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# A model for allocation of sewage treatment systems to the Naval Fleet

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



A MODEL FOR ALLOCATION OF SEWAGE TREATMENT SYSTEMS

TO THE NAVAL FLEET

by

Edward A. Brill

July 1972

Approved for public release; distribution unlimited.



NAVAL POSTGRADUATE SCHOOL  
Monterey, California

Rear Admiral M. B. Freeman  
Superintendent

M. U. Clauser  
Provost

ABSTRACT

The following paper discusses the problem of planning the allocation of sewage treatment systems to the Naval Fleet. We assume that the obsolescence or deterioration process of a ship is a semi-Markov process. For the special case of deterministic deterioration, the total discounted cost of revamping a group of ships is determined as a function of the time horizon chosen to do so. This function is graphed for a set of representative cost factors. Finally, the semi-Markov and Markov models are discussed with a view towards a future simulation study.

Prepared by:



## Executive Summary

The following paper discusses the problem of planning the allocation of sewage treatment systems to the Naval fleet. In addition to important considerations such as scheduling constraints and cries for clean water, there is the matter of cost. It is the purpose of this paper to ascertain the cost of revamping the fleet as a function of the time horizon chosen to do so. Once this is out of the way, it is assumed that the conflicting pressure groups may compromise intelligently based on their knowledge of the costs involved.

The deterministic model set forth assumes ships to have a fixed and known life expectancy. When cost is the only criterion for choosing between the alternatives of repiping an old ship or replacement, it is more cost effective to repipe provided the ship has another year of useful life. This is due to the time discounting factor of money. The total discounted cost of revamping 100 ships as a function of the time horizon is tabulated and graphed in Figure 1 for representative cost parameters. Finally, a stochastic model is introduced with implications to future analysis.



## 1. Introduction.

Disposal of sanitary wastes onboard U. S. Navy ships has been dealt with somewhat cavalierly for a long time. Waste has been simply dumped overboard, both on the high seas and in restricted harbors. Strong social pressure, threat of economic sanctions, congressional directives and, of course, real fear of ecological damage have made it mandatory for the Navy to initiate a widespread policy of sewage treatment. At present the Navy has contracted several major R&D projects in an effort to develop and implement operational sewage treatment systems onboard its ships (Kinney [1]). Progress on this front has been slow and erratic, but soon choices will be made among several proposed sewage treatment systems, with a corresponding assignment to each type of ship in the Naval fleet. These choices will take into account such factors as:

- a. weight and space requirements of system.
- b. chemical composition of effluent discharged.
- c. flexibility of system to changes in pollution standards.
- d. installation costs.
- e. reliability and maintenance factors.
- f. power requirements of the system.

Once this choice is made, another important policy will have to be determined. This policy basically will deal with the following questions:

1. How quickly should the Navy revamp its fleet (i.e. install sewage systems)?

2. Which ships should be replaced by new ships with sewage systems, and which should be repiped to accommodate a sewage system?

The purpose of this paper is to formulate the major issues at the heart of these complex problems and to model the revamping process in such a way that policymakers may obtain "qualitative" insights into the nature of these complexities. As for quantitative results, we conclude these for the case where deterministic deterioration is assumed for all ships. To find similar results for stochastic deterioration will require a large-scale simulation study. We allude to such a model in the latter sections.

## 2. Basic Problem Formulation.

In order to get some handle on how to effectively manage the Navy's revamping process, it is first necessary to understand the operational characteristics of two polarized policies, both of which assume that a choice of sewage system has already been made for each of several homogeneous groups of ships (e.g. destroyers, cruisers, aircraft carriers) in the fleet. These policies are:

1. Let a group of ships phase out "naturally," ultimately to be replaced by the next generation equipped with sewage treatment systems.
2. Revamp all ships in a group within one year.

These policies represent the viewpoints of two polarized interest groups--avid ecologists versus conservative budgeters--and it is of course clear that neither is feasible. However, in order to play the game of compromise, we should be able to ascertain the costs and scheduling operations of each policy to give us the bounds within which a compromise is to be reached. We shall assume that a compromise is in the form of a specified time horizon. As for other relatively unquantifiable considerations such as fear of ecological disaster and fear of a low military posture, we should debate these vigorously once the mundane matters of cost and scheduling are conquered.

Consider now policy number 1. To analyze the operational characteristics of this policy we must in some fashion model the deterioration or obsolescence process of a ship. We assume, therefore,

that there exists an ordinal classification of ships according to each ship's general "state of health." Let the classification be summarized by a state space  $S = \{0,1,2,\dots,d\}$  where  $d$  is some number which signifies "too old, or sick, or out of date to keep as a functional vessel." State 0 corresponds to a new ship. We shall view the state of a ship the instant prior to the decision to repipe or replace.

As a first approximation we could assume a deterministic deterioration process where in effect each ship is assumed to be the average ship with life expectancy of about 25 years, let us say. In this case, we would simply let  $d = 24$  and proceed with the analysis as described below. However, we choose to describe the deterioration process, at present as a random process. In the final section, we will in particular discuss the general structure of a semi-Markov model in which case both the deterministic model and Markov model are special cases. This will provide us with a general and unified treatment which will point the way to a general simulation study. In this paper, however, all numerical results will be for the deterministic model. We also assume that any action on a ship in state  $i$  is to be taken only if the ship has just entered state  $i$  from another state.

Let us consider the deterioration process of a ship within a particular group as a realization of a stochastic process  $\{X_n, n = 0,1,\dots\}$  with members of a single group being independent of each other. Let  $C = (c_{ij})$  denote a cost matrix, where  $c_{ij} \equiv$  annual maintenance and operational cost of a ship without onboard treatment when moving from  $i$  to  $j$ . Also let  $C^* = (c_{ij}^*)$  denote

the matrix of costs associated with the ships with onboard treatment. We shall assume that  $c_{ij}^* > c_{ij}$  for all  $i$  and  $j$  since any sewage system only adds to the maintenance and operational costs of a ship.

We also need to define the following quantities:

- $C_N$   $\equiv$  cost of building a new ship with a sewage treatment system.
- $C_R$   $\equiv$  cost of repiping a ship (and installing sewage system).
- $\alpha$   $\equiv$  discount factor.
- $\beta_i$   $\equiv$  expected total discounted maintenance cost of a ship (without sewage treatment) until entering state 0, given it just entered state  $i$ .
- $\beta_i^*$   $\equiv$  same as  $\beta_i$  but for a ship with sewage treatment.
- $K_i$   $\equiv$  expected total discounted cost of letting a ship, which has just entered state  $i$ , reach state 0 without treatment to be subsequently replaced, generation after generation, by ships with sewage systems.
- $K_i^*$   $\equiv$  expected total discounted cost of repiping a ship, which has just entered state  $i$ , and replacing it, generation after generation, by ships with sewage systems.
- $Y(i)$   $\equiv$  number of years until entrance into state 0, given the ship has just entered state  $i$ .

Thus, to repipe a ship which has just entered state  $i$  has an associated overall expected discounted cost

$$(1) \quad C_R + \beta_i^* + E\alpha^{Y(i)} [C_N + \beta_0^*] + E\alpha^{Y(i)+Y(0)} [C_N + \beta_0^*] \\ + E\alpha^{Y(i)+Y(0)+Y'(0)} [C_N + \beta_0^*] + \dots$$

where  $Y(i), Y(0), Y'(0), \dots$  are independent random variables and  $Y(0), Y'(0), \dots$  are also identically distributed. Equation (1) may be simplified to

$$(2) \quad C_R + \beta_i^* + E\alpha^{Y(i)} [C_N + \beta_0^*] / [1 - E\alpha^{Y(0)}].$$

To build a new ship to "instantly" replace a ship having just entered state  $i$  has an associated overall expected discounted cost

$$C_N + \beta_0^* + (C_N + \beta_0^*)E\alpha^{Y(0)} + (C_N + \beta_0^*)E\alpha^{Y(0)+Y'(0)} + \dots$$

or simply

$$(3) \quad (C_N + \beta_0^*) / [1 - E\alpha^{Y(0)}].$$

Thus, given only the two choices "repipe" or "replace," we should repipe a ship which has just entered state  $i$  iff (2) is less than (3). That is, we repipe iff

$$(4) \quad C_R + \beta_i^* < (C_N + \beta_0^*) [1 - E\alpha^{Y(i)}] / [1 - E\alpha^{Y(0)}].$$

We shall henceforth refer to the deterministic case as the one in which  $P(X_{n+1} = i + 1 \mid X_n = i) \equiv P_{i,i+1} = 1$  for  $i = 0, \dots, d-1$  and  $P_{d0} = 1$ . If, for example, we let  $d = 24$ , then  $Y(i) \equiv 25 - i$  and

$$\beta_i^* = c_{i,i+1} + \alpha c_{i+1,i+2} + \dots + \alpha^{24-i} c_{24,0}^*$$

for  $i = 0, 1, 2, \dots, 24$ . Thus the decision here is to repipe iff

$$(5) \quad C_R + \beta_i^* < (C_N + \beta_0^*) (1 - \alpha^{25-i}) / (1 - \alpha^{25}).$$

If we further assume that  $c_{i,i+1}^* = c^*$  for all  $i$  then condition (5) reduces to

$$(6) \quad C_R < C_N (1-\alpha)^{25-i} / (1-\alpha^{25})$$

after some elementary algebraic manipulations.

### 3. The Deterministic Case.

Let us investigate the case where  $d = 24$ ,  $P_{i,i+1} = 1$  for  $i = 0, \dots, 24$  and  $P_{24,0} = 1$ . Our goal is to obtain a cost curve which relates the total discounted cost of the entire revamping process, to the time horizon intended to revamp the entire Naval fleet. In order to do so we must attack two problems. First, we must decide which ships, if any, should be replaced, and which should be repiped. When  $c_{i,i+1}^* = c^*$  for all  $i$  equation (6) gives us the answer. However, note that when  $\alpha = .95$  and  $i = 24$ , the right hand side of (6) is approximately  $0.07 C_N$ . Since recent figures ([2]) place  $C_R$  at about \$300,000 and  $C_N$  at ten million to one billion dollars, depending on the ship, (6) is very easily satisfied so that we should repipe a ship instead of replace it even if its remaining expected life is just one year. There is really nothing deep about this fact. It simply points out that it pays to defer spending  $C_N$  dollars for a year and instead pay  $C_R$  at present provided  $C_R$  is significantly less than  $C_N$ . We emphasize that this fact does not preclude the third alternative which is simply to let the ship die.

Let  $TDC(h)$  be the total discounted cost of revamping a group of one hundred ships in a specified time horizon of  $h$  years. It is also necessary to specify the distribution of ships among the various states. For purposes of illustration we shall at present assume that we have a group of 100 ships equally distributed over the 25 states.

For a single ship in state  $i$ ,

$$\begin{aligned} K_i &= \beta_i + \alpha^{25-i}(\beta_0^* + C_N) + \alpha^{50-i}(\beta_0^* + C_N) + \dots \\ &= \beta_i + \alpha^{25-i}(\beta_0^* + C_N) / (1 - \alpha^{25}) \quad \text{for } i = 0, 1, \dots, 24 \end{aligned}$$

If we assume that  $c_{i,i+1} = c$  and  $c_{i,i+1}^* = c^*$  for all  $i$ , then

$$\beta_i = c(1 - \alpha^{25-i}) / (1 - \alpha) \quad \text{and} \quad \beta_i^* = c^*(1 - \alpha^{25-i}) / (1 - \alpha)$$

thus,

$$(7) \quad K_i = c(1 - \alpha^{25-i}) / (1 - \alpha) + c^* \alpha^{25-i} / (1 - \alpha) + C_N \alpha^{25-i} / (1 - \alpha^{25}).$$

The point is that for a time horizon of 25 years (policy 1) the total discounted cost of revamping 100 ships uniformly distributed over the state space is simply  $4 \sum_{i=1}^{25} K_i$ . Using (7) we find that this cost is

$$(8) \quad \text{TDC}(25) = 100c / (1 - \alpha) + 4(c^* - c)(1 - \alpha^{25}) / (1 - \alpha)^2 + 4 C_N / (1 - \alpha)$$

after simple algebraic manipulations.

At the other extreme we have a time horizon of 1 year (policy 2). Assuming, as before, that  $C_R$  is significantly less than  $C_N$ , we may assume that all ships are to be repiped in one year. Absurd as this may be, it does give us an upper bound on the total cost. The total cost is

$$\text{TDC}(1) = 4 \sum_{i=1}^{25} (C_R + \beta_i^* + \alpha^{25-i} K_0^*)$$

which is equivalent to

$$(9) \quad 4 \sum_{i=1}^{25} [C_R + c^*(1-\alpha^{25-i}) / (1-\alpha) + \alpha^{25-i} (C_N + \beta_0^*) / (1-\alpha^{25})].$$

After substituting  $\beta_0^* = c^*(1-\alpha^{25}) / (1-\alpha)$  into (9) we get

$$(10) \quad 4 \sum_{i=1}^{25} [C_R + C_N \alpha^{25-i} / (1-\alpha^{25}) + c^* / (1-\alpha)]$$

which in turn reduces to

$$(11) \quad \text{TDC}(1) = 100 C_R + 4 C_N / (1-\alpha) + 100 c^* / (1-\alpha).$$

The difference between the costs of the two policies is

$$(12) \quad 100 C_R + (c^* - c) [100 / (1-\alpha) - 4(1-\alpha^{25}) / (1-\alpha)^2]$$

In Table I, we have tabulated the costs of policies 1 and 2 for various values of  $C_R$ ,  $C_N$ ,  $c$ ,  $c^*$  and  $\alpha$ . Included also is the difference in cost between these policies, which is of course, more significant. From (12) and Table I one sees that this difference is not a function of  $C_N$  nor of the magnitudes of  $c$  and  $c^*$ , but rather is a function of  $c^* - c$ ,  $C_R$  and  $\alpha$ .

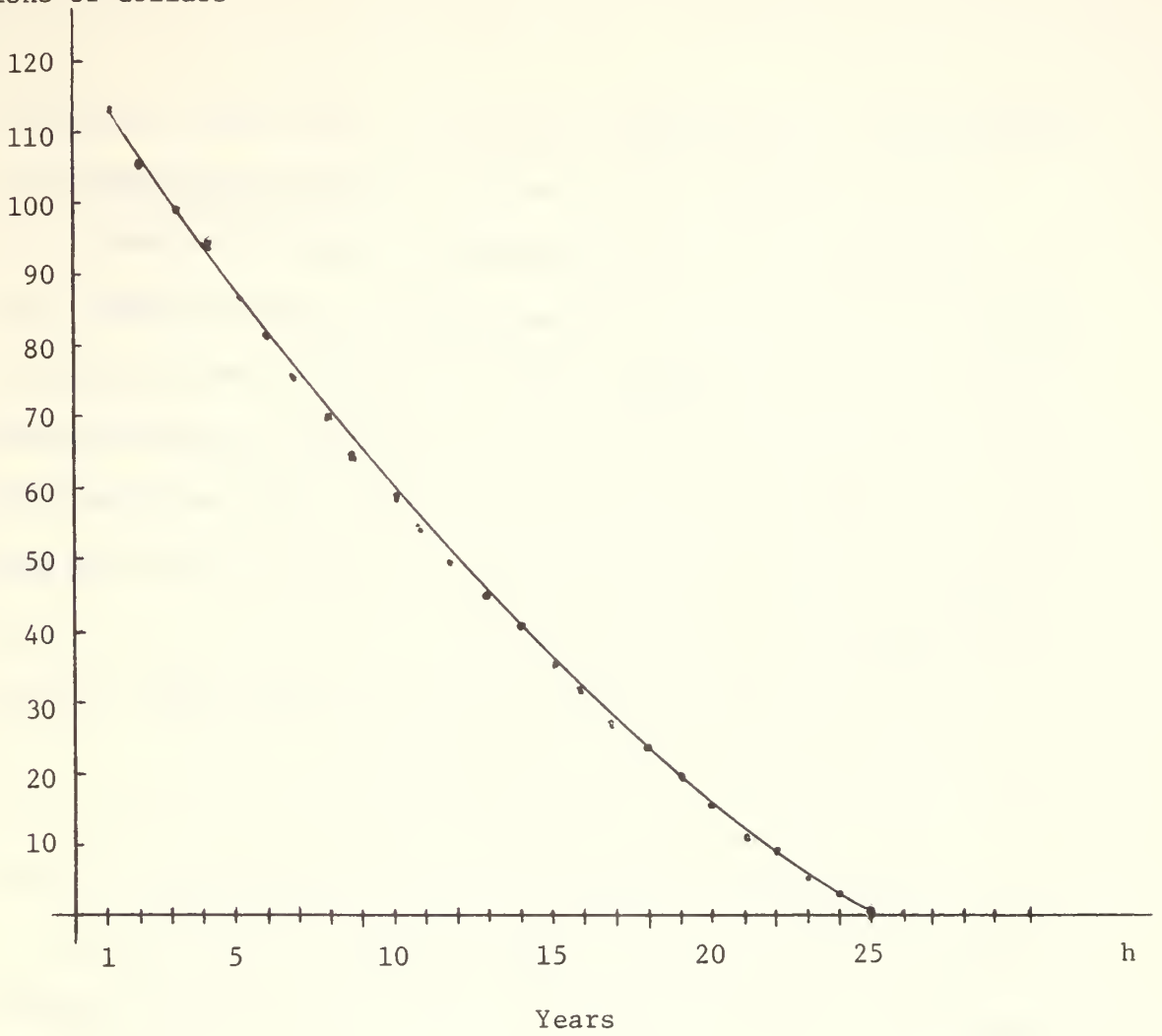
Table 1

$\alpha$	$c$	$c^*$	$C_R$	$C_N$	TDC(1)	TDC(25)	TDC(1)-TDC(25)
.95	1.0	1.1	0.3	50	6,228.8	6,115.6	113.2
.95	1.0	1.1	0.6	50	6,257.6	6,115.6	142.0
.95	1.0	1.3	0.3	50	6,628.8	6,346.8	282.0
.95	1.0	1.3	0.6	50	6,657.6	6,346.8	310.8
.95	1.0	1.1	0.3	100	10,228.8	10,115.6	113.2
.95	1.0	1.1	0.6	100	10,256.6	10,115.6	142.0
.95	1.0	1.3	0.3	100	10,628.8	10,346.8	282.0
.95	1.0	1.3	0.6	100	10,657.6	10,346.8	310.8
.95	1.0	1.1	0.3	200	18,228.8	18,115.6	113.2
.95	1.0	1.1	0.6	200	18,257.6	18,115.6	142.0
.95	1.0	1.3	0.3	200	18,628.8	18,346.8	282.0
.95	1.0	1.3	0.6	200	18,657.6	18,346.8	310.8

The total discounted cost, in millions of dollars, is tabulated for both a horizon of 1 and 25 years for various values of  $c$ ,  $c^*$ ,  $C_R$ , and  $C_N$ . Recall that  $TDC(1)-TDC(25)$  is a function of  $C_R$ ,  $c^*-c$ , and  $\alpha$  but does not depend on  $C_N$ . Also recall that  $TDC(1)$  and  $TDC(25)$  is calculated for a group of 100 ships.

Once these bounds are determined it behooves us to find  $TDC(h)$  for intermediate values of  $h$  between 1 and 25 years. Again, we begin with the assumption that ships are distributed uniformly among the states. For illustration, suppose we want to find  $TDC(8)$ . This implies that we must revamp 12.5 ships every year for eight years. To sidestep the problem of fractional ships we simply choose a number which is divisible by both  $h$  and 25, in this case 200, and later adjust the cost to reflect a group of 100 ships. Figure 1 is a typical graph of  $TDC(h)$ . Other graphs (not included) have the same general shape--namely, convex and decreasing. In every case drawing a straight line from  $TDC(1)$  to  $TDC(25)$  appears to be a good approximation to the true graph.

TDC(h)-TDC(25)  
millions of dollars



h	TDC(h)-TDC(25)	h	TDC(h)-TDC(25)
1	113.2	14	40.4
2	106.3	15	36.1
3	99.7	16	31.9
4	93.2	17	27.9
5	87.0	18	24.0
6	81.1	19	20.2
7	75.3	20	16.5
8	69.8	21	12.9
9	64.5	22	9.5
10	59.3	23	6.2
11	54.3	24	3.0
12	49.5	25	0
13	44.9		

#### 4. Stochastic Deterioration.

Whether extension of this model to stochastic deterioration is really essential to the decisionmakers or merely of academic interest is a moot point. In other words, it is quite possible that by generalizing to a stochastic model, we may find that the function  $TDC(h)$ , interpreted as an expected cost function, is very nearly the same as its deterministic counterpart. On the other hand, there is the matter of variability of costs in the stochastic model with the usual implications of risk and utility.

Let us briefly outline the steps that could be taken in the study of the stochastic version of this model:

If we consider the deterioration process  $\{X_n, n = 0, 1, \dots\}$  to be a semi-Markov process, then we are saying that  $T_i$ , the total time spent in state  $i$ , is a random variable whose distribution depends on  $i$ , and that changes of state, as perceived at times of transition, are governed by a one-step transition matrix  $P$ . If we assume that either  $T_i$  is geometrically distributed or that  $P(T_i=1) = 1$  for all  $i$ , then we have a Markov chain. So it certainly appears that we are gaining some generality by using the semi-Markov model. However, this generality is gained at the expense of greater complexity. It will be argued below that an appropriate redefinition of the state space gives us a Markov chain which is simpler to handle.

In the meantime assume  $\{X_n, n = 0, 1, \dots\}$  is a semi-Markov process. For states  $i$  and  $j$  ( $i \neq j$ ) let  $q_{ij}(n)$  denote the

probability that a ship, having just entered state  $i$ , will make its next transition to  $j$  in exactly  $n$  years. Then

$$q_{ij} \equiv \sum_{n=1}^{\infty} q_{ij}(n) \quad (i \neq j)$$

denotes the probability that a ship, having just entered state  $i$ , will make the next transition to  $j$ . We do not interpret the process staying in state  $i$  as a "transition." Thus,  $q_{ii} \equiv 0$ . In semi-Markov parlance,  $(q_{ij})$  is the one-step transition matrix of the imbedded Markov chain. If  $T_i$  denotes the time a ship spends in state  $i$ , then

$$P(T_i = n) \equiv h_i(n) = \sum_{j=0}^d q_{ij}(n), \quad n = 1, 2, 3, \dots$$

Finally, we let

$$f_{ij}(n) \equiv q_{ij}(n)/q_{ij}, \quad n = 1, 2, 3, \dots$$

denote the conditional probability that the next transition out of state  $i$  is to state  $j$  in  $n$  years, given that the next transition is to  $j$ . Clearly  $\sum_{n=1}^{\infty} f_{ij}(n) = 1$ . For this problem, we would assume  $d$  to be a reflecting barrier in the sense that  $P(T_d = 1) = q_{d0}(1) = q_{d0} = 1$ .

For the special case where  $T_i$  is geometrically distributed,  $\{X_n\}$  is a Markov chain, i.e.

$$P(T_i=n) = P_{ii}^{n-1}(1-P_{ii}), \quad n = 1, 2, \dots$$

where  $P_{ii} \equiv 1 - \sum_{j \neq i} q_{ij}(1)$ . Here,  $P_{ii}$  is the probability of staying in state  $i$  for one year. The lack of memory property of the geometric assures us that we have a Markov chain.

In general we must determine  $\beta_i, \beta_i^*, K_i, K_i^*$  and  $E\alpha^{Y(i)} \equiv \gamma_i(\alpha)$  for  $i = 0, 1, \dots, d$ . We proceed as follows: Let  $T_{ij} \equiv$  total time spent in  $i$ , given the next transition is to  $j$ . Then

$$P[T_{ij}=n] = f_{ij}(n), \quad n = 1, 2, \dots \text{ and } j \neq i.$$

Thus, subject to the condition  $\beta_d = c_{d0}$ , the sequence  $\beta_i$  ( $i = 0, 1, \dots, d-1$ ) satisfies the system of equations

$$\begin{aligned} \beta_i = & \sum_{j \neq i} q_{ij} \{ f_{ij}(1) [c_{ij} + \alpha \beta_j] + \sum_{n=2}^{\infty} f_{ij}(n) [\alpha^n \beta_j + \alpha^{n-1} c_{ij}] \\ & + c_{ii} (1 - \alpha^{n-1}) / (1 - \alpha) \}. \end{aligned}$$

This may be rewritten as

$$\begin{aligned} (13) \quad \beta_i = & \sum_{j \neq i} \beta_j \left[ \sum_{n=1}^{\infty} \alpha^n q_{ij}(n) \right] + \sum_{j \neq i} c_{ij} q_{ij}(1) + \sum_{j \neq i} \sum_{n=2}^{\infty} c_{ij} \alpha^{n-1} q_{ij}(n) \\ & + c_{ii} (1 - \alpha^{n-1}) / (1 - \alpha) \sum_{j \neq i} [q_{ij} - q_{ij}(1)]. \end{aligned}$$

The system of equations for  $\beta_i^*$  is of the same form with  $c_{ij}^*$  replacing  $c_{ij}$ , and condition  $\beta_d^* = c_{d0}^*$ .

To determine  $K_i$  and  $K_i^*$  we note that

$$(14) \quad K_i = \beta_i + (C_N + \beta_0^*) E\alpha^{Y(i)} / [1 - E\alpha^{Y(0)}], \quad i = 0, 1, \dots, d$$

with the same equations for  $K_i^*$  provided we replace  $\beta_i$  by  $\beta_i^*$ .

It remains to determine  $E\alpha^{Y(i)} \equiv \gamma_i(\alpha)$  for  $i = 0, 1, \dots, d$ . It

follows that

$$E\alpha^{Y(i)} = \sum_{j \neq i} q_{ij} \sum_{n=1}^{\infty} f_{ij}^{(n)} E\alpha^{n+Y(j)}$$

whence

$$(15) \quad \gamma_i(\alpha) = \sum_{j \neq i} \gamma_j(\alpha) \sum_{n=1}^{\infty} q_{ij}^{(n)} \alpha^n \quad \text{for } i = 0, 1, \dots, d-1$$

and  $\gamma_d(\alpha) = \alpha$ .

Although we have assumed that the action of repiping or replacing is to be taken only after a change of state, the semi-Markov model implies that ships in a given state should be viewed differently depending on how long each ship has been in that state. Unless  $T_i$  is distributed geometric or  $P(T_i=1) = 1$  (as in a Markov model), two ships in state  $i$  have remaining stays in state  $i$  which depend on how long they have already been in state  $i$ . To take this fact into account we may redefine the state space by the pair  $(i, \tau_i)$  where  $\tau_i$  is the time the ship has already spent in  $i$ . If the range of  $T_i$  is finite for every  $i$ , then such a redefinition is equivalent

to enlarging the original state space by adding intermediary states between each pair of original states, and this results in a process which changes state at each epoch. In general then, a Markov model is a sufficiently general description provided the state space is suitably defined.

Motivated by the above comments, we may assume without loss of generality that the one-step transition matrix satisfies  $P_{ii} = 0$  for all  $i$  and  $P_{d0} = 1$ . To determine  $\beta_i$ ,  $\beta_i^*$ ,  $K_i$ ,  $K_i^*$  and  $\gamma_i(\alpha)$  we proceed as follows:

To determine  $\{\beta_i\}$  we solve the system of equations

$$\beta_i = \sum_j P_{ij} (c_{ij} + \alpha \beta_j) \quad i = 0, 1, \dots, d-1$$

with condition  $\beta_d = c_{d0}$ .

To determine  $\beta_i^*$  we solve

$$\beta_i^* = \sum_j P_{ij} (c_{ij}^* + \alpha \beta_j^*) \quad i = 0, 1, \dots, d-1$$

with condition  $\beta_d^* = c_{d0}^*$ .

To find  $K_i$  and  $K_i^*$  we refer to (14). As for  $\gamma_i(\alpha)$  we solve

$$\gamma_i(\alpha) = \alpha \sum_j P_{ij} \gamma_j(\alpha) \quad i = 0, 1, \dots, d-1$$

with condition  $\gamma_d(\alpha) = \alpha$ .

The above equations are best solved by computer.

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<p>The following paper discusses the problem of planning the allocation of sewage treatment systems to the Naval Fleet. We assume that the obsolescence or deterioration process of a ship is a semi-Markov process. For the special case of deterministic deterioration, the total discounted cost of revamping a group of ships is determined as a function of the time horizon chosen to do so. This function is graphed for a set of representative cost factors. Finally, the semi-Markov and Markov models are discussed with a view towards a future simulation study.</p>			

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