	39	18	180	1100	.40
Wadi Al Khirr	31	25	43	11	150	1035	.85
Tallil Air Base	30	56	46	05	200	1200	.85
Rasheed	33	16	44	29	80	1040	.50

There are eight targets, it is 0800, and one hour is the minimum preparation time for the first departure. Figure 6 shows the result of the first run.

Now suppose it is 0815 and we want to run the model again. All the data is the same, except for a change in the weather forecast from good to marginal, which affects the pk's. Two of the launches from the 0800 solution were planned for 0900, so it is too late to change these decisions. These fixed decisions are indicated by asterisks in the 0815 solution, which appears in Figure 7.

```
Air Tasking Order (generated at:  800)

TARGET          TOT  ETD  A/C     BASE  DIST CONFIG   WEAPON  PK
------------------------------------------------------------------
Safwan          1000  930 2 FA18  OEDF   194 config02 3 MK-83 0.81
Al Asad         1015  900 2 FA18  CV71   587 config01 2 MK-84 0.75
H2              1140  900 2 A10   OERY   703 config01 4 MK-82 0.90
H3 Airbase      1055  915 1 FA18  CV71   679 config01 2 MK-84 0.50
H3 Highway      1100  930 1 F16   OERK   705 config02 2 MK-84 0.45
Wadi Al Khirr   1035  915 2 FA18  CV71   453 config02 4 MK-83 0.88
Tallil Air Base 1200 1030 2 A10   OERY   432 config02 2 MK-83 0.85
Rasheed         1040  930 1 FA18  CV71   479 config01 2 MK-84 0.50
```

Figure 6. Solution to the dynamic model run at 0800

The output for the dynamic model is an ATO with the following information: name of the target, required time on target (TOT), estimated time of departure (ETD), number and type of aircraft participating on the attack (A/C), distance (in nautical miles) from the base to the target, configuration to be used, number and type of the weapon to be delivered and the expected probability of kill.

```
Air Tasking Order (generated at:  815)

TARGET          TOT  ETD  A/C     BASE  DIST CONFIG   WEAPON  PK  FIX
---------------------------------------------------------------------
Safwan          1000  930 2 FA18  OEDF   194 config02 3 MK-83 0.81
Al Asad         1015  900 2 FA18  CV71   587 config01 2 MK-84       *
H2              1140  900 2 A10   OERY   703 config01 4 MK-82       *
H3 Airbase      1055  915 1 F16   OERK   693 config01 2 MK-83 0.53
H3 Highway      1100  930 1 FA18  CV71   699 config02 2 MK-83 0.46
Wadi Al Khirr   1035  930 2 FA18  CV71   453 config02 4 MK-83 0.88
Tallil Air Base 1200 1030 2 A10   OERY   432 config02 2 MK-83 0.85
Rasheed         1040  915 1 F16   OEDF   567 config01 2 MK-83 0.53
```

Figure 7. Solution to the dynamic model run at 0815 without persistence

Sorties marked with an asterisk are fixed. Differences from the 0800 solution are highlighted in bold.

The pk's for the fixed sorties are left blank. This is because the previously assigned weapon may be ineffective in the marginal weather state, but it is too late for this run of the model to consider changing weapons.

The version of the dynamic model used to obtain this solution does not encourage persistence. Different bases and aircraft are assigned for three of the six targets not constrained by the earlier run. This is a major disruption of the plan in only 15 minutes.

The next step is to implement the persistence penalties in the same way we did for the static model in section A. Penalty parameters route_pen and pkg_pen(n,n',t) are applied to the new optimal value of STRIKE(n,i,j,w,nw,t), for changing routes and packages. We define the parameter pkg_pen(n,n',t) using the following criteria. Assume package n uses nac aircraft of type a in configuration c; similarly define n', a', nac', and c'. Then pkg_pen(n,n',t) = 0 if $n = n'$ otherwise, and pkg_pen(n,n',t) is the sum of penalties as follows:

p1	persistence penalty for different aircraft type
p2	persistence penalty for smaller number of aircraft
p3	persistence penalty for greater number of aircraft
p4	persistence penalty for different configuration
prev_forecast(t)	previous weather forecast for period t

$$\text{p1 if } a \neq a'$$
$$\text{p2} \cdot (nac' - nac) \ \text{ if } a \neq a' \text{ and } nac < nac'$$
$$\text{p3} \cdot (nac - nac') \ \text{ if } a \neq a' \text{ and } nac > nac'$$
$$\text{p4 if } a \neq a', c \neq c' \text{ and forecast}(t) \neq \text{prev\_forecast}(t)$$

In other words, there is no penalty when the same package is assigned in the second solution as in the first but there are cumulative penalties for using different types of aircraft, different number of aircraft or different weapon configurations. However, there is no penalty for changing weapons when the weather changes.

The persistence penalties applied to STRIKE(n,i,j,w,nw,t) variables in the dynamic model are exactly the same as we had for the static model:

$$\text{pers\_pen}(n,i,j,t) = \begin{cases} \text{route\_pen} \ \text{ if } \ \text{previj}(i,j)=0 \ \text{ and } \ \text{prev\_nogo}(j)=0 \\ \text{pkg\_pen}(n,n',t) \ \text{ if } \ \text{prev\_nogo}(j)=0 \ \text{ and } \ \text{prev}(n',i,j)=1 \\ 0 \quad \text{otherwise} \end{cases}$$

Finally, the dynamic persistent model has the following objective function:

$$\begin{aligned} &\text{Minimize} \\ &\sum_{n,i,j,w,nw,t} \text{pers\_pen}(n,i,j,t)\cdot \text{STRIKE}(n,i,j,w,nw,t) \\ &+\sum_{j} \text{tval}(j)\cdot \text{NOGO}(j) \\ &+\sum_{n,i,a,j,w,nw,t} \begin{bmatrix} \text{dpen}(i,a,j)+\text{pkpen}(n,j,w,nw) \\ +\text{attrition}(n,j)+\text{dnac}(n,a)\cdot \text{acval}(a) \end{bmatrix} \cdot \\ &\qquad\qquad\qquad\qquad\qquad \text{STRIKE}(n,i,j,w,nw,t) \end{aligned}$$

Using this model, we are able to create the persistent solution presented in Figure 8.

```
Air Tasking Order (generated at:  815)/p

TARGET          TOT  ETD  A/C     BASE  DIST CONFIG   WEAPON  PK  FIX
---------------------------------------------------------------------
Safwan          1000  930 2 FA18  OEDF   194 config02 3 MK-83 0.81
Al Asad         1015  900 2 FA18  CV71   587 config01 2 MK-84       *
H2              1140  900 2 A10   OERY   703 config01 4 MK-82       *
H3 Airbase      1055  915 1 FA18  CV71   679 config02 2 MK-83 0.57
H3 Highway      1100  930 1 F16   OERK   705 config01 2 MK-83 0.53
Wadi Al Khirr   1035  915 2 FA18  CV71   453 config02 4 MK-83 0.88
Tallil Air Base 1200 1030 2 A10   OERY   432 config02 2 MK-83 0.85
Rasheed         1040  930 1 FA18  CV71   479 config02 2 MK-83 0.57
```

Figure 8. Solution to the dynamic model run at 0815 with persistence

In this persistent solution, only three minor changes occur; they are noted in bold type.

All bases remain the same for this solution. The three configuration changes were expected because in the original plan the MK-84 weapon is used many times but is less effective under the new weather forecast. As before, all targets are struck. Table 15 summarizes and compares the results for each run.

Table 15. Summary of the results for the dynamic model example

	original run	subsequent run without persistence	subsequent run with persistence
Major changes	-	6	0
Change of base	-	3	0
Change of aircraft type	-	3	0
Objective function value	33.978	34.271	35.726

The objective function values reported in Table 15 exclude persistence penalties and the terms attributed to fixed decisions. The original run has the best objective function value, because it assumes better weather.

V. CONCLUSION

This research has developed models for optimally composing air strike packages and assigning these packages to strike a set of prioritized targets. A package consists of aircraft from various bases or carriers along with appropriate weapons for the assigned target. The models ensure that the aircraft have sufficient range, time on target and the right weapons so that a sufficiently high probability of kill is achieved for each assigned target.

Two basic models are created, a static one and a dynamic one. The static model covers a short time frame during which an aircraft would fly at most one sortie. This model allows the creation of heterogeneous strike packages with different sorts of strike aircraft. The dynamic model covers a longer time frame during which an aircraft may be involved in multiple strikes. For computational reasons, this model only considers homogeneous packages, i.e., packages containing a single type of aircraft.

Both models have "persistent" variants. A persistent model is important when a strike plan is already in place, new high-priority targets arise, and a new plan must be developed. The persistent model creates a new strike plan that disrupts the old one as little as possible — this is important because much time-consuming work has probably already been invested in implementing the original plan — yet is still near-optimal in the mathematical sense.

All models precompute nonlinear probabilities of kill for potential strike packages. Binary variables represent the assignment of these potential packages to targets so that a mixed-integer linear program results. An instance of the dynamic model with three weapon types, 36 aircraft from seven bases and 100 potential targets is solved in less than three minutes on a personal computer.

A. REFINEMENTS

1. Weapons Effectiveness Calculations

Our models use simple, fast and well-known algorithms to calculate the probability of kill for a combination of assets, from information such as a weapon's lethal radius and circular error probable. However our algorithms do not cover all weapon and target types, and depending on the nature of the application, other algorithms should be used. The Joint Technical Coordinating Group for Munitions Effectiveness (JTCG/ME) Program, for example, develops analytical methods for measuring and predicting munitions effectiveness [USAF 1998]. This group has also produced a large body of scientifically valid data related to specific weapons, munitions, and appropriate targets. Their models are more sophisticated than ours and include such details as aircraft capabilities and configurations; target characteristics, such as size, shape, and hardness; and delivery parameters such as altitudes, speeds, and dive angles. These models and algorithms, or other options, should be considered for refining our models' probability-of-kill calculations.

2. Weather Effects

The way we handle weather is perhaps too simple. We only modify weapons-effectiveness calculations based on three weather states, but the real situation is more complex. Even slight changes in humidity, ambient light, or intensity of precipitation can significantly degrade weapons systems performance. Sometimes, a pilot will detect a target, but his weapons systems cannot "see" the target because of weather conditions. The results are an increase in ammunition expenditure, and degradation in time-on-station performance and mission accomplishment [Cohen 1993]. Consequently, we believe that more refined weather information could be usefully incorporated into our models.

B. FURTHER RESEARCH PROJECTS

There are several potential projects that can be pursued to extend the models presented in this thesis. This thesis has only addressed the strike aircraft of an air strike package. Additional work, in both the static and dynamic models, will be required to include specialized escort aircraft and aircraft for suppression of enemy air defenses. However, our basic model paradigm should extend directly: we precompute the set of potential strike packages and the model optimally selects a feasible subset of those packages and assigns them to targets.

An important extension of the dynamic model would allow the model to choose the optimal time on target. The target list should specify a time window for the strike and let the model choose time and the best assets to use. A more difficult extension of the dynamic model would incorporate heterogeneous packages.

Another challenging project would permit the redirection of previously committed sorties. In an extension of the current dynamic model, aircraft enroute to targets could possibly be redirected to a just-identified, higher-priority target, or to a secondary target if the original target is no longer appropriate for some reason (because of changes in local weather conditions, new battle-damage information shows the target has already been destroyed, etc.). New variables will be necessary to monitor each aircraft's position and to determine which targets are close enough to be struck and how much air refueling is needed.

Another extension of this work would allow strike package sorties to strike multiple targets. One way this could happen is if the package stays intact and visits two targets with weapon loadouts sufficient for both. A more difficult scenario to model and solve would allow a package to strike a primary target and then divide into "sub-packages" to strike secondary targets.

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APPENDIX A. CALCULATIONS OF THE PROBABILITY OF KILL

This appendix presents the formulation used to calculate the probability of kill of the packages created for the static and dynamic models. The reference for this subject is "Notes on Firing Theory" [Washburn 2002].

To calculate the probability of kill we should consider solving an independent sequence of *n* aircraft delivering sets of weapons arranged in dependent shots. The independent part is easily formulated as

$P_{kill\ of\ \mathrm{n}\ indep\ \mathrm{salvos}} = 1 - \left[(1-P_{k_1})\cdot(1-P_{k_2})\ldots(1-P_{k_n})\right] = 1 - \prod_{i=1}^{n}(1-P_{k_i})$ where P_{k_i} is the probability of kill of the *i*-th aircraft. If the package has only one type of aircraft and each aircraft delivers the same quantity and type of weapons, the formula simplifies to

$P_{kill\ of\ n\ \mathrm{identical}\ indep\ \mathrm{salvos}} = 1-(1-P_k)^n$ where n is the number of aircraft and P_k is constant among them.

In both cases we still need to calculate the probability of kill for each aircraft. In the case where we have different types of aircraft and weapons in the package, the problem is more complex: each aircraft has its own delivery profile (angle, velocity, etc.) for some type of weapon that will be different from another aircraft, even if all aircraft are launching the same type of weapon. Those particularities will be reflected as different parameters for each aircraft-weapon pair.

However, we can have all the values of pk calculated in advance for each package against each type of target. For the purpose of this thesis, we apply a simplified assumption described in Washburn [2002]. We consider the weapon having a "diffuse Gaussian" (DG) damage function which, more realistically, should be applied only to weapons that kill by fragmentation. The DG assumption produces a simple expression for P_{k_i}:

$P_{k_i} = 1 - E\left(\prod_{q=1}^{s}\left\{1 - P_q(U,V)\right\}\right)$ is the probability of kill of a collection of s shots from a given aircraft, where the expected value is with respect to the normal distribution of the random common error (U,V). Each aircraft's shot can potentially have distinct parameters for dispersion and lethality which allows us to determine the result for a heterogeneous package.

For the homogeneous package, we start with the simple case of calculating the probability of kill of a single shot:

$P_{kill} = 1 - (.5)^{(R^2/CEP^2)}$ where R is the lethal radius of a "cookie cutter" air-to-ground weapon and CEP is the circular error probable ($CEP = 1.1774 \cdot \sigma$ where σ is the standard deviation associated to the weapon).

The first parameter is dependent on the type of weapon and the nature of the target. The second is a function of the weapon's aerodynamics and aircraft's delivery profile.

Now, *b* weapons launched from the same platform represent dependent shots subject to the same bias and pattern of the distribution of the hits on the ground, so we use the result that Washburn calls the "confetti approximation:"

$$P_k = 1 - (1 + \sqrt{2 \cdot Z}) \cdot \exp(-\sqrt{2 \cdot Z}) \text{ where } Z = \frac{bR^2}{2\sigma^2}$$

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