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Transistors in non-linear circuits

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GLENN AUSTIN REIFF

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TRANSISTORS IN NON-LINEAR CIRCUITS

by

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PREFACE

About four years ago a new electrical device, the transistor, was announced. Since this announcement, and with the realization of its potential utility, transistor developmental activities have been expanding at an extraordinary rate. When the evolution of the transistor is compared with that of the vacuum tube, we can see that the development of the transistor has been amazingly fast. This does not mean to imply that transistors have reached the same stage of development as vacuum tubes, but it does indicate the rate of development. There is still much to be learned about transistors and their circuit applications.

The current development work can be divided into two interrelated fields. One is a study to improve the transistor itself and the means of uniform production; while the other is the study of its circuit application with a view towards circuit optimization and improving the device characteristics for the particular application. These circuit studies may be further divided into two categories. The first is the application to circuits which utilize the transistor in its linear range of operation, an example of which is the amplifier. The second category is the group of circuits which utilize the non-linearities of the transistor, examples of which are the oscillator and switching circuit. This paper is a small part of the latter category of circuit application studies.

The circuits and experimental transistor characteristics described herein were investigated by the author while working at the Hewlett-Packard Company of Palo Alto, California. It is a part of an investigation of

the application of transistors to Frequency Meter FR 65 under the U. S. Army Signal Corps Contract Number DA 36-039 SC-42483. The author wishes to express his appreciation to Mr. Wm. Johnson, Jr. and the other members of the research staff at the Hewlett-Packard Company with whom he consulted for their advice and assistance.

This work was done under the supervision of Associate Professor A. Sheingold, whose criticism and editorial comment is gratefully acknowledged.

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r_e	Emitter resistance in transistor	2
r_b	Base resistance in transistor	2
r_c	Collector resistance in transistor	2
r_m	Net mutual resistance in transistor	2
α	Current gain in transistor	4
R_e	External emitter impedance	6
R_b	Total impedance in Base lead	6
R_c	External collector impedance	6
R_f	External feedback impedance between collector and emitter	6
D	Determinate of a network	6
B	Cofactor of a network	9
R_{in}	Input emitter impedance	13
V_{ee}	Emitter bias voltage	20
V_{cc}	Collector bias voltage	20
e_e	Voltage between emitter and ground	24

CHAPTER I

INTRODUCTION

1. Summary

Many of today's transistor circuit studies are being conducted with two objectives. One is to determine the circuits which will operate satisfactorily with the present production transistors and to optimize these circuits. The second is to provide information for the transistor designer so that a more satisfactory group of transistor specifications and ratings may be established. With a view toward these objectives, several non-linear circuits will be presented along with the results of investigations of some of these circuits. However, before these circuits can be described, a general theoretical study of the instability requirements of transistor circuits to produce either sinusoidal or relaxation type oscillations will be necessary. The experimental circuits will then follow. Finally, experimental data on the characteristics of a number of different transistors will be discussed along with an indication of how the circuits may be optimized and additional transistor specifications or ratings established.

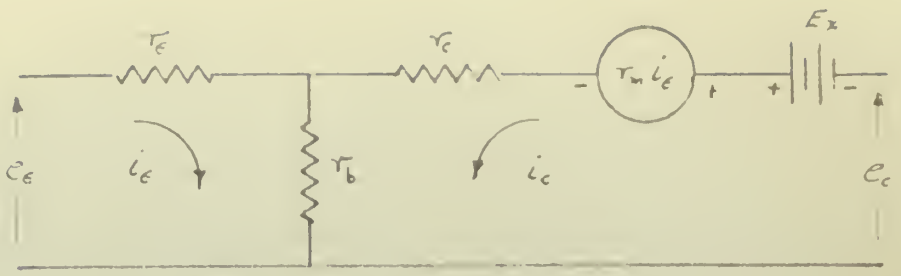
2. Current Status of Transistor Circuits

Today transistors are being produced in sufficient quantities, and with sufficiently uniform small signal characteristics, to be used in one full-scale commercial application.[1] However, this application utilizes only the linear range of operation. Many other experimental applications have been announced, [2,3] but most of these applications are not yet considered practical because of either financial or technical difficulties.

Some of the technical difficulties restrict the transistor's capabilities in linear operation. These include the availability of transistors themselves, poor temperature characteristics, and limited frequency response. In addition to these, other technical difficulties restrict its practical application in non-linear circuits. Non-linear circuits are, in general, more complex than those used in linear operation, and an additional set of transistor parameters are needed to completely define its operation. These needs are not yet completely known; consequently, the transistor's specifications are not complete. Pulsed currents are used in the production process to "form" point contact transistors and produce the desired small signal characteristics. However, in switching circuits pulse currents are again present, and in some cases they alter the transistor's characteristics. To date maximum pulse current specifications have not been established.

3. Equivalent Circuit and Parameters

The transistor, as a circuit element, may be analyzed as a four terminal, active network.[4] When this is done, many different equivalent circuits may be drawn. The one selected for this analysis is shown in Figure 1. r_e, r_b and r_c are the internal resistances of the emitter, base, and collector respectively, and r_m is the net mutual resistance in the transistor. Although these parameters are non-linear, the usual approximations divide its range of operation into three linear regions.[5] In region I the transistor is said to be cutoff, i.e., it is not active and both r_e and r_c are high. In region II the transistor is said to be active. In region III the transistor is said to be saturated, i.e., it is not active and both r_e and r_c are



TRANSISTOR EQUIVALENT CKT.

Figure 1

TABLE I

Region		I (Cutoff)	II (Active)	III (Saturated)
r_e	Max.	∞	~ 300 Ω	~ 200 Ω
	Min	100 K		
r_b	Max	~ 500 Ω	~ 500 Ω	~ 300 Ω
	Min.			
r_c	Max	∞	-	~ 1000 Ω
	Min.	20 K	10 K	
r_m	Max	0	-	~ 100 Ω
	Min		15 K	
α	Max	0	-	~ 0
	Min		1.5	

WESTERN ELECTRIC TYPE 169E TRANSISTOR PARAMETERS

low. The voltage E_x shown in the equivalent circuit is due to the fact that there is a finite current, I_{co} in the collector loop when i_e is zero.[5] In the first approximation and in the analysis to follow it is to be neglected. Another pertinent parameter is α, the current gain of the transistor, or $\left.\frac{\partial i_c}{\partial i_e}\right|_{v_c}$. Table I gives the specification values of these parameters for the Western Electric Type 1698 transistor in the three regions. This transistor type was used in all of the experiments. However, the theoretical work and general circuits presented will apply to any point contact transistor. For the open circuit condition as shown in Table I, α is approximately equal to r_m/r_c and is greater than unity for most point contact type transistors.

CHAPTER II

STABILITY

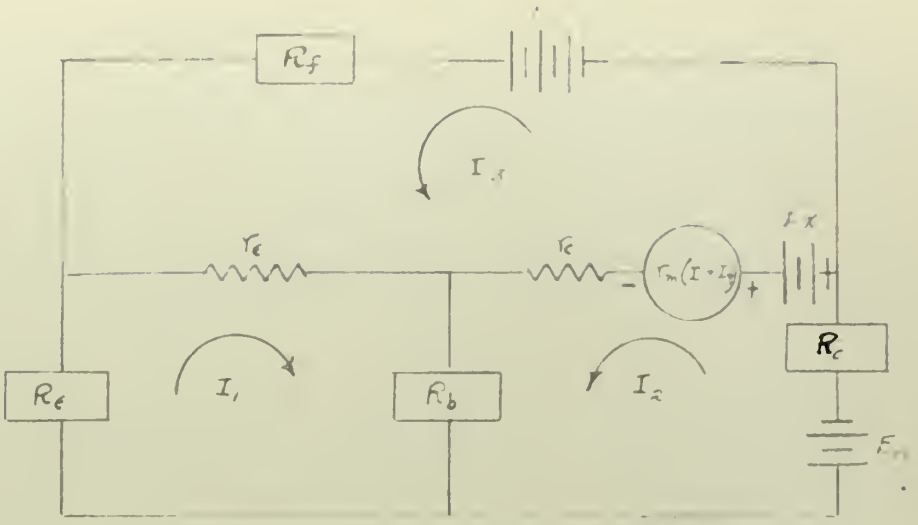
One of the fundamental differences between a transistor and a vacuum tube is that there is no phase reversal through the transistor. Consequently, positive feedback is possible with a single transistor and no phase inverting transformers. The addition of resistance in the base lead can be used to achieve an unstable circuit. As will be shown later, the addition of this resistance has some undesirable effects upon a circuit which is to operate with many different transistors. Therefore, it is desirable to investigate the various means by which an unstable circuit may be achieved. Both sinusoidal and relaxation types of oscillations are of interest.

Any complete examination of the conditions for oscillation would have to include the non-linearities of the device. However, as a starting point, and to establish a criterion for the conditions of oscillation, the circuits may first be examined on a linear basis.

1. General Circuit

Consider the three-loop network given in Figure 2. The external impedances shown may be reactive or resistive while the internal impedances of the transistor will be considered as purely resistive. This is the equivalent circuit of a transistor with both series and shunt feedback, either one of which can produce an oscillatory condition. Also included in Figure 2 are the mesh equations and the system determinant, Equation1.

The regeneration in the circuit will be provided by series feedback only if R_f goes to infinity. Equation 1 then becomes:



$$(R_e + r_e + R_b)\, I_1 \quad + R_b\, I_2 \quad + r_e\, I_3 \quad = 0$$

$$(R_b + r_m)\; I_1 + (R_c + r_c + R_e)\, I_2 + (r_m - r_c)\, I_3 = -E_{cc}$$

$$(r_e - r_m)\; I_1 \quad - r_c\, I_2 + (R_f + r_e + r_c - r_m)\, I_3 = E_x$$

$$\begin{aligned} D = {} & R_f \left[R_e R_b + R_e r_c + R_e R_c + r_e R_b + r_e r_c + r_e R_c + R_b r_c + R_b R_c \right] \\ & + r_e \left[R_e R_b + R_e r_c + R_e R_c + R_b r_c \right] \\ & + r_c \left[R_e R_b + R_e R_c + r_e R_c + R_b R_c \right] \\ & - r_m \left[R_e R_b + R_e R_c + R_b R_c + R_b R_f \right] \end{aligned} \qquad \text{Eq. 1}$$

Generalized Feedback Circuit

Figure 2

$$D_c = R_b[R_e + r_e + r_c + R_c - r_m] + R_e[r_c + R_c] + r_e[r_c + R_c] \qquad \text{Eq. 2}$$

To obtain an essentially shunt feedback circuit arrangement, let R_b be much smaller than both R_c and R_e. For this case the determinant is:

$$D_v = R_f[R_c + r_c][R_e + r_e] + r_e R_e[R_c + r_c] + r_c R_c[R_e + r_e] - r_m[R_c R_e + R_b R_f] \qquad \text{Eq. 3}$$

These determinants may be either positive, zero, or negative depending upon the relative magnitudes of the resistances. The resistance at any opened branch is directly proportional to the value of the determinant; consequently, if the determinant is negative this resistance will be negative.[6] Now if a tuned circuit is connected across this opened branch, the negative resistance will supply the losses and oscillations can result. If at least one of the external resistances shown is considered as the resonant impedance of a tuned circuit and if this impedance as a function of a complex frequency is included in the determinant, then the real part of the roots of the resulting determinant will be directly proportional, with the algebraic sign reversed, to the value of the determinant given in Equations 2 or 3. Therefore, the circuit will be stable if the applicable determinant, Equation 2 or 3, is positive; stable, sinusoidally oscillatory at one frequency if the determinant is zero, and unstable oscillatory if the determinant is negative. The latter case is the practical one in oscillators for then oscillations can build up until limited by the non-linearities of the device.

2. Criterion for Oscillation

For the series feedback case, if the following inequality holds, the determinant, Equation 2, will be either negative or zero:

$$\alpha \left[\frac{r_c}{r_c + R_c} \right] \geq 1 + \frac{R_e + r_e}{R_c + r_c} + \frac{R_e + r_e}{R_b} \qquad \text{Eq. 4}$$

There are many ways in which this inequality may be satisfied. Conversely, an inappropriate variation of any one of the parameters may upset the desired relationship. As an example, suppose that a circuit is adjusted to oscillate, and that the last two terms on the right are very small. One typical variation in the transistor when it heats is for r_c to decrease. It can be seen that this one change can upset the oscillation.

For the shunt feedback case, if the following inequality holds, its determinant, Equation 3, will be either negative or zero:

$$\alpha \left[1 + \frac{R_f R_b}{R_c R_e} \right] \geq \left[\frac{R_f}{r_c} + \frac{R_f}{r_c} + 1 \right] \left[1 + \frac{r_e}{R_e} \right] + r_e \left[\frac{1}{r_c} + \frac{1}{R_c} \right] \qquad \text{Eq. 5}$$

Like the previous expression, this inequality can be upset by a variation of any one of the parameters. It has already been assumed that R_b is much smaller than R_c or R_e. If it is further assumed that R_c is much larger than r_e and r_c is much larger than r_e, the expression simplifies to the expression given in Equation 4. These last two assumptions are reasonable since r_e in a transistor is in the order of hundreds of ohms.

$$\alpha \geq \left[\frac{R_f}{r_c} + \frac{R_f}{R_c} + 1 \right] \left[1 + \frac{r_e}{R_e} \right] \qquad \text{Eq. 6}$$

The two equations, 4 and 6, can now be generalized into two circuits which offer a design criterion. It must be emphasized that this is only a

criterion, for the parameters are all interrelated to give a certain desired strength of inequality and type of oscillation. These circuits are shown in Figure 3.

3. Negative Resistance

The application of any device in a multivibrator or switching type circuit depends upon it displaying a negative resistance characteristic.[7] The transistor will display such a characteristic. A resume of the theoretical characteristics for the transistor and the circuits to which they apply are shown in Figure 4 along with the equations for the characteristic curves. The approximate slopes of the characteristics in the three regions are shown on the curves*. Note that these curves are achieved through the use of series feedback alone. Let us now consider the effects of shunt feedback on the emitter characteristic.

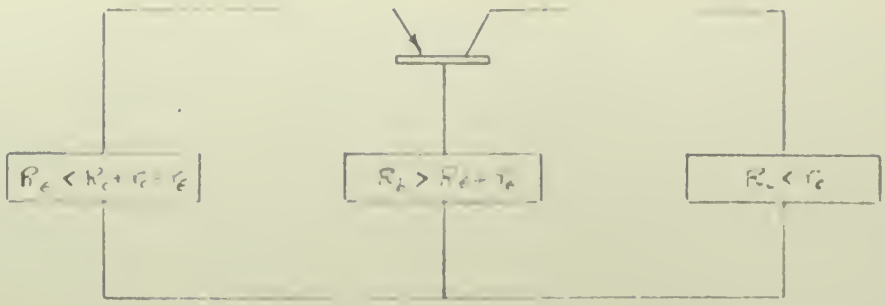
In the circuit of Figure 2 if we open the connection between the emitter and ground, assume a voltage E_1 between emitter and ground, and let R_e equal zero we can write an expression for E_1 in terms of I_1 and E_2.[8] This again assumes E_x equal to zero. The expression can then be simplified for the three regions in a manner similar to that used by Anderson. Thus

$$E_1 = \frac{D}{B_{11}} I_1 + \frac{B_{21}}{B_{11}} E_2$$

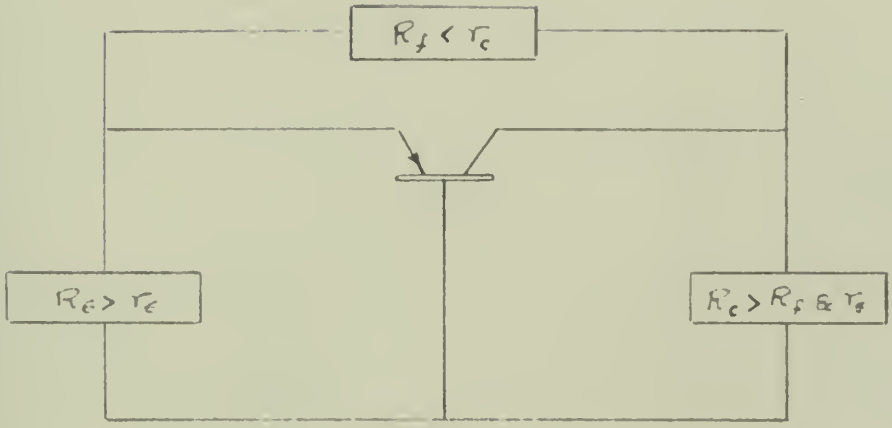
The results of the indicated algebraic manipulation, with r_b assumed much smaller than r_c, are shown in Figure 5.

Only one aspect of the effects of R_f on the negative resistance characteristic will be discussed theoretically. Let us consider the slope of the

*For a complete discussion of these curves and their use in circuit application the reader is referred to Anderson's article, reference 5.



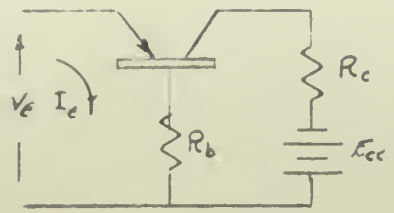
Series Feedback

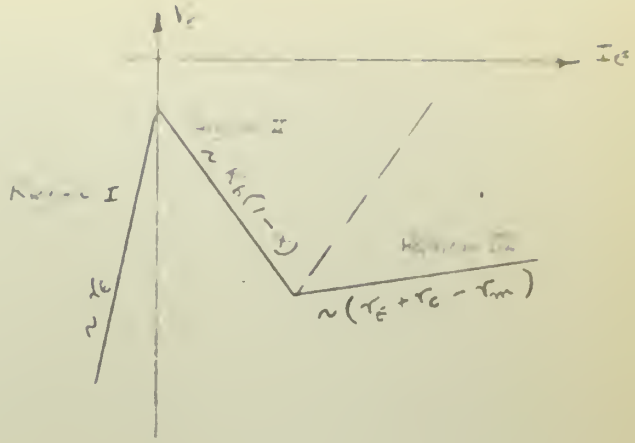


Shunt Feedback

Oscillator Design Criterion

Figure 3



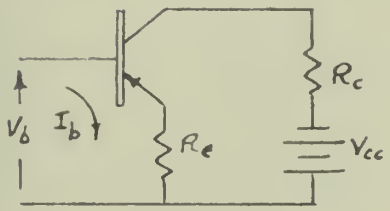


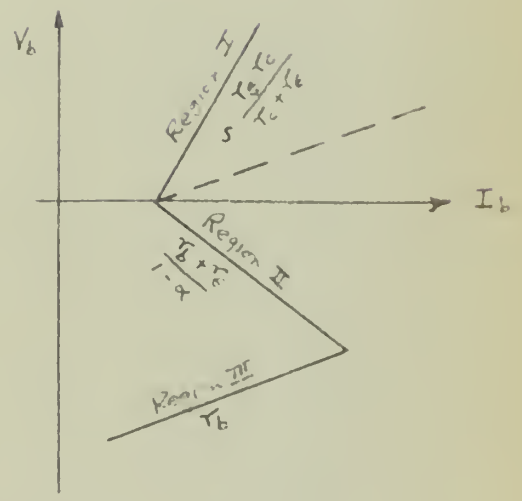
In General:

$$V_e = I_e \left[(r_e + r_b + R_b) - \frac{(r_b + R_b)(r_b + R_b + r_m)}{r_b + R_b + r_c + R_c} \right]$$

$$+ \frac{(V_{cc} + E_x)(r_b + R_b)}{r_b + R_b + r_c + R_c}$$

Isolated Emitter Characteristic





In General:

$$V_b = I_b \left[r_b + R_e + r_e - \frac{(r_e + R_e)(r_e + R_c - r_m)}{r_e + R_e + r_c + R_c - r_m} \right]$$

$$+ \frac{(V_{cc} + E_x)(r_e + R_e)}{r_e + R_e + r_c + R_c - r_m}$$

Isolated Base Characteristic

Figure 4

GENERAL IF $R_c \gg R_b$ & $R_e = 0$

$$E_1 = \frac{R_f[r_e + R_b][r_c + R_c] + R_c[R_b(r_e + r_c) + r_c r_e] - r_m R_b(R_c + R_f)}{R_c[R_f + r_e + r_c - r_m] + r_c[R_f + r_e]} I_1$$

$$- \frac{R_b[R_f + r_e + r_c - r_m] + r_e r_c}{R_c[R_f + r_e + r_c - r_m] + r_c[R_f + r_e]} E_2$$

REGION I WHERE $r_e \gg R_b$ & $r_c \gg R_b$

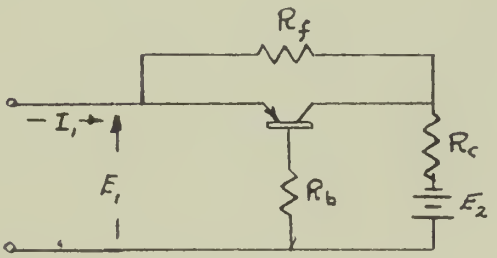
$$E_1 = \frac{R_f r_e[r_c + R_c] + R_c r_c r_e}{R_c[R_f + r_e + r_c] + r_c[R_f + r_e]} I_1 - \frac{R_b[R_f + r_e + r_c] + r_e r_c}{R_c[R_f + r_e + r_c] + r_c[R_f + r_e]} E_2$$

REGION II WHERE $r_e \ll R_b$ & r_c & R_c

$$E_1 = R_b\left[1 - \frac{R_f r_m}{R_f[R_c + r_c] + R_c[r_c - r_m]}\right] I_1 - \frac{R_b[R_f + r_c - r_m]}{R_c[R_f + r_c - r_m] + r_c R_f} E_2$$

REGION III WHERE $r_e \ll R_b$, $r_c \ll R_c$, & $r_m = 0$

$$E_1 = R_b I_1 - \frac{R_b}{R_c} E_2$$



Equations for Shunt Feedback Negative Resistance Characteristic

Figure 5

curve in region II. This slope is equal to the amount of negative resistance which the transistor will present, and it affects the transition time between states in a switching circuit. This resistance, R_{in}, is equal to the coefficient multiplying the I_1 term in the equation applicable to region II and is of the form

$$\frac{R_{in}}{R_b} = \frac{-AR_f - B}{CR_f - D}$$

A qualitative plot of R_{in}/R_b as a function of R_f is shown in Figure 6. From this it can be seen that as R_f is decreased from some high value the negative resistance increases.

When using a negative resistance characteristic to design a multivibrator or switching type circuit the slopes of the curve in all three regions are important, for these slopes help to determine the cycling time. Also of importance are the values of current and voltage at the transition points between the three regions. On the emitter characteristic the transition between regions I and II is known as the "peak point" while the transition between regions II and III is known as the "valley point". These points influence the cycling time, trigger sensitivity, and output of the trigger circuit. Although it may be deduced from the equations given in Figures 4 and 5, the effects of the variation of a circuit parameter may be more easily understood by plotting the characteristic for different values of the parameter. Figure 7 shows two such curves which were determined experimentally. The first shows that a variation in series feedback affects both the negative resistance and valley point on the characteristic. The

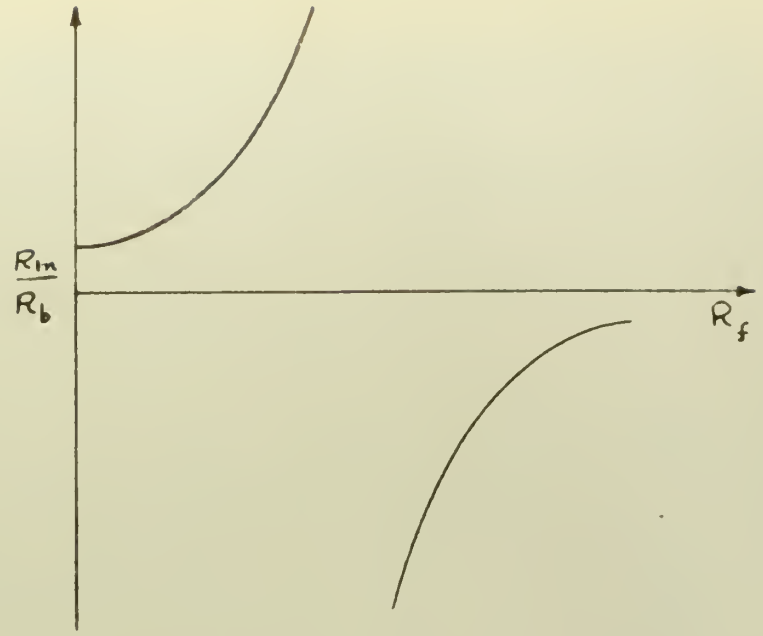
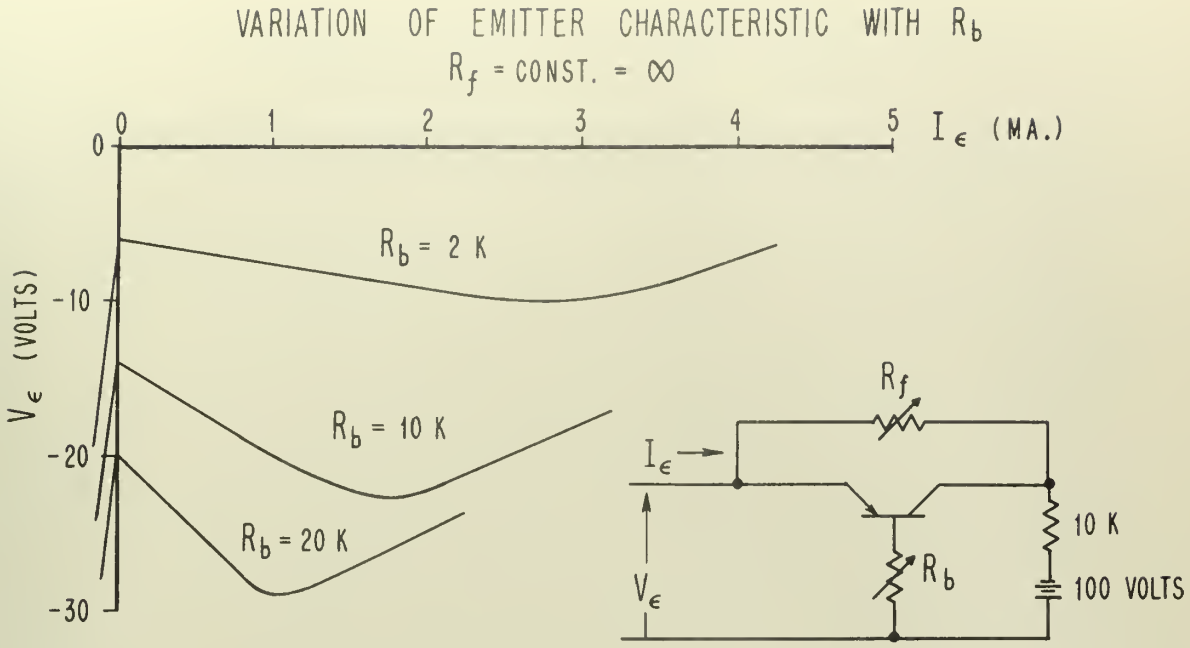


Figure 6

second indicates that shunt feedback affects both the negative resistance and peak point. Therefore, it may be possible with the use of both series and shunt feedback to alter the characteristic for optimum performance in a specific circuit application.

The design of a transistor oscillator or switching circuit requires that rather stringent requirements be imposed upon the various external impedances. Once these requirements are met, any variation in the transistor parameters will affect the operation of the circuit. However, it should be possible to minimize the effects of transistor parameter variations by the proper choice of the external parameters and by a selection of the correct type of feedback.


VARIATION OF EMITTER CHARACTERISTIC WITH R_b
R_f = CONST. = ∞
0
1
2
3
4
5
I_ϵ (MA.)
V_ϵ (VOLTS)
-10
-20
-30
R_b = 2 K
R_b = 10 K
R_b = 20 K
R_f
I_ϵ
V_ϵ
R_b
10 K
100 VOLTS

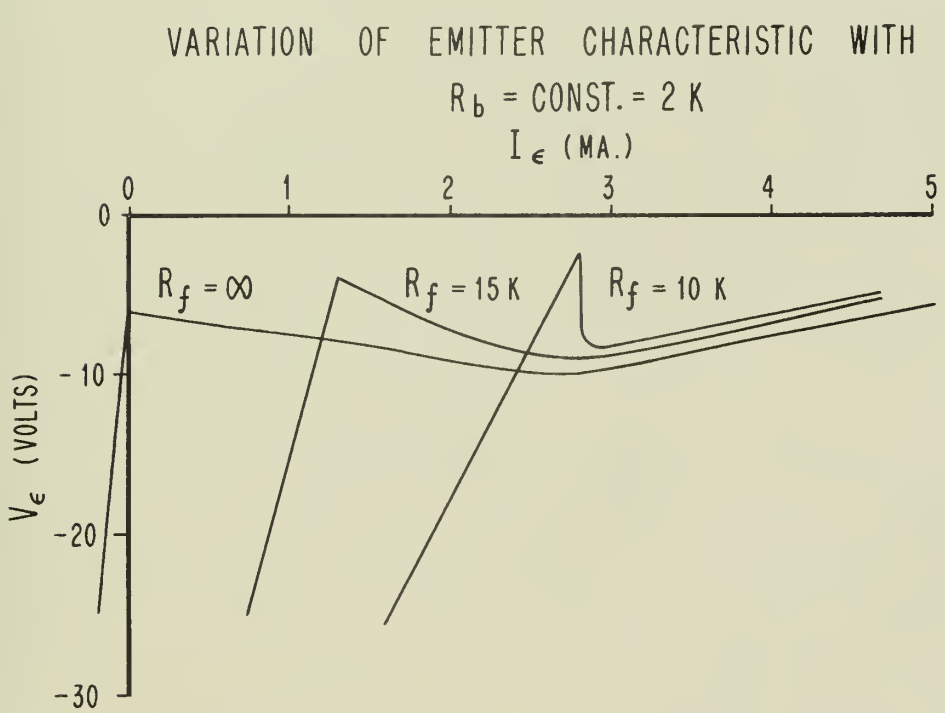

VARIATION OF EMITTER CHARACTERISTIC WITH R_f
R_b = CONST. = 2 K
I_ϵ (MA.)
0
1
2
3
4
5
V_ϵ (VOLTS)
-10
-20
-30
R_f = ∞
R_f = 15 K
R_f = 10 K

Figure 7

CHAPTER III

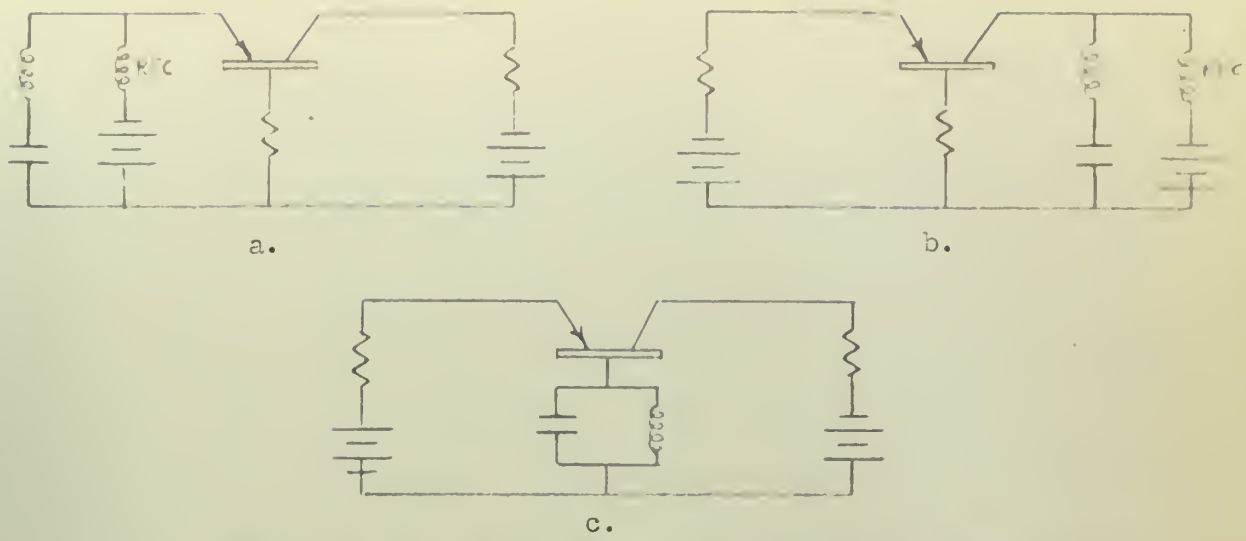
INVESTIGATIONS OF NON-LINEAR CIRCUITS

In this chapter several sinusoidal oscillators will be presented with the experimental results of an investigation of one of these circuits. Following this will be a discussion of the experimental results of the operation of two relaxation oscillators. The author's objectives in conducting these experiments were to determine whether or not interchangeability of today's production transistors is practical in simple types of non-linear circuits, to determine the effects of the circuit upon the transistor, and to obtain an indication of how the effects of a variation in transistor parameters may be minimized by proper choice of the external parameters.

1. Sinusoidal Oscillators

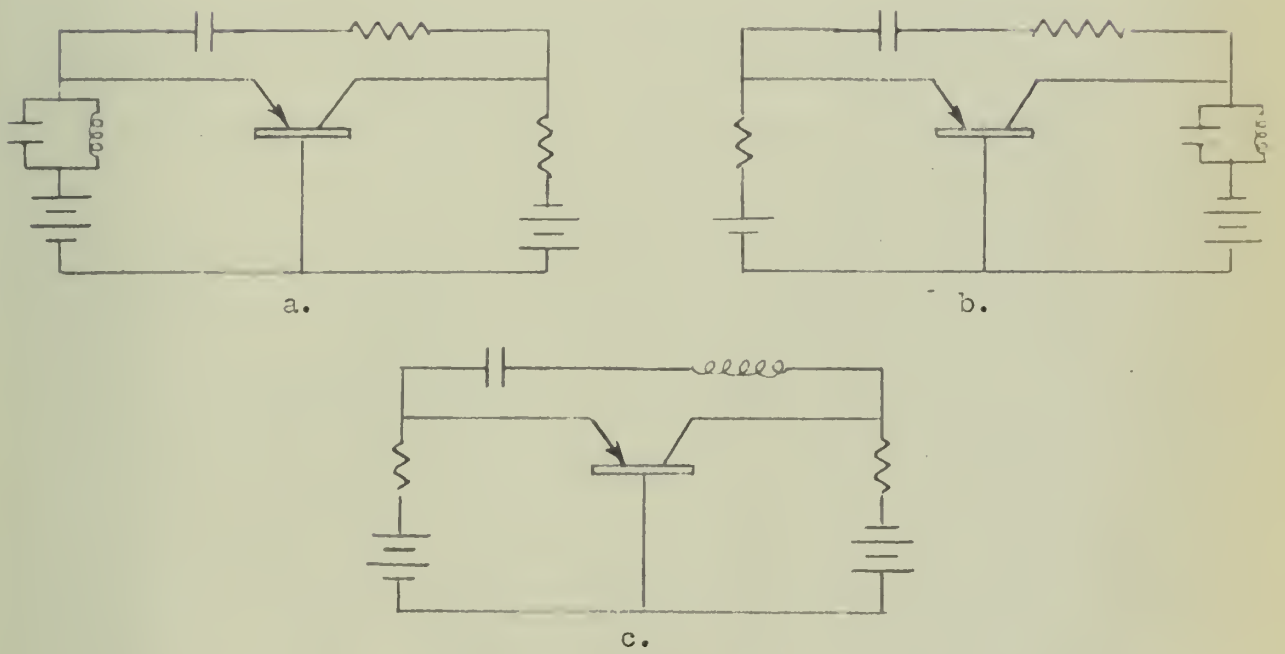
The design criterion established in Chapter II shows that several different sinusoidal oscillator circuits are possible. Many of the circuits presented here have appeared elsewhere in the literature.[9,10] As might be suspected, it has been found that with an oscillator circuit adjusted for non-distorted sinusoidal oscillation with one transistor, the same circuit may produce no oscillation or produce a more non-linear type oscillation with another transistor.

Three of the possible series feedback sinusoidal oscillator configurations are shown in Figure 8. Combinations of these circuits can also be used. The circuits shown in Figure 8, (a) and (b) are very critical of supply voltages since the base resistor and r_c govern the bias on both the collector and emitter. The frequency is also dependent



Examples of [illegible] Sinusoidal Oscillators

Figure 8



Examples of Shunt Fed Tuned Sinusoidal Oscillators

Figure 9

upon these voltages. The circuit of Figure 8(c) does not suffer from the bias difficulties. However, its frequency is also influenced to some extent by the supply voltages. There are definite minimum requirements on the Q of the coils in all three of these circuits, and the tank in Figure 8(c) may have to be center tapped to match the low impedance of the emitter. By using a combination of these circuits, say one with a parallel resonant circuit in the base and a series resonant circuit in the emitter, an oscillator which is less critical of supply voltages results.

Three of the possible shunt feedback oscillators are shown in Figure 9. All of these circuits are less dependent upon supply voltages since the base resistance has been minimized. Again there are definite requirements on the Q's of the resonant circuits involved.

It is possible to use any of the circuits shown as crystal controlled oscillators by replacing the tuned LC circuits with the proper crystal. One such crystal controlled oscillator has been tested and is shown in Figure 10. This circuit produced oscillations at about 100KC with 9 out of 10 transistors. This interchangeability was obtained by making R_e relatively high. It operated as a class C oscillator with the degree of non-linearity dependent upon the transistor used. Consequently, the frequency changed with a change in transistors. This frequency variation was about three parts in 10^5 with the ten transistors tested. The frequency changed about two parts in 10^5 as the collector voltage was varied between 70 and 90 volts, and it changed one part in 10^5 as the emitter voltage was varied between 4 and 9 volts. These results indicate that the circuit as shown cannot be used as a precision oscillator without additional

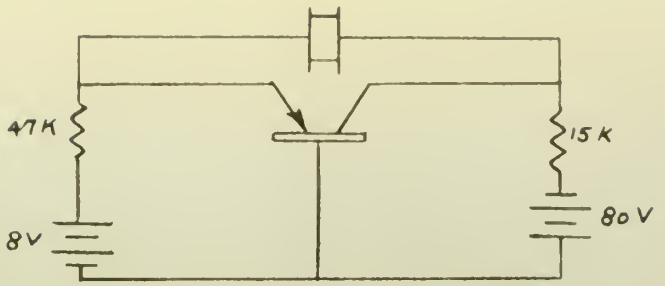


Figure 10

means of compensation to allow for the variation of the transistor's parameters. It is believed that the inclusion of a parallel resonant circuit in place of the 15K would improve the performance.

2. Multivibrators

It has been shown in Chapter II that it is possible to obtain a negative resistance characteristic by the use of either series or shunt feedback. Therefore, as with sinusoidal oscillators, it is possible to build transistor multivibrators using several different circuit configurations.

A basic series feedback multivibrator is shown in Figure 11. With the switch open the circuit is monostable, and with the switch closed it is monostable if V_{ee} is negative and astable if V_{ee} is positive. The operation of these configurations have been discussed in the literature, and McDuffie has derived theoretical expressions[11] for the cycling times in the astable configuration. The operation of this circuit is dependent upon the emitter negative resistance characteristic as discussed in connection with Figure 4, and it suffers from the same bias difficulties as previously mentioned. The cycling times for this multivibrator are very dependent upon the slopes, peak point, and valley point of the characteristic. Several

methods for compensating this circuit have been suggested.[12,13] One of these places a biased diode in the base lead. Another uses a peaking coil in the base lead. To obtain an indication of the possibilities of interchanging transistors in this circuit, the frequency of operation of this type of circuit was measured using twenty-five different transistors. The circuit and results are shown in Figure 12. 1.822 KC was the lowest frequency of operation. This curve is a statistical average of the frequencies. Thus, if a frequency deviation of 15% is allowable in a specific application of this circuit, only 20% of the present production transistors would be satisfactory. Some form of compensation would undoubtedly improve this

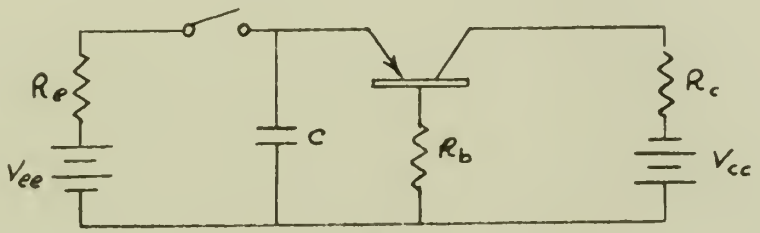
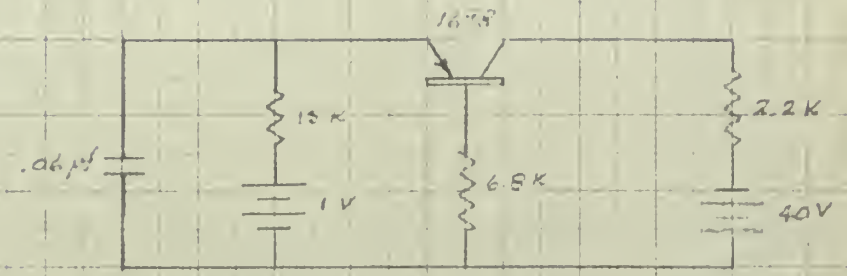


Figure 11

performance; however, it was desired to test the transistor without auxiliary compensation.

The circuit shown in Figure 12 was also operated as a monostable multivibrator. The one volt battery was replaced by a negative eight volts, and the trigger sensitivity measured with ten different transistors. Six of these transistors produced an astable circuit, while with the remaining four the trigger sensitivity varied from 1.8 to 9 volts. The lead containing the 15K was then opened, and the trigger sensitivity again measured using the same ten transistors. One of the transistors did not work, one


.06 μf
10 K
1 V
6.8 K
2.2 K
40 V

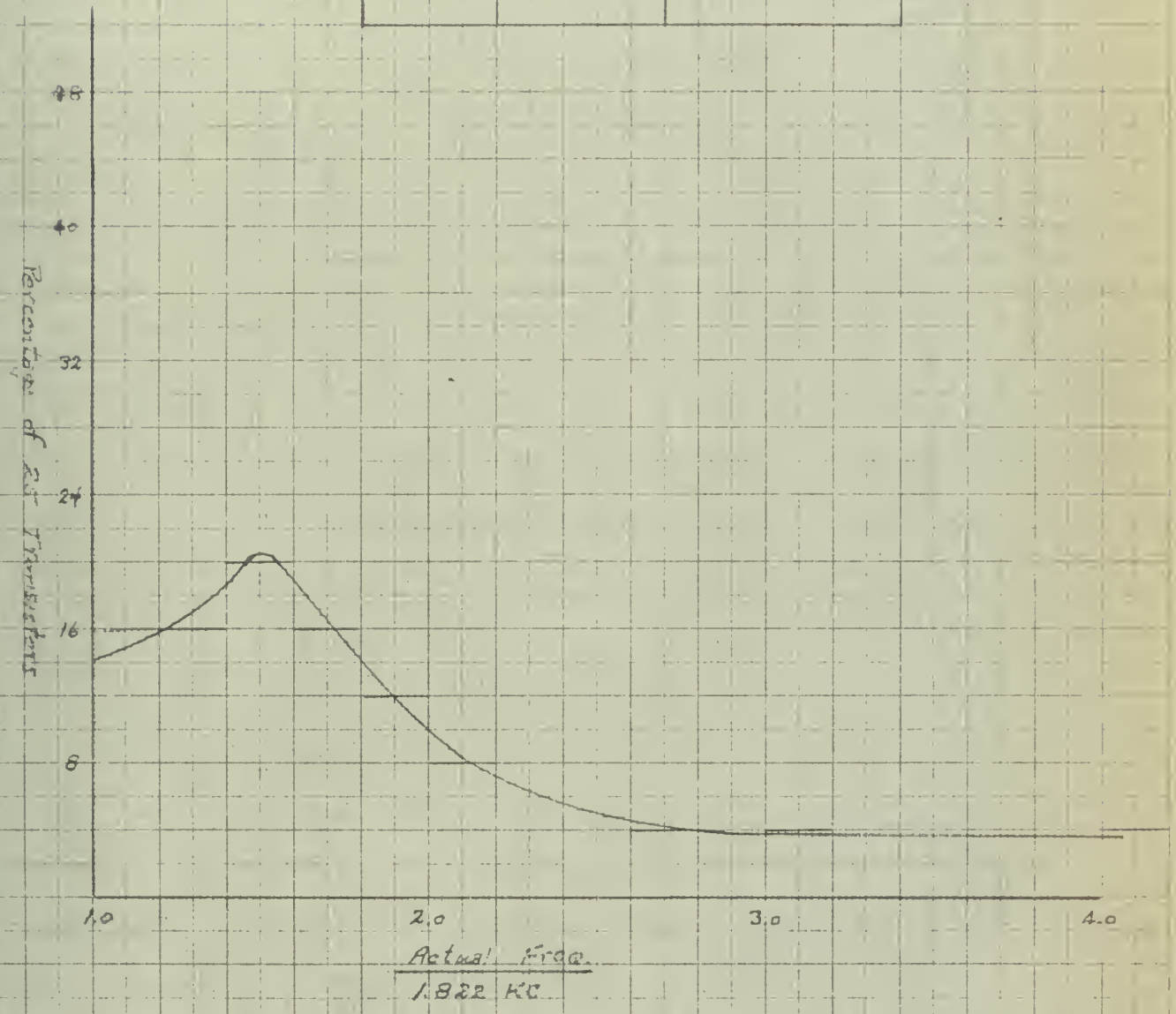
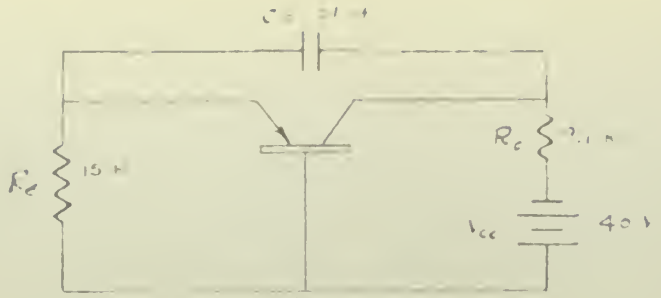

48
40
32
24
16
8
Percentage of 25 Transients
1.0
2.0
3.0
4.0
Actual Freq.
1822 KC

Figure 12

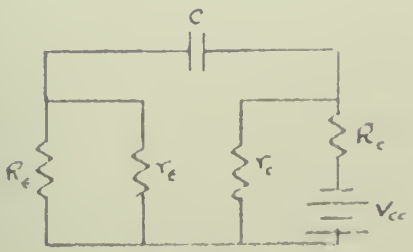
was astable, and with the remaining eight the sensitivity varied from 0.5 to 15 volts. Again compensating circuits using diodes have been suggested, but were not tried.[14]

A basic shunt feedback multivibrator is shown in Figure 13(a). From a qualitative standpoint several things about this circuit may be deduced. First, it should be highly regenerative since during the rapid switch time in the active region the condenser provides a virtual short circuit. Hence, an alpha of only slightly greater than unity is required. Secondly, R_e and R_c may be made as large as desired without decreasing the negative resistance. However, from a consideration of the time constants, their values are limited. Thirdly, the effects in region I of the cutoff collector current, I_{co}, will be reduced since the only voltage induced in the emitter circuit by this current is due to the internal base resistance of the transistor. To an approximation, the circuit may be reduced to two equivalent circuits, Figures 13(b) and 13(c). Figure 13(b) applies when the transistor is cutoff, and Figure 13(c) applies when the transistor is saturated. Note that in the equivalent circuit, Figure 13(c), the condenser is shorted. This would give a zero time in the saturated region. However, since there is actually some resistance in both the emitter and collector, this time, as determined experimentally, is in the order of 2 microseconds. The waveforms are shown in Figure 13(d).

The circuit operation may be described as follows: When the transistor cuts off, the emitter voltage drops to a large negative value and begins an exponential rise towards zero as the condenser charges through a combination of R_e, r_e, r_c, and R_c. As this voltage nears zero, the

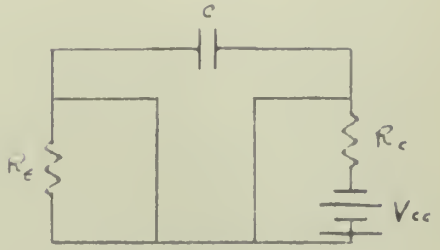


(a)



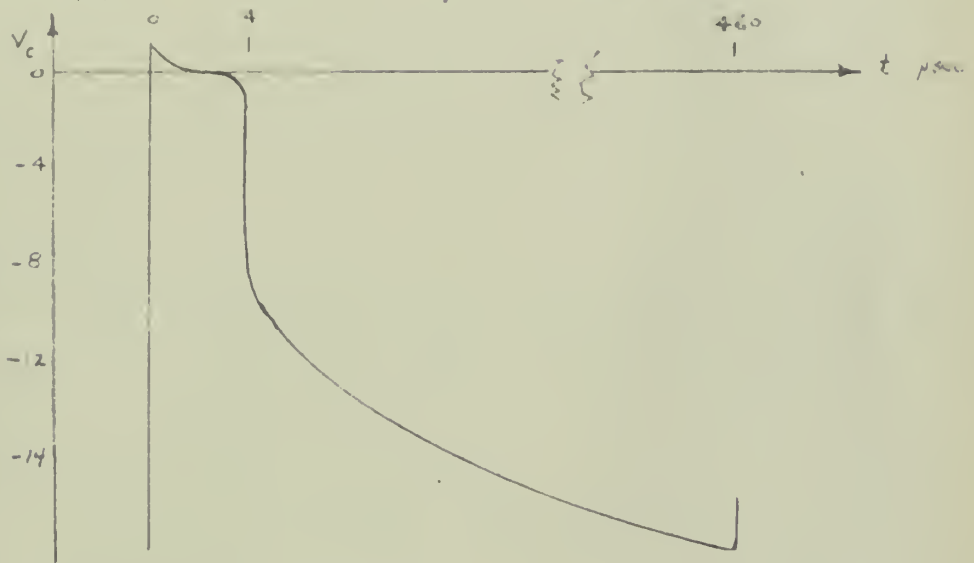
Cutoff

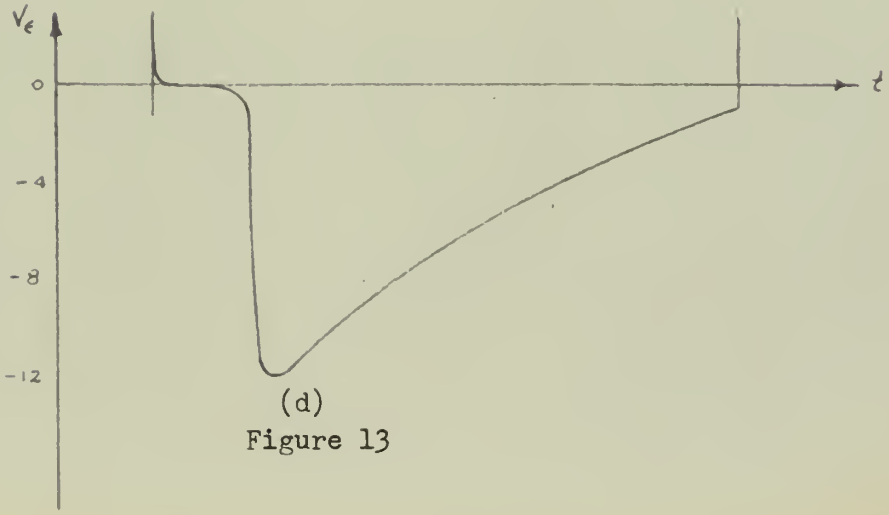
(b)



Saturated

(c)





(d)

Figure 13

transistor moves into the active region so that a further increase in the emitter voltage appears, amplified, at the collector and is coupled back through the condenser to the emitter. This regenerative process continues until the collector becomes saturated, and at this time the collector voltage drops. This drop in voltage is coupled back to the emitter, and the transistor again enters the regenerative region. Thus both the emitter and collector voltages drop until the emitter cuts off. When the emitter cuts off, its voltage drops further as the condenser begins to recharge.

In the cutoff and saturated regions r_m is approximately zero. Also, let us assume that the internal base resistance of the transistor is negligible. Then from the circuit of Figure 2, with R_f replaced by a condenser, the expression for the voltage between the emitter terminal and ground, written in terms of the Laplace transform, is:

$$e_e = \frac{r_e\left[(R_c + r_c)E_g - r_c E_{cc}\right]}{r_e\left[R_c + r_c\right] + r_c R_c\left[1 + \frac{r_e}{R_e}\right]} \times \frac{1}{\frac{(R_e + r_e)(r_c + R_c)}{\left[r_e R_e (r_c + R_c) + r_c R_c (r_c + R_e)\right]C} + s} \qquad \text{Eq. 7}$$

Taking the inverse

$$e_e = K \epsilon^{-\frac{t}{\left[\frac{r_e R_e}{r_e + R_e} + \frac{r_c R_c}{r_c + R_c}\right]C}} \qquad \text{Eq. 8}$$

If the proper values of transistor parameters and initial conditions are used, this equation will apply to both the saturation and cutoff regions. However, since the time in the saturated region is very short, the period is determined primarily by the exponential decay when the transistor is

cutoff. Only this time will be considered. If the internal base resistance is zero, then the end of the cutoff region will occur when the emitter voltage reaches zero. From Equation 8 the expression for the decay time then becomes:

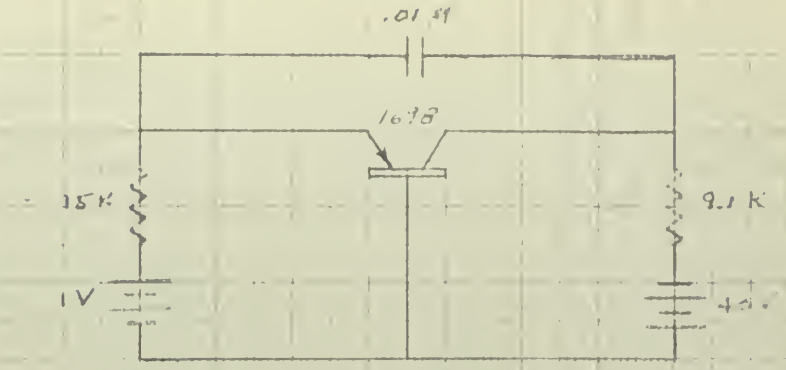
$$t = \left[\frac{r_e R_e}{r_e + R_e} + \frac{r_c R_c}{r_c + R_c} \right] C \qquad \text{Eq. 9}$$

Now if R_e is much smaller than r_e and R_c is much smaller than r_c the period will be determined primarily by the external parameters. Table II is a summary of the approximations involved in both the cutoff and saturation regions. Since there are many approximations involved in this analysis it cannot be used for computing the frequency. However, it does indicate the requirements imposed upon R_e and R_c to make the frequency less dependent upon the transistor parameters.

TABLE II

Region	I	III
r_e	$> R_e$	0
r_b	0	0
r_c	$\gg R_c$	0
r_m	0	0
Time Const	$(R_e + R_c)\,C \gg (r_e + r_c)\,C$	

The frequency and sensitivity of this type of circuit were measured using the same twenty-five transistors as were used in the series feedback circuit. The experimental, astable circuit and results are shown in Figure 14. (2.118 KC was the lowest frequency of operation.) Although


.01 μf
1618
15K
1V
9.1 K
4 v.

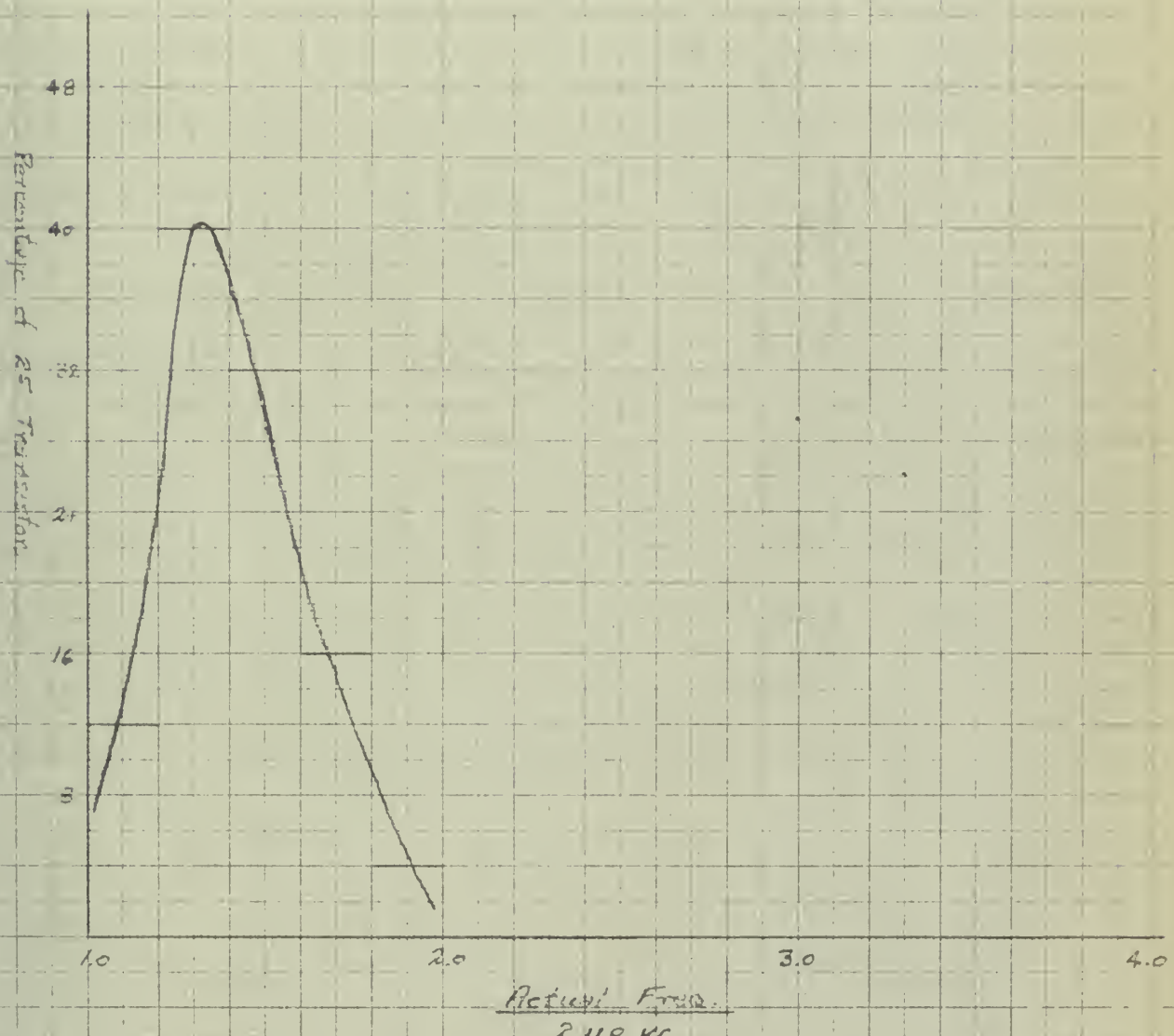

48
40
32
24
16
8
Percentage of Rectification
1.0
2.0
3.0
4.0
Actual Freq.
2.118 KC.

Figure 14

the emitter bias was not theoretically necessary for this circuit it was included for two reasons. One is that it was necessary with some transistors to make them astable, and the other is that it was desired to make this circuit similar to the one shown in Figure 12. The external collector resistance does not satisfy the condition that R_c be much smaller than r_c. However, this value was necessary with the bias conditions for proper operation. The statistical curve indicates that, if a frequency deviation of 15% is allowable in a specific application, 40% of the present production transistors would be satisfactory. For the trigger sensitivity test the positive one volt battery was replaced by a negative eight volts and an 800 ohm resistor was added to the base lead. The trigger pulse was applied across this resistor. With the same ten transistors as were used in the previous sensitivity test the trigger sensitivity varied about 10%.

CHAPTER IV

THE VARIATIONS IN TRANSISTOR PARAMETERS

In the previous chapters the variations in transistor parameters have been mentioned in general terms, and some of their effects upon the operation of specific circuits have been described. In this chapter we shall examine some of these variations a little more closely.

1. Small Signal, Region II, Parameters

The small signal, Region II, parameters, although insufficient to completely define the operation of a non-linear circuit, are still of prime importance. If they are not within specifications, the circuit may not have enough regeneration for satisfactory operation.

Coblenz and Owens[15] have investigated the variation of the region II parameters of the type 1698 transistor with temperature and between different transistors. They found, with 20 units tested, that the minimum values as given in Table I were met with 100% of the units at 25° C. For many applications it would be desirable to have both maximum and minimum tolerances specified. As the temperature was varied between 25° and 55° C (the maximum rated ambient temperature for this transistor) the average variation of the parameters were as much as:

Parameter	Variation
r_e	160 to 170 Ohms
r_b	110 to 150 Ohms
r_c	21 to 13 Kohms
r_m	52 to 33 Kohms
Alpha	2.1 to 2.4

Note that r_c is highly temperature dependent and may vary by a factor of nearly two, while alpha is relatively constant over the temperature range.

This variation in r_c can modify the inequalities given in Equations 4 and 6, and hence change the mode of non-linear circuit operation.

Other parameters which were measured by Coblenz and Owens as a function of temperature were I_{co} and V_{cl}. I_{co} is the collector current under conditions of zero emitter current and 40 volts collector potential. This current helps determine E_x in the equivalent circuit, Figure 1, the value of which influences the position of the peak point. It was found that I_{co} varies about plus or minus 35% of its mean value over the rated temperature range. V_{cl} is the collector voltage for the transistor when in the saturated condition and will influence the valley point. It varied about plus or minus 10% over the temperature range. These two factors are of importance in determining the mode of operation of an oscillator since the non-linear limiting of an oscillator occurs at these points.

2. Large Signal, Non-Linear, Parameters of New Transistors

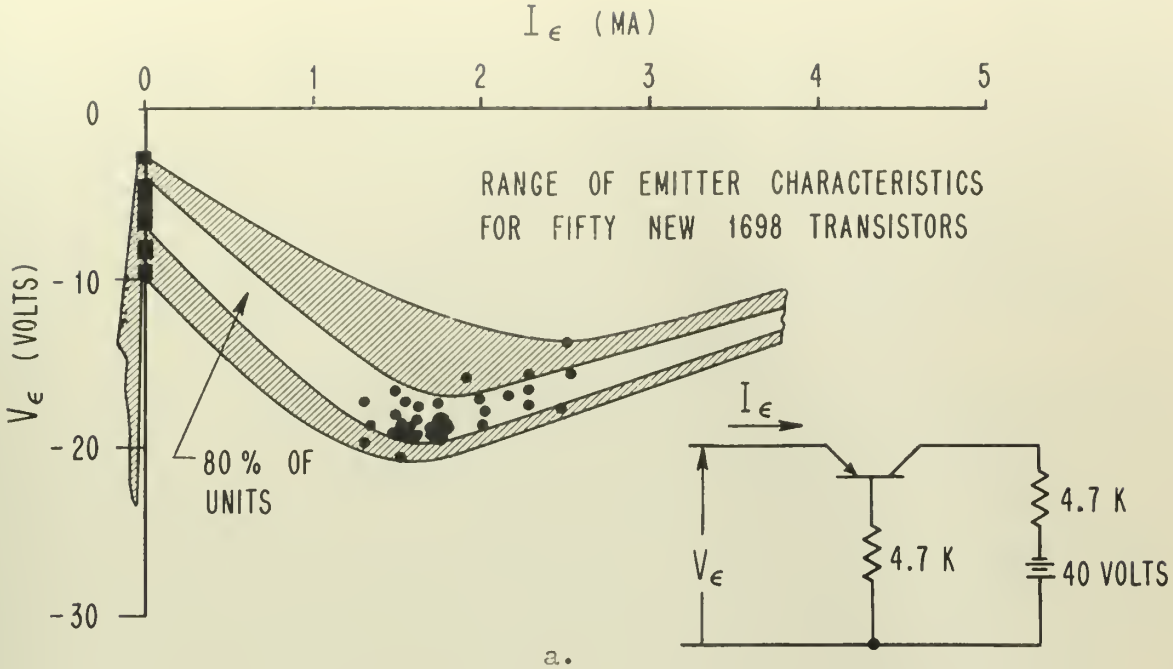
The N-type negative resistance characteristic which the emitter input of a transistor will display under proper conditions is useful in describing the operation of many non-linear circuits. If this characteristic is known, the operation of a circuit can be completely described. Therefore, an investigation of the variation of this characteristic with different transistors will indicate the practicability of interchanging transistors in a circuit application.[16] This characteristic will not define the basic equivalent T-network parameters explicitly, and it can be changed by changing the external parameters. However, the overall characteristic is of paramount interest to the circuit designer, and from it he can obtain an indication of the basic parameter variations.

The emitter input characteristic is not the only non-linear characteristic obtainable from a transistor, for a similar characteristic will be displayed between any two terminals under the proper conditions. The emitter characteristic was chosen since it was thought to be the most easily visualized, and because the same factors which cause its variation will cause similar variations in the other characteristics.

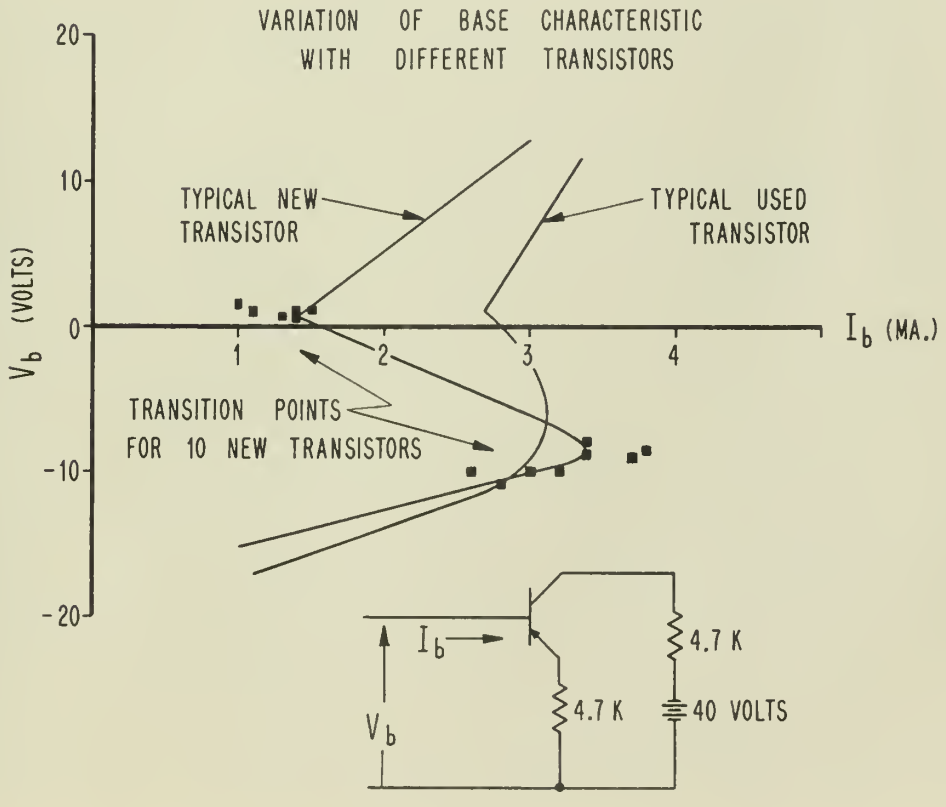
Figure 15 shows both the emitter and base characteristics for a number of new type 1698 transistors. The base characteristic is included to show that its variation is not unlike that of the emitter. On both characteristics there is a sharp transition between regions I and II. The transition between regions II and III is less well defined, and the linear approximations are not completely valid. The emitter characteristic has been taken for enough transistors to give a good indication of the uniformity of different transistors. For eighty percent of the units the variation of the resistance in each of the three regions is about plus or minus 30% of their mean values. The voltage at which the peak point occurs also varies by about plus or minus 30%, while the voltage and current of the valley point each vary by about plus or minus 10%. These variations make it extremely difficult to design practical circuits; furthermore, they do not present the only difficulty. These parameters undoubtedly show temperature variations similar to those of the small signal parameters and sometimes show variations after being used in particular circuit applications.

3. Large Signal, Non-Linear, Parameters of Used Transistors

A "forming" technique is used in the production of point contact transistors in order to achieve the desired small signal parameters.[17] This technique involves the application of current pulses to the collector. The


I_ϵ (MA)
0
1
2
3
4
5
0
-10
-20
-30
V_ϵ (VOLTS)
RANGE OF EMITTER CHARACTERISTICS
FOR FIFTY NEW 1698 TRANSISTORS
80 % OF
UNITS
I_ϵ
V_ϵ
4.7 K
4.7 K
40 VOLTS

a.

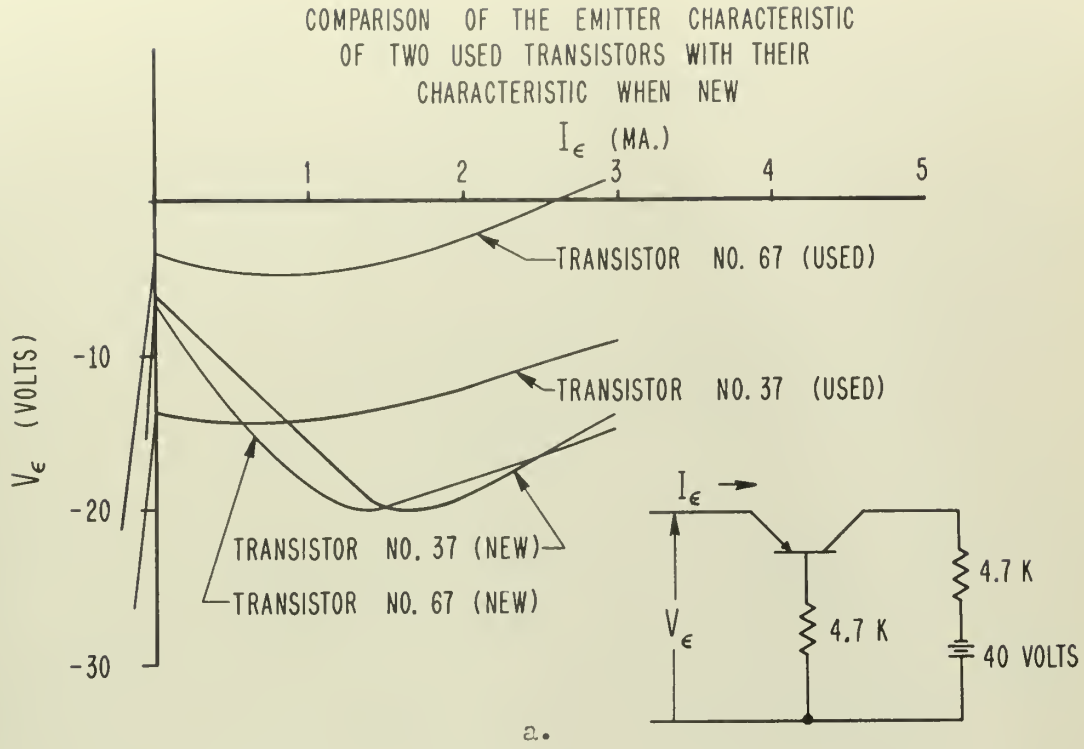

VARIATION OF BASE CHARACTERISTIC
WITH DIFFERENT TRANSISTORS
20
10
0
-10
-20
V_b (VOLTS)
TYPICAL NEW
TRANSISTOR
TYPICAL USED
TRANSISTOR
1
2
3
4
I_b (MA.)
TRANSITION POINTS
FOR 10 NEW TRANSISTORS
I_b
V_b
4.7 K
4.7 K
40 VOLTS

b.

Figure 15

intensity and width of these pulses have not become standard in the industry, for the physical process involved is not completely understood. This fact produces another difficulty in the design of non-linear circuits, for if the pulse handling capacities of the transistor are exceeded the negative resistance characteristic will change. These capacities were evidently exceeded in the circuits previously described, for the characteristic did change.

Figure 16 is included to show how the emitter characteristic changed after the transistors had been operated in the multivibrator circuits. It should be pointed out that at no time were the average current-voltage ratings of the transistor exceeded. Consequently, it is likely that this variation was caused by the pulse currents. Figure 16(a) shows how the individual characteristics for two transistors changed, while Figure 16(b and c) show that the spread in the characteristic for a number of different transistors was greatly increased after usage. From an examination of these curves and a comparison with the theoretical curves shown in Figure 4, a qualitative indication of the variation in basic transistor parameters may be obtained. One noticeable variation is that the voltage at which the peak transition point occurs changes considerably. This indicates that the cut-off collector resistance has changed. Theoretically, for the perfect switching transistor, the current gain, alpha, as a function of emitter current would be a rectangular curve; that is, in region I it would be zero, then as the emitter current is increased to region II alpha would jump to some finite value, and then as the emitter current is increased further alpha would remain constant until region III is reached, at which time alpha would again drop to zero. Figure 17 shows a qualitative curve, drawn from an


COMPARISON OF THE EMITTER CHARACTERISTIC
OF TWO USED TRANSISTORS WITH THEIR
CHARACTERISTIC WHEN NEW
I_ϵ (MA.)
1
2
3
4
5
TRANSISTOR NO. 67 (USED)
TRANSISTOR NO. 37 (USED)
V_ϵ (VOLTS)
-10
-20
-30
TRANSISTOR NO. 37 (NEW)
TRANSISTOR NO. 67 (NEW)
I_ϵ
V_ϵ
4.7 K
4.7 K
40 VOLTS
a.

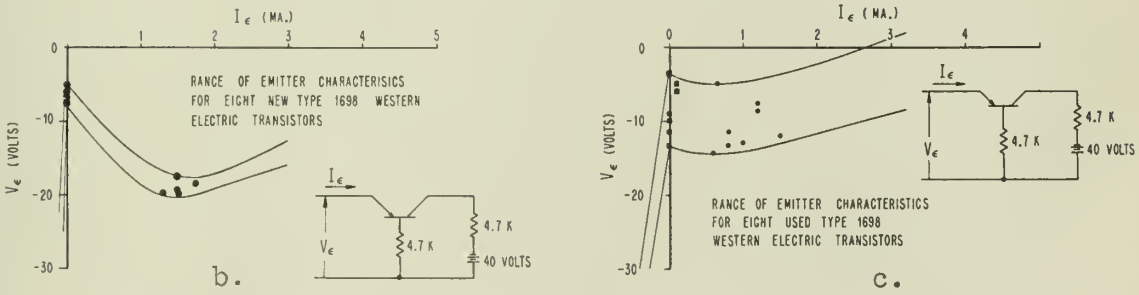

I_ϵ (MA.)
RANCE OF EMITTER CHARACTERISICS
FOR EIGHT NEW TYPE 1698 WESTERN
ELECTRIC TRANSISTORS
V_ϵ (VOLTS)
4.7 K
4.7 K
40 VOLTS
b.
RANCE OF EMITTER CHARACTERISTICS
FOR EIGHT USED TYPE 1698
WESTERN ELECTRIC TRANSISTORS
c.

Figure 16

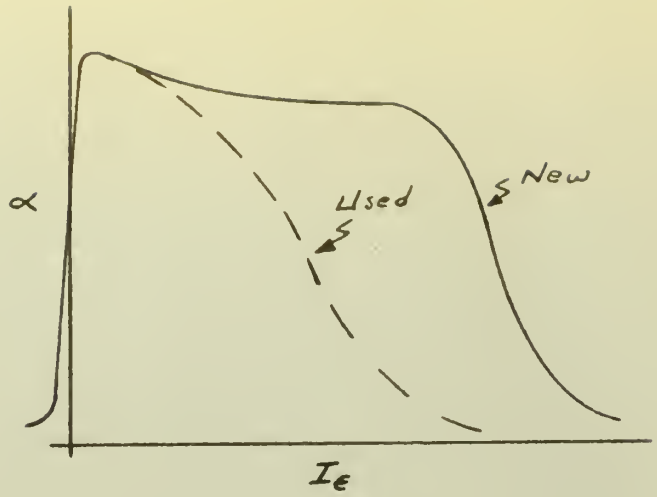


Figure 17

inspection of Figure 16, of alpha as a function of emitter current for a new and used transistor. These curves show an effect analagous to the cutoff characteristic of a vacuum tube. In the new condition it presents a sharp ~~cutoff~~ saturation while in the used condition it indicates a remote ~~cutoff~~ saturation.

These variations require that additional transistor ratings be established. Maximum pulse ratings, within which the transistor will maintain a sharp cutoff, are needed.

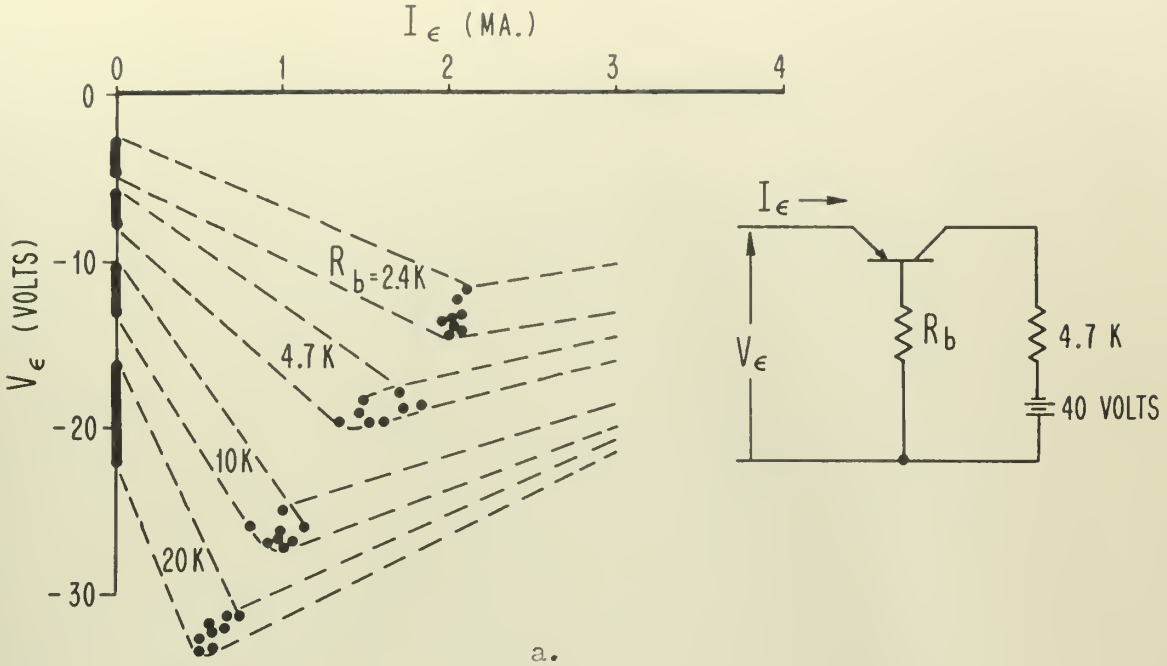
The used transistors whose characteristics are depicted are still satisfactory for use in the multivibrators; however, with their modified characteristics it would be very difficult to use them in bistable circuits and to design circuits for interchangeability of transistors.

4. Circuit Optimization with Present Production Transistors

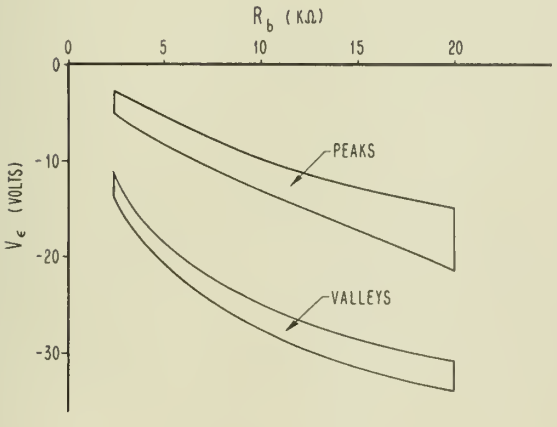
As previously mentioned, the negative resistance characteristic can be altered by a change in external parameters. Therefore, it should be possible

to select the external parameters of a circuit in such a way as to minimize the effects of transistor parameter variations. One means of such optimization will be described.

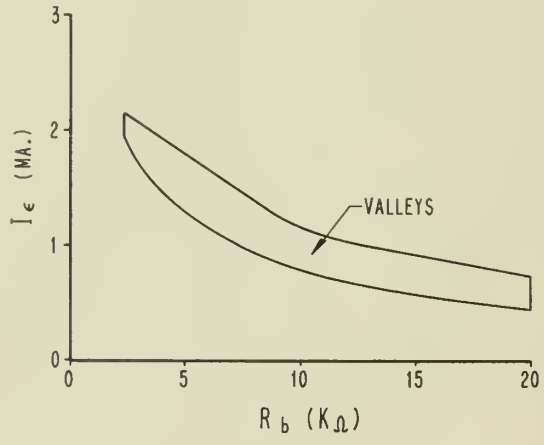
In a bistable circuit the separation of the peak and valley points is of particular importance, for this separation determines both the allowable trigger sensitivity and output pulse. Let us consider a means by which the external base resistance could be optimized to produce a circuit which would allow maximum interchangeability of transistors. Figure 18(a) shows the emitter characteristic for eight new, selected transistors with the external base resistance as a parameter. Figure 18(b) shows the peak and valley point voltages as a function of base resistance. An inspection of this curve reveals that for a base resistance of about 10K the voltage separation of the peaks and valleys is the greatest while the variations in peak voltage due to different transistors is smaller than at higher base resistance. Furthermore, the valley point emitter currents are higher than at the larger values of base resistance, and the variations in valley point emitter currents due to different transistors are less than at smaller base resistance. This then indicates that, for the circuit shown, an optimized value of base resistance of about 10K could be chosen as a compromise between reducing the effects of transistor parameter variations and relatively good trigger sensitivity and output pulse. It should be emphasized that this only demonstrates a method of optimization, for it does not include the effects of new and used transistor parameter variations and is for only one circuit configuration.



a.



b.



c.

Figure 18

CHAPTER V

CONCLUSIONS

The results of this work show that simple, experimental non-linear transistor circuits are easily built. However, the design of practical circuits for application in production equipment is indeed a difficult, if not impossible, task with the present day transistor. It has also been shown that this difficulty is caused not only by the variation in the transistor parameters, but also by a parameter variation which can occur in certain circuit applications.

Production type sinusoidal oscillators are within the realm of practicability providing the stability requirements are not too great. However, precision oscillators will require more refined and complicated circuits.

With multivibrators it is easy to exceed the pulse handling capacities of the transistor, and thus modify the characteristic. This may or may not be a serious limitation in production type circuits, but it does complicate the design. Definite pulse ratings should be established. The shunt feedback multivibrator described shows promise for use in frequency divider type applications where the width of the output pulse is not critical. However, its long time stability was not investigated.

The present production transistors can be used only in restricted, practical applications. However, the advantages of light weight, small size, and low power dissipation make them particularly desirable, and in demand, for switching applications. Consequently, with this demand and the great amount of developmental activity, it is believed that the transistor designer and application engineer will ultimately find the answers to the problems discussed.

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